

TermFactor = Binomial[i + j, i] (-1)^k Pochhammer[-α - i - j, k] / Factorial[k]

$$\frac{(-1)^k \binom{i+j}{i} (-i-j-\alpha)_k}{k!}$$

NonStandardLaguerreterm = TermFactor Pochhammer[-k, r + s]

Pochhammer[-β - i, r] Pochhammer[-i, r] Pochhammer[-γ - j, s] Pochhammer[-j, s] / (Factorial[r] Factorial[s] Pochhammer[-α - i - j, r + s] Pochhammer[-i - j, r + s])

$$\frac{(-1)^k \binom{i+j}{i} (-i)_r (-j)_s (-i-\beta)_r (-j-\gamma)_s (-k)_{r+s} (-i-j-\alpha)_k}{k! r! s! (-i-j)_{r+s} (-i-j-\alpha)_{r+s}}$$

Timing[rec = SumCertificate[FindCertificate[NonStandardLaguerreterm, k, {r, s}, 1]]]

{669.766,

$$\begin{aligned} & \{-(i+j-k+\alpha+2)(2j-k+\alpha-\beta+2)(2i-k+\alpha-\gamma+2)(2\alpha i-2\beta i+2i-2j-2j\alpha+k\beta+2j\gamma-k\gamma) \text{SUM}(k-2) + \\ & (5\beta k^4 - 5\gamma k^4 + 5\beta^2 k^3 - 5\gamma^2 k^3 + 10ik^3 - 10jk^3 + 10i\alpha k^3 - 10j\alpha k^3 - 23i\beta k^3 - 13j\beta k^3 - 12\alpha\beta k^3 - 20\beta k^3 + \\ & 13i\gamma k^3 + 23j\gamma k^3 + 12\alpha\gamma k^3 + 20\gamma k^3 + \beta^3 k^2 - \gamma^3 k^2 - 26i^2 k^2 + 26j^2 k^2 - 24i\alpha^2 k^2 + 24j\alpha^2 k^2 - 21i\beta^2 k^2 - \\ & 5j\beta^2 k^2 - 8\alpha\beta^2 k^2 - 13\beta^2 k^2 + 5i\gamma^2 k^2 + 21j\gamma^2 k^2 + 8\alpha\gamma^2 k^2 - 4\beta\gamma^2 k^2 + 13\gamma^2 k^2 - 38ik^2 + 38jk^2 - \\ & 26i^2\alpha k^2 + 26j^2\alpha k^2 - 62i\alpha k^2 + 62j\alpha k^2 + 34i^2\beta k^2 + 8j^2\beta k^2 + 9\alpha^2\beta k^2 + 82i\beta k^2 + 54ij\beta k^2 + \\ & 24j\beta k^2 + 54i\alpha\beta k^2 + 10j\alpha\beta k^2 + 31\alpha\beta k^2 + 26\beta k^2 - 8i^2\gamma k^2 - 34j^2\gamma k^2 - 9\alpha^2\gamma k^2 + 4\beta^2\gamma k^2 - \\ & 24i\gamma k^2 - 54ij\gamma k^2 - 82j\gamma k^2 - 10i\alpha\gamma k^2 - 54j\alpha\gamma k^2 - 31\alpha\gamma k^2 - 4i\beta\gamma k^2 + 4j\beta\gamma k^2 - 26\gamma k^2 + 16i^3 k - \\ & 16j^3 k + 18i\alpha^3 k - 18j\alpha^3 k - 3i\beta^3 k - \alpha\beta^3 k - \beta^3 k + 3j\gamma^3 k + \alpha\gamma^3 k - \beta\gamma^3 k + \gamma^3 k + 62i^2 k - 40ij^2 k - \\ & 62j^2 k + 40i^2\alpha^2 k - 40j^2\alpha^2 k + 76i\alpha^2 k - 76j\alpha^2 k + 28i^2\beta^2 k + 3\alpha^2\beta^2 k + 42i\beta^2 k + 16ij\beta^2 k + 3j\beta^2 k + \\ & 26i\alpha\beta^2 k + 2j\alpha\beta^2 k + 10\alpha\beta^2 k + 8\beta^2 k - 28j^2\gamma^2 k - 3\alpha^2\gamma^2 k - 3i\gamma^2 k - 16ij\gamma^2 k - 42j\gamma^2 k - 2i\alpha\gamma^2 k - \\ & 26j\alpha\gamma^2 k - 10\alpha\gamma^2 k + 2i\beta\gamma^2 k + 13j\beta\gamma^2 k + 3\alpha\beta\gamma^2 k + 6\beta\gamma^2 k - 8\gamma^2 k + 48ik + 40i^2jk - 48jk + \\ & 16i^3\alpha k - 16j^3\alpha k + 102i^2\alpha k - 40ij^2\alpha k - 102j^2\alpha k + 106i\alpha k + 40i^2j\alpha k - 106j\alpha k - 16i^3\beta k - \\ & 2\alpha^3\beta k - 94i^2\beta k - 28ij^2\beta k - 41i\alpha^2\beta k + 9j\alpha^2\beta k - 11\alpha^2\beta k - 95i\beta k - 68i^2j\beta k - 90ij\beta k + 3j\beta k - \\ & 68i^2\alpha\beta k + 4j^2\alpha\beta k - 125i\alpha\beta k - 48ij\alpha\beta k + 15j\alpha\beta k - 19\alpha\beta k - 11\beta k + 16j^3\gamma k + 2\alpha^3\gamma k + \beta^3\gamma k + \\ & 68ij^2\gamma k + 94j^2\gamma k - 9i\alpha^2\gamma k + 41j\alpha^2\gamma k + 11\alpha^2\gamma k - 13i\beta^2\gamma k - 2j\beta^2\gamma k - 3\alpha\beta^2\gamma k - 6\beta^2\gamma k - 3i\gamma k + \\ & 28i^2j\gamma k + 90ij\gamma k + 95j\gamma k - 4i^2\alpha\gamma k + 68j^2\alpha\gamma k - 15i\alpha\gamma k + 48ij\alpha\gamma k + 125j\alpha\gamma k + 19\alpha\gamma k + \\ & 4i^2\beta\gamma k - 4j^2\beta\gamma k + 25i\beta\gamma k - 25j\beta\gamma k + 22i\alpha\beta\gamma k - 22j\alpha\beta\gamma k + 11\gamma k - 4i\alpha^4 + 4j\alpha^4 - 16i^3 + \\ & 24ij^3 + 16j^3 - 14i^2\alpha^3 + 14j^2\alpha^3 - 24i\alpha^3 + 24j\alpha^3 + 2i^2\beta^3 + 2j^2\beta^3 + 2i\alpha\beta^3 - 2j^2\gamma^3 - 2j\gamma^3 - 2j\alpha\gamma^3 + \\ & 2j\beta\gamma^3 - 36i^2 + 48ij^2 + 36j^2 - 12i^3\alpha^2 + 12j^3\alpha^2 - 58i^2\alpha^2 + 28ij^2\alpha^2 + 58j^2\alpha^2 - 54i\alpha^2 - 28i^2j\alpha^2 + \\ & 54j\alpha^2 - 12i^3\beta^2 - 30i^2\beta^2 - 8i\alpha^2\beta^2 + 2j\alpha^2\beta^2 - 18i\beta^2 - 12i^2j\beta^2 - 8ij\beta^2 + 2j\beta^2 - 18i^2\alpha\beta^2 - 24i\alpha\beta^2 - \\ & 6ij\alpha\beta^2 + 4j\alpha\beta^2 + 12j^3\gamma^2 + 12ij^2\gamma^2 + 30j^2\gamma^2 - 2i\alpha^2\gamma^2 + 8j\alpha^2\gamma^2 - 2i\beta^2\gamma^2 + 2j\beta^2\gamma^2 - 2i\gamma^2 + \\ & 8ij\gamma^2 + 18j\gamma^2 + 18j^2\alpha\gamma^2 - 4i\alpha\gamma^2 + 6ij\alpha\gamma^2 + 24j\alpha\gamma^2 - 8j^2\beta\gamma^2 + 4i\beta\gamma^2 - 6ij\beta\gamma^2 - 16j\beta\gamma^2 + \\ & 4i\alpha\beta\gamma^2 - 10j\alpha\beta\gamma^2 - 20i - 24i^3j - 48i^2j + 20j - 28i^3\alpha + 24ij^3\alpha + 28j^3\alpha - 80i^2\alpha + 76ij^2\alpha + \end{aligned}$$

$$\begin{aligned}
& 80 j^2 \alpha - 54 i \alpha - 24 i^3 j \alpha - 76 i^2 j \alpha + 54 j \alpha + 28 i^3 \beta + 10 i \alpha^3 \beta - 6 j \alpha^3 \beta + 64 i^2 \beta + 24 i^2 j^2 \beta + 8 i j^2 \beta - \\
& 8 j^2 \beta + 30 i^2 \alpha^2 \beta - 8 j^2 \alpha^2 \beta + 46 i \alpha^2 \beta + 6 i j \alpha^2 \beta - 24 j \alpha^2 \beta + 36 i \beta + 24 i^3 j \beta + 68 i^2 j \beta + 24 i j \beta - 14 j \beta + \\
& 24 i^3 \alpha \beta + 88 i^2 \alpha \beta - 16 j^2 \alpha \beta + 70 i \alpha \beta + 40 i^2 j \alpha \beta + 24 i j \alpha \beta - 32 j \alpha \beta - 24 i j^3 \gamma - 28 j^3 \gamma + 6 i \alpha^3 \gamma - \\
& 10 j \alpha^3 \gamma - 2 i \beta^3 \gamma + 8 i^2 \gamma - 24 i^2 j^2 \gamma - 68 i j^2 \gamma - 64 j^2 \gamma + 8 i^2 \alpha^2 \gamma - 30 j^2 \alpha^2 \gamma + 24 i \alpha^2 \gamma - 6 i j \alpha^2 \gamma - \\
& 46 j \alpha^2 \gamma + 8 i^2 \beta^2 \gamma + 16 i \beta^2 \gamma + 6 i j \beta^2 \gamma - 4 j \beta^2 \gamma + 10 i \alpha \beta^2 \gamma - 4 j \alpha \beta^2 \gamma + 14 i \gamma - 8 i^2 j \gamma - 24 i j \gamma - \\
& 36 j \gamma - 24 j^3 \alpha \gamma + 16 i^2 \alpha \gamma - 40 i j^2 \alpha \gamma - 88 j^2 \alpha \gamma + 32 i \alpha \gamma - 24 i j \alpha \gamma - 70 j \alpha \gamma - 16 i^2 \beta \gamma + 16 j^2 \beta \gamma - \\
& 14 i \alpha^2 \beta \gamma + 14 j \alpha^2 \beta \gamma - 28 i \beta \gamma + 28 j \beta \gamma - 16 i^2 \alpha \beta \gamma + 16 j^2 \alpha \beta \gamma - 40 i \alpha \beta \gamma + 40 j \alpha \beta \gamma) \text{SUM}(k-1) + \\
& (9 \beta k^4 - 9 \gamma k^4 + 7 \beta^2 k^3 - 7 \gamma^2 k^3 + 18 i k^3 - 18 j k^3 + 18 i \alpha k^3 - 18 j \alpha k^3 - 39 i \beta k^3 - 21 j \beta k^3 - 15 \alpha \beta k^3 - \\
& 18 \beta k^3 + 21 i \gamma k^3 + 39 j \gamma k^3 + 15 \alpha \gamma k^3 + 18 \gamma k^3 + \beta^3 k^2 - \gamma^3 k^2 - 42 i^2 k^2 + 42 j^2 k^2 - 30 i \alpha^2 k^2 + 30 j \alpha^2 k^2 - \\
& 27 i \beta^2 k^2 - 7 j \beta^2 k^2 - 6 \alpha \beta^2 k^2 - 8 \beta^2 k^2 + 7 i \gamma^2 k^2 + 27 j \gamma^2 k^2 + 6 \alpha \gamma^2 k^2 - 3 \beta \gamma^2 k^2 + 8 \gamma^2 k^2 - 28 i k^2 + \\
& 28 j k^2 - 42 i^2 \alpha k^2 + 42 j^2 \alpha k^2 - 58 i \alpha k^2 + 58 j \alpha k^2 + 54 i^2 \beta k^2 + 12 j^2 \beta k^2 + 6 \alpha^2 \beta k^2 + 68 i \beta k^2 + 78 i j \beta k^2 + \\
& 12 j \beta k^2 + 66 i \alpha \beta k^2 + 8 j \alpha \beta k^2 + 17 \alpha \beta k^2 + 7 \beta k^2 - 12 i^2 \gamma k^2 - 54 j^2 \gamma k^2 - 6 \alpha^2 \gamma k^2 + 3 \beta^2 \gamma k^2 - 12 i \gamma k^2 - \\
& 78 i j \gamma k^2 - 68 j \gamma k^2 - 8 i \alpha \gamma k^2 - 66 j \alpha \gamma k^2 - 17 \alpha \gamma k^2 - 8 i \beta \gamma k^2 + 8 j \beta \gamma k^2 - 7 \gamma k^2 + 24 i^3 k - 24 j^3 k + \\
& 12 i \alpha^3 k - 12 j \alpha^3 k - 3 i \beta^3 k + 3 j \gamma^3 k + 32 i^2 k - 48 i j^2 k - 32 j^2 k + 44 i^2 \alpha^2 k - 44 j^2 \alpha^2 k + 34 i \alpha^2 k - \\
& 34 j \alpha^2 k + 32 i^2 \beta^2 k + 23 i \beta^2 k + 20 i j \beta^2 k + 19 i \alpha \beta^2 k + j \alpha \beta^2 k + 2 \alpha \beta^2 k - 32 j^2 \gamma^2 k - 20 i j \gamma^2 k - \\
& 23 j \gamma^2 k - i \alpha \gamma^2 k - 19 j \alpha \gamma^2 k - 2 \alpha \gamma^2 k + i \beta \gamma^2 k + 10 j \beta \gamma^2 k + 2 \beta \gamma^2 k + 12 i k + 48 i^2 j k - 12 j k + \\
& 24 i^3 \alpha k - 24 j^3 \alpha k + 76 i^2 \alpha k - 48 i j^2 \alpha k - 76 j^2 \alpha k + 34 i \alpha k + 48 i^2 j \alpha k - 34 j \alpha k - 24 i^3 \beta k - 64 i^2 \beta k - \\
& 36 i j^2 \beta k + 8 j^2 \beta k - 28 i \alpha^2 \beta k + 8 j \alpha^2 \beta k - 2 \alpha^2 \beta k - 24 i \beta k - 84 i^2 j \beta k - 48 i j \beta k + 10 j \beta k - 76 i^2 \alpha \beta k + \\
& 8 j^2 \alpha \beta k - 59 i \alpha \beta k - 48 i j \alpha \beta k + 9 j \alpha \beta k + 24 j^3 \gamma k - 8 i^2 \gamma k + 84 i j^2 \gamma k + 64 j^2 \gamma k - 8 i \alpha^2 \gamma k + \\
& 28 j \alpha^2 \gamma k + 2 \alpha^2 \gamma k - 10 i \beta^2 \gamma k - j \beta^2 \gamma k - 2 \beta^2 \gamma k - 10 i \gamma k + 36 i^2 j \gamma k + 48 i j \gamma k + 24 j \gamma k - 8 i^2 \alpha \gamma k + \\
& 76 j^2 \alpha \gamma k - 9 i \alpha \gamma k + 48 i j \alpha \gamma k + 59 j \alpha \gamma k + 8 i^2 \beta \gamma k - 8 j^2 \beta \gamma k + 11 i \beta \gamma k - 11 j \beta \gamma k + 18 i \alpha \beta \gamma k - \\
& 18 j \alpha \beta \gamma k + 24 i j^3 - 8 i^2 \alpha^3 + 8 j^2 \alpha^3 + 2 i^2 \beta^3 - 2 j^2 \gamma^3 + 16 i j^2 - 12 i^3 \alpha^2 + 12 j^3 \alpha^2 - 16 i^2 \alpha^2 + 20 i j^2 \alpha^2 + \\
& 16 j^2 \alpha^2 - 20 i^2 j \alpha^2 - 12 i^3 \beta^2 - 14 i^2 \beta^2 - 12 i^2 j \beta^2 - 12 i^2 \alpha \beta^2 - 2 i \alpha \beta^2 - 4 i j \alpha \beta^2 - 2 j \alpha \beta^2 + 12 j^3 \gamma^2 + \\
& 12 i j^2 \gamma^2 + 14 j^2 \gamma^2 + 12 j^2 \alpha \gamma^2 + 2 i \alpha \gamma^2 + 4 i j \alpha \gamma^2 + 2 j \alpha \gamma^2 - 6 j^2 \beta \gamma^2 - 2 i \beta \gamma^2 - 4 i j \beta \gamma^2 - 2 j \beta \gamma^2 - \\
& 24 i^3 j - 16 i^2 j - 12 i^3 \alpha + 24 i j^3 \alpha + 12 j^3 \alpha - 8 i^2 \alpha + 36 i j^2 \alpha + 8 j^2 \alpha - 24 i^3 j \alpha - 36 i^2 j \alpha + 12 i^3 \beta + 8 i^2 \beta + \\
& 24 i^2 j^2 \beta - 4 i j^2 \beta + 18 i^2 \alpha^2 \beta - 6 j^2 \alpha^2 \beta + 2 i \alpha^2 \beta + 4 i j \alpha^2 \beta + 2 j \alpha^2 \beta + 24 i^3 j \beta + 24 i^2 j \beta - 8 i j \beta + \\
& 24 i^3 \alpha \beta + 30 i^2 \alpha \beta - 2 j^2 \alpha \beta + 32 i^2 j \alpha \beta + 4 i j \alpha \beta - 24 i j^3 \gamma - 12 j^3 \gamma - 24 i^2 j^2 \gamma - 24 i j^2 \gamma - 8 j^2 \gamma + \\
& 6 i^2 \alpha^2 \gamma - 18 j^2 \alpha^2 \gamma - 2 i \alpha^2 \gamma - 4 i j \alpha^2 \gamma - 2 j \alpha^2 \gamma + 6 i^2 \beta^2 \gamma + 2 i \beta^2 \gamma + 4 i j \beta^2 \gamma + 2 j \beta^2 \gamma + 4 i^2 j \gamma + 8 i j \gamma - \\
& 24 j^3 \alpha \gamma + 2 i^2 \alpha \gamma - 32 i j^2 \alpha \gamma - 30 j^2 \alpha \gamma - 4 i j \alpha \gamma - 2 i^2 \beta \gamma + 2 j^2 \beta \gamma - 12 i^2 \alpha \beta \gamma + 12 j^2 \alpha \beta \gamma) \text{SUM}(k) - \\
& (i + j - k)(7 \beta k^3 - 7 \gamma k^3 + 3 \beta^2 k^2 - 3 \gamma^2 k^2 + 14 i k^2 - 14 j k^2 + 14 i \alpha k^2 - 14 j \alpha k^2 - 22 i \beta k^2 - 8 j \beta k^2 - \\
& 6 \alpha \beta k^2 + 8 i \gamma k^2 + 22 j \gamma k^2 + 6 \alpha \gamma k^2 - 16 i^2 k + 16 j^2 k - 12 i \alpha^2 k + 12 j \alpha^2 k - 8 i \beta^2 k + \beta^2 k + 8 j \gamma^2 k - \\
& \gamma^2 k + 10 i k - 10 j k - 16 i^2 \alpha k + 16 j^2 \alpha k - 2 i \alpha k + 2 j \alpha k + 16 i^2 \beta k - 6 i \beta k + 20 i j \beta k - 8 j \beta k + \\
& 20 i \alpha \beta k - 4 j \alpha \beta k - 2 \alpha \beta k - 9 \beta k - 16 j^2 \gamma k + 8 i \gamma k - 20 i j \gamma k + 6 j \gamma k + 4 i \alpha \gamma k - 20 j \alpha \gamma k + \\
& 2 \alpha \gamma k - 4 i \beta \gamma k + 4 j \beta \gamma k + 9 \gamma k - 16 i^2 - 8 i j^2 + 16 j^2 + 4 i^2 \alpha^2 - 4 j^2 \alpha^2 - 12 i \alpha^2 + 12 j \alpha^2 + 4 i^2 \beta^2 - \\
& 4 i \beta^2 - 2 \beta^2 - 4 j^2 \gamma^2 + 4 j \gamma^2 + 2 \gamma^2 - 4 i + 8 i^2 j + 4 j - 12 i^2 \alpha - 8 i j^2 \alpha + 12 j^2 \alpha - 16 i \alpha + 8 i^2 j \alpha + \\
& 16 j \alpha + 12 i^2 \beta + 16 i \beta - 8 i^2 j \beta + 16 i j \beta - 8 i^2 \alpha \beta + 16 i \alpha \beta - 8 j \alpha \beta + 4 \alpha \beta - 2 \beta + 8 i j^2 \gamma - \\
& 12 j^2 \gamma - 16 i j \gamma - 16 j \gamma + 8 j^2 \alpha \gamma + 8 i \alpha \gamma - 16 j \alpha \gamma - 4 \alpha \gamma - 8 i \beta \gamma + 8 j \beta \gamma + 2 \gamma) \text{SUM}(k+1) + \\
& 2(i + j - k - 1)(i + j - k)(k + 2)(2 \alpha i - 2 \beta i + 2 i - 2 j - 2 j \alpha + k \beta - \beta + 2 j \gamma - k \gamma + \gamma) \text{SUM}(k+2) = 0\}
\end{aligned}$$

res = Map[Factor, rec [[1, 1]] /. {k → k + 2, SUM → S}]

$$\begin{aligned}
& -(i + j - k + \alpha)(2j - k + \alpha - \beta)(2i - k + \alpha - \gamma)(2\alpha i - 2\beta i + 2i - 2j - 2j\alpha + k\beta + 2\beta + 2j\gamma - k\gamma - 2\gamma)S(k) - \\
& (-5\beta k^4 + 5\gamma k^4 - 5\beta^2 k^3 + 5\gamma^2 k^3 - 10ik^3 + 10jk^3 - 10i\alpha k^3 + 10j\alpha k^3 + 23i\beta k^3 + 13j\beta k^3 + 12\alpha\beta k^3 - 20\beta k^3 -
\end{aligned}$$

$$\begin{aligned}
& 13 i \gamma k^3 - 23 j \gamma k^3 - 12 \alpha \gamma k^3 + 20 \gamma k^3 - \beta^3 k^2 + \gamma^3 k^2 + 26 i^2 k^2 - 26 j^2 k^2 + 24 i \alpha^2 k^2 - 24 j \alpha^2 k^2 + 21 i \beta^2 k^2 + \\
& 5 j \beta^2 k^2 + 8 \alpha \beta^2 k^2 - 17 \beta^2 k^2 - 5 i \gamma^2 k^2 - 21 j \gamma^2 k^2 - 8 \alpha \gamma^2 k^2 + 4 \beta \gamma^2 k^2 + 17 \gamma^2 k^2 - 22 i k^2 + 22 j k^2 + \\
& 26 i^2 \alpha k^2 - 26 j^2 \alpha k^2 + 2 i \alpha k^2 - 2 j \alpha k^2 - 34 i^2 \beta k^2 - 8 j^2 \beta k^2 - 9 \alpha^2 \beta k^2 + 56 i \beta k^2 - 54 i j \beta k^2 + 54 j \beta k^2 - \\
& 54 i \alpha \beta k^2 - 10 j \alpha \beta k^2 + 41 \alpha \beta k^2 - 26 \beta k^2 + 8 i^2 \gamma k^2 + 34 j^2 \gamma k^2 + 9 \alpha^2 \gamma k^2 - 4 \beta^2 \gamma k^2 - 54 i \gamma k^2 + 54 i j \gamma k^2 - \\
& 56 j \gamma k^2 + 10 i \alpha \gamma k^2 + 54 j \alpha \gamma k^2 - 41 \alpha \gamma k^2 + 4 i \beta \gamma k^2 - 4 j \beta \gamma k^2 + 26 \gamma k^2 - 16 i^3 k + 16 j^3 k - 18 i \alpha^3 k + \\
& 18 j \alpha^3 k + 3 i \beta^3 k + \alpha \beta^3 k - 3 \beta^3 k - 3 j \gamma^3 k - \alpha \gamma^3 k + \beta \gamma^3 k + 3 \gamma^3 k + 42 i^2 k + 40 i j^2 k - 42 j^2 k - 40 i^2 \alpha^2 k + \\
& 40 j^2 \alpha^2 k + 20 i \alpha^2 k - 20 j \alpha^2 k - 28 i^2 \beta^2 k - 3 \alpha^2 \beta^2 k + 42 i \beta^2 k - 16 i j \beta^2 k + 17 j \beta^2 k - 26 i \alpha \beta^2 k - 2 j \alpha \beta^2 k + \\
& 22 \alpha \beta^2 k - 16 \beta^2 k + 28 j^2 \gamma^2 k + 3 \alpha^2 \gamma^2 k - 17 i \gamma^2 k + 16 i j \gamma^2 k - 42 j \gamma^2 k + 2 i \alpha \gamma^2 k + 26 j \alpha \gamma^2 k - 22 \alpha \gamma^2 k - \\
& 2 i \beta \gamma^2 k - 13 j \beta \gamma^2 k - 3 \alpha \beta \gamma^2 k + 10 \beta \gamma^2 k + 16 \gamma^2 k - 16 i k - 40 i^2 j k + 16 j k - 16 i^3 \alpha k + 16 j^3 \alpha k + 2 i^2 \alpha k + \\
& 40 i j^2 \alpha k - 2 j^2 \alpha k + 22 i \alpha k - 40 i^2 j \alpha k - 22 j \alpha k + 16 i^3 \beta k + 2 \alpha^3 \beta k - 42 i^2 \beta k + 28 i j^2 \beta k - 32 j^2 \beta k + \\
& 41 i \alpha^2 \beta k - 9 j \alpha^2 \beta k - 25 \alpha^2 \beta k + 43 i \beta k + 68 i^2 j \beta k - 126 i j \beta k + 57 j \beta k + 68 i^2 \alpha \beta k - 4 j^2 \alpha \beta k - \\
& 91 i \alpha \beta k + 48 i j \alpha \beta k - 55 j \alpha \beta k + 39 \alpha \beta k - 13 \beta k - 16 j^3 \gamma k - 2 \alpha^3 \gamma k - \beta^3 \gamma k + 32 i^2 \gamma k - 68 i j^2 \gamma k + \\
& 42 j^2 \gamma k + 9 i \alpha^2 \gamma k - 41 j \alpha^2 \gamma k + 25 \alpha^2 \gamma k + 13 i \beta^2 \gamma k + 2 j \beta^2 \gamma k + 3 \alpha \beta^2 \gamma k - 10 \beta^2 \gamma k - 57 i \gamma k - 28 i^2 j \gamma k + \\
& 126 i j \gamma k - 43 j \gamma k + 4 i^2 \alpha \gamma k - 68 j^2 \alpha \gamma k + 55 i \alpha \gamma k - 48 i j \alpha \gamma k + 91 j \alpha \gamma k - 39 \alpha \gamma k - 4 i^2 \beta \gamma k + \\
& 4 j^2 \beta \gamma k - 9 i \beta \gamma k + 9 j \beta \gamma k - 22 i \alpha \beta \gamma k + 22 j \alpha \beta \gamma k + 13 \gamma k + 4 i \alpha^4 - 4 j \alpha^4 - 16 i^3 - 24 i j^3 + 16 j^3 + \\
& 14 i^2 \alpha^3 - 14 j^2 \alpha^3 - 12 i \alpha^3 + 12 j \alpha^3 - 2 i^2 \beta^3 + 4 i \beta^3 - 2 i \alpha \beta^3 + 2 \alpha \beta^3 - 2 \beta^3 + 2 j^2 \gamma^3 - 4 j \gamma^3 + 2 j \alpha \gamma^3 - \\
& 2 \alpha \gamma^3 - 2 j \beta \gamma^3 + 2 \beta \gamma^3 + 2 \gamma^3 + 16 i^2 + 32 i j^2 - 16 j^2 + 12 i^3 \alpha^2 - 12 j^3 \alpha^2 - 22 i^2 \alpha^2 - 28 i j^2 \alpha^2 + 22 j^2 \alpha^2 - \\
& 2 i \alpha^2 + 28 i^2 j \alpha^2 + 2 j \alpha^2 + 12 i^3 \beta^2 - 26 i^2 \beta^2 + 8 i \alpha^2 \beta^2 - 2 j \alpha^2 \beta^2 - 6 \alpha^2 \beta^2 + 18 i \beta^2 + 12 i^2 j \beta^2 - 24 i j \beta^2 + \\
& 12 j \beta^2 + 18 i^2 \alpha \beta^2 - 28 i \alpha \beta^2 + 6 i j \alpha \beta^2 - 8 j \alpha \beta^2 + 12 \alpha \beta^2 - 4 \beta^2 - 12 j^3 \gamma^2 - 12 i j^2 \gamma^2 + 26 j^2 \gamma^2 + 2 i \alpha^2 \gamma^2 - \\
& 8 j \alpha^2 \gamma^2 + 6 \alpha^2 \gamma^2 + 2 i \beta^2 \gamma^2 - 2 j \beta^2 \gamma^2 - 12 i \gamma^2 + 24 i j \gamma^2 - 18 j \gamma^2 - 18 j^2 \alpha \gamma^2 + 8 i \alpha \gamma^2 - 6 i j \alpha \gamma^2 + 28 j \alpha \gamma^2 - \\
& 12 \alpha \gamma^2 + 8 j^2 \beta \gamma^2 - 8 i \beta \gamma^2 + 6 i j \beta \gamma^2 - 10 j \beta \gamma^2 - 4 i \alpha \beta \gamma^2 + 10 j \alpha \beta \gamma^2 - 6 \alpha \beta \gamma^2 + 4 \beta \gamma^2 + 4 \gamma^2 - 4 i + \\
& 24 i^3 j - 32 i^2 j + 4 j - 4 i^3 \alpha - 24 i j^3 \alpha + 4 j^3 \alpha - 20 i^2 \alpha + 4 i j^2 \alpha + 20 j^2 \alpha + 10 i \alpha + 24 i^3 j \alpha - 4 i^2 j \alpha - 10 j \alpha + \\
& 4 i^3 \beta - 10 i \alpha^3 \beta + 6 j \alpha^3 \beta + 4 \alpha^3 \beta - 12 i^2 \beta - 24 i^2 j^2 \beta + 48 i j^2 \beta - 24 j^2 \beta - 30 i^2 \alpha^2 \beta + 8 j^2 \alpha^2 \beta + 36 i \alpha^2 \beta - \\
& 6 i j \alpha^2 \beta + 6 j \alpha^2 \beta - 14 \alpha^2 \beta + 10 i \beta - 24 i^3 j \beta + 68 i^2 j \beta - 60 i j \beta + 16 j \beta - 24 i^3 \alpha \beta + 48 i^2 \alpha \beta + 8 j^2 \alpha \beta - \\
& 36 i \alpha \beta - 40 i^2 j \alpha \beta + 72 i j \alpha \beta - 38 j \alpha \beta + 10 \alpha \beta - 2 \beta + 24 i j^3 \gamma - 4 j^3 \gamma - 6 i \alpha^3 \gamma + 10 j \alpha^3 \gamma - 4 \alpha^3 \gamma + \\
& 2 i \beta^3 \gamma - 2 \beta^3 \gamma + 24 i^2 \gamma + 24 i^2 j^2 \gamma - 68 i j^2 \gamma + 12 j^2 \gamma - 8 i^2 \alpha^2 \gamma + 30 j^2 \alpha^2 \gamma - 6 i \alpha^2 \gamma + 6 i j \alpha^2 \gamma - 36 j \alpha^2 \gamma + \\
& 14 \alpha^2 \gamma - 8 i^2 \beta^2 \gamma + 10 i \beta^2 \gamma - 6 i j \beta^2 \gamma + 8 j \beta^2 \gamma - 10 i \alpha \beta^2 \gamma + 4 j \alpha \beta^2 \gamma + 6 \alpha \beta^2 \gamma - 4 \beta^2 \gamma - 16 i \gamma - 48 i^2 j \gamma + \\
& 60 i j \gamma - 10 j \gamma + 24 j^3 \alpha \gamma - 8 i^2 \alpha \gamma + 40 i j^2 \alpha \gamma - 48 j^2 \alpha \gamma + 38 i \alpha \gamma - 72 i j \alpha \gamma + 36 j \alpha \gamma - 10 \alpha \gamma + 8 i^2 \beta \gamma - \\
& 8 j^2 \beta \gamma + 14 i \alpha^2 \beta \gamma - 14 j \alpha^2 \beta \gamma - 6 i \beta \gamma + 6 j \beta \gamma + 16 i^2 \alpha \beta \gamma - 16 j^2 \alpha \beta \gamma - 4 i \alpha \beta \gamma + 4 j \alpha \beta \gamma + 2 \gamma) S(k+1) - \\
& (-9 \beta k^4 + 9 \gamma k^4 - 7 \beta^2 k^3 + 7 \gamma^2 k^3 - 18 i k^3 + 18 j k^3 - 18 i \alpha k^3 + 18 j \alpha k^3 + 39 i \beta k^3 + 21 j \beta k^3 + 15 \alpha \beta k^3 - \\
& 54 \beta k^3 - 21 i \gamma k^3 - 39 j \gamma k^3 - 15 \alpha \gamma k^3 + 54 \gamma k^3 - \beta^3 k^2 + \gamma^3 k^2 + 42 i^2 k^2 - 42 j^2 k^2 + 30 i \alpha^2 k^2 - 30 j \alpha^2 k^2 + \\
& 27 i \beta^2 k^2 + 7 j \beta^2 k^2 + 6 \alpha \beta^2 k^2 - 34 \beta^2 k^2 - 7 i \gamma^2 k^2 - 27 j \gamma^2 k^2 - 6 \alpha \gamma^2 k^2 + 3 \beta \gamma^2 k^2 + 34 \gamma^2 k^2 - 80 i k^2 + \\
& 80 j k^2 + 42 i^2 \alpha k^2 - 42 j^2 \alpha k^2 - 50 i \alpha k^2 + 50 j \alpha k^2 - 54 i^2 \beta k^2 - 12 j^2 \beta k^2 - 6 \alpha^2 \beta k^2 + 166 i \beta k^2 - 78 i j \beta k^2 + \\
& 114 j \beta k^2 - 66 i \alpha \beta k^2 - 8 j \alpha \beta k^2 + 73 \alpha \beta k^2 - 115 \beta k^2 + 12 i^2 \gamma k^2 + 54 j^2 \gamma k^2 + 6 \alpha^2 \gamma k^2 - 3 \beta^2 \gamma k^2 - 114 i \gamma k^2 + \\
& 78 i j \gamma k^2 - 166 j \gamma k^2 + 8 i \alpha \gamma k^2 + 66 j \alpha \gamma k^2 - 73 \alpha \gamma k^2 + 8 i \beta \gamma k^2 - 8 j \beta \gamma k^2 + 115 \gamma k^2 - 24 i^3 k + 24 j^3 k - \\
& 12 i \alpha^3 k + 12 j \alpha^3 k + 3 i \beta^3 k - 4 \beta^3 k - 3 j \gamma^3 k + 4 \gamma^3 k + 136 i^2 k + 48 i j^2 k - 136 j^2 k - 44 i^2 \alpha^2 k + 44 j^2 \alpha^2 k + \\
& 86 i \alpha^2 k - 86 j \alpha^2 k - 32 i^2 \beta^2 k + 85 i \beta^2 k - 20 i j \beta^2 k + 28 j \beta^2 k - 19 i \alpha \beta^2 k - j \alpha \beta^2 k + 22 \alpha \beta^2 k - 52 \beta^2 k + \\
& 32 j^2 \gamma^2 k - 28 i \gamma^2 k + 20 i j \gamma^2 k - 85 j \gamma^2 k + i \alpha \gamma^2 k + 19 j \alpha \gamma^2 k - 22 \alpha \gamma^2 k - i \beta \gamma^2 k - 10 j \beta \gamma^2 k + 10 \beta \gamma^2 k + \\
& 52 \gamma^2 k - 116 i k - 48 i^2 j k + 116 j k - 24 i^3 \alpha k + 24 j^3 \alpha k + 92 i^2 \alpha k + 48 i j^2 \alpha k - 92 j^2 \alpha k - 18 i \alpha k - 48 i^2 j \alpha k + \\
& 18 j \alpha k + 24 i^3 \beta k - 152 i^2 \beta k + 36 i j^2 \beta k - 56 j^2 \beta k + 28 i \alpha^2 \beta k - 8 j \alpha^2 \beta k - 22 \alpha^2 \beta k + 220 i \beta k + 84 i^2 j \beta k - \\
& 264 i j \beta k + 194 j \beta k + 76 i^2 \alpha \beta k - 8 j^2 \alpha \beta k - 205 i \alpha \beta k + 48 i j \alpha \beta k - 41 j \alpha \beta k + 112 \alpha \beta k - 100 \beta k - \\
& 24 j^3 \gamma k + 56 i^2 \gamma k - 84 i j^2 \gamma k + 152 j^2 \gamma k + 8 i \alpha^2 \gamma k - 28 j \alpha^2 \gamma k + 22 \alpha^2 \gamma k + 10 i \beta^2 \gamma k + j \beta^2 \gamma k - 10 \beta^2 \gamma k -
\end{aligned}$$

$$\begin{aligned}
& 194 i \gamma k - 36 i^2 j \gamma k + 264 i j \gamma k - 220 j \gamma k + 8 i^2 \alpha \gamma k - 76 j^2 \alpha \gamma k + 41 i \alpha \gamma k - 48 i j \alpha \gamma k + 205 j \alpha \gamma k - \\
& 112 \alpha \gamma k - 8 i^2 \beta \gamma k + 8 j^2 \beta \gamma k + 21 i \beta \gamma k - 21 j \beta \gamma k - 18 i \alpha \beta \gamma k + 18 j \alpha \beta \gamma k + 100 \gamma k - 48 i^3 - 24 i j^3 + \\
& 48 j^3 + 8 i^2 \alpha^3 - 8 j^2 \alpha^3 - 24 i \alpha^3 + 24 j \alpha^3 - 2 i^2 \beta^3 + 6 i \beta^3 - 4 \beta^3 + 2 j^2 \gamma^3 - 6 j \gamma^3 + 4 \gamma^3 + 104 i^2 + 80 i j^2 - 104 j^2 + \\
& 12 i^3 \alpha^2 - 12 j^3 \alpha^2 - 72 i^2 \alpha^2 - 20 i j^2 \alpha^2 + 72 j^2 \alpha^2 + 52 i \alpha^2 + 20 i^2 j \alpha^2 - 52 j \alpha^2 + 12 i^3 \beta^2 - 50 i^2 \beta^2 + 62 i \beta^2 + \\
& 12 i^2 j \beta^2 - 40 i j \beta^2 + 28 j \beta^2 + 12 i^2 \alpha \beta^2 - 36 i \alpha \beta^2 + 4 i j \alpha \beta^2 + 20 \alpha \beta^2 - 24 \beta^2 - 12 j^3 \gamma^2 - 12 i j^2 \gamma^2 + 50 j^2 \gamma^2 - \\
& 28 i \gamma^2 + 40 i j \gamma^2 - 62 j \gamma^2 - 12 j^2 \alpha \gamma^2 - 4 i j \alpha \gamma^2 + 36 j \alpha \gamma^2 - 20 \alpha \gamma^2 + 6 j^2 \beta \gamma^2 + 4 i j \beta \gamma^2 - 18 j \beta \gamma^2 + 8 \beta \gamma^2 + \\
& 24 \gamma^2 - 56 i + 24 i^3 j - 80 i^2 j + 56 j - 36 i^3 \alpha - 24 i j^3 \alpha + 36 j^3 \alpha + 24 i^2 \alpha + 60 i j^2 \alpha - 24 j^2 \alpha + 20 i \alpha + 24 i^3 j \alpha - \\
& 60 i^2 j \alpha - 20 j \alpha + 36 i^3 \beta - 96 i^2 \beta - 24 i^2 j^2 \beta + 76 i j^2 \beta - 64 j^2 \beta - 18 i^2 \alpha^2 \beta + 6 j^2 \alpha^2 \beta + 54 i \alpha^2 \beta - 4 i j \alpha^2 \beta - \\
& 18 j \alpha^2 \beta - 20 \alpha^2 \beta + 88 i \beta - 24 i^3 j \beta + 144 i^2 j \beta - 208 i j \beta + 100 j \beta - 24 i^3 \alpha \beta + 122 i^2 \alpha \beta - 14 j^2 \alpha \beta - 146 i \alpha \beta - \\
& 32 i^2 j \alpha \beta + 92 i j \alpha \beta - 50 j \alpha \beta + 52 \alpha \beta - 28 \beta + 24 i j^3 \gamma - 36 j^3 \gamma + 64 i^2 \gamma + 24 i^2 j^2 \gamma - 144 i j^2 \gamma + 96 j^2 \gamma - \\
& 6 i^2 \alpha^2 \gamma + 18 j^2 \alpha^2 \gamma + 18 i \alpha^2 \gamma + 4 i j \alpha^2 \gamma - 54 j \alpha^2 \gamma + 20 \alpha^2 \gamma - 6 i^2 \beta^2 \gamma + 18 i \beta^2 \gamma - 4 i j \beta^2 \gamma - 8 \beta^2 \gamma - 100 i \gamma - \\
& 76 i^2 j \gamma + 208 i j \gamma - 88 j \gamma + 24 j^3 \alpha \gamma + 14 i^2 \alpha \gamma + 32 i j^2 \alpha \gamma - 122 j^2 \alpha \gamma + 50 i \alpha \gamma - 92 i j \alpha \gamma + 146 j \alpha \gamma - \\
& 52 \alpha \gamma - 14 i^2 \beta \gamma + 14 j^2 \beta \gamma + 10 i \beta \gamma - 10 j \beta \gamma + 12 i^2 \alpha \beta \gamma - 12 j^2 \alpha \beta \gamma - 36 i \alpha \beta \gamma + 36 j \alpha \beta \gamma + 28 \gamma) S(k+2) - \\
& (i+j-k-2)(7 \beta k^3 - 7 \gamma k^3 + 3 \beta^2 k^2 - 3 \gamma^2 k^2 + 14 i k^2 - 14 j k^2 + 14 i \alpha k^2 - 14 j \alpha k^2 - 22 i \beta k^2 - 8 j \beta k^2 - \\
& 6 \alpha \beta k^2 + 42 \beta k^2 + 8 i \gamma k^2 + 22 j \gamma k^2 + 6 \alpha \gamma k^2 - 42 \gamma k^2 - 16 i^2 k + 16 j^2 k - 12 i \alpha^2 k + 12 j \alpha^2 k - 8 i \beta^2 k + \\
& 13 \beta^2 k + 8 j \gamma^2 k - 13 \gamma^2 k + 66 i k - 66 j k - 16 i^2 \alpha k + 16 j^2 \alpha k + 54 i \alpha k - 54 j \alpha k + 16 i^2 \beta k - 94 i \beta k + \\
& 20 i j \beta k - 40 j \beta k + 20 i \alpha \beta k - 4 j \alpha \beta k - 26 \alpha \beta k + 75 \beta k - 16 j^2 \gamma k + 40 i \gamma k - 20 i j \gamma k + 94 j \gamma k + 4 i \alpha \gamma k - \\
& 20 j \alpha \gamma k + 26 \alpha \gamma k - 4 i \beta \gamma k + 4 j \beta \gamma k - 75 \gamma k - 48 i^2 - 8 i j^2 + 48 j^2 + 4 i^2 \alpha^2 - 4 j^2 \alpha^2 - 36 i \alpha^2 + 36 j \alpha^2 + \\
& 4 i^2 \beta^2 - 20 i \beta^2 + 12 \beta^2 - 4 j^2 \gamma^2 + 20 j \gamma^2 - 12 \gamma^2 + 72 i + 8 i^2 j - 72 j - 44 i^2 \alpha - 8 i j^2 \alpha + 44 j^2 \alpha + 36 i \alpha + 8 i^2 j \alpha - \\
& 36 j \alpha + 44 i^2 \beta - 84 i \beta - 8 i^2 j \beta + 56 i j \beta - 48 j \beta - 8 i^2 \alpha \beta + 56 i \alpha \beta - 16 j \alpha \beta - 24 \alpha \beta + 36 \beta + 8 i j^2 \gamma - \\
& 44 j^2 \gamma + 48 i \gamma - 56 i j \gamma + 84 j \gamma + 8 j^2 \alpha \gamma + 16 i \alpha \gamma - 56 j \alpha \gamma + 24 \alpha \gamma - 16 i \beta \gamma + 16 j \beta \gamma - 36 \gamma) S(k+3) + \\
& 2(i+j-k-3)(i+j-k-2)(k+4)(2 \alpha i - 2 \beta i + 2 i - 2 j - 2 j \alpha + k \beta + \beta + 2 j \gamma - k \gamma - \gamma) S(k+4)
\end{aligned}$$