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[> restart:
> read "ODE3solve.mpl":
    Package "Solving third-order holonomic differential equations", Maple 16
    Copyright 2017, Mouafo Wouodjie Merlin, University of Kassel
    Package "Hypergeometric Summation", Maple V - Maple 17
    Copyright 1998-2013, Wolfram Koepf, University of Kassel
]

```

(1)

[Here are the Maple implementations in chapter 5 related just to the 1F1 square functions.

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[> ##### THE EXPONENT DIFFERENCES #####

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[In chapter 5, section 5.2 which is called "Exponent differences", we have the following Maple implementations:

```

[>
> L11 := x^2*Dx^3+3*x*(-x+b+1)*Dx^2-(-2*x^2+4*x*(a+b)-b*(2*b-1)-1)*
    Dx-2*a*(-2*x+2*b-1);
L11 := x^2 Dx^3 + 3 x (-x + b + 1) Dx^2 - (-2 x^2 + 4 x (a + b) - b (2 b - 1) - 1) Dx - 2 a (-2 x + 2 b - 1)
]

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(2)

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[> gen_exp(L11,t,x=0);
    [[0,t=x],[RootOf(_Z^2+3 b _Z+2 b^2-4 b),t=x]]
]

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[> gen_exp(L11,t,x=infinity);
    [[2 a,t=1/x],[-1/t-3+b,t=1/x],[-2/t+6-2 a+2 b,t=1/x]]
]

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[> ##### EXAMPLE IN THE THESIS #####

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[In chapter 5, section 5.4.4 which is called "Examples", those are the Maple implementations for the example that we have used in the 1F1 square solutions:

```

[> LA:=MinOp(hypergeom([a],[b],x)^2);
LA := Dx^3 + 3 (-x + b) Dx^2 / x - (4 a x - 2 b^2 + 4 b x - 2 x^2 + b) Dx / x^2
    - 2 a (-2 x + 2 b - 1) / x^2
]

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[> L1:=subs({a=1/7,b=1/3},LA);
    LI := Dx^3 + 3 (-x + 1/3) Dx^2 / x - ((40/21 x + 1/9 - 2 x^2) Dx / x^2 - 2 (-2 x - 1/3) / 7 x^2)
]

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(6)

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[> f:=(x-1)^6/(x-12);
]

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(7)

$$f := \frac{(x-1)^6}{x-12} \quad (7)$$

> L:=ChangeOfVariables(L1,f);

$$L := 63 Dx^3 (x-12)^5 (x-1)^2 (5x-71)^2 - 63 (75x^8 - 2580x^7 + 29028x^6 - 124188x^5 + 270570x^4 - 334825x^3 + 238286x^2 - 73447x - 74793) Dx^2 (x-12)^3 (x-1) (5x-71) + (78750x^{16} - 5418000x^{15} + 154148400x^{14} - 2357777520x^{13} + 21336285096x^{12} - 121187182692x^{11} + 460524395496x^{10} - 1230477210270x^9 + 2390095153908x^8 - 3439319623344x^7 + 3677984618849x^6 - 2867623517288x^5 + 1536998515098x^4 - 481721327852x^3 + 41643716165x^2 - 26597677398x + 124928729910) Dx (x-12) + 6 (6x^6 - 36x^5 + 90x^4 - 120x^3 + 90x^2 - 35x - 6) (x-1)^5 (5x-71)^5 \quad (8)$$

> ext:=indets(L,{RootOf,name}) minus {x,Dx};

$$ext := \emptyset \quad (9)$$

> ext:= indets(map(s-> ReplirrRoot(s,{ }),ext),{RootOf,name});

$$ext := \emptyset \quad (10)$$

> extppp:={};

$$extppp := \emptyset \quad (11)$$

> E:= Singular(L,extppp);

$$E := \left[\left[x - \frac{71}{5}, \frac{71}{5} \right], [x-1, 1], [x-12, 12], [\infty, \infty] \right] \quad (12)$$

> F:= NotAppSing(L,E,ext);

$$F := [[\infty, \infty], [x-12, 12]] \quad (13)$$

> Sirr:=irrsglFlsq(L,t,F,ext);

$$Sirr := \left[[[\infty, \infty], [x-12, 12]], \left[\left[\frac{10}{7}, -\frac{10}{t^5} - \frac{48}{t^4} - \frac{522}{t^3} - \frac{4096}{t^2} - \frac{24606}{t} + \frac{40}{21}, -\frac{5}{t^5} - \frac{24}{t^4} - \frac{261}{t^3} - \frac{2048}{t^2} - \frac{12303}{t} + \frac{5}{3} \right], \left[\frac{2}{7}, -\frac{3543122}{t} + \frac{8}{21}, -\frac{1771561}{t} + \frac{1}{3} \right] \right], \left[\left[-\frac{5}{t^5} - \frac{24}{t^4} - \frac{261}{t^3} - \frac{2048}{t^2} - \frac{12303}{t} + \frac{5}{21}, \frac{5}{t^5} + \frac{24}{t^4} + \frac{261}{t^3} + \frac{2048}{t^2} + \frac{12303}{t} - \frac{5}{21}, -\frac{10}{t^5} - \frac{48}{t^4} - \frac{522}{t^3} - \frac{4096}{t^2} - \frac{24606}{t} + \frac{10}{21} \right], \left[-\frac{1771561}{t} + \frac{1}{21}, \frac{1771561}{t} - \frac{1}{21}, -\frac{3543122}{t} + \frac{2}{21} \right] \right], [5, 1], [1, 1], \left[\left[\left[-\frac{5}{t^5} - \frac{24}{t^4} - \frac{261}{t^3} - \frac{2048}{t^2} - \frac{12303}{t} + \frac{5}{3}, \frac{10}{7} \right], \left[-\frac{5}{t^5} - \frac{24}{t^4} - \frac{261}{t^3} - \frac{2048}{t^2} - \frac{12303}{t} + \frac{5}{3}, -\frac{10}{t^5} - \frac{48}{t^4} - \frac{522}{t^3} - \frac{4096}{t^2} - \frac{24606}{t} + \frac{40}{21} \right], \left[-\frac{10}{t^5} - \frac{48}{t^4} - \frac{522}{t^3} - \frac{4096}{t^2} - \frac{24606}{t} + \frac{40}{21}, \frac{40}{21}, \frac{10}{7} \right] \right], \left[\left[-\frac{1771561}{t} + \frac{1}{3}, \frac{2}{7} \right], \left[-\frac{1771561}{t} + \frac{1}{3}, -\frac{3543122}{t} + \frac{8}{21} \right], \left[-\frac{3543122}{t} + \frac{8}{21}, \frac{2}{7} \right] \right], [[-5t^5 - 24t^4 - 261t^3 - 2048t^2 - 12303t, 5t^5 + 24t^4$$

$$+ 261 t^3 + 2048 t^2 + 12303 t, -10 t^5 - 48 t^4 - 522 t^3 - 4096 t^2 - 24606 t], [-1771561 t, 1771561 t, -3543122 t], \left[\left[\frac{5}{21}, -\frac{5}{21}, \frac{10}{21} \right], \left[\frac{1}{21}, -\frac{1}{21}, \frac{2}{21} \right] \right], [[], []]$$

> Sreg:=regsingtruelFlsq(L,t,Sirr[-1],ext);

$$Sreg := []$$

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> RSreg:=SregseptrueFlsq(L,Sreg,ext);

$$RSreg := [[], [], []]$$

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> R1:=IrrRegAppsinglFlsq(L,t,E,ext);

$$R1 := \left[\left[[[\infty, \infty], [x - 12, 12]], \left[\left[\frac{10}{7}, -\frac{10}{t^5} - \frac{48}{t^4} - \frac{522}{t^3} - \frac{4096}{t^2} - \frac{24606}{t} + \frac{40}{21}, -\frac{5}{t^5} - \frac{24}{t^4} - \frac{261}{t^3} - \frac{2048}{t^2} - \frac{12303}{t} + \frac{5}{3} \right], \left[\frac{2}{7}, -\frac{3543122}{t} + \frac{8}{21}, -\frac{1771561}{t} + \frac{1}{3} \right] \right], \left[\left[-\frac{5}{t^5} - \frac{24}{t^4} - \frac{261}{t^3} - \frac{2048}{t^2} - \frac{12303}{t} + \frac{5}{21}, \frac{5}{t^5} + \frac{24}{t^4} + \frac{261}{t^3} + \frac{2048}{t^2} + \frac{12303}{t} - \frac{5}{21}, -\frac{10}{t^5} - \frac{48}{t^4} - \frac{522}{t^3} - \frac{4096}{t^2} - \frac{24606}{t} + \frac{10}{21} \right], \left[-\frac{1771561}{t} + \frac{1}{21}, \frac{1771561}{t} - \frac{1}{21}, -\frac{3543122}{t} + \frac{2}{21} \right] \right], [5, 1], [1, 1], \left[\left[\left[-\frac{5}{t^5} - \frac{24}{t^4} - \frac{261}{t^3} - \frac{2048}{t^2} - \frac{12303}{t} + \frac{5}{3}, \frac{10}{7} \right], \left[-\frac{5}{t^5} - \frac{24}{t^4} - \frac{261}{t^3} - \frac{2048}{t^2} - \frac{12303}{t} + \frac{5}{3}, -\frac{10}{t^5} - \frac{48}{t^4} - \frac{522}{t^3} - \frac{4096}{t^2} - \frac{24606}{t} + \frac{40}{21} \right], \left[-\frac{10}{t^5} - \frac{48}{t^4} - \frac{522}{t^3} - \frac{4096}{t^2} - \frac{24606}{t} + \frac{40}{21}, \frac{10}{7} \right] \right], \left[\left[-\frac{1771561}{t} + \frac{1}{3}, \frac{2}{7} \right], \left[-\frac{1771561}{t} + \frac{1}{3}, -\frac{3543122}{t} + \frac{8}{21} \right], \left[-\frac{3543122}{t} + \frac{8}{21}, \frac{2}{7} \right] \right] \right], [[-5 t^5 - 24 t^4 - 261 t^3 - 2048 t^2 - 12303 t, 5 t^5 + 24 t^4$$

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$$+ 261 t^3 + 2048 t^2 + 12303 t, -10 t^5 - 48 t^4 - 522 t^3 - 4096 t^2 - 24606 t], [-1771561 t, 1771561 t, -3543122 t], \left[\left[\frac{5}{21}, -\frac{5}{21}, \frac{10}{21} \right], \left[\frac{1}{21}, -\frac{1}{21}, \frac{2}{21} \right] \right], [], [[], [], []], \left[\left[[x - 1, 1], \left[x - \frac{71}{5}, \frac{71}{5} \right] \right], [[0, 4, 8], [0, 2, 4]], [[4, 8, 4], [2, 4, 2]], [[4, 0], [8, 0], [8, 4]], [[2, 0], [4, 0], [4, 2]] \right], \left[[[\infty, \infty], [x - 12, 12]], \left[\left[\frac{10}{7}, -\frac{10}{t^5} - \frac{48}{t^4} - \frac{522}{t^3} - \frac{4096}{t^2} - \frac{24606}{t} + \frac{40}{21}, -\frac{5}{t^5} - \frac{24}{t^4} - \frac{261}{t^3} - \frac{2048}{t^2} - \frac{12303}{t} + \frac{5}{3} \right], \left[\frac{2}{7}, -\frac{3543122}{t} + \frac{8}{21}, -\frac{1771561}{t} + \frac{1}{3} \right] \right], \left[\left[-\frac{5}{t^5} - \frac{24}{t^4} - \frac{261}{t^3} - \frac{2048}{t^2} - \frac{12303}{t} + \frac{5}{21}, \frac{5}{t^5} + \frac{24}{t^4} + \frac{261}{t^3} + \frac{2048}{t^2} + \frac{12303}{t} - \frac{5}{21}, -\frac{10}{t^5} - \frac{48}{t^4} - \frac{522}{t^3} - \frac{4096}{t^2} - \frac{24606}{t} + \frac{40}{21} \right], \left[-\frac{10}{t^5} - \frac{48}{t^4} - \frac{522}{t^3} - \frac{4096}{t^2} - \frac{24606}{t} + \frac{40}{21}, \frac{10}{7} \right] \right], \left[\left[-\frac{1771561}{t} + \frac{1}{3}, \frac{2}{7} \right], \left[-\frac{1771561}{t} + \frac{1}{3}, -\frac{3543122}{t} + \frac{8}{21} \right], \left[-\frac{3543122}{t} + \frac{8}{21}, \frac{2}{7} \right] \right] \right]$$

$$\begin{aligned} & -\frac{24606}{t} + \frac{10}{21} \Big], \Big[-\frac{1771561}{t} + \frac{1}{21}, \frac{1771561}{t} - \frac{1}{21}, -\frac{3543122}{t} + \frac{2}{21} \Big], \Big[\Big[\Big[-\frac{5}{t^5} \\ & -\frac{24}{t^4} - \frac{261}{t^3} - \frac{2048}{t^2} - \frac{12303}{t} + \frac{5}{3}, \frac{10}{7} \Big], \Big[-\frac{5}{t^5} - \frac{24}{t^4} - \frac{261}{t^3} - \frac{2048}{t^2} \\ & -\frac{12303}{t} + \frac{5}{3}, -\frac{10}{t^5} - \frac{48}{t^4} - \frac{522}{t^3} - \frac{4096}{t^2} - \frac{24606}{t} + \frac{40}{21} \Big], \Big[-\frac{10}{t^5} - \frac{48}{t^4} \\ & -\frac{522}{t^3} - \frac{4096}{t^2} - \frac{24606}{t} + \frac{40}{21}, \frac{10}{7} \Big] \Big], \Big[\Big[-\frac{1771561}{t} + \frac{1}{3}, \frac{2}{7} \Big], \Big[-\frac{1771561}{t} \\ & + \frac{1}{3}, -\frac{3543122}{t} + \frac{8}{21} \Big], \Big[-\frac{3543122}{t} + \frac{8}{21}, \frac{2}{7} \Big] \Big], [[1, 1, 1], [1, 1, 1]] \Big] \end{aligned}$$

> F1:=Hyp1FlsqSubst(L,x,t,R1[1],ext);

$$\begin{aligned} F1 := & \left[-\frac{x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 147636x + 1771561}{x - 12}, \right. \\ & -\frac{x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 147636x - 1771561}{x - 12}, \\ & \frac{x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 147636x - 1771561}{x - 12}, \\ & \left. \frac{x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 147636x + 1771561}{x - 12} \right] \end{aligned} \quad (18)$$

> find1FlsqInt(L,R1,F1,x,t,ext);

$$\begin{aligned} & \Big[\Big[\Big[\Big\{ \frac{11}{28}, \frac{25}{28}, \frac{37}{84}, \frac{79}{84} \Big\}, \Big[\frac{5}{6} \Big], \Big[\Big\{ \frac{5}{14}, \frac{6}{7}, \frac{13}{42}, \frac{17}{21} \Big\}, \Big[\frac{2}{3} \Big], \Big[\Big\{ \frac{19}{84}, \frac{23}{84}, \frac{61}{84}, \frac{65}{84} \Big\}, \right. \\ & \left. \Big[\frac{1}{2} \Big], \Big[\Big\{ \frac{3}{28}, \frac{5}{84}, \frac{17}{28}, \frac{47}{84} \Big\}, \Big[\frac{1}{6} \Big], \Big[\Big\{ \frac{1}{7}, \frac{4}{21}, \frac{9}{14}, \frac{29}{42} \Big\}, \Big[\frac{1}{3} \Big], -\frac{(x-1)^6}{x-12} \right], \\ & \left[\Big[\Big\{ \frac{11}{28}, \frac{25}{28}, \frac{37}{84}, \frac{79}{84} \Big\}, \Big[\frac{5}{6} \Big], \Big[\Big\{ \frac{5}{14}, \frac{6}{7}, \frac{13}{42}, \frac{17}{21} \Big\}, \Big[\frac{2}{3} \Big], \Big[\Big\{ \frac{19}{84}, \frac{23}{84}, \frac{61}{84}, \frac{65}{84} \Big\}, \right. \\ & \left. \Big[\frac{1}{2} \Big], \Big[\Big\{ \frac{3}{28}, \frac{5}{84}, \frac{17}{28}, \frac{47}{84} \Big\}, \Big[\frac{1}{6} \Big], \Big[\Big\{ \frac{1}{7}, \frac{4}{21}, \frac{9}{14}, \frac{29}{42} \Big\}, \Big[\frac{1}{3} \Big], \frac{(x-1)^6}{x-12} \right] \Big] \end{aligned} \quad (19)$$

> TIME :=time();

Hyp1FlsqSolutions(L);

time() - TIME;

TIME := 16.359

$$\begin{aligned} & \Big\{ \Big[\Big[\Big[\Big[\frac{6}{7} \Big], \frac{2}{3}, \Big[\frac{2(15x^6 - 288x^5 + 1215x^4 - 2280x^3 + 2205x^2 - 1082x + 237)}{3(x-12)^2} \Big], \right. \\ & \left[\frac{25(x-12)^4 Dx^2}{(x-1)^2(5x-71)^2} - \frac{1}{(5x^2-76x+71)(x-1)^2(5x-71)^2} (25(25x^{10} \right. \\ & - 1460x^9 + 33916x^8 - 397460x^7 + 2477038x^6 - 8237214x^5 + 15746608x^4 \\ & \left. \left. - 18029416x^3 + 12294913x^2 - 4493976x - 359280) Dx) \right] \right] \right] \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{63 (5 x^2 - 76 x + 71) (x - 1)^2 (5 x - 71)^2} (100 (225 x^9 - 20340 x^8 + 629424 x^7 \\
& - 8886528 x^6 + 60186690 x^5 - 188269955 x^4 + 308035936 x^3 - 275763515 x^2 \\
& + 129142605 x - 26899308))]], -\frac{(x-1)^6}{x-12} \Big], \Big[\Big[\Big[\Big[\frac{17}{21} \Big], \frac{2}{3}, \Big[-\frac{4}{3(x-12)} \Big], \\
& \Big[\frac{25 (x-12)^4 D x^2}{(x-1)^2 (5 x-71)^2} - \frac{1}{(5 x^2 - 76 x + 71) (x-1)^2 (5 x-71)^2} (25 (75 x^9 \\
& - 3480 x^8 + 59988 x^7 - 472524 x^6 + 1760826 x^5 - 3581710 x^4 + 4258368 x^3 \\
& - 2972672 x^2 + 1130019 x - 91044) (x-12) D x) \\
& + \frac{1}{63 (5 x^2 - 76 x + 71) (x-1)^2 (5 x-71)^2} (50 (7875 x^{14} - 422100 x^{13} \\
& + 8887095 x^{12} - 94700088 x^{11} + 567990423 x^{10} - 2140534065 x^9 + 5451674505 x^8 \\
& - 9810846966 x^7 + 12853710555 x^6 - 12735490620 x^5 + 10209362165 x^4 \\
& - 7036659298 x^3 + 3937674029 x^2 - 1458754863 x + 244411821))]], \frac{(x-1)^6}{x-12} \Big] \Big\}
\end{aligned}$$

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(20)

[Here are another examples related to the 1F1 square type solutions. Those examples are not in my PhD thesis.

[> ##### THE INTEGER CASE #####

$$\begin{aligned}
& > \text{LA} := \text{MinOp}(\text{hypergeom}([a],[b],x)^2); \\
& \text{LA} := D x^3 + \frac{3 (-x+b) D x^2}{x} - \frac{(4 a x - 2 b^2 + 4 b x - 2 x^2 + b) D x}{x^2} \\
& \quad - \frac{2 a (-2 x + 2 b - 1)}{x^2}
\end{aligned}$$

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$$\begin{aligned}
& > \text{L1} := \text{subs}(\{a=1/3, b=1/2\}, \text{LA}); \\
& \text{L1} := D x^3 + \frac{3 \left(-x + \frac{1}{2}\right) D x^2}{x} - \frac{\left(\frac{10}{3} x - 2 x^2\right) D x}{x^2} + \frac{4}{3 x}
\end{aligned}$$

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[> f:=(x-1)^2/x;

$$f := \frac{(x-1)^2}{x} \quad (23)$$

> L:=ChangeOfVariables(L1,f);

$$L := 6 D x^3 (x+1)^2 x^5 - 9 (2 x^3 + x^2 - 5 x - 2) D x^2 (x+1) x^3 + 2 (6 x^6 + 2 x^5 - 46 x^4 - 66 x^3 - x^2 + 20 x + 6) D x x + 8 (x-1) (x+1)^5 \quad (24)$$

> ext:=indets(L,{RootOf,name}) minus {x,Dx};

$$ext := \emptyset \quad (25)$$

> ext:= indets(map(s-> ReplirrRoot(s,{ }),ext),{RootOf,name});

$$ext := \emptyset \quad (26)$$

> extppp:={};

$$extppp := \emptyset \quad (27)$$

> E:= Singular(L,extppp);

$$E := [[x, 0], [x + 1, -1], [\infty, \infty]] \quad (28)$$

> F:= NotAppSing(L,E,ext);

$$F := [[\infty, \infty], [x, 0]] \quad (29)$$

> Sirr:=irrsinglFlsq(L,t,F,ext);

$$Sirr := \left[[[\infty, \infty], [x, 0]], \left[\left[\frac{2}{3}, -\frac{1}{t} + \frac{1}{2}, -\frac{2}{t} + \frac{1}{3} \right], \left[\frac{2}{3}, -\frac{1}{t} + \frac{1}{2}, -\frac{2}{t} + \frac{1}{3} \right], \left[-\frac{1}{t} - \frac{1}{6}, -\frac{1}{t} - \frac{1}{6}, -\frac{2}{t} - \frac{1}{3} \right], \left[-\frac{1}{t} - \frac{1}{6}, -\frac{1}{t} - \frac{1}{6}, -\frac{2}{t} - \frac{1}{3} \right], [1, 1], [1, 1], \left[\left[-\frac{1}{t} + \frac{1}{2}, \frac{2}{3} \right], \left[-\frac{2}{t} + \frac{1}{3}, -\frac{1}{t} + \frac{1}{2} \right], \left[-\frac{2}{t} + \frac{1}{3}, \frac{2}{3} \right], \left[-\frac{1}{t} + \frac{1}{2}, \frac{2}{3} \right], \left[-\frac{2}{t} + \frac{1}{3}, -\frac{1}{t} + \frac{1}{2} \right], \left[-\frac{2}{t} + \frac{1}{3}, \frac{2}{3} \right] \right], [[-t, -t, -2t], [-t, -t, -2t]], \left[-\frac{1}{6}, -\frac{1}{6}, -\frac{1}{3} \right], \left[-\frac{1}{6}, -\frac{1}{6}, -\frac{1}{3} \right], [[], []] \right] \quad (30)$$

> Sreg:=regsingtruelFlsq(L,t,Sirr[-1],ext);

$$Sreg := [] \quad (31)$$

> RSreg:=SregseptrueFlsq(L,Sreg,ext);

$$RSreg := [[], [], []] \quad (32)$$

> R1:=IrrRegAppsinglFlsq(L,t,E,ext);

$$R1 := \left[[[\infty, \infty], [x, 0]], \left[\left[\frac{2}{3}, -\frac{1}{t} + \frac{1}{2}, -\frac{2}{t} + \frac{1}{3} \right], \left[\frac{2}{3}, -\frac{1}{t} + \frac{1}{2}, -\frac{2}{t} + \frac{1}{3} \right], \left[-\frac{1}{t} - \frac{1}{6}, -\frac{1}{t} - \frac{1}{6}, -\frac{2}{t} - \frac{1}{3} \right], \left[-\frac{1}{t} - \frac{1}{6}, -\frac{1}{t} - \frac{1}{6}, -\frac{2}{t} - \frac{1}{3} \right], [1, 1], [1, 1], \left[\left[-\frac{1}{t} + \frac{1}{2}, \frac{2}{3} \right], \left[-\frac{2}{t} + \frac{1}{3}, -\frac{1}{t} + \frac{1}{2} \right], \left[-\frac{2}{t} + \frac{1}{3}, \frac{2}{3} \right], \left[-\frac{1}{t} + \frac{1}{2}, \frac{2}{3} \right], \left[-\frac{2}{t} + \frac{1}{3}, -\frac{1}{t} + \frac{1}{2} \right], \left[-\frac{2}{t} + \frac{1}{3}, \frac{2}{3} \right] \right], [[-t, -t, -2t], [-t, -t, -2t]], \left[-\frac{1}{6}, -\frac{1}{6}, -\frac{1}{3} \right], \left[-\frac{1}{6}, -\frac{1}{6}, -\frac{1}{3} \right], [], [], [], [], [[x + 1, -1], [0, 2, 4], [2, 4, 2], [[2, 0], [4, 0], [4, 2]]], \left[[[\infty, \infty], [x, 0]], \left[\left[\frac{2}{3}, -\frac{1}{t} + \frac{1}{2}, -\frac{2}{t} + \frac{1}{3} \right], \left[\frac{2}{3}, -\frac{1}{t} + \frac{1}{2}, -\frac{2}{t} + \frac{1}{3} \right], \left[-\frac{1}{t} - \frac{1}{6}, -\frac{1}{t} - \frac{1}{6}, -\frac{2}{t} - \frac{1}{3} \right], \left[-\frac{1}{t} - \frac{1}{6}, -\frac{1}{t} - \frac{1}{6}, -\frac{2}{t} - \frac{1}{3} \right], [1, 1], [1, 1], \left[\left[-\frac{1}{t} + \frac{1}{2}, \frac{2}{3} \right], \left[-\frac{2}{t} + \frac{1}{3}, -\frac{1}{t} + \frac{1}{2} \right], \left[-\frac{2}{t} + \frac{1}{3}, \frac{2}{3} \right], \left[-\frac{1}{t} + \frac{1}{2}, \frac{2}{3} \right], \left[-\frac{2}{t} + \frac{1}{3}, -\frac{1}{t} + \frac{1}{2} \right], \left[-\frac{2}{t} + \frac{1}{3}, \frac{2}{3} \right] \right], [[-t, -t, -2t], [-t, -t, -2t]], \left[-\frac{1}{6}, -\frac{1}{6}, -\frac{1}{3} \right], \left[-\frac{1}{6}, -\frac{1}{6}, -\frac{1}{3} \right], [], [], [], [], [[x + 1, -1], [0, 2, 4], [2, 4, 2], [[2, 0], [4, 0], [4, 2]]] \right] \right] \quad (33)$$


```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
ext := ∅ (41)
```

```
> ext:= indets(map(s-> ReplirrRoot(s,{ } ),ext),{RootOf,name});
ext := ∅ (42)
```

```
> extppp:={ };
extppp := ∅ (43)
```

```
> E:= Singular(L,extppp);
E := [[x-14, 14], [x-3, 3]] (44)
```

```
> F:= NotAppSing(L,E,ext);
F := [[x-3, 3], [x-14, 14]] (45)
```

```
> Sirr:=irrSinglFlsq(L,t,F,ext);
Sirr := [[ [x-14, 14]], [[  $\frac{2}{3}, -\frac{11}{t} + 1, -\frac{22}{t} + \frac{4}{3}$  ]], [[  $-\frac{11}{t} + \frac{1}{3}, -\frac{11}{t} + \frac{1}{3}, -\frac{22}{t} + \frac{2}{3}$  ]], [1], [1], [[  $[-\frac{11}{t} + 1, \frac{2}{3}], [-\frac{22}{t} + \frac{4}{3}, -\frac{11}{t} + 1]$  ], [[  $[-\frac{22}{t} + \frac{4}{3}, \frac{2}{3}]$  ]], [[ -11 t, -11 t, -22 t]], [[  $\frac{1}{3}, \frac{1}{3}, \frac{2}{3}$  ]], [[ [x-3, 3]], [[ [0, 0, 0], [0, 0, 0], [1, 1, 1], [0, 0], [0, 0], [0, 0]], 4]]] (46)
```

```
> Sreg:=regSingtruelFlsq(L,t,Sirr[-1],ext);
Sreg := [[ [x-3, 3]], [[ [0, 0, 0]], [[ [0, 0, 0]], [[ [0, 0], [0, 0], [0, 0]]]]] (47)
```

```
> RSreg:=SregseptruelFlsq(L,Sreg,ext);
RSreg := [[ ], [ ], [[ [x-3, 3]], [[ [0, 0, 0]], [[ [ ], [0, 0, 0]]]]] (48)
```

```
> R1:=IrrRegAppsinglFlsq(L,t,E,ext);
R1 := [[ [ [x-14, 14]], [[  $\frac{2}{3}, -\frac{11}{t} + 1, -\frac{22}{t} + \frac{4}{3}$  ]], [[  $-\frac{11}{t} + \frac{1}{3}, -\frac{11}{t} + \frac{1}{3}, -\frac{22}{t} + \frac{2}{3}$  ]], [1], [1], [[  $[-\frac{11}{t} + 1, \frac{2}{3}], [-\frac{22}{t} + \frac{4}{3}, -\frac{11}{t} + 1]$  ], [[  $[-\frac{22}{t} + \frac{4}{3}, \frac{2}{3}]$  ]], [[ -11 t, -11 t, -22 t]], [[  $\frac{1}{3}, \frac{1}{3}, \frac{2}{3}$  ]], [[ [x-3, 3]], [[ [0, 0, 0]], [[ [0, 0, 0]], [[ [0, 0], [0, 0], [0, 0]]]], [[ [ ], [ ], [[ [x-3, 3]], [[ [0, 0, 0]], [[ [ ], [0, 0, 0]]]]], [ ], [[ [x-3, 3], [x-14, 14]], [[ [0, 0, 0], [[  $\frac{2}{3}, -\frac{11}{t} + 1, -\frac{22}{t} + \frac{4}{3}$  ]], [[ [0, 0, 0], [[  $-\frac{11}{t} + \frac{1}{3}, -\frac{11}{t} + \frac{1}{3}, -\frac{22}{t} + \frac{2}{3}$  ]], [[ [0, 0], [0, 0], [0, 0]], [[  $[-\frac{11}{t} + 1, \frac{2}{3}], [-\frac{22}{t} + \frac{4}{3}, -\frac{11}{t} + 1]$  ], [[  $[-\frac{22}{t} + \frac{4}{3}, \frac{2}{3}]$  ]], [[ [1, 1, 1], [1, 1, 1]]]]] (49)
```

```
> F1:=Hyp1FlsqSubst(L,x,t,R1[1],ext);
F1 := [  $-\frac{11}{x-14}, \frac{11}{x-14}$  ] (50)
```

```
> find1FlsqIn(L,R1,F1,x,t,ext);
[[ [[  $\frac{1}{3}, \frac{1}{6}, \frac{2}{3}, \frac{5}{6}$  ], [2]], [  $-\frac{x-3}{x-14}$  ]], [[ [[  $\frac{1}{3}, \frac{1}{6}, \frac{2}{3}, \frac{5}{6}$  ], [2]], [  $\frac{x-3}{x-14}$  ]]] (51)
```

```
> TIME :=time();
```



```
Hyp1FlsqSolutions(L);
time() - TIME;
```

TIME := 29.250

$$\left\{ \left[\left[\left[\left[\frac{1}{3} \right], 2, \left[\frac{1}{x-3} \right], \left[(x-14)^2 (x-3) Dx^2 + (-11x+275) Dx + \frac{11(3x-64)}{3(x-14)} \right] \right] \right], \right. \right. \\ \left. \left. \left[\frac{x-3}{x-14} \right], \left[\left[\left[\frac{2}{3} \right], 2, \left[\frac{x^2-61x+416}{(x-14)^2 (x-3)} \right], \left[(x-14)^3 Dx^2 + (33x-462) Dx \right. \right. \right. \right. \\ \left. \left. \left. - \frac{11(9x^2-197x+268)}{3(x^2-17x+42)} \right] \right] \right], -\frac{x-3}{x-14} \right] \right\}$$

0.671 (52)

[> ##### THE RATIONAL AND IRRATIONAL CASE #####

```
> LA:=MinOp(hypergeom([a],[b],x)^2);
```

$$LA := Dx^3 + \frac{3(-x+b) Dx^2}{x} - \frac{(4ax-2b^2+4bx-2x^2+b) Dx}{x^2} - \frac{2a(-2x+2b-1)}{x^2} \quad (53)$$

```
> L1:=subs({a=1/3,b=1/2},LA);
```

$$L1 := Dx^3 + \frac{3\left(-x+\frac{1}{2}\right) Dx^2}{x} - \frac{\left(\frac{10}{3}x-2x^2\right) Dx}{x^2} + \frac{4}{3x} \quad (54)$$

```
> f:=(x-3)/(x-7);
```

$$f := \frac{x-3}{x-7} \quad (55)$$

```
> L:=ChangeOfVariables(L1,f);
```

$$L := -256 + 3 Dx^3 (x-3) (x-7)^5 + 18 (x^2-9x+22) Dx^2 (x-7)^3 + 2 (9x^3-135x^2+679x-1033) Dx (x-7) \quad (56)$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \emptyset \quad (57)$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{ }),ext),{RootOf,name});
```

$$ext := \emptyset \quad (58)$$

```
> extppp:={};
```

$$extppp := \emptyset \quad (59)$$

```
> E:= Singular(L,extppp);
```

$$E := [[x-3, 3], [x-7, 7]] \quad (60)$$

```
> F:= NotAppSing(L,E,ext);
```

$$F := [[x-7, 7], [x-3, 3]] \quad (61)$$

```
> Sirr:=irrsinglFlsq(L,t,F,ext);
```

$$Sirr := \left[[[x-7, 7]], \left[\left[\frac{2}{3}, -\frac{8}{t} + \frac{1}{3}, -\frac{4}{t} + \frac{1}{2} \right], \left[-\frac{4}{t} - \frac{1}{6}, \frac{4}{t} + \frac{1}{6}, -\frac{8}{t} - \frac{1}{3} \right] \right] \right], \quad (62)$$

$$[1], [1], \left[\left[\left[-\frac{4}{t} + \frac{1}{2}, \frac{2}{3} \right], \left[-\frac{4}{t} + \frac{1}{2}, -\frac{8}{t} + \frac{1}{3} \right], \left[-\frac{8}{t} + \frac{1}{3}, \frac{2}{3} \right] \right], [-4t, 4t, -8t], \left[\left[-\frac{1}{6}, \frac{1}{6}, -\frac{1}{3} \right], \left[[x-3, 3], \left[\left[0, 1, \frac{1}{2} \right], \left[1, \frac{1}{2}, -\frac{1}{2} \right], [1, 1, 1], \left[[1, 0], \left[\frac{1}{2}, 0 \right], \left[\frac{1}{2}, 1 \right] \right], 3 \right] \right] \right]$$

> Sreg:=regsingtruelFlsq(L,t,Sirr[-1],ext);

$$Sreg := \left[[x-3, 3], \left[\left[0, 1, \frac{1}{2} \right], \left[\left[\frac{1}{2}, -\frac{1}{2}, 1 \right], \left[\left[\left[\frac{1}{2}, 0 \right], \left[\frac{1}{2}, 1 \right], [1, 0] \right] \right] \right] \right] \right] \quad (63)$$

> RSreg:=Sregseptrue1Flsq(L,Sreg,ext);

$$RSreg := \left[\left[[x-3, 3], \left[\left[0, 1, \frac{1}{2} \right], \left[\left[\left[\frac{1}{2}, -\frac{1}{2} \right], [1] \right] \right], [], [] \right] \right] \right] \quad (64)$$

> R1:=IrrRegAppsing1Flsq(L,t,E,ext);

$$R1 := \left[\left[[x-7, 7], \left[\left[\frac{2}{3}, -\frac{8}{t} + \frac{1}{3}, -\frac{4}{t} + \frac{1}{2} \right], \left[\left[-\frac{4}{t} - \frac{1}{6}, \frac{4}{t} + \frac{1}{6}, -\frac{8}{t} - \frac{1}{3} \right], [1], [1], \left[\left[\left[-\frac{4}{t} + \frac{1}{2}, \frac{2}{3} \right], \left[-\frac{4}{t} + \frac{1}{2}, -\frac{8}{t} + \frac{1}{3} \right], \left[-\frac{8}{t} + \frac{1}{3}, \frac{2}{3} \right] \right], [-4t, 4t, -8t], \left[\left[-\frac{1}{6}, \frac{1}{6}, -\frac{1}{3} \right], \left[[x-3, 3], \left[\left[0, 1, \frac{1}{2} \right], \left[\left[\frac{1}{2}, -\frac{1}{2}, 1 \right], \left[\left[\left[\frac{1}{2}, 0 \right], \left[\frac{1}{2}, 1 \right], [1, 0] \right] \right], \left[\left[[x-3, 3], \left[\left[0, 1, \frac{1}{2} \right], \left[\left[\left[\frac{1}{2}, -\frac{1}{2} \right], [1] \right] \right], [], [] \right], [], \left[[x-7, 7], [x-3, 3], \left[\left[\frac{2}{3}, -\frac{8}{t} + \frac{1}{3}, -\frac{4}{t} + \frac{1}{2} \right], \left[0, 1, \frac{1}{2} \right], \left[\left[-\frac{4}{t} - \frac{1}{6}, \frac{4}{t} + \frac{1}{6}, -\frac{8}{t} - \frac{1}{3} \right], \left[1, \frac{1}{2}, -\frac{1}{2} \right], \left[\left[\left[-\frac{4}{t} + \frac{1}{2}, \frac{2}{3} \right], \left[-\frac{4}{t} + \frac{1}{2}, -\frac{8}{t} + \frac{1}{3} \right], \left[-\frac{8}{t} + \frac{1}{3}, \frac{2}{3} \right], \left[[1, 0], \left[\frac{1}{2}, 0 \right], \left[\frac{1}{2}, 1 \right] \right], [1, 1, 1], [1, 1, 1] \right] \right] \right] \right] \right] \right] \quad (65)$$

> F1:=Hyp1FlsqSubst(L,x,t,R1[1],ext);

$$F1 := \left[-\frac{4}{x-7}, \frac{4}{x-7} \right] \quad (66)$$

> find1FlsqRatIrr(L,R1,F1,x,t,ext);

$$\left[\left[\left[\left\{ \frac{1}{3}, \frac{1}{6}, \frac{2}{3}, \frac{5}{6} \right\}, \left[\frac{1}{2} \right], \frac{x-3}{x-7} \right], \left[\left[\left\{ \frac{1}{3}, \frac{1}{6}, \frac{2}{3}, \frac{5}{6} \right\}, \left[\frac{1}{2} \right], -\frac{x-3}{x-7} \right] \right] \right] \right] \quad (67)$$

> TIME :=time();

Hyp1FlsqSolutions(L);

time() - TIME;

$$TIME := 31.062$$

$$\left\{ \left[\left[\left[\left[\frac{1}{3} \right], \frac{1}{2}, [0], [1] \right], \frac{x-3}{x-7} \right], \left[\left[\left[\frac{1}{6} \right], \frac{1}{2}, \left[\frac{x^2 - 22x + 73}{(x-7)^2 (x-3)} \right], \left[\frac{1}{x-3} \right] \right], -\frac{x-3}{x-7} \right] \right\}$$

$$0.125$$

(68)

$$\begin{aligned}
& \text{> LA:=MinOp(hypergeom([a],[b],x)^2);} \\
& LA := Dx^3 + \frac{3(-x+b) Dx^2}{x} - \frac{(4ax - 2b^2 + 4bx - 2x^2 + b) Dx}{x^2} \\
& \quad - \frac{2a(-2x+2b-1)}{x^2}
\end{aligned} \tag{69}$$

$$\begin{aligned}
& \text{> L1:=subs(\{a=1/7,b=1/2\},LA);} \\
& L1 := Dx^3 + \frac{3\left(-x + \frac{1}{2}\right) Dx^2}{x} - \frac{\left(\frac{18}{7}x - 2x^2\right) Dx}{x^2} + \frac{4}{7x}
\end{aligned} \tag{70}$$

$$\begin{aligned}
& \text{> f:=(x-1)/(x-12);} \\
& f := \frac{x-1}{x-12}
\end{aligned} \tag{71}$$

$$\begin{aligned}
& \text{> L:=ChangeOfVariables(L1,f);} \\
& L := -10648 + 14 Dx^3 (x-1) (x-12)^5 + 21 (4x^2 - 41x + 158) Dx^2 (x-12)^3 + 2 (42x^3 \\
& \quad - 819x^2 + 6110x - 9326) Dx (x-12)
\end{aligned} \tag{72}$$

$$\begin{aligned}
& \text{> ext:=indets(L,\{RootOf,name\}) minus \{x,Dx\};} \\
& ext := \emptyset
\end{aligned} \tag{73}$$

$$\begin{aligned}
& \text{> ext:= indets(map(s-> ReplirrRoot(s,\{ \}),ext),\{RootOf,name\});} \\
& ext := \emptyset
\end{aligned} \tag{74}$$

$$\begin{aligned}
& \text{> extppp:=\{ \};} \\
& extppp := \emptyset
\end{aligned} \tag{75}$$

$$\begin{aligned}
& \text{> E:= Singular(L,extppp);} \\
& E := [[x-1, 1], [x-12, 12]]
\end{aligned} \tag{76}$$

$$\begin{aligned}
& \text{> F:= NotAppSing(L,E,ext);} \\
& F := [[x-12, 12], [x-1, 1]]
\end{aligned} \tag{77}$$

$$\begin{aligned}
& \text{> Sirr:=irrsinglFlsq(L,t,F,ext);} \\
& Sirr := \left[[[x-12, 12]], \left[\left[\frac{2}{7}, -\frac{11}{t} + \frac{1}{2}, -\frac{22}{t} + \frac{5}{7} \right], \left[-\frac{11}{t} + \frac{3}{14}, -\frac{11}{t} + \frac{3}{14}, \right. \right. \right. \\
& \quad \left. \left. -\frac{22}{t} + \frac{3}{7} \right], [1], [1], \left[\left[-\frac{11}{t} + \frac{1}{2}, \frac{2}{7} \right], \left[-\frac{22}{t} + \frac{5}{7}, -\frac{11}{t} + \frac{1}{2} \right], \left[-\frac{22}{t} + \frac{5}{7}, \right. \right. \right. \\
& \quad \left. \left. \frac{2}{7} \right] \right], [[-11t, -11t, -22t]], \left[\left[\frac{3}{14}, \frac{3}{14}, \frac{3}{7} \right], \left[[x-1, 1], \left[\left[0, 1, \frac{1}{2} \right], \left[1, \frac{1}{2}, \right. \right. \right. \right. \right. \\
& \quad \left. \left. -\frac{1}{2} \right], [1, 1, 1], \left[[1, 0], \left[\frac{1}{2}, 0 \right], \left[\frac{1}{2}, 1 \right], 3 \right] \right] \right]
\end{aligned} \tag{78}$$

$$\begin{aligned}
& \text{> Sreg:=regsingtruelFlsq(L,t,Sirr[-1],ext);} \\
& Sreg := \left[[[x-1, 1]], \left[\left[0, 1, \frac{1}{2} \right], \left[\left[\frac{1}{2}, -\frac{1}{2}, 1 \right], \left[\left[\frac{1}{2}, 0 \right], \left[\frac{1}{2}, 1 \right], [1, 0] \right] \right] \right] \right]
\end{aligned} \tag{79}$$

$$\begin{aligned}
& \text{> RSreg:=SregseptruelFlsq(L,Sreg,ext);} \\
& RSreg := \left[\left[[[x-1, 1]], \left[\left[0, 1, \frac{1}{2} \right], \left[\left[\frac{1}{2}, -\frac{1}{2} \right], [1] \right] \right] \right], [], [] \right]
\end{aligned} \tag{80}$$

$$\begin{aligned}
& \text{> R1:=IrrRegAppsinglFlsq(L,t,E,ext);} \\
& R1 := \left[\left[[[x-12, 12]], \left[\left[\frac{2}{7}, -\frac{11}{t} + \frac{1}{2}, -\frac{22}{t} + \frac{5}{7} \right], \left[-\frac{11}{t} + \frac{3}{14}, -\frac{11}{t} + \frac{3}{14}, \right. \right. \right. \right.
\end{aligned} \tag{81}$$

$$\begin{aligned} & -\frac{22}{t} + \frac{3}{7} \Big], [1], [1], \Big[\Big[-\frac{11}{t} + \frac{1}{2}, \frac{2}{7} \Big], \Big[-\frac{22}{t} + \frac{5}{7}, -\frac{11}{t} + \frac{1}{2} \Big], \Big[-\frac{22}{t} + \frac{5}{7}, \\ & \frac{2}{7} \Big] \Big], [[-11t, -11t, -22t], \Big[\Big[\frac{3}{14}, \frac{3}{14}, \frac{3}{7} \Big] \Big], \Big[[x-1, 1], \Big[0, 1, \frac{1}{2} \Big], \Big[\Big[\frac{1}{2}, \\ & -\frac{1}{2}, 1 \Big], \Big[\Big[\Big[\frac{1}{2}, 0 \Big], \Big[\frac{1}{2}, 1 \Big], [1, 0] \Big] \Big], \Big[[x-1, 1], \Big[0, 1, \frac{1}{2} \Big], \Big[\Big[\frac{1}{2}, -\frac{1}{2} \Big], \\ & [1] \Big] \Big], [], [] \Big], [], \Big[[x-12, 12], [x-1, 1], \Big[\frac{2}{7}, -\frac{11}{t} + \frac{1}{2}, -\frac{22}{t} + \frac{5}{7} \Big], \Big[0, 1, \\ & \frac{1}{2} \Big], \Big[-\frac{11}{t} + \frac{3}{14}, -\frac{11}{t} + \frac{3}{14}, -\frac{22}{t} + \frac{3}{7} \Big], \Big[1, \frac{1}{2}, -\frac{1}{2} \Big], \Big[\Big[-\frac{11}{t} + \frac{1}{2}, \frac{2}{7} \Big], \Big[\\ & -\frac{22}{t} + \frac{5}{7}, -\frac{11}{t} + \frac{1}{2} \Big], \Big[-\frac{22}{t} + \frac{5}{7}, \frac{2}{7} \Big], \Big[[1, 0], \Big[\frac{1}{2}, 0 \Big], \Big[\frac{1}{2}, 1 \Big] \Big], \Big[[1, 1, 1], [1, \\ & 1, 1] \Big] \Big] \end{aligned}$$

> F1:=Hyp1F1sqSubst(L,x,t,R1[1],ext);

$$F1 := \left[-\frac{11}{x-12}, \frac{11}{x-12} \right] \quad (82)$$

> find1F1sqRatIrr(L,R1,F1,x,t,ext);

$$\left[\left[\left[\left[\frac{1}{7}, \frac{5}{14}, \frac{6}{7}, \frac{9}{14} \right], \left[\frac{1}{2} \right], \frac{x-1}{x-12} \right], \left[\left[\left[\frac{1}{7}, \frac{5}{14}, \frac{6}{7}, \frac{9}{14} \right], \left[\frac{1}{2} \right], -\frac{x-1}{x-12} \right] \right] \right] \quad (83)$$

**> TIME :=time();
Hyp1F1sqSolutions(L);
time() - TIME;**

$$TIME := 32.312$$

$$\left\{ \left[\left[\left[\left[\frac{1}{7} \right], \frac{1}{2}, [0], [1] \right], \frac{x-1}{x-12} \right], \left[\left[\left[\frac{5}{14} \right], \frac{1}{2}, \left[\frac{x^2-46x+166}{(x-12)^2(x-1)} \right], \left[\frac{1}{x-1} \right] \right], \right. \right. \\ \left. \left. -\frac{x-1}{x-12} \right] \right\}$$

$$0.125 \quad (84)$$

> LA:=MinOp(hypergeom([a],[b],x)^2);

$$LA := Dx^3 + \frac{3(-x+b)Dx^2}{x} - \frac{(4ax-2b^2+4bx-2x^2+b)Dx}{x^2} \\ - \frac{2a(-2x+2b-1)}{x^2} \quad (85)$$

> L1:=subs({a=1/7,b=RootOf(x^2+2)},LA);

$$L1 := Dx^3 + \frac{3(-x+RootOf(_Z^2+2))Dx^2}{x} \\ - \frac{\left(\frac{4x}{7} - 2RootOf(_Z^2+2)^2 + 4RootOf(_Z^2+2)x - 2x^2 + RootOf(_Z^2+2) \right) Dx}{x^2} \quad (86)$$

$$- \frac{2(-2x + 2 \operatorname{RootOf}(_Z^2 + 2) - 1)}{7x^2}$$

> f:=(x^2+1)/x;

$$f := \frac{x^2 + 1}{x} \quad (87)$$

> L:=ChangeOfVariables(L1,f);

$$L := 7x^5(x-1)^2(x+1)^2(x^2+1)^2Dx^3 + 21(\operatorname{RootOf}(_Z^2 + 2)x^5 - x^6 - 2\operatorname{RootOf}(_Z^2 + 2)x^3 + x^4 - 2x^3 + \operatorname{RootOf}(_Z^2 + 2)x + x^2 - 2x - 1)(x^2+1)(x+1)(x-1)x^3Dx^2 - x(28\operatorname{RootOf}(_Z^2 + 2)x^{11} - 14x^{12} + 7\operatorname{RootOf}(_Z^2 + 2)x^{10} + 4x^{11} - 84\operatorname{RootOf}(_Z^2 + 2)x^9 + 56x^{10} + 14\operatorname{RootOf}(_Z^2 + 2)x^8 - 54x^9 + 56\operatorname{RootOf}(_Z^2 + 2)x^7 - 140x^8 + 8x^7 + 56\operatorname{RootOf}(_Z^2 + 2)x^5 - 14x^6 - 70\operatorname{RootOf}(_Z^2 + 2)x^4 + 92x^5 - 84\operatorname{RootOf}(_Z^2 + 2)x^3 - 224x^4 + 49\operatorname{RootOf}(_Z^2 + 2)x^2 - 12x^3 + 28\operatorname{RootOf}(_Z^2 + 2)x + 14x^2 - 38x - 14)Dx - 2(2\operatorname{RootOf}(_Z^2 + 2)x - 2x^2 - x - 2)(x+1)^5(x-1)^5 \quad (88)$$

> ext:=indets(L,{RootOf,name}) minus {x,Dx};

$$ext := \{\operatorname{RootOf}(_Z^2 + 2)\} \quad (89)$$

> ext:= indets(map(s-> ReplirrRoot(s,{ }),ext),{RootOf,name});

$$ext := \{\operatorname{RootOf}(_Z^2 + 2)\} \quad (90)$$

> extppp:={};

$$extppp := \emptyset \quad (91)$$

> E:= Singular(L,extppp);

$$E := [[x, 0], [x^2 + 1, \operatorname{RootOf}(_Z^2 + 1)], [x - 1, 1], [x + 1, -1], [\infty, \infty]] \quad (92)$$

> F:= NotAppSing(L,E,ext);

$$F := [[\infty, \infty], [x, 0], [x^2 + 1, \operatorname{RootOf}(_Z^2 + 1)]] \quad (93)$$

> Sirr:=irrsinglFlsq(L,t,F,ext);

$$Sirr := \left[[[\infty, \infty], [x, 0]], \left[\left[\frac{2}{7}, -\frac{1}{t} + \operatorname{RootOf}(_Z^2 + 2), -\frac{2}{t} - \frac{2}{7} + 2\operatorname{RootOf}(_Z^2 + 2) \right], \left[\frac{2}{7}, -\frac{1}{t} + \operatorname{RootOf}(_Z^2 + 2), -\frac{2}{t} - \frac{2}{7} + 2\operatorname{RootOf}(_Z^2 + 2) \right] \right], \left[\left[-\frac{1}{t} + \operatorname{RootOf}(_Z^2 + 2) - \frac{2}{7}, -\frac{1}{t} + \operatorname{RootOf}(_Z^2 + 2) - \frac{2}{7}, -\frac{2}{t} - \frac{4}{7} + 2\operatorname{RootOf}(_Z^2 + 2) \right], \left[-\frac{1}{t} + \operatorname{RootOf}(_Z^2 + 2) - \frac{2}{7}, -\frac{1}{t} + \operatorname{RootOf}(_Z^2 + 2) - \frac{2}{7}, -\frac{2}{t} - \frac{4}{7} + 2\operatorname{RootOf}(_Z^2 + 2) \right] \right], \left[\left[-\frac{1}{t} + \operatorname{RootOf}(_Z^2 + 2), \frac{2}{7} \right], \left[-\frac{2}{t} - \frac{2}{7} + 2\operatorname{RootOf}(_Z^2 + 2), -\frac{1}{t} + \operatorname{RootOf}(_Z^2 + 2) \right], \left[-\frac{2}{t} - \frac{2}{7} + 2\operatorname{RootOf}(_Z^2 + 2), \frac{2}{7} \right] \right], \left[\left[-\frac{1}{t} + \operatorname{RootOf}(_Z^2 + 2), \frac{2}{7} \right], \left[-\frac{2}{t} - \frac{2}{7} + 2\operatorname{RootOf}(_Z^2 + 2), -\frac{1}{t} + \operatorname{RootOf}(_Z^2 + 2) \right], \left[-\frac{2}{t} - \frac{2}{7} + 2\operatorname{RootOf}(_Z^2 + 2), \frac{2}{7} \right] \right], [[-t, -t, -2t], [-t, -t, \frac{2}{7}]] \right] \quad (94)$$

$$-2t]], \left[\left[\text{RootOf}(_Z^2 + 2) - \frac{2}{7}, \text{RootOf}(_Z^2 + 2) - \frac{2}{7}, -\frac{4}{7} + 2 \text{RootOf}(_Z^2 + 2) \right], \right. \\ \left. \left[\text{RootOf}(_Z^2 + 2) - \frac{2}{7}, \text{RootOf}(_Z^2 + 2) - \frac{2}{7}, -\frac{4}{7} + 2 \text{RootOf}(_Z^2 + 2) \right] \right], [[x^2 \\ + 1, \text{RootOf}(_Z^2 + 1)]], [[0, 2 - 2 \text{RootOf}(_Z^2 + 2), 1 - \text{RootOf}(_Z^2 + 2)], [2 \\ - 2 \text{RootOf}(_Z^2 + 2), 1 - \text{RootOf}(_Z^2 + 2), \text{RootOf}(_Z^2 + 2) - 1], [1, 1, 1], [2 \\ - 2 \text{RootOf}(_Z^2 + 2), 0], [1 - \text{RootOf}(_Z^2 + 2), 0], [1 - \text{RootOf}(_Z^2 + 2), 2 \\ - 2 \text{RootOf}(_Z^2 + 2)]], 2]]]]$$

> Sreg:=regsingtruelFlsq(L,t,Sirr[-1],ext);

$$Sreg := [[[x^2 + 1, \text{RootOf}(_Z^2 + 1)]], [[0, 2 - 2 \text{RootOf}(_Z^2 + 2), 1 - \text{RootOf}(_Z^2 + 2)]], [[1 - \text{RootOf}(_Z^2 + 2), \text{RootOf}(_Z^2 + 2) - 1, 2 - 2 \text{RootOf}(_Z^2 + 2)]], [[1 - \text{RootOf}(_Z^2 + 2), 0], [1 - \text{RootOf}(_Z^2 + 2), 2 - 2 \text{RootOf}(_Z^2 + 2)], [2 - 2 \text{RootOf}(_Z^2 + 2), 0]]] \quad (95)$$

> RSreg:=SregseptruelFlsq(L,Sreg,ext);

$$RSreg := [[[x^2 + 1, \text{RootOf}(_Z^2 + 1)]], [[0, 2 - 2 \text{RootOf}(_Z^2 + 2), 1 - \text{RootOf}(_Z^2 + 2)]], [[1 - \text{RootOf}(_Z^2 + 2), \text{RootOf}(_Z^2 + 2) - 1, 2 - 2 \text{RootOf}(_Z^2 + 2)], []]], [], [] \quad (96)$$

> R1:=IrrRegAppsinglFlsq(L,t,E,ext);

$$R1 := \left[\left[[\infty, \infty], [x, 0] \right], \left[\left[\frac{2}{7}, -\frac{1}{t} + \text{RootOf}(_Z^2 + 2), -\frac{2}{t} - \frac{2}{7} + 2 \text{RootOf}(_Z^2 + 2) \right], \left[\frac{2}{7}, -\frac{1}{t} + \text{RootOf}(_Z^2 + 2), -\frac{2}{t} - \frac{2}{7} + 2 \text{RootOf}(_Z^2 + 2) \right] \right], \left[\left[-\frac{1}{t} + \text{RootOf}(_Z^2 + 2) - \frac{2}{7}, -\frac{1}{t} + \text{RootOf}(_Z^2 + 2) - \frac{2}{7}, -\frac{2}{t} - \frac{4}{7} + 2 \text{RootOf}(_Z^2 + 2) \right], \left[-\frac{1}{t} + \text{RootOf}(_Z^2 + 2) - \frac{2}{7}, -\frac{1}{t} + \text{RootOf}(_Z^2 + 2) - \frac{2}{7}, -\frac{2}{t} - \frac{4}{7} + 2 \text{RootOf}(_Z^2 + 2) \right] \right], [1, 1], [1, 1], \left[\left[\left[-\frac{1}{t} + \text{RootOf}(_Z^2 + 2), \frac{2}{7} \right], \left[-\frac{2}{t} - \frac{2}{7} + 2 \text{RootOf}(_Z^2 + 2), -\frac{1}{t} + \text{RootOf}(_Z^2 + 2) \right], \left[-\frac{2}{t} - \frac{2}{7} + 2 \text{RootOf}(_Z^2 + 2), \frac{2}{7} \right] \right], \left[\left[-\frac{1}{t} + \text{RootOf}(_Z^2 + 2), \frac{2}{7} \right], \left[-\frac{2}{t} - \frac{2}{7} + 2 \text{RootOf}(_Z^2 + 2), -\frac{1}{t} + \text{RootOf}(_Z^2 + 2) \right], \left[-\frac{2}{t} - \frac{2}{7} + 2 \text{RootOf}(_Z^2 + 2), \frac{2}{7} \right] \right], [[-t, -t, -2t], [-t, -t, -2t]], \left[\left[\text{RootOf}(_Z^2 + 2) - \frac{2}{7}, \text{RootOf}(_Z^2 + 2) - \frac{2}{7}, -\frac{4}{7} + 2 \text{RootOf}(_Z^2 + 2) \right], \left[\text{RootOf}(_Z^2 + 2) - \frac{2}{7}, \text{RootOf}(_Z^2 + 2) - \frac{2}{7}, -\frac{4}{7} + 2 \text{RootOf}(_Z^2 + 2) \right] \right], [[x^2 \\ + 1, \text{RootOf}(_Z^2 + 1)]], [[0, 2 - 2 \text{RootOf}(_Z^2 + 2), 1 - \text{RootOf}(_Z^2 + 2)]], [[1$$

$$\begin{aligned}
& -\text{RootOf}(_Z^2 + 2), \text{RootOf}(_Z^2 + 2) - 1, 2 - 2 \text{RootOf}(_Z^2 + 2) \], [[[1 \\
& -\text{RootOf}(_Z^2 + 2), 0], [1 - \text{RootOf}(_Z^2 + 2), 2 - 2 \text{RootOf}(_Z^2 + 2)], [2 \\
& - 2 \text{RootOf}(_Z^2 + 2), 0]]], [[[[x^2 + 1, \text{RootOf}(_Z^2 + 1)]], [[0, 2 - 2 \text{RootOf}(_Z^2 \\
& + 2), 1 - \text{RootOf}(_Z^2 + 2)]], [[[2 - 2 \text{RootOf}(_Z^2 + 2), 1 - \text{RootOf}(_Z^2 + 2), \\
& \text{RootOf}(_Z^2 + 2) - 1], []]]], [], []], [[[x + 1, -1], [x - 1, 1]], [[0, 2, 4], [0, 2, 4]], \\
& [[2, 4, 2], [2, 4, 2]], [[[2, 0], [4, 0], [4, 2]], [[2, 0], [4, 0], [4, 2]]]], \left[[[\infty, \infty], [x, 0], \right. \\
& [x^2 + 1, \text{RootOf}(_Z^2 + 1)]], \left[\left[\frac{2}{7}, -\frac{1}{t} + \text{RootOf}(_Z^2 + 2), -\frac{2}{t} - \frac{2}{7} + 2 \text{RootOf}(_Z^2 \right. \right. \\
& + 2) \left. \right], \left[\frac{2}{7}, -\frac{1}{t} + \text{RootOf}(_Z^2 + 2), -\frac{2}{t} - \frac{2}{7} + 2 \text{RootOf}(_Z^2 + 2) \right], [0, 2 \\
& - 2 \text{RootOf}(_Z^2 + 2), 1 - \text{RootOf}(_Z^2 + 2)] \left. \right], \left[\left[-\frac{1}{t} + \text{RootOf}(_Z^2 + 2) - \frac{2}{7}, -\frac{1}{t} \right. \right. \\
& + \text{RootOf}(_Z^2 + 2) - \frac{2}{7}, -\frac{2}{t} - \frac{4}{7} + 2 \text{RootOf}(_Z^2 + 2) \left. \right], \left[-\frac{1}{t} + \text{RootOf}(_Z^2 + 2) \right. \\
& - \frac{2}{7}, -\frac{1}{t} + \text{RootOf}(_Z^2 + 2) - \frac{2}{7}, -\frac{2}{t} - \frac{4}{7} + 2 \text{RootOf}(_Z^2 + 2) \left. \right], [2 \\
& - 2 \text{RootOf}(_Z^2 + 2), 1 - \text{RootOf}(_Z^2 + 2), \text{RootOf}(_Z^2 + 2) - 1] \left. \right], \left[\left[\left[-\frac{1}{t} \right. \right. \right. \\
& + \text{RootOf}(_Z^2 + 2), \frac{2}{7} \left. \right], \left[-\frac{2}{t} - \frac{2}{7} + 2 \text{RootOf}(_Z^2 + 2), -\frac{1}{t} + \text{RootOf}(_Z^2 + 2) \right], \\
& \left[-\frac{2}{t} - \frac{2}{7} + 2 \text{RootOf}(_Z^2 + 2), \frac{2}{7} \right], \left[\left[-\frac{1}{t} + \text{RootOf}(_Z^2 + 2), \frac{2}{7} \right], \left[-\frac{2}{t} - \frac{2}{7} \right. \right. \\
& + 2 \text{RootOf}(_Z^2 + 2), -\frac{1}{t} + \text{RootOf}(_Z^2 + 2) \left. \right], \left[-\frac{2}{t} - \frac{2}{7} + 2 \text{RootOf}(_Z^2 + 2), \right. \\
& \left. \frac{2}{7} \right] \left. \right], [[[2 - 2 \text{RootOf}(_Z^2 + 2), 0], [1 - \text{RootOf}(_Z^2 + 2), 0], [1 - \text{RootOf}(_Z^2 + 2), 2 \\
& - 2 \text{RootOf}(_Z^2 + 2)] \left. \right], [[[1, 1, 1], [1, 1, 1], [1, 1, 1]]]]]
\end{aligned}$$

> F1:=Hyp1FlsqSubst(L,x,t,R1[1],ext);

$$F1 := \left[-\frac{x^2+1}{x}, -\frac{x^2-1}{x}, \frac{x^2-1}{x}, \frac{x^2+1}{x} \right] \quad (98)$$

> find1FlsqRatIrr(L,R1,F1,x,t,ext);

$$\begin{aligned}
& \left[\left[\left[\left\{ \frac{1}{7}, \frac{9}{14}, \text{RootOf}(_Z^2 + 2) + \frac{5}{14}, \text{RootOf}(_Z^2 + 2) + \frac{6}{7} \right\}, [\text{RootOf}(_Z^2 + 2)] \right], \right. \right. \\
& \left. \left. -\frac{x^2+1}{x} \right], \left[\left[\left\{ \frac{1}{7}, \frac{9}{14}, \text{RootOf}(_Z^2 + 2) + \frac{5}{14}, \text{RootOf}(_Z^2 + 2) + \frac{6}{7} \right\}, [\text{RootOf}(_Z^2 \right. \right. \right.
\end{aligned} \quad (99)$$

$$+ 2) \Big] \Big], \frac{x^2 + 1}{x} \Big] \Big]$$

```
> TIME :=time();
Hyp1FlsqSolutions(L);
time() - TIME;
```

TIME := 75.593

$$\left\{ \left[\left[\left[\left[\frac{1}{7} \right], \text{RootOf}(_Z^2 + 2), [0], [1] \right] \right], \frac{x^2 + 1}{x} \right], \left[\left[\left[\left[\text{RootOf}(_Z^2 + 2) + \frac{6}{7} \right], \text{RootOf}(_Z^2 + 2), \left[\frac{2(x^2 - x - 1)}{x^2} \right], \left[\frac{x^4(x^2 + 1)^2 Dx^2}{(x - 1)^2 (x + 1)^2} - \frac{1}{7(x - 1)^2 (x + 1)^2 (x^2 - 1)} \left((-7 \text{RootOf}(_Z^2 + 2) x^7 + 7 x^8 - 16 x^7 + 7 \text{RootOf}(_Z^2 + 2) x^5 - 26 x^5 + 7 \text{RootOf}(_Z^2 + 2) x^3 - 14 x^4 + 44 x^3 - 7 \text{RootOf}(_Z^2 + 2) x + 54 x + 7) x^2 Dx \right) - \frac{1}{49(x - 1)^2 (x + 1)^2 (x^2 - 1)} (2(-49 \text{RootOf}(_Z^2 + 2) x^7 + 7 x^8 - x^7 + 49 \text{RootOf}(_Z^2 + 2) x^5 + 84 x^6 + 73 x^5 + 49 \text{RootOf}(_Z^2 + 2) x^3 - 98 x^4 + 53 x^3 - 49 \text{RootOf}(_Z^2 + 2) x - 84 x^2 + 267 x + 91) x) \right] \right], -\frac{x^2 + 1}{x} \right] \right\}$$

18.797

(100)