```
> restart:
> read "ODE3solve.mpl":
```

Package "Solving third-order holonomic differential equations", Maple 16
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Package "Hypergeometric Summation", Maple V - Maple 17
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Here are the Maple implementations in chapter 5 related just to the 0F2 functions.

```
> ####### THE EXPONENT DIFFERENCES #######
```

In chapter 5, section 5.2 which is called "Exponent differences", we have the following Maple implementations:

```
> L02 := x^2*Dx^3+(x*b2+x+x*b1)*Dx^2+b2*b1*Dx-1;
```

$$L02 := x^2\,Dx^3 + (x\,b1 + x\,b2 + x)\,Dx^2 + b2\,b1\,Dx - 1 \quad \textbf{(2)}$$

```
> gen_exp(L02,t,x=0);
```

$$[[0, t = x], [1 - b1, t = x], [1 - b2, t = x]] \quad \textbf{(3)}$$

```
> gen_exp(L02,t,x=infinity);
```

$$\left[\left[\frac{1}{t} - \frac{1}{3} + \frac{b1}{3} + \frac{b2}{3}, -t^3 = \frac{1}{x}\right]\right] \quad \textbf{(4)}$$

```
> ####### EXAMPLE IN THE THESIS #######
```

In chapter 5, section 5.5.7 which is called "Examples", those are the Maple implementations for the example that we have used in the 0F2 type solutions:

```
> F:=sumdiffeq(hyperterm([],[b1,b2],x,k),k,J(x));
```

$$F := \left(\frac{d^3}{dx^3}\,J(x)\right) x^2 + (b1 + b2 + 1)\left(\frac{d^2}{dx^2}\,J(x)\right) x + b1\,b2\left(\frac{d}{dx}\,J(x)\right) - J(x) = 0 \quad \textbf{(5)}$$

```
> LA:=de2diffop(F,J(x));
```

$$LA := x^2\,Dx^3 + (x\,b1 + x\,b2 + x)\,Dx^2 + b2\,b1\,Dx - 1 \quad \textbf{(6)}$$

```
> L12:=subs({b1=1/3,b2=1/7},LA);
```

$$L12 := x^2\,Dx^3 + \frac{31}{21}\,x\,Dx^2 + \frac{1}{21}\,Dx - 1 \quad \textbf{(7)}$$

```
> f:=(2*(x-1)^2*(x-3)*(x-7)^3)/((x-9)^2*(x-12)^3);
```

$$f := \frac{2\,(x-1)^2\,(x-3)\,(x-7)^3}{(x-9)^2\,(x-12)^3} \quad \textbf{(8)}$$

```
> L:=ChangeOfVariables(L12,f);
```

$$L := 21\,Dx^3\,(x-1)^2\,(x-3)^2\,(x-7)^2\,(x-9)^5\,(x-12)^6\,(x^4 - 60\,x^3 + 830\,x^2 - 3852\,x \quad \textbf{(9)}$$

$$+5193)^2+(31x^8-3720x^7+86200x^6-649176x^5-635762x^4+32319144x^3-159987168x^2+300375864x-173735685)Dx^2(x-1)(x-3)(x-7)(x-9)^4(x-12)^5(x^4-60x^3+830x^2-3852x+5193)+(x^{16}-240x^{15}+63960x^{14}-3904976x^{13}+130520372x^{12}-3065641808x^{11}+54803306488x^{10}-746349293552x^9+7599004335182x^8-56984531313168x^7+311300882943048x^6-1223256100618800x^5+3388591949109444x^4-6395117622870960x^3+7748517717658728x^2-5387188885607952x+1632102637284153)Dx(x-9)^3(x-12)^4-42(x^4-60x^3+830x^2-3852x+5193)^5(x-1)(x-7)^2$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \varnothing \tag{10}$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```

$$ext := \varnothing \tag{11}$$

```
> extppp:={};
```

$$extppp := \varnothing \tag{12}$$

```
> E:= Singular(L,extppp);
```

$$E := [[x-1,1],[x-12,12],[\infty,\infty],[x-3,3],[x^4-60x^3+830x^2-3852x+5193, RootOf(\_Z^4-60\_Z^3+830\_Z^2-3852\_Z+5193)],[x-9,9],[x-7,7]] \tag{13}$$

```
> F:=NotAppSing(L,E,ext);
```

$$F := [[x-1,1],[x-3,3],[x-9,9],[x-12,12],[\infty,\infty],[x-7,7]] \tag{14}$$

```
> Sirr:=irrsing0F2(L,t,F,ext);
```

$$Sirr := \left[[[x-9,9],[x-12,12],[\infty,\infty]],\left[\left[\frac{16\,12^{1/3}}{3t^{2/3}}-\frac{22}{63},\frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2t^{2/3}}-\frac{22}{63}\right],\left[\frac{15\,RootOf(\_Z^3+242,index=1)}{t}-\frac{11}{21},\frac{15\,RootOf(\_Z^3+242,index=2)}{t}-\frac{11}{21},\frac{15\,RootOf(\_Z^3+242,index=3)}{t}-\frac{11}{21}\right],\left[-\frac{2^{1/3}}{t^{1/3}}-\frac{11}{63},-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}-\frac{11}{63},\frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}-\frac{11}{63}\right]\right],\left[\left[\frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2t^{2/3}}-\frac{16\,12^{1/3}}{3t^{2/3}},\frac{64\,12^{1/3}}{3\left(I\sqrt{3}+1\right)^2t^{2/3}}-\frac{16\,12^{1/3}}{3t^{2/3}},\frac{64\,12^{1/3}}{3\left(I\sqrt{3}+1\right)^2t^{2/3}}-\frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2t^{2/3}}\right],\left[\frac{15\,RootOf(\_Z^3+242,index=2)}{t}-\frac{15\,RootOf(\_Z^3+242,index=1)}{t},\frac{15\,RootOf(\_Z^3+242,index=3)}{t}\right],\left[-\frac{15\,RootOf(\_Z^3+242,index=1)}{t},\frac{15\,RootOf(\_Z^3+242,index=3)}{t}\right]\right] \tag{15}$$

$$-\frac{15\,RootOf\left(\_Z^3+242,\,index=2\right)}{t}\Bigg],\left[-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}+\frac{2^{1/3}}{t^{1/3}},\frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}+\frac{2^{1/3}}{t^{1/3}},\frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}+\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}\right]\Bigg],\left[\frac{2}{3},1,\frac{1}{3}\right],[3,1,3],$$

$$\left[\left[\left[\frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2t^{2/3}}-\frac{22}{63},\frac{16\,12^{1/3}}{3\,t^{2/3}}-\frac{22}{63}\right],\left[\frac{64\,12^{1/3}}{3\left(I\sqrt{3}+1\right)^2t^{2/3}}-\frac{22}{63},\frac{16\,12^{1/3}}{3\,t^{2/3}}-\frac{22}{63}\right],\left[\frac{64\,12^{1/3}}{3\left(I\sqrt{3}+1\right)^2t^{2/3}}-\frac{22}{63},\frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2t^{2/3}}-\frac{22}{63}\right]\right],\right.$$

$$\left[\left[\frac{15\,RootOf\left(\_Z^3+242,\,index=2\right)}{t}-\frac{11}{21},\frac{15\,RootOf\left(\_Z^3+242,\,index=1\right)}{t}-\frac{11}{21}\right],\right.$$

$$\left[\frac{15\,RootOf\left(\_Z^3+242,\,index=3\right)}{t}-\frac{11}{21},\frac{15\,RootOf\left(\_Z^3+242,\,index=1\right)}{t}-\frac{11}{21}\right],$$

$$\left.\left[\frac{15\,RootOf\left(\_Z^3+242,\,index=3\right)}{t}-\frac{11}{21},\frac{15\,RootOf\left(\_Z^3+242,\,index=2\right)}{t}-\frac{11}{21}\right]\right],$$

$$\left[\left[-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}-\frac{11}{63},-\frac{2^{1/3}}{t^{1/3}}-\frac{11}{63}\right],\left[\frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}-\frac{11}{63},-\frac{2^{1/3}}{t^{1/3}}-\frac{11}{63}\right],\left[\frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}-\frac{11}{63},-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}-\frac{11}{63}\right]\right]\right],$$

$$\left[\left[\frac{8\left(I\sqrt{3}\,12^{1/3}-3\,12^{1/3}\right)t^2}{3},-\frac{8\left(I\sqrt{3}\,12^{1/3}+3\,12^{1/3}\right)t^2}{3},-\frac{16\,I}{3}\sqrt{3}\,12^{1/3}t^2\right],\right.$$

$$\left[15\left(RootOf\left(\_Z^3+242,\,index=2\right)-RootOf\left(\_Z^3+242,\,index=1\right)\right)t,15\left(RootOf\left(\_Z^3\right.\right.\right.$$

$$\left.\left.\left.+242,\,index=3\right)-RootOf\left(\_Z^3+242,\,index=1\right)\right)t,15\left(RootOf\left(\_Z^3+242,\,index=3\right)\right.\right.$$

$$\left.\left.-RootOf\left(\_Z^3+242,\,index=2\right)\right)t\right],\left[\frac{\left(I\sqrt{3}\,2^{1/3}+3\,2^{1/3}\right)t}{2},\right.$$

$$\left.\left.-\frac{\left(I\sqrt{3}\,2^{1/3}-3\,2^{1/3}\right)t}{2},-I\,2^{1/3}\sqrt{3}\,t\right]\right],[[0,0,0],[0,0,0],[0,0,0]],\left[[x-1,1],\right.$$

$$[x-3,3],[x-7,7]],\left[\left[\left[0,\frac{12}{7},\frac{4}{3}\right],\left[\frac{12}{7},\frac{4}{3},-\frac{8}{21}\right],[1,1,1],\left[\left[\frac{12}{7},0\right],\left[\frac{4}{3},0\right],\right.\right.\right.$$

$$\left.\left.\left.\left[\frac{4}{3},\frac{12}{7}\right]\right],2\right],\left[\left[0,\frac{6}{7},\frac{2}{3}\right],\left[\frac{6}{7},\frac{2}{3},-\frac{4}{21}\right],[1,1,1],\left[\left[\frac{6}{7},0\right],\left[\frac{2}{3},0\right],\left[\frac{2}{3},\frac{6}{7}\right]\right],2\right],$$

$$\left.\left[\left[0,2,\frac{18}{7}\right],\left[2,\frac{18}{7},\frac{4}{7}\right],[1,1,1],\left[[2,0],\left[\frac{18}{7},0\right],\left[\frac{18}{7},2\right]\right],3\right]\right]\right]$$

```
> Sreg:=regsingtrue0F2(L,t,Sirr[-1],ext);
```

$$Sreg := \left[[[x-1,1],[x-3,3],[x-7,7]],\left[\left[0,\frac{12}{7},\frac{4}{3}\right],\left[0,\frac{6}{7},\frac{2}{3}\right],\left[0,2,\frac{18}{7}\right]\right],\left[\left[\frac{12}{7},\right.\right.\right. \quad \textbf{(16)}$$

$$\left[\frac{4}{3}, -\frac{8}{21}\right], \left[\frac{6}{7}, \frac{2}{3}, -\frac{4}{21}\right], \left[2, \frac{18}{7}, \frac{4}{7}\right]\right], \left[\left[\left[\frac{12}{7}, 0\right], \left[\frac{4}{3}, 0\right], \left[\frac{4}{3}, \frac{12}{7}\right]\right], \left[\left[\frac{6}{7}, 0\right], \left[\frac{2}{3}, 0\right], \left[\frac{2}{3}, \frac{6}{7}\right]\right], \left[[2, 0], \left[\frac{18}{7}, 0\right], \left[\frac{18}{7}, 2\right]\right]\right]\right]$$

```
> RSreg:=Sregseptrue0F2(L,Sreg,ext);
```

$$RSreg := \left[\left[[[x-1, 1], [x-3, 3], [x-7, 7]], \left[\left[0, \frac{12}{7}, \frac{4}{3}\right], \left[0, \frac{6}{7}, \frac{2}{3}\right], \left[0, 2, \frac{18}{7}\right]\right], \left[\left[\left[\frac{12}{7}, \frac{4}{3}, -\frac{8}{21}\right], [\,]\right], \left[\left[\frac{6}{7}, \frac{2}{3}, -\frac{4}{21}\right], [\,]\right], \left[\left[\frac{18}{7}, \frac{4}{7}\right], [2]\right]\right], [\,], [\,]\right] \quad (17)$$

```
> R1:=IrrRegAppsing0F2(L,t,E,ext);
```

$$R1 := \left[\left[[[x-9, 9], [x-12, 12], [\infty, \infty]], \left[\left[\frac{16\,12^{1/3}}{3\,t^{2/3}} - \frac{22}{63}, \frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2 t^{2/3}} - \frac{22}{63}, \frac{64\,12^{1/3}}{3\left(I\sqrt{3}+1\right)^2 t^{2/3}} - \frac{22}{63}\right], \left[\frac{15\,RootOf\left(\_Z^3+242, index=1\right)}{t} - \frac{11}{21}, \frac{15\,RootOf\left(\_Z^3+242, index=2\right)}{t} - \frac{11}{21}, \frac{15\,RootOf\left(\_Z^3+242, index=3\right)}{t} - \frac{11}{21}\right], \left[-\frac{2^{1/3}}{t^{1/3}} - \frac{11}{63}, -\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}} - \frac{11}{63}, \frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}} - \frac{11}{63}\right]\right], \left[\left[\frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2 t^{2/3}} - \frac{16\,12^{1/3}}{3\,t^{2/3}}, \frac{64\,12^{1/3}}{3\left(I\sqrt{3}+1\right)^2 t^{2/3}} - \frac{16\,12^{1/3}}{3\,t^{2/3}}, \frac{64\,12^{1/3}}{3\left(I\sqrt{3}+1\right)^2 t^{2/3}} - \frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2 t^{2/3}}\right], \left[\frac{15\,RootOf\left(\_Z^3+242, index=2\right)}{t} - \frac{15\,RootOf\left(\_Z^3+242, index=1\right)}{t}, \frac{15\,RootOf\left(\_Z^3+242, index=3\right)}{t} - \frac{15\,RootOf\left(\_Z^3+242, index=1\right)}{t}, \frac{15\,RootOf\left(\_Z^3+242, index=3\right)}{t} - \frac{15\,RootOf\left(\_Z^3+242, index=2\right)}{t}\right], \left[-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}} + \frac{2^{1/3}}{t^{1/3}}, \frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}} + \frac{2^{1/3}}{t^{1/3}}, \frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}} + \frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}\right]\right], \left[\frac{2}{3}, 1, \frac{1}{3}\right], [3, 1, 3], \left[\left[\left[\frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2 t^{2/3}} - \frac{22}{63}, \frac{16\,12^{1/3}}{3\,t^{2/3}} - \frac{22}{63}\right], \left[\frac{64\,12^{1/3}}{3\left(I\sqrt{3}+1\right)^2 t^{2/3}} - \frac{22}{63}, \frac{16\,12^{1/3}}{3\,t^{2/3}} - \frac{22}{63}\right], \left[\frac{64\,12^{1/3}}{3\left(I\sqrt{3}+1\right)^2 t^{2/3}} - \frac{22}{63}, \frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2 t^{2/3}} - \frac{22}{63}\right]\right], \quad (18)$$

$$\left[\left[\frac{15\,RootOf\left(\_Z^3+242,\,index=2\right)}{t}-\frac{11}{21},\frac{15\,RootOf\left(\_Z^3+242,\,index=1\right)}{t}-\frac{11}{21}\right],\left[\frac{15\,RootOf\left(\_Z^3+242,\,index=3\right)}{t}-\frac{11}{21},\frac{15\,RootOf\left(\_Z^3+242,\,index=1\right)}{t}-\frac{11}{21}\right],\left[\frac{15\,RootOf\left(\_Z^3+242,\,index=3\right)}{t}-\frac{11}{21},\frac{15\,RootOf\left(\_Z^3+242,\,index=2\right)}{t}-\frac{11}{21}\right]\right],$$

$$\left[\left[-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}-\frac{11}{63},-\frac{2^{1/3}}{t^{1/3}}-\frac{11}{63}\right],\left[\frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}-\frac{11}{63},-\frac{2^{1/3}}{t^{1/3}}-\frac{11}{63}\right],\left[\frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}-\frac{11}{63},-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}-\frac{11}{63}\right]\right]\right],$$

$$\left[\left[\frac{8\left(I\sqrt{3}\,12^{1/3}-3\,12^{1/3}\right)t^2}{3},-\frac{8\left(I\sqrt{3}\,12^{1/3}+3\,12^{1/3}\right)t^2}{3},-\frac{16\,I}{3}\sqrt{3}\,12^{1/3}t^2\right],\right.$$

$$\left[15\left(RootOf\left(\_Z^3+242,\,index=2\right)-RootOf\left(\_Z^3+242,\,index=1\right)\right)t,\,15\left(RootOf\left(\_Z^3\right.\right.\right.$$

$$\left.\left.\left.+242,\,index=3\right)-RootOf\left(\_Z^3+242,\,index=1\right)\right)t,\,15\left(RootOf\left(\_Z^3+242,\,index=3\right)\right.\right.$$

$$\left.\left.-RootOf\left(\_Z^3+242,\,index=2\right)\right)t\right],\left[\frac{\left(I\sqrt{3}\,2^{1/3}+3\,2^{1/3}\right)t}{2},\right.$$

$$\left.\left.-\frac{\left(I\sqrt{3}\,2^{1/3}-3\,2^{1/3}\right)t}{2},\,-I\,2^{1/3}\sqrt{3}\,t\right]\right],\left[\left[0,0,0\right],\left[0,0,0\right],\left[0,0,0\right]\right]\right],\left[\left[\left[x-1,\right.\right.\right.$$

$$\left.\left.\left.1\right],\left[x-3,3\right],\left[x-7,7\right]\right],\left[\left[0,\frac{12}{7},\frac{4}{3}\right],\left[0,\frac{6}{7},\frac{2}{3}\right],\left[0,2,\frac{18}{7}\right]\right],\left[\left[\frac{12}{7},\frac{4}{3},-\frac{8}{21}\right],\right.\right.$$

$$\left.\left.\left[\frac{6}{7},\frac{2}{3},-\frac{4}{21}\right],\left[2,\frac{18}{7},\frac{4}{7}\right]\right],\left[\left[\left[\frac{12}{7},0\right],\left[\frac{4}{3},0\right],\left[\frac{4}{3},\frac{12}{7}\right]\right],\left[\left[\frac{6}{7},0\right],\left[\frac{2}{3},0\right],\left[\frac{2}{3},\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{6}{7}\right]\right],\left[\left[2,0\right],\left[\frac{18}{7},0\right],\left[\frac{18}{7},2\right]\right]\right]\right],\left[\left[\left[\left[x-1,1\right],\left[x-3,3\right],\left[x-7,7\right]\right],\left[\left[0,\frac{12}{7},\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{4}{3}\right],\left[0,\frac{6}{7},\frac{2}{3}\right],\left[0,2,\frac{18}{7}\right]\right],\left[\left[\left[\frac{12}{7},\frac{4}{3},-\frac{8}{21}\right],\left[\,\right]\right],\left[\left[\frac{6}{7},\frac{2}{3},-\frac{4}{21}\right],\left[\,\right]\right],\left[\left[\frac{18}{7},\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{4}{7}\right],\left[2\right]\right]\right]\right],\left[\,\right],\left[\,\right]\right],\left[\left[\left[x^4-60\,x^3+830\,x^2-3852\,x+5193,\,RootOf\left(\_Z^4-60\,\_Z^3\right.\right.\right.\right.$$

$$\left.\left.\left.\left.+830\,\_Z^2-3852\,\_Z+5193\right)\right]\right],\left[\left[0,2,4\right]\right],\left[\left[2,4,2\right]\right],\left[\left[\left[2,0\right],\left[4,0\right],\left[4,2\right]\right]\right]\right],\left[\left[x\right.\right.$$

$$-1, 1], [x-3, 3], [x-9, 9], [x-12, 12], [\infty, \infty], [x-7, 7]], \left[\left[0, \frac{12}{7}, \frac{4}{3}\right], \left[0, \frac{6}{7}, \frac{2}{3}\right], \left[\frac{16\,12^{1/3}}{3\,t^{2/3}} - \frac{22}{63}, \frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2 t^{2/3}} - \frac{22}{63}, \frac{64\,12^{1/3}}{3\left(I\sqrt{3}+1\right)^2 t^{2/3}} - \frac{22}{63}\right], \left[\frac{15\,RootOf\left(\_Z^3+242, index=1\right)}{t} - \frac{11}{21}, \frac{15\,RootOf\left(\_Z^3+242, index=2\right)}{t} - \frac{11}{21}, \frac{15\,RootOf\left(\_Z^3+242, index=3\right)}{t} - \frac{11}{21}\right], \left[-\frac{2^{1/3}}{t^{1/3}} - \frac{11}{63}, -\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}} - \frac{11}{63}, \frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}} - \frac{11}{63}\right], \left[0, 2, \frac{18}{7}\right]\right], \left[\left[\frac{12}{7}, \frac{4}{3}, -\frac{8}{21}\right], \left[\frac{6}{7}, \frac{2}{3}, -\frac{4}{21}\right], \left[\frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2 t^{2/3}} - \frac{16\,12^{1/3}}{3\,t^{2/3}}, \frac{64\,12^{1/3}}{3\left(I\sqrt{3}+1\right)^2 t^{2/3}} - \frac{16\,12^{1/3}}{3\,t^{2/3}}, \frac{64\,12^{1/3}}{3\left(I\sqrt{3}+1\right)^2 t^{2/3}} - \frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2 t^{2/3}}\right], \left[\frac{15\,RootOf\left(\_Z^3+242, index=2\right)}{t} - \frac{15\,RootOf\left(\_Z^3+242, index=1\right)}{t}, \frac{15\,RootOf\left(\_Z^3+242, index=3\right)}{t} - \frac{15\,RootOf\left(\_Z^3+242, index=1\right)}{t}, \frac{15\,RootOf\left(\_Z^3+242, index=3\right)}{t} - \frac{15\,RootOf\left(\_Z^3+242, index=2\right)}{t}\right], \left[-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}} + \frac{2^{1/3}}{t^{1/3}}, \frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}} + \frac{2^{1/3}}{t^{1/3}}, \frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}} + \frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}\right], \left[2, \frac{18}{7}, \frac{4}{7}\right]\right], \left[\left[\left[\frac{12}{7}, 0\right], \left[\frac{4}{3}, 0\right], \left[\frac{4}{3}, \frac{12}{7}\right]\right], \left[\left[\frac{6}{7}, 0\right], \left[\frac{2}{3}, 0\right], \left[\frac{2}{3}, \frac{6}{7}\right]\right], \left[\left[\frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2 t^{2/3}} - \frac{22}{63}, \frac{16\,12^{1/3}}{3\,t^{2/3}} - \frac{22}{63}\right], \left[\frac{64\,12^{1/3}}{3\left(I\sqrt{3}+1\right)^2 t^{2/3}} - \frac{22}{63}, \frac{16\,12^{1/3}}{3\,t^{2/3}} - \frac{22}{63}\right], \left[\frac{64\,12^{1/3}}{3\left(I\sqrt{3}+1\right)^2 t^{2/3}} - \frac{22}{63}, \frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2 t^{2/3}} - \frac{22}{63}\right]\right], \left[\left[\frac{15\,RootOf\left(\_Z^3+242, index=2\right)}{t} - \frac{11}{21}, \frac{15\,RootOf\left(\_Z^3+242, index=1\right)}{t} - \frac{11}{21}\right], \left[\frac{15\,RootOf\left(\_Z^3+242, index=3\right)}{t} - \frac{11}{21}, \frac{15\,RootOf\left(\_Z^3+242, index=1\right)}{t} - \frac{11}{21}\right], \left[\frac{15\,RootOf\left(\_Z^3+242, index=3\right)}{t} - \frac{11}{21}, \right.$$

$$\frac{15\,RootOf\left(\_Z^3+242,\,index=2\right)}{t}-\frac{11}{21}\Bigg]\Bigg],\left[\left[-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}-\frac{11}{63},-\frac{2^{1/3}}{t^{1/3}}-\frac{11}{63}\right],\left[\frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}-\frac{11}{63},-\frac{2^{1/3}}{t^{1/3}}-\frac{11}{63}\right],\left[\frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}-\frac{11}{63},-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}-\frac{11}{63}\right]\Bigg],\left[[2,0],\left[\frac{18}{7},0\right],\left[\frac{18}{7},2\right]\right]\Bigg],[[1,1,1],[1,1,1],[3,3,3],[1,1,1],[3,3,3],[1,1,1]]\Bigg]$$

```
> F1:= Sirr0F2info1(L,R1[1],R1[2],x,t,ext);
```

$$F1:=\left[\left[\left[9,x-9,\left[\frac{2048}{9\,(x-9)^2}\right],2,\varnothing,\varnothing\right],\left[12,x-12,\left[-\frac{30250}{(x-12)^3}\right],3,\{RootOf\left(\_Z^3+242,index=1\right),RootOf\left(\_Z^3+242,index=2\right)\},\{RootOf\left(\_Z^3+242,index=1\right),RootOf\left(\_Z^3+242,index=2\right)\}\right],\left[\infty,\frac{1}{x},[2\,x],1,\varnothing,\varnothing\right]\right],6,6,(x-9)^2\,(x-12)^3,(x-9)\,(x-12)^2\right] \quad (19)$$

```
> find0F2Rat(L,R1,F1,x,t,T,ext);
```

$$\left[\left[\left[\left[\frac{1}{3},\frac{1}{7}\right]\right],-\frac{2\,(x-1)^2\,(x-3)\,(x-7)^3}{(x-9)^2\,(x-12)^3}\right]\right] \quad (20)$$

```
> TIME := time();
  Hyp0F2Solutions(L);
  time()-TIME;
```

$$TIME := 4.375$$

$$\left\{\left[\left\{\left[\left[\frac{1}{3},\frac{1}{7}\right],[0],[1]\right]\right\},\frac{2\,(x-1)^2\,(x-3)\,(x-7)^3}{(x-9)^2\,(x-12)^3}\right]\right\}$$

$$0.828 \quad (21)$$

Here are another examples related to the 0F2 type solutions. Those examples are not in my PhD thesis.

```
> ####### THE EASY CASE #######
```

```
> F:=sumdiffeq(hyperterm([],[b1,b2],x,k),k,J(x));
```

$$F:=\left(\frac{d^3}{dx^3}J(x)\right)x^2+(b1+b2+1)\left(\frac{d^2}{dx^2}J(x)\right)x+b1\,b2\left(\frac{d}{dx}J(x)\right)-J(x)=0 \quad (22)$$

```
> LA:=de2diffop(F,J(x));
```

$$LA:=x^2\,Dx^3+(x\,b1+x\,b2+x)\,Dx^2+b2\,b1\,Dx-1 \quad (23)$$

```
> L12:=subs({b1=1/7,b2=1/5},LA);
```

$$L12 := x^2\,Dx^3 + \frac{47}{35}\,x\,Dx^2 + \frac{1}{35}\,Dx - 1 \tag{24}$$

```
> f:=(2*(x-1)*(x-3)*(x-7)^3)/((x-9)*(x-12)^3);
```

$$f := \frac{2\,(x-1)\,(x-3)\,(x-7)^3}{(x-9)\,(x-12)^3} \tag{25}$$

```
> L:=ChangeOfVariables(L12,f);
```

$$\begin{aligned} L := {} & 35\,Dx^3\,(x-1)^2\,(x-3)^2\,(x-7)^2\,(x-9)^4\,(x-12)^6\,\left(x^4-52\,x^3+654\,x^2-2724\,x\right. \\ & \left.+3177\right)^2+\left(47\,x^8-4888\,x^7+109184\,x^6-887448\,x^5+1577382\,x^4+13791960\,x^3\right. \\ & \left.-66454056\,x^2+74432088\,x+29847123\right)\,Dx^2\,(x-1)\,(x-3)\,(x-7)\,(x-9)^3\,(x \\ & -12)^5\,\left(x^4-52\,x^3+654\,x^2-2724\,x+3177\right)+\left(x^{16}-208\,x^{15}+62688\,x^{14}\right. \\ & -4003552\,x^{13}+136634380\,x^{12}-3121053072\,x^{11}+51346447872\,x^{10} \\ & -618655004064\,x^9+5448848277222\,x^8-34817324526576\,x^7+159415702939296\,x^6 \\ & -511984588809888\,x^5+1109983631731596\,x^4-1509287444297136\,x^3 \\ & \left.+1091269894148352\,x^2-222950742289632\,x-85264151958783\right)\,Dx\,(x-9)^2\,(x \\ & -12)^4-70\,\left(x^4-52\,x^3+654\,x^2-2724\,x+3177\right)^5\,(x-7)^2 \end{aligned} \tag{26}$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \varnothing \tag{27}$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```

$$ext := \varnothing \tag{28}$$

```
> extppp:={};
```

$$extppp := \varnothing \tag{29}$$

```
> E:= Singular(L,extppp);
```

$$E := \left[\left[x-1,\,1\right],\,\left[x-12,\,12\right],\,\left[\infty,\,\infty\right],\,\left[x-3,\,3\right],\,\left[x^4-52\,x^3+654\,x^2-2724\,x+3177,\right.\right.$$
$$\left.\left.RootOf\left(\_Z^4-52\,\_Z^3+654\,\_Z^2-2724\,\_Z+3177\right)\right],\,\left[x-9,\,9\right],\,\left[x-7,\,7\right]\right] \tag{30}$$

```
> F:=NotAppSing(L,E,ext);
```

$$F := \left[\left[x-1,\,1\right],\,\left[x-3,\,3\right],\,\left[x-9,\,9\right],\,\left[x-12,\,12\right],\,\left[\infty,\,\infty\right],\,\left[x-7,\,7\right]\right] \tag{31}$$

```
> Sirr:=irrsing0F2(L,t,F,ext);
```

$$\begin{aligned} Sirr := {} & \left[\left[x-9,\,9\right],\,\left[x-12,\,12\right],\,\left[\infty,\,\infty\right]\right],\,\left[\left[\frac{4\,18^{2/3}}{9\,t^{1/3}}-\frac{23}{105},\,\frac{8\,18^{2/3}}{9\,\left(I\sqrt{3}-1\right)\,t^{1/3}}\right.\right. \\ & \left.-\frac{23}{105},\,-\frac{8\,18^{2/3}}{9\,\left(I\sqrt{3}+1\right)\,t^{1/3}}-\frac{23}{105}\right],\,\left[\frac{15\,RootOf\left(\_Z^3+66,\,index=1\right)}{t}-\frac{23}{35},\right. \\ & \left.\frac{15\,RootOf\left(\_Z^3+66,\,index=2\right)}{t}-\frac{23}{35},\,\frac{15\,RootOf\left(\_Z^3+66,\,index=3\right)}{t}-\frac{23}{35}\right],\,\left[\\ & -\frac{2^{1/3}}{t^{1/3}}-\frac{23}{105},\,-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)\,t^{1/3}}-\frac{23}{105},\,\frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)\,t^{1/3}}-\frac{23}{105}\left.\right]\right], \\ & \left[\left[\frac{8\,18^{2/3}}{9\,\left(I\sqrt{3}-1\right)\,t^{1/3}}-\frac{4\,18^{2/3}}{9\,t^{1/3}},\,-\frac{8\,18^{2/3}}{9\,\left(I\sqrt{3}+1\right)\,t^{1/3}}-\frac{4\,18^{2/3}}{9\,t^{1/3}},\right.\right. \end{aligned} \tag{32}$$

$$-\frac{8\,18^{2/3}}{9\left(I\sqrt{3}+1\right)t^{1/3}}-\frac{8\,18^{2/3}}{9\left(I\sqrt{3}-1\right)t^{1/3}}\Bigg],\left[\frac{15\,RootOf\left(\_Z^3+66,\,index=2\right)}{t}\right.$$

$$-\frac{15\,RootOf\left(\_Z^3+66,\,index=1\right)}{t},\frac{15\,RootOf\left(\_Z^3+66,\,index=3\right)}{t}$$

$$-\frac{15\,RootOf\left(\_Z^3+66,\,index=1\right)}{t},\frac{15\,RootOf\left(\_Z^3+66,\,index=3\right)}{t}$$

$$\left.-\frac{15\,RootOf\left(\_Z^3+66,\,index=2\right)}{t}\right],\left[-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}+\frac{2^{1/3}}{t^{1/3}},\frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}\right.$$

$$\left.+\frac{2^{1/3}}{t^{1/3}},\frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}+\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}\right]\Bigg],\left[\frac{1}{3},1,\frac{1}{3}\right],[3,1,3],$$

$$\left[\left[\left[\frac{8\,18^{2/3}}{9\left(I\sqrt{3}-1\right)t^{1/3}}-\frac{23}{105},\frac{4\,18^{2/3}}{9\,t^{1/3}}-\frac{23}{105}\right],\left[-\frac{8\,18^{2/3}}{9\left(I\sqrt{3}+1\right)t^{1/3}}-\frac{23}{105},\right.\right.\right.$$

$$\left.\left.\left.\frac{4\,18^{2/3}}{9\,t^{1/3}}-\frac{23}{105}\right],\left[-\frac{8\,18^{2/3}}{9\left(I\sqrt{3}+1\right)t^{1/3}}-\frac{23}{105},\frac{8\,18^{2/3}}{9\left(I\sqrt{3}-1\right)t^{1/3}}-\frac{23}{105}\right]\right],$$

$$\left[\left[\frac{15\,RootOf\left(\_Z^3+66,\,index=2\right)}{t}-\frac{23}{35},\frac{15\,RootOf\left(\_Z^3+66,\,index=1\right)}{t}-\frac{23}{35}\right],\right.$$

$$\left[\frac{15\,RootOf\left(\_Z^3+66,\,index=3\right)}{t}-\frac{23}{35},\frac{15\,RootOf\left(\_Z^3+66,\,index=1\right)}{t}-\frac{23}{35}\right],$$

$$\left.\left[\frac{15\,RootOf\left(\_Z^3+66,\,index=3\right)}{t}-\frac{23}{35},\frac{15\,RootOf\left(\_Z^3+66,\,index=2\right)}{t}-\frac{23}{35}\right]\right],\left[\left[\right.\right.$$

$$-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}-\frac{23}{105},-\frac{2^{1/3}}{t^{1/3}}-\frac{23}{105}\Bigg],\left[\frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}-\frac{23}{105},-\frac{2^{1/3}}{t^{1/3}}\right.$$

$$\left.\left.-\frac{23}{105}\right],\left[\frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}-\frac{23}{105},-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}-\frac{23}{105}\right]\right]\Bigg],\left[\left[\right.\right.$$

$$\left.-\frac{2\left(3\,18^{2/3}+I\sqrt{3}\,18^{2/3}\right)t}{9},\frac{2\left(I\sqrt{3}\,18^{2/3}-3\,18^{2/3}\right)t}{9},\frac{4\,I}{9}\,18^{2/3}\sqrt{3}\,t\right],$$

$$[15\left(RootOf\left(\_Z^3+66,\,index=2\right)-RootOf\left(\_Z^3+66,\,index=1\right)\right)t,15\left(RootOf\left(\_Z^3\right.\right.$$

$$\left.+66,\,index=3\right)-RootOf\left(\_Z^3+66,\,index=1\right)\right)t,15\left(RootOf\left(\_Z^3+66,\,index=3\right)$$

$$\left.-RootOf\left(\_Z^3+66,\,index=2\right)\right)t],\left[\frac{\left(I\sqrt{3}\,2^{1/3}+3\,2^{1/3}\right)t}{2},\right.$$

$$\left.-\frac{\left(I\sqrt{3}\,2^{1/3}-3\,2^{1/3}\right)t}{2},-I\,2^{1/3}\sqrt{3}\,t\right]\Bigg],[[0,0,0],[0,0,0],[0,0,0]],\Bigg[[x-1,1],$$

$$[x-3,3],[x-7,7]],\left[\left[\left[0,\frac{6}{7},\frac{4}{5}\right],\left[\frac{6}{7},\frac{4}{5},-\frac{2}{35}\right],[1,1,1],\left[\left[\frac{6}{7},0\right],\left[\frac{4}{5},0\right],\left[\frac{4}{5},\right.\right.\right.\right.$$

$$\left[\frac{6}{7}\right]\right], 2\Big], \Big[\Big[0, \frac{6}{7}, \frac{4}{5}\Big], \Big[\frac{6}{7}, \frac{4}{5}, -\frac{2}{35}\Big], [1, 1, 1], \Big[\Big[\frac{6}{7}, 0\Big], \Big[\frac{4}{5}, 0\Big], \Big[\frac{4}{5}, \frac{6}{7}\Big]\Big], 2\Big], \Big[\Big[0, \frac{18}{7}, \frac{12}{5}\Big], \Big[\frac{18}{7}, \frac{12}{5}, -\frac{6}{35}\Big], [1, 1, 1], \Big[\Big[\frac{18}{7}, 0\Big], \Big[\frac{12}{5}, 0\Big], \Big[\frac{12}{5}, \frac{18}{7}\Big]\Big], 2\Big]\Big]\Big]$$

```
> Sreg:=regsingtrue0F2(L,t,Sirr[-1],ext);
```

$$Sreg := \Big[[[x-1, 1], [x-3, 3], [x-7, 7]], \Big[\Big[0, \frac{6}{7}, \frac{4}{5}\Big], \Big[0, \frac{6}{7}, \frac{4}{5}\Big], \Big[0, \frac{18}{7}, \frac{12}{5}\Big]\Big], \tag{33}$$
$$\Big[\Big[\frac{6}{7}, \frac{4}{5}, -\frac{2}{35}\Big], \Big[\frac{6}{7}, \frac{4}{5}, -\frac{2}{35}\Big], \Big[\frac{18}{7}, \frac{12}{5}, -\frac{6}{35}\Big]\Big], \Big[\Big[\Big[\frac{6}{7}, 0\Big], \Big[\frac{4}{5}, 0\Big], \Big[\frac{4}{5}, \frac{6}{7}\Big]\Big], \Big[\Big[\frac{6}{7}, 0\Big], \Big[\frac{4}{5}, 0\Big], \Big[\frac{4}{5}, \frac{6}{7}\Big]\Big], \Big[\Big[\frac{18}{7}, 0\Big], \Big[\frac{12}{5}, 0\Big], \Big[\frac{12}{5}, \frac{18}{7}\Big]\Big]\Big]\Big]$$

```
> RSreg:=Sregseptrue0F2(L,Sreg,ext);
```

$$RSreg := \Big[\Big[[[x-1, 1], [x-3, 3], [x-7, 7]], \Big[\Big[0, \frac{6}{7}, \frac{4}{5}\Big], \Big[0, \frac{6}{7}, \frac{4}{5}\Big], \Big[0, \frac{18}{7}, \frac{12}{5}\Big]\Big], \tag{34}$$
$$\Big[\Big[\Big[\frac{6}{7}, \frac{4}{5}, -\frac{2}{35}\Big], [\,]\Big], \Big[\Big[\frac{6}{7}, \frac{4}{5}, -\frac{2}{35}\Big], [\,]\Big], \Big[\Big[\frac{18}{7}, \frac{12}{5}, -\frac{6}{35}\Big], [\,]\Big]\Big], [\,], [\,]\Big]$$

```
> R1:=IrrRegAppsing0F2(L,t,E,ext);
```

$$R1 := \Big[\Big[[[x-9, 9], [x-12, 12], [\infty, \infty]], \Big[\Big[\frac{4\,18^{2/3}}{9\,t^{1/3}} - \frac{23}{105}, \frac{8\,18^{2/3}}{9\,(I\sqrt{3}-1)\,t^{1/3}} \tag{35}$$
$$-\frac{23}{105}, -\frac{8\,18^{2/3}}{9\,(I\sqrt{3}+1)\,t^{1/3}} - \frac{23}{105}\Big], \Big[\frac{15\,RootOf(\_Z^3+66, index=1)}{t} - \frac{23}{35},$$
$$\frac{15\,RootOf(\_Z^3+66, index=2)}{t} - \frac{23}{35}, \frac{15\,RootOf(\_Z^3+66, index=3)}{t} - \frac{23}{35}\Big], \Big[$$
$$-\frac{2^{1/3}}{t^{1/3}} - \frac{23}{105}, -\frac{2\,2^{1/3}}{(I\sqrt{3}-1)\,t^{1/3}} - \frac{23}{105}, \frac{2\,2^{1/3}}{(I\sqrt{3}+1)\,t^{1/3}} - \frac{23}{105}\Big]\Big],$$
$$\Big[\Big[\frac{8\,18^{2/3}}{9\,(I\sqrt{3}-1)\,t^{1/3}} - \frac{4\,18^{2/3}}{9\,t^{1/3}}, -\frac{8\,18^{2/3}}{9\,(I\sqrt{3}+1)\,t^{1/3}} - \frac{4\,18^{2/3}}{9\,t^{1/3}},$$
$$-\frac{8\,18^{2/3}}{9\,(I\sqrt{3}+1)\,t^{1/3}} - \frac{8\,18^{2/3}}{9\,(I\sqrt{3}-1)\,t^{1/3}}\Big], \Big[\frac{15\,RootOf(\_Z^3+66, index=2)}{t}$$
$$-\frac{15\,RootOf(\_Z^3+66, index=1)}{t}, \frac{15\,RootOf(\_Z^3+66, index=3)}{t}$$
$$-\frac{15\,RootOf(\_Z^3+66, index=1)}{t}, \frac{15\,RootOf(\_Z^3+66, index=3)}{t}$$
$$-\frac{15\,RootOf(\_Z^3+66, index=2)}{t}\Big], \Big[-\frac{2\,2^{1/3}}{(I\sqrt{3}-1)\,t^{1/3}} + \frac{2^{1/3}}{t^{1/3}}, \frac{2\,2^{1/3}}{(I\sqrt{3}+1)\,t^{1/3}}$$
$$+\frac{2^{1/3}}{t^{1/3}}, \frac{2\,2^{1/3}}{(I\sqrt{3}+1)\,t^{1/3}} + \frac{2\,2^{1/3}}{(I\sqrt{3}-1)\,t^{1/3}}\Big]\Big], \Big[\frac{1}{3}, 1, \frac{1}{3}\Big], [3, 1, 3],$$

$$\left[\left[\left[\frac{8\,18^{2/3}}{9\left(I\sqrt{3}-1\right)t^{1/3}}-\frac{23}{105},\frac{4\,18^{2/3}}{9\,t^{1/3}}-\frac{23}{105}\right],\left[-\frac{8\,18^{2/3}}{9\left(I\sqrt{3}+1\right)t^{1/3}}-\frac{23}{105},\frac{4\,18^{2/3}}{9\,t^{1/3}}-\frac{23}{105}\right],\left[-\frac{8\,18^{2/3}}{9\left(I\sqrt{3}+1\right)t^{1/3}}-\frac{23}{105},\frac{8\,18^{2/3}}{9\left(I\sqrt{3}-1\right)t^{1/3}}-\frac{23}{105}\right]\right],\left[\left[\frac{15\,RootOf\left(\_Z^3+66,index=2\right)}{t}-\frac{23}{35},\frac{15\,RootOf\left(\_Z^3+66,index=1\right)}{t}-\frac{23}{35}\right],\left[\frac{15\,RootOf\left(\_Z^3+66,index=3\right)}{t}-\frac{23}{35},\frac{15\,RootOf\left(\_Z^3+66,index=1\right)}{t}-\frac{23}{35}\right],\left[\frac{15\,RootOf\left(\_Z^3+66,index=3\right)}{t}-\frac{23}{35},\frac{15\,RootOf\left(\_Z^3+66,index=2\right)}{t}-\frac{23}{35}\right]\right],\left[\left[-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}-\frac{23}{105},-\frac{2^{1/3}}{t^{1/3}}-\frac{23}{105}\right],\left[\frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}-\frac{23}{105},-\frac{2^{1/3}}{t^{1/3}}-\frac{23}{105}\right],\left[\frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}-\frac{23}{105},-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}-\frac{23}{105}\right]\right],\left[\left[-\frac{2\left(3\,18^{2/3}+I\sqrt{3}\,18^{2/3}\right)t}{9},\frac{2\left(I\sqrt{3}\,18^{2/3}-3\,18^{2/3}\right)t}{9},\frac{4\,I}{9}\,18^{2/3}\sqrt{3}\,t\right],\left[15\left(RootOf\left(\_Z^3+66,index=2\right)-RootOf\left(\_Z^3+66,index=1\right)\right)t,15\left(RootOf\left(\_Z^3+66,index=3\right)-RootOf\left(\_Z^3+66,index=1\right)\right)t,15\left(RootOf\left(\_Z^3+66,index=3\right)-RootOf\left(\_Z^3+66,index=2\right)\right)t\right],\left[\frac{\left(I\sqrt{3}\,2^{1/3}+3\,2^{1/3}\right)t}{2},-\frac{\left(I\sqrt{3}\,2^{1/3}-3\,2^{1/3}\right)t}{2},-I\,2^{1/3}\sqrt{3}\,t\right]\right],\left[[0,0,0],[0,0,0],[0,0,0]\right]\right],\left[\left[[x-1,1],[x-3,3],[x-7,7]\right],\left[\left[0,\frac{6}{7},\frac{4}{5}\right],\left[0,\frac{6}{7},\frac{4}{5}\right],\left[0,\frac{18}{7},\frac{12}{5}\right]\right],\left[\left[\frac{6}{7},\frac{4}{5},-\frac{2}{35}\right],\left[\frac{6}{7},\frac{4}{5},-\frac{2}{35}\right],\left[\frac{18}{7},\frac{12}{5},-\frac{6}{35}\right]\right],\left[\left[\left[\frac{6}{7},0\right],\left[\frac{4}{5},0\right],\left[\frac{4}{5},\frac{6}{7}\right]\right],\left[\left[\frac{6}{7},0\right],\left[\frac{4}{5},0\right],\left[\frac{4}{5},\frac{6}{7}\right]\right],\left[\left[\frac{18}{7},0\right],\left[\frac{12}{5},0\right],\left[\frac{12}{5},\frac{18}{7}\right]\right]\right]\right],\left[\left[\left[[x-1,1],[x-3,3],[x-7,7]\right],\left[\left[0,\frac{6}{7},\frac{4}{5}\right],\left[0,\frac{6}{7},\frac{4}{5}\right],\left[0,\frac{18}{7},\frac{12}{5}\right]\right],\left[\left[\left[\frac{6}{7},\frac{4}{5},-\frac{2}{35}\right],[\,]\right],\left[\left[\frac{6}{7},\frac{4}{5},-\frac{2}{35}\right],[\,]\right],\left[\left[\frac{18}{7},\frac{12}{5},-\frac{6}{35}\right],[\,]\right]\right]\right],[\,],[\,]\right],\left[\left[\left[x^4-52\,x^3+654\,x^2-2724\,x+3177,\right.\right.\right.$$

$$\mathit{RootOf}\left(\_Z^4 - 52\,\_Z^3 + 654\,\_Z^2 - 2724\,\_Z + 3177\right)]], [[0, 2, 4]], [[2, 4, 2]], [[[2, 0], [4, 0], [4, 2]]]], \left[[[x-1, 1], [x-3, 3], [x-9, 9], [x-12, 12], [\infty, \infty], [x-7, 7]], \left[\left[0, \frac{6}{7}, \frac{4}{5}\right], \left[0, \frac{6}{7}, \frac{4}{5}\right], \left[\frac{4\,18^{2/3}}{9\,t^{1/3}} - \frac{23}{105}, \frac{8\,18^{2/3}}{9\left(I\sqrt{3}-1\right)t^{1/3}} - \frac{23}{105}, -\frac{8\,18^{2/3}}{9\left(I\sqrt{3}+1\right)t^{1/3}} - \frac{23}{105}\right], \left[\frac{15\,\mathit{RootOf}\left(\_Z^3+66, \mathit{index}=1\right)}{t} - \frac{23}{35}, \frac{15\,\mathit{RootOf}\left(\_Z^3+66, \mathit{index}=2\right)}{t} - \frac{23}{35}, \frac{15\,\mathit{RootOf}\left(\_Z^3+66, \mathit{index}=3\right)}{t} - \frac{23}{35}\right], \left[-\frac{2^{1/3}}{t^{1/3}} - \frac{23}{105}, -\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}} - \frac{23}{105}, \frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}} - \frac{23}{105}\right], \left[0, \frac{18}{7}, \frac{12}{5}\right]\right], \left[\left[\frac{6}{7}, \frac{4}{5}, -\frac{2}{35}\right], \left[\frac{6}{7}, \frac{4}{5}, -\frac{2}{35}\right], \left[\frac{8\,18^{2/3}}{9\left(I\sqrt{3}-1\right)t^{1/3}} - \frac{4\,18^{2/3}}{9\,t^{1/3}}, -\frac{8\,18^{2/3}}{9\left(I\sqrt{3}+1\right)t^{1/3}} - \frac{4\,18^{2/3}}{9\,t^{1/3}}, -\frac{8\,18^{2/3}}{9\left(I\sqrt{3}+1\right)t^{1/3}} - \frac{8\,18^{2/3}}{9\left(I\sqrt{3}-1\right)t^{1/3}}\right], \left[\frac{15\,\mathit{RootOf}\left(\_Z^3+66, \mathit{index}=2\right)}{t} - \frac{15\,\mathit{RootOf}\left(\_Z^3+66, \mathit{index}=1\right)}{t}, \frac{15\,\mathit{RootOf}\left(\_Z^3+66, \mathit{index}=3\right)}{t} - \frac{15\,\mathit{RootOf}\left(\_Z^3+66, \mathit{index}=1\right)}{t}, \frac{15\,\mathit{RootOf}\left(\_Z^3+66, \mathit{index}=3\right)}{t} - \frac{15\,\mathit{RootOf}\left(\_Z^3+66, \mathit{index}=2\right)}{t}\right], \left[-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}} + \frac{2^{1/3}}{t^{1/3}}, \frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}} + \frac{2^{1/3}}{t^{1/3}}, \frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}} + \frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}\right], \left[\frac{18}{7}, \frac{12}{5}, -\frac{6}{35}\right]\right], \left[\left[\left[\frac{6}{7}, 0\right], \left[\frac{4}{5}, 0\right], \left[\frac{4}{5}, \frac{6}{7}\right]\right], \left[\left[\frac{6}{7}, 0\right], \left[\frac{4}{5}, 0\right], \left[\frac{4}{5}, \frac{6}{7}\right]\right], \left[\left[\frac{8\,18^{2/3}}{9\left(I\sqrt{3}-1\right)t^{1/3}} - \frac{23}{105}, \frac{4\,18^{2/3}}{9\,t^{1/3}} - \frac{23}{105}\right], \left[-\frac{8\,18^{2/3}}{9\left(I\sqrt{3}+1\right)t^{1/3}} - \frac{23}{105}, \frac{4\,18^{2/3}}{9\,t^{1/3}} - \frac{23}{105}\right], \left[-\frac{8\,18^{2/3}}{9\left(I\sqrt{3}+1\right)t^{1/3}} - \frac{23}{105}, \frac{8\,18^{2/3}}{9\left(I\sqrt{3}-1\right)t^{1/3}} - \frac{23}{105}\right]\right], \left[\left[\frac{15\,\mathit{RootOf}\left(\_Z^3+66, \mathit{index}=2\right)}{t} - \frac{23}{35}, \frac{15\,\mathit{RootOf}\left(\_Z^3+66, \mathit{index}=1\right)}{t} - \frac{23}{35}\right], \left[\frac{15\,\mathit{RootOf}\left(\_Z^3+66, \mathit{index}=3\right)}{t} - \frac{23}{35}, \frac{15\,\mathit{RootOf}\left(\_Z^3+66, \mathit{index}=1\right)}{t} - \frac{23}{35}\right], \left[\frac{15\,\mathit{RootOf}\left(\_Z^3+66, \mathit{index}=3\right)}{t} - \frac{23}{35},\right.$$

$$\frac{15\,RootOf(\_Z^3+66,\,index=2)}{t}-\frac{23}{35}\Big]\Big],\Big[\Big[-\frac{2\,2^{1/3}}{(I\sqrt{3}-1)\,t^{1/3}}-\frac{23}{105},-\frac{2^{1/3}}{t^{1/3}}-\frac{23}{105}\Big],\Big[\frac{2\,2^{1/3}}{(I\sqrt{3}+1)\,t^{1/3}}-\frac{23}{105},-\frac{2^{1/3}}{t^{1/3}}-\frac{23}{105}\Big],\Big[\frac{2\,2^{1/3}}{(I\sqrt{3}+1)\,t^{1/3}}-\frac{23}{105},-\frac{2\,2^{1/3}}{(I\sqrt{3}-1)\,t^{1/3}}-\frac{23}{105}\Big]\Big],\Big[\Big[\frac{18}{7},0\Big],\Big[\frac{12}{5},0\Big],\Big[\frac{12}{5},\frac{18}{7}\Big]\Big]\Big],[[1,1,1],[1,1,1],[3,3,3],[1,1,1],[3,3,3],[1,1,1]]\Big]\Big]$$

```
> F1:= Sirr0F2info1(L,R1[1],R1[2],x,t,ext);
```

$$F1 := \Big[\Big[\Big[9,\,x-9,\,\Big[-\frac{256}{9\,(x-9)}\Big],\,1,\,\varnothing,\,\varnothing\Big],\Big[12,\,x-12,\,\Big[-\frac{8250}{(x-12)^3}\Big],\,3,\,\{RootOf(\_Z^3+66,\,index=1),\,RootOf(\_Z^3+66,\,index=2)\},\,\{RootOf(\_Z^3+66,\,index=1),\,RootOf(\_Z^3+66,\,index=2)\}\Big],\Big[\infty,\,\frac{1}{x},\,[2\,x],\,1,\,\varnothing,\,\varnothing\Big]\Big],\,5,\,6,\,(x-9)\,(x-12)^3,\,(x-12)^2\Big] \quad (36)$$

```
> easy0F2(L,R1,F1,x,t,ext);
```

$$\Big[\Big[\Big[\Big[\Big[\frac{1}{5},\frac{1}{7}\Big]\Big],\frac{2\,(x-1)\,(x-3)\,(x-7)^3}{(x-9)\,(x-12)^3}\Big],\Big[\Big[\Big[\frac{1}{5},\frac{1}{7}\Big]\Big],-\frac{2\,(x-1)\,(x-3)\,(x-7)^3}{(x-9)\,(x-12)^3}\Big]\Big] \quad (37)$$

```
> find0F2Rat(L,R1,F1,x,t,T,ext);
```

$$\Big[\Big[\Big[\Big[\Big[\frac{1}{5},\frac{1}{7}\Big]\Big],\frac{2\,(x-1)\,(x-3)\,(x-7)^3}{(x-9)\,(x-12)^3}\Big]\Big] \quad (38)$$

```
> TIME := time();
  Hyp0F2Solutions(L);
  time()-TIME;
```

$$TIME := 7.218$$

$$\Big\{\Big[\Big\{\Big[\Big[\frac{1}{5},\frac{1}{7}\Big],[0],[1]\Big]\Big\},\frac{2\,(x-1)\,(x-3)\,(x-7)^3}{(x-9)\,(x-12)^3}\Big]\Big\}$$

$$0.641 \quad (39)$$

```
> F:=sumdiffeq(hyperterm([],[b1,b2],x,k),k,J(x));
```

$$F := \Big(\frac{d^3}{dx^3}\,J(x)\Big)\,x^2+(b1+b2+1)\Big(\frac{d^2}{dx^2}\,J(x)\Big)\,x+b1\,b2\Big(\frac{d}{dx}\,J(x)\Big)-J(x)=0 \quad (40)$$

```
> LA:=de2diffop(F,J(x));
```

$$LA := x^2\,Dx^3+(x\,b1+x\,b2+x)\,Dx^2+b2\,b1\,Dx-1 \quad (41)$$

```
> L12:=subs({b1=1,b2=1+RootOf(x^2+1)},LA);
```

$$L12 := x^2\,Dx^3+(2\,x+x\,(1+RootOf(\_Z^2+1)))\,Dx^2+(1+RootOf(\_Z^2+1))\,Dx-1 \quad (42)$$

```
> f:=(2*(x-1)^2*(x-3)*(x-7)^2)/((x-9)*(x-12));
```

$$f := \frac{2\,(x-1)^2\,(x-3)\,(x-7)^2}{(x-9)\,(x-12)} \tag{43}$$

```
> L:=ChangeOfVariables(L12,f);
```

$$L := Dx^3\,(x-1)^2\,(x-3)^2\,(x-7)^2\,(x-9)^4\,(x-12)^4\,(3x^4-98x^3+1058x^2-4350x+5499)^2+(1+RootOf(\_Z^2+1))\,(9x^8+176\,RootOf(\_Z^2+1)\,x^6-588x^7-5806\,RootOf(\_Z^2+1)\,x^5+15776x^6+72218\,RootOf(\_Z^2+1)\,x^4-227662x^5-411684\,RootOf(\_Z^2+1)\,x^3+1932740x^4+1000188\,RootOf(\_Z^2+1)\,x^2-9870720x^3-434862\,RootOf(\_Z^2+1)\,x+29558196x^2-1335366\,RootOf(\_Z^2+1)-47406438x+31574367)\,(3x^4-98x^3+1058x^2-4350x+5499)\,(x-7)\,(x-3)\,(x-1)\,(x-9)^3\,(x-12)^3\,Dx^2-\frac{1}{5}((1+3\,RootOf(\_Z^2+1))\,(-547157401526799+1640315296209240x+1877136788751552x^3-350016x^{14}+12594984x^{13}-306602642x^{12}+5353573328x^{11}-69354920712x^{10}+679945841960x^9-45x^{16}+5880x^{15}-5097833895956x^8+29323504806456x^7-128971487877984x^6-2253196066142232x^2-17123543914158\,RootOf(\_Z^2+1)-1059559724735838x^4+429181878089880x^5+528\,RootOf(\_Z^2+1)\,x^{14}-34132\,RootOf(\_Z^2+1)\,x^{13}+856206\,RootOf(\_Z^2+1)\,x^{12}-7848464\,RootOf(\_Z^2+1)\,x^{11}-89708864\,RootOf(\_Z^2+1)\,x^{10}+3652659780\,RootOf(\_Z^2+1)\,x^9-53915229562\,RootOf(\_Z^2+1)\,x^8+488313786912\,RootOf(\_Z^2+1)\,x^7-3003412628448\,RootOf(\_Z^2+1)\,x^6+12913918449300\,RootOf(\_Z^2+1)\,x^5-38725548708006\,RootOf(\_Z^2+1)\,x^4+78674056953264\,RootOf(\_Z^2+1)\,x^3-101103717757104\,RootOf(\_Z^2+1)\,x^2+70417349719740\,RootOf(\_Z^2+1)\,x)\,(x-9)^2\,(x-12)^2\,Dx)-2\,(3x^4-98x^3+1058x^2-4350x+5499)^5\,(x-1)\,(x-7) \tag{44}$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \{RootOf(\_Z^2+1)\} \tag{45}$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```

$$ext := \{RootOf(\_Z^2+1)\} \tag{46}$$

```
> extppp:={};
```

$$extppp := \varnothing \tag{47}$$

```
> E:= Singular(L,extppp);
```

$$E := \left[\left[x^4-\frac{98}{3}x^3+\frac{1058}{3}x^2-1450x+1833,\ RootOf(3\_Z^4-98\_Z^3+1058\_Z^2-4350\_Z+5499)\right],[x-1,1],[x-12,12],[\infty,\infty],[x-3,3],[x-9,9],[x-7,7]\right] \tag{48}$$

```
> F:=NotAppSing(L,E,ext);
```

$$F := [[x-1,1],[x-3,3],[x-9,9],[x-12,12],[\infty,\infty],[x-7,7]] \tag{49}$$

```
> Sirr:=irrsing0F2(L,t,F,ext);
```

$$Sirr := \left[[[x-9,9],[x-12,12],[\infty,\infty]],\left[\left[\frac{8\,2^{1/3}}{t^{1/3}}+\frac{1}{3}+\frac{RootOf(\_Z^2+1)}{3},\right.\right.\right. \tag{50}$$

$$\frac{16\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3},-\frac{16\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}+\frac{1}{3}$$
$$+\frac{RootOf\left(\_Z^2+1\right)}{3}\Bigg],\left[-\frac{1980^{2/3}}{6\,t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3},-\frac{1980^{2/3}}{3\left(I\sqrt{3}-1\right)t^{1/3}}\right.$$
$$\left.+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3},\frac{1980^{2/3}}{3\left(I\sqrt{3}+1\right)t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3}\right],$$
$$\left[\frac{3\,RootOf\left(\_Z^3+2,index=1\right)}{t}+1+RootOf\left(\_Z^2+1\right),\frac{3\,RootOf\left(\_Z^3+2,index=2\right)}{t}\right.$$
$$\left.+1+RootOf\left(\_Z^2+1\right),\frac{3\,RootOf\left(\_Z^3+2,index=3\right)}{t}+1+RootOf\left(\_Z^2+1\right)\right]\Bigg],$$
$$\left[\left[\frac{16\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}-\frac{8\,2^{1/3}}{t^{1/3}},-\frac{16\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}-\frac{8\,2^{1/3}}{t^{1/3}},-\frac{16\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}\right.\right.$$
$$\left.-\frac{16\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}\right],\left[-\frac{1980^{2/3}}{3\left(I\sqrt{3}-1\right)t^{1/3}}+\frac{1980^{2/3}}{6\,t^{1/3}},\frac{1980^{2/3}}{3\left(I\sqrt{3}+1\right)t^{1/3}}\right.$$
$$\left.+\frac{1980^{2/3}}{6\,t^{1/3}},\frac{1980^{2/3}}{3\left(I\sqrt{3}+1\right)t^{1/3}}+\frac{1980^{2/3}}{3\left(I\sqrt{3}-1\right)t^{1/3}}\right],$$
$$\left[\frac{3\,RootOf\left(\_Z^3+2,index=2\right)}{t}-\frac{3\,RootOf\left(\_Z^3+2,index=1\right)}{t},\right.$$
$$\frac{3\,RootOf\left(\_Z^3+2,index=3\right)}{t}-\frac{3\,RootOf\left(\_Z^3+2,index=1\right)}{t},$$
$$\left.\left.\frac{3\,RootOf\left(\_Z^3+2,index=3\right)}{t}-\frac{3\,RootOf\left(\_Z^3+2,index=2\right)}{t}\right]\right],\left[\frac{1}{3},\frac{1}{3},1\right],[3,3,$$
$$1],\left[\left[\left[\frac{16\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3},\frac{8\,2^{1/3}}{t^{1/3}}+\frac{1}{3}\right.\right.\right.$$
$$\left.+\frac{RootOf\left(\_Z^2+1\right)}{3}\right],\left[-\frac{16\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3},\frac{8\,2^{1/3}}{t^{1/3}}+\frac{1}{3}\right.$$
$$\left.+\frac{RootOf\left(\_Z^2+1\right)}{3}\right],\left[-\frac{16\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3},\right.$$
$$\left.\left.\frac{16\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3}\right]\right],\left[\left[-\frac{1980^{2/3}}{3\left(I\sqrt{3}-1\right)t^{1/3}}+\frac{1}{3}\right.\right.$$
$$\left.+\frac{RootOf\left(\_Z^2+1\right)}{3},-\frac{1980^{2/3}}{6\,t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3}\right],\left[\frac{1980^{2/3}}{3\left(I\sqrt{3}+1\right)t^{1/3}}\right.$$
$$\left.+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3},-\frac{1980^{2/3}}{6\,t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3}\right],$$

$$\left[\frac{1980^{2/3}}{3\left(I\sqrt{3}+1\right)t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3},-\frac{1980^{2/3}}{3\left(I\sqrt{3}-1\right)t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3}\right]\right],\left[\left[\frac{3\,RootOf\left(\_Z^3+2,index=2\right)}{t}+1+RootOf\left(\_Z^2+1\right),\frac{3\,RootOf\left(\_Z^3+2,index=1\right)}{t}+1+RootOf\left(\_Z^2+1\right)\right],\left[\frac{3\,RootOf\left(\_Z^3+2,index=3\right)}{t}+1+RootOf\left(\_Z^2+1\right),\frac{3\,RootOf\left(\_Z^3+2,index=1\right)}{t}+1+RootOf\left(\_Z^2+1\right)\right],\left[\frac{3\,RootOf\left(\_Z^3+2,index=3\right)}{t}+1+RootOf\left(\_Z^2+1\right),\frac{3\,RootOf\left(\_Z^3+2,index=2\right)}{t}+1+RootOf\left(\_Z^2+1\right)\right]\right]\right],\left[\left[-4\left(I\sqrt{3}\,2^{1/3}+3\,2^{1/3}\right)t,4\left(I\sqrt{3}\,2^{1/3}-3\,2^{1/3}\right)t,8\,I\,2^{1/3}\sqrt{3}\,t\right],\left[\frac{\left(3\,1980^{2/3}+I\sqrt{3}\,1980^{2/3}\right)t}{12},-\frac{\left(I\sqrt{3}\,1980^{2/3}-3\,1980^{2/3}\right)t}{12},-\frac{I}{6}\,1980^{2/3}\sqrt{3}\,t\right],\left[3\left(RootOf\left(\_Z^3+2,index=2\right)-RootOf\left(\_Z^3+2,index=1\right)\right)t,3\left(RootOf\left(\_Z^3+2,index=3\right)-RootOf\left(\_Z^3+2,index=2\right)\right)t,3\left(RootOf\left(\_Z^3+2,index=3\right)-RootOf\left(\_Z^3+2,index=2\right)\right)t\right]\right],\left[\left[\left[x-1,1\right],\left[x-3,3\right],\left[x-7,7\right]\right],\left[\left[\left[0,0,-2\,RootOf\left(\_Z^2+1\right)\right],\left[0,-2\,RootOf\left(\_Z^2+1\right),-2\,RootOf\left(\_Z^2+1\right)\right],\left[1,1,1\right],\left[\left[0,0\right],\left[-2\,RootOf\left(\_Z^2+1\right),0\right],\left[-2\,RootOf\left(\_Z^2+1\right),0\right]\right],3\right],\left[\left[0,0,-RootOf\left(\_Z^2+1\right)\right],\left[0,-RootOf\left(\_Z^2+1\right),-RootOf\left(\_Z^2+1\right)\right],\left[1,1,1\right],\left[\left[0,0\right],\left[-RootOf\left(\_Z^2+1\right),0\right],\left[-RootOf\left(\_Z^2+1\right),0\right]\right],3\right],\left[\left[0,0,-2\,RootOf\left(\_Z^2+1\right)\right],\left[0,-2\,RootOf\left(\_Z^2+1\right),-2\,RootOf\left(\_Z^2+1\right)\right],\left[1,1,1\right],\left[\left[0,0\right],\left[-2\,RootOf\left(\_Z^2+1\right),0\right],\left[-2\,RootOf\left(\_Z^2+1\right),0\right]\right],3\right]\right]\right]$$

```
> Sreg:=regsingtrue0F2(L,t,Sirr[-1],ext);
```

$$Sreg := \left[\left[\left[x-1,1\right],\left[x-3,3\right],\left[x-7,7\right]\right],\left[\left[0,0,-2\,RootOf\left(\_Z^2+1\right)\right],\left[0,0,-RootOf\left(\_Z^2+1\right)\right],\left[0,0,-2\,RootOf\left(\_Z^2+1\right)\right]\right],\left[\left[0,-2\,RootOf\left(\_Z^2+1\right),-2\,RootOf\left(\_Z^2+1\right)\right],\left[0,-RootOf\left(\_Z^2+1\right),-RootOf\left(\_Z^2+1\right)\right],\left[0,-2\,RootOf\left(\_Z^2+1\right),-2\,RootOf\left(\_Z^2+1\right)\right]\right],\left[\left[\left[0,0\right],\left[-2\,RootOf\left(\_Z^2+1\right),0\right],\left[-2\,RootOf\left(\_Z^2+1\right),0\right]\right],\left[\left[0,0\right],\left[-RootOf\left(\_Z^2+1\right),0\right],\left[-RootOf\left(\_Z^2+1\right),0\right]\right],\left[\left[0,0\right],\left[-2\,RootOf\left(\_Z^2+1\right),0\right],\left[-2\,RootOf\left(\_Z^2+1\right),0\right]\right]\right]\right] \quad (51)$$

```
> RSreg:=Sregseptrue0F2(L,Sreg,ext);
```

$$RSreg := \left[\left[\right],\left[\right],\left[\left[\left[x-1,1\right],\left[x-3,3\right],\left[x-7,7\right]\right],\left[\left[0,0,-2\,RootOf\left(\_Z^2+1\right)\right],\left[0,0,-RootOf\left(\_Z^2+1\right)\right],\left[0,0,-2\,RootOf\left(\_Z^2+1\right)\right]\right],\left[\left[\left[-2\,RootOf\left(\_Z^2+1\right),-2\,RootOf\left(\_Z^2+1\right)\right],\left[0\right]\right],\left[\left[-RootOf\left(\_Z^2+1\right),-RootOf\left(\_Z^2+1\right)\right],\left[0\right]\right],\left[\left[-2\,RootOf\left(\_Z^2+1\right),-2\,RootOf\left(\_Z^2+1\right)\right],\left[0\right]\right]\right]\right] \quad (52)$$

```
> R1:=IrrRegAppsing0F2(L,t,E,ext);
```

```
> l:=1;
```

$$l := 1 \tag{53}$$

```
> F1:= Sirr0F2info1(L,R1[1],R1[2],l,x,t,ext);
```

$$F1 := \left[\left[\left[\infty, \frac{1}{x}, [-2x^3], 3, \{RootOf(2+\_Z^3, index=1), RootOf(2+\_Z^3, index=2)\}, \{RootOf(\_Z^2+1), RootOf(2+\_Z^3, index=1), RootOf(2+\_Z^3, index=2)\}\right], \left[9, x-9, \left[-\frac{1024}{x-9}\right], 1, \{\}, \{RootOf(\_Z^2+1)\}\right], \left[12, x-12, \left[\frac{18150}{x-12}\right], 1, \{\}, \{RootOf(\_Z^2+1)\}\right]\right], 5, 6, (x-9)(x-12), 1\right] \tag{54}$$

```
> easy0F2(L,R1,F1,x,t,ext);
```

$$\left[\left[[[1, RootOf(\_Z^2+1)]], \frac{2(x-1)^2(x-3)(x-7)^2}{(x-9)(x-12)}\right], \left[[[1, RootOf(\_Z^2+1)]], -\frac{2(x-1)^2(x-3)(x-7)^2}{(x-9)(x-12)}\right]\right] \tag{55}$$

```
> find0F2ln(L,R1,F1,x,t,ext);
```

$$\left[\left[[[1, RootOf(\_Z^2+1)]], \frac{2(x-1)^2(x-3)(x-7)^2}{(x-9)(x-12)}\right], \left[[[1, RootOf(\_Z^2+1)]], -\frac{2(x-1)^2(x-3)(x-7)^2}{(x-9)(x-12)}\right]\right] \tag{56}$$

```
> TIME := time();
  Hyp0F2Solutions(L);
  time()-TIME;
```

$$TIME := 34.312$$

$$\left\{\left[\left\{\left[\left[1, RootOf(\_Z^2+1)\right], \left[-\frac{2}{x-7}\right], \left[\frac{9(x-3)(x-7)^2(x-9)^3(x-12)^3 Dx^2}{(5499-4350x+1058x^2-98x^3+3x^4)^2} + (9(15x^{13}-1623x^{12}+79349x^{11}-2318073x^{10}+45075498x^9-614907602x^8+6040382598x^7-43 + 223351175871x^5-824867387271x^4+2104961242149x^3-3505189872969x^2 + 3413344871016x-1477552421520)Dx)\Big/((5499-4350x+1058x^2-98x^3 + 3x^4)^2(3x^5-101x^4+1156x^3-5408x^2+9849x-5499))+(18(6x^{12} - 621x^{11}+28733x^{10}-785302x^9+14104058x^8-175093272x^7+1537782840x^6 - 9607222674x^5+42285234816x^4-127655542107x^3+250793603163x^2 - 288655394040x+148734510768))\Big/((5499-4350x+1058x^2-98x^3\right]\right\}\right]$$

$$+3\,x^4)^2\,(3\,x^5-101\,x^4+1156\,x^3-5408\,x^2+9849\,x-5499)\,)\,]]\},$$

$$\frac{2\,(x-1)^2\,(x-3)\,(x-7)^2}{(x-9)\,(x-12)}\,]]\}$$

26.328 **(57)**

```
> F:=sumdiffeq(hyperterm([],[b1,b2],x,k),k,J(x));
```

$$F := \left(\frac{d^3}{dx^3}\,J(x)\right)x^2+(b2+1+b1)\left(\frac{d^2}{dx^2}\,J(x)\right)x-J(x)+b2\,b1\left(\frac{d}{dx}\,J(x)\right)=0 \quad \textbf{(58)}$$

```
> LA:=de2diffop(F,J(x));
```

$$LA := x^2\,Dx^3+(x\,b2+x+x\,b1)\,Dx^2+b2\,b1\,Dx-1 \quad \textbf{(59)}$$

```
> L12:=subs({b1=1,b2=1/7},LA);
```

$$L12 := x^2\,Dx^3+\frac{15}{7}\,x\,Dx^2+\frac{1}{7}\,Dx-1 \quad \textbf{(60)}$$

```
> f:=(2*(x-1)*(x-3)*(x-7)^3)/((x-9)*(x-12)^3);
```

$$f := \frac{2\,(x-1)\,(x-3)\,(x-7)^3}{(x-9)\,(x-12)^3} \quad \textbf{(61)}$$

```
> L:=ChangeOfVariables(L12,f);
```

$$L := 7\,Dx^3\,(x-1)^2\,(x-3)^2\,(x-7)^2\,(x-9)^4\,(x-12)^6\,(3177-2724\,x+654\,x^2-52\,x^3+x^4)^2+3\,(20830689-27346680\,x+17177688\,x^2-6348216\,x^3+1444242\,x^4+5\,x^8-196296\,x^5+14768\,x^6-520\,x^7)\,Dx^2\,(x-1)\,(x-3)\,(x-7)\,(x-9)^3\,(x-12)^5\,(3177-2724\,x+654\,x^2-52\,x^3+x^4)+(564228922154400\,x-777883985371440\,x^2+624680810940240\,x^3-328646471988996\,x^4-208\,x^{15}+x^{16}+51096393696\,x^9-1231064592\,x^{10}-172695696\,x^{11}+20647516\,x^{12}-1008736\,x^{13}+23376\,x^{14}-681599791818\,x^8+119460916666080\,x^5-30650023473552\,x^6+5538208328208\,x^7-174852961459983)\,Dx\,(x-9)^2\,(x-12)^4-14\,(3177-2724\,x+654\,x^2-52\,x^3+x^4)^5\,(x-7)^2 \quad \textbf{(62)}$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \{\} \quad \textbf{(63)}$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```

$$ext := \{\} \quad \textbf{(64)}$$

```
> extppp:={};
```

$$extppp := \{\} \quad \textbf{(65)}$$

```
> E:= Singular(L,extppp);
```

$$E := [[x-3,3],[\infty,\infty],[x-1,1],[x-9,9],[x-12,12],[3177-2724\,x+654\,x^2-52\,x^3+x^4, RootOf(3177-2724\,\_Z+654\,\_Z^2-52\,\_Z^3+\_Z^4)],[x-7,7]] \quad \textbf{(66)}$$

```
> F:=NotAppSing(L,E,ext);
```

$$F := [[x-1,1],[x-7,7],[x-3,3],[\infty,\infty],[x-9,9],[x-12,12]] \quad \textbf{(67)}$$

```
> Sirr:=irrsing0F2(L,t,F,ext);
```

$$Sirr := \left[[[\infty, \infty], [x-9, 9], [x-12, 12]], \left[\left[-\frac{2^{1/3}}{t^{1/3}} + \frac{1}{21}, \frac{1}{\left(\frac{1}{4}\,2^{2/3} - \frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right) t^{1/3}} \right.\right.\right. \tag{68}$$

$$\left. + \frac{1}{21}, \frac{1}{\left(\frac{1}{4}\,2^{2/3} + \frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right) t^{1/3}} + \frac{1}{21} \right], \left[\frac{4}{9}\,\frac{18^{2/3}}{t^{1/3}} + \frac{1}{21}, \right.$$

$$\frac{1}{\left(-\frac{1}{16}\,18^{1/3} + \frac{1}{16}\,I\sqrt{3}\,18^{1/3}\right) t^{1/3}} + \frac{1}{21}, \frac{1}{\left(-\frac{1}{16}\,18^{1/3} - \frac{1}{16}\,I\sqrt{3}\,18^{1/3}\right) t^{1/3}}$$

$$\left. + \frac{1}{21} \right], \left[\frac{15\,RootOf\left(66 + \_Z^3, index = 1\right)}{t} + \frac{1}{7}, \frac{15\,RootOf\left(66 + \_Z^3, index = 2\right)}{t} \right.$$

$$\left. + \frac{1}{7}, \frac{15\,RootOf\left(66 + \_Z^3, index = 3\right)}{t} + \frac{1}{7} \right] \Bigg], \left[\left[\left[\frac{1}{\left(\frac{1}{4}\,2^{2/3} - \frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right) t^{1/3}} \right.\right.\right.$$

$$+ \frac{2^{1/3}}{t^{1/3}}, \frac{1}{\left(\frac{1}{4}\,2^{2/3} + \frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right) t^{1/3}} + \frac{2^{1/3}}{t^{1/3}}, \frac{1}{\left(\frac{1}{4}\,2^{2/3} + \frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right) t^{1/3}}$$

$$\left. - \frac{1}{\left(\frac{1}{4}\,2^{2/3} - \frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right) t^{1/3}} \right], \left[\frac{1}{\left(-\frac{1}{16}\,18^{1/3} + \frac{1}{16}\,I\sqrt{3}\,18^{1/3}\right) t^{1/3}} - \frac{4}{9}\,\frac{18^{2/3}}{t^{1/3}}, \right.$$

$$\frac{1}{\left(-\frac{1}{16}\,18^{1/3} - \frac{1}{16}\,I\sqrt{3}\,18^{1/3}\right) t^{1/3}} - \frac{4}{9}\,\frac{18^{2/3}}{t^{1/3}}, \frac{1}{\left(-\frac{1}{16}\,18^{1/3} - \frac{1}{16}\,I\sqrt{3}\,18^{1/3}\right) t^{1/3}}$$

$$\left. - \frac{1}{\left(-\frac{1}{16}\,18^{1/3} + \frac{1}{16}\,I\sqrt{3}\,18^{1/3}\right) t^{1/3}} \right], \left[\frac{15\,RootOf\left(66 + \_Z^3, index = 2\right)}{t} \right.$$

$$- \frac{15\,RootOf\left(66 + \_Z^3, index = 1\right)}{t}, \frac{15\,RootOf\left(66 + \_Z^3, index = 3\right)}{t}$$

$$- \frac{15\,RootOf\left(66 + \_Z^3, index = 1\right)}{t}, \frac{15\,RootOf\left(66 + \_Z^3, index = 3\right)}{t}$$

$$\left. - \frac{15\,RootOf\left(66 + \_Z^3, index = 2\right)}{t} \right] \Bigg], \left[\frac{1}{3}, \frac{1}{3}, 1 \right], [3, 3, 1],$$

$$\left[\left[\left[\frac{1}{\left(\frac{1}{4}\,2^{2/3} - \frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right) t^{1/3}} + \frac{1}{21}, -\frac{2^{1/3}}{t^{1/3}} + \frac{1}{21} \right] \right.\right.$$

$$\left[\frac{1}{\left(\frac{1}{4}\,2^{2/3} + \frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right) t^{1/3}} + \frac{1}{21}, -\frac{2^{1/3}}{t^{1/3}} + \frac{1}{21} \right], \left[\frac{1}{\left(\frac{1}{4}\,2^{2/3} + \frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right) t^{1/3}} \right.$$

$$+\frac{1}{21},\frac{1}{\left(\frac{1}{4}\,2^{2/3}-\frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right)t^{1/3}}+\frac{1}{21}\Bigg]\Bigg],\Bigg[\Bigg[\frac{1}{\left(-\frac{1}{16}\,18^{1/3}+\frac{1}{16}\,I\sqrt{3}\,18^{1/3}\right)t^{1/3}}$$
$$+\frac{1}{21},\frac{4}{9}\,\frac{18^{2/3}}{t^{1/3}}+\frac{1}{21}\Bigg],\Bigg[\frac{1}{\left(-\frac{1}{16}\,18^{1/3}-\frac{1}{16}\,I\sqrt{3}\,18^{1/3}\right)t^{1/3}}+\frac{1}{21},\frac{4}{9}\,\frac{18^{2/3}}{t^{1/3}}$$
$$+\frac{1}{21}\Bigg],\Bigg[\frac{1}{\left(-\frac{1}{16}\,18^{1/3}-\frac{1}{16}\,I\sqrt{3}\,18^{1/3}\right)t^{1/3}}+\frac{1}{21},$$
$$\frac{1}{\left(-\frac{1}{16}\,18^{1/3}+\frac{1}{16}\,I\sqrt{3}\,18^{1/3}\right)t^{1/3}}+\frac{1}{21}\Bigg]\Bigg],\Bigg[\Bigg[\frac{15\,RootOf\left(66+\_Z^3,index=2\right)}{t}$$
$$+\frac{1}{7},\frac{15\,RootOf\left(66+\_Z^3,index=1\right)}{t}+\frac{1}{7}\Bigg],\Bigg[\frac{15\,RootOf\left(66+\_Z^3,index=3\right)}{t}$$
$$+\frac{1}{7},\frac{15\,RootOf\left(66+\_Z^3,index=1\right)}{t}+\frac{1}{7}\Bigg],\Bigg[\frac{15\,RootOf\left(66+\_Z^3,index=3\right)}{t}$$
$$+\frac{1}{7},\frac{15\,RootOf\left(66+\_Z^3,index=2\right)}{t}+\frac{1}{7}\Bigg]\Bigg]\Bigg],\Bigg[\Bigg[\frac{1}{2}\,\left(3\,2^{1/3}+I\,2^{1/3}\sqrt{3}\right)t,-\frac{1}{2}\,($$
$$-3\,2^{1/3}+I\,2^{1/3}\sqrt{3}\,)\,t,-I\sqrt{3}\,t\,2^{1/3}\Bigg],\Bigg[-\frac{2}{9}\,\left(3\,18^{2/3}+I\,18^{2/3}\sqrt{3}\right)t,\frac{2}{9}\,\left(-3\,18^{2/3}\right.$$
$$\left.+I\,18^{2/3}\sqrt{3}\right)t,\frac{4}{9}\,I\sqrt{3}\,t\,18^{2/3}\Bigg],\left[15\,\left(RootOf\left(66+\_Z^3,index=2\right)-RootOf\left(66\right.\right.\right.$$
$$\left.\left.\left.+\_Z^3,index=1\right)\right)t,15\,\left(RootOf\left(66+\_Z^3,index=3\right)-RootOf\left(66+\_Z^3,index\right.\right.\right.$$
$$\left.\left.\left.=1\right)\right)t,15\,\left(RootOf\left(66+\_Z^3,index=3\right)-RootOf\left(66+\_Z^3,index=2\right)\right)t\right]\Bigg],\Bigg[\Bigg[0,0,$$
$$0\Bigg],\Bigg[0,0,0\Bigg],\Bigg[0,0,0\Bigg]\Bigg],\Bigg[\Bigg[\left[x-1,1\right],\left[x-7,7\right],\left[x-3,3\right]\Bigg],\Bigg[\Bigg[\Bigg[0,0,\frac{6}{7}\Bigg],\Bigg[0,\frac{6}{7},\frac{6}{7}\Bigg],$$
$$\left[1,1,1\right],\Bigg[\left[0,0\right],\left[\frac{6}{7},0\right],\left[\frac{6}{7},0\right]\Bigg],3\Bigg],\Bigg[\Bigg[0,0,\frac{18}{7}\Bigg],\Bigg[0,\frac{18}{7},\frac{18}{7}\Bigg],\left[1,1,1\right],\Bigg[\left[0,0\right],$$
$$\left[\frac{18}{7},0\right],\left[\frac{18}{7},0\right]\Bigg],3\Bigg],\Bigg[\Bigg[0,0,\frac{6}{7}\Bigg],\Bigg[0,\frac{6}{7},\frac{6}{7}\Bigg],\left[1,1,1\right],\Bigg[\left[0,0\right],\left[\frac{6}{7},0\right],\left[\frac{6}{7},0\right]\Bigg],3\Bigg]\Bigg]\Bigg]$$

```
> Sreg:=regsingtrue0F2(L,t,Sirr[-1],ext);
```

$$Sreg:=\Bigg[\left[\left[x-1,1\right],\left[x-7,7\right],\left[x-3,3\right]\right],\Bigg[\Bigg[0,0,\frac{6}{7}\Bigg],\Bigg[0,0,\frac{18}{7}\Bigg],\Bigg[0,0,\frac{6}{7}\Bigg]\Bigg],\Bigg[\Bigg[0,\frac{6}{7},\frac{6}{7}\Bigg],\Bigg[0,\frac{18}{7},\frac{18}{7}\Bigg],\Bigg[0,\frac{6}{7},\frac{6}{7}\Bigg]\Bigg],\Bigg[\Bigg[\left[0,0\right],\left[\frac{6}{7},0\right],\left[\frac{6}{7},0\right]\Bigg],\Bigg[\left[0,0\right],\left[\frac{18}{7},0\right],\left[\frac{18}{7},0\right]\Bigg],\Bigg[\left[0,0\right],\left[\frac{6}{7},0\right],\left[\frac{6}{7},0\right]\Bigg]\Bigg]\Bigg] \quad (69)$$

```
> RSreg:=Sregseptrue0F2(L,Sreg,ext);
```

(70)

$$RSreg := \left[[\,], [\,], \left[[[x-1, 1], [x-7, 7], [x-3, 3]], \left[\left[0, 0, \frac{6}{7}\right], \left[0, 0, \frac{18}{7}\right], \left[0, 0, \frac{6}{7}\right] \right], \left[\left[\left[\frac{6}{7}, \frac{6}{7} \right], [0] \right], \left[\left[\frac{18}{7}, \frac{18}{7} \right], [0] \right], \left[\left[\frac{6}{7}, \frac{6}{7} \right], [0] \right] \right] \right] \right] \quad (70)$$

```
> R1:=IrrRegAppsing0F2(L,t,E,ext);
```

```
> l:=1;
```

$$l := 1 \quad (71)$$

```
> F1:= Sirr0F2info1(L,R1[1],R1[2],l,x,t,ext);
```

$$F1 := \left[\left[\left[\infty, \frac{1}{x}, [2\,x], 1, \{\,\}, \{\,\} \right], \left[9, x-9, \left[-\frac{256}{9\,(x-9)} \right], 1, \{\,\}, \{\,\} \right], \left[12, x-12, \left[-\frac{8250}{(x-12)^3} \right], 3, \{RootOf(66+\_Z^3, index=1), RootOf(66+\_Z^3, index=2)\}, \{RootOf(66+\_Z^3, index=1), RootOf(66+\_Z^3, index=2)\} \right] \right], 5, 6, (x-9)\,(x-12)^3, (x-12)^2 \right] \quad (72)$$

```
> easy0F2(L,R1,F1,x,t,ext);
```

$$\left[\left[\left[\left[1, \frac{1}{7} \right] \right], \frac{2\,(x-1)\,(x-3)\,(x-7)^3}{(x-9)\,(x-12)^3} \right], \left[\left[\left[1, \frac{1}{7} \right] \right], -\frac{2\,(x-1)\,(x-3)\,(x-7)^3}{(x-9)\,(x-12)^3} \right] \right] \quad (73)$$

```
> find0F2ln(L,R1,F1,x,t,ext);
```

$$\left[\left[\left[\left[1, \frac{1}{7} \right] \right], \frac{2\,(x-1)\,(x-3)\,(x-7)^3}{(x-9)\,(x-12)^3} \right], \left[\left[\left[1, \frac{1}{7} \right] \right], -\frac{2\,(x-1)\,(x-3)\,(x-7)^3}{(x-9)\,(x-12)^3} \right] \right] \quad (74)$$

```
> TIME := time();
  Hyp0F2Solutions(L);
  time()-TIME;
```

$$TIME := 65.578$$

$$\left\{ \left[\left\{ \left[\left[1, \frac{1}{7} \right], [0], [1] \right] \right\}, \frac{2\,(x-1)\,(x-3)\,(x-7)^3}{(x-9)\,(x-12)^3} \right] \right\}$$

$$1.359 \quad (75)$$

```
> ####### THE LOGARITHMIC CASE #######
```

```
> F:=sumdiffeq(hyperterm([],[b1,b2],x,k),k,J(x));
```

$$F := \left(\frac{d^3}{dx^3} J(x) \right) x^2 + (b2+1+b1) \left(\frac{d^2}{dx^2} J(x) \right) x - J(x) + b2\,b1 \left(\frac{d}{dx} J(x) \right) = 0 \quad (76)$$

```
> LA:=de2diffop(F,J(x));
```

$$LA := x^2\,Dx^3 + (x\,b2 + x + x\,b1)\,Dx^2 + b2\,b1\,Dx - 1 \quad (77)$$

```
> L02:=subs({b1=1,b2=1/7},LA);
```

$$L02 := x^2\,Dx^3 + \frac{15}{7}\,x\,Dx^2 + \frac{1}{7}\,Dx - 1 \quad (78)$$

```
> f:=(2*(x-1)^2*(x-3)*(x-7)^3)/((x-9)^2*(x-12)^3);
```

$$f := \frac{2\,(x-1)^2\,(x-3)\,(x-7)^3}{(x-9)^2\,(x-12)^3} \tag{79}$$

```
> L:=ChangeOfVariables(L02,f);
```

$$L := 7\,Dx^3\,(x-1)^2\,(x-3)^2\,(x-7)^2\,(x-9)^5\,(x-12)^6\,(5193 - 3852\,x + 830\,x^2 - 60\,x^3 + x^4)^2 + 3\,(22645089 - 28857816\,x + 18714312\,x^2 - 7325064\,x^3 + 1736178\,x^4 + 5\,x^8 - 239048\,x^5 + 17760\,x^6 - 600\,x^7)\,Dx^2\,(x-1)\,(x-3)\,(x-7)\,(x-9)^4\,(x-12)^5\,(5193 - 3852\,x + 830\,x^2 - 60\,x^3 + x^4) + (2835488467971504\,x - 3598922709698256\,x^2 + 2694809695467600\,x^3 - 1330495042610628\,x^4 - 240\,x^{15} + x^{16} + 198921889104\,x^9 - 9212730352\,x^{10} + 28636848\,x^{11} + 24210204\,x^{12} - 1404816\,x^{13} + 32240\,x^{14} - 2416016455178\,x^8 + 456801809131152\,x^5 - 111773530633392\,x^6 + 19582335010800\,x^7 - 988135334786511)\,Dx\,(x-9)^3\,(x-12)^4 - 14\,(5193 - 3852\,x + 830\,x^2 - 60\,x^3 + x^4)^5\,(x-1)\,(x-7)^2 \tag{80}$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \{\,\} \tag{81}$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```

$$ext := \{\,\} \tag{82}$$

```
> extppp:={};
```

$$extppp := \{\,\} \tag{83}$$

```
> E:= Singular(L,extppp);
```

$$E := [\,[x-3,\,3\,],\,[\,\infty,\,\infty\,],\,[x-1,\,1\,],\,[5193 - 3852\,x + 830\,x^2 - 60\,x^3 + x^4,\,RootOf(5193 - 3852\,\_Z + 830\,\_Z^2 - 60\,\_Z^3 + \_Z^4)\,],\,[x-9,\,9\,],\,[x-12,\,12\,],\,[x-7,\,7\,]\,] \tag{84}$$

```
> F:=NotAppSing(L,E,ext);
```

$$F := [\,[x-1,\,1\,],\,[x-7,\,7\,],\,[x-3,\,3\,],\,[\,\infty,\,\infty\,],\,[x-9,\,9\,],\,[x-12,\,12\,]\,] \tag{85}$$

```
> Sirr:=irrsing0F2(L,t,F,ext);
```

$$Sirr := \left[\,[\,[\,\infty,\,\infty\,],\,[x-9,\,9\,],\,[x-12,\,12\,]\,],\,\left[\left[-\frac{2^{1/3}}{t^{1/3}} + \frac{1}{21},\,\frac{1}{\left(\frac{1}{4}\,2^{2/3} - \frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right)t^{1/3}} + \frac{1}{21}\right],\left[\frac{16}{3}\,\frac{12^{1/3}}{t^{2/3}} + \frac{2}{21},\,\frac{16384}{9\left(-\frac{8}{3}\,12^{1/3} + \frac{8}{3}\,I\sqrt{3}\,12^{1/3}\right)^2 t^{2/3}} + \frac{2}{21}\right],\left[\frac{16384}{9\left(-\frac{8}{3}\,12^{1/3} - \frac{8}{3}\,I\sqrt{3}\,12^{1/3}\right)^2 t^{2/3}} + \frac{2}{21}\right],\left[\frac{15\,RootOf(242 + \_Z^3,\,index=1)}{t} + \frac{1}{7},\,\frac{15\,RootOf(242 + \_Z^3,\,index=2)}{t} + \frac{1}{7},\,\frac{15\,RootOf(242 + \_Z^3,\,index=3)}{t} + \frac{1}{7}\right]\right],\left[\left[\frac{1}{\left(\frac{1}{4}\,2^{2/3} - \frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right)t^{1/3}}\right.\right.\right. \tag{86}$$

$$+\frac{2^{1/3}}{t^{1/3}},\ \frac{1}{\left(\frac{1}{4}\,2^{2/3}+\frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right)t^{1/3}}+\frac{2^{1/3}}{t^{1/3}},\ \frac{1}{\left(\frac{1}{4}\,2^{2/3}+\frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right)t^{1/3}}$$

$$-\frac{1}{\left(\frac{1}{4}\,2^{2/3}-\frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right)t^{1/3}}\Bigg],\left[\frac{16384}{9\left(-\frac{8}{3}\,12^{1/3}+\frac{8}{3}\,I\sqrt{3}\,12^{1/3}\right)^2t^{2/3}}\right.$$

$$-\frac{16}{3}\,\frac{12^{1/3}}{t^{2/3}},\ \frac{16384}{9\left(-\frac{8}{3}\,12^{1/3}-\frac{8}{3}\,I\sqrt{3}\,12^{1/3}\right)^2t^{2/3}}-\frac{16}{3}\,\frac{12^{1/3}}{t^{2/3}},$$

$$\left.\frac{16384}{9\left(-\frac{8}{3}\,12^{1/3}-\frac{8}{3}\,I\sqrt{3}\,12^{1/3}\right)^2t^{2/3}}-\frac{16384}{9\left(-\frac{8}{3}\,12^{1/3}+\frac{8}{3}\,I\sqrt{3}\,12^{1/3}\right)^2t^{2/3}}\right],$$

$$\left[\frac{15\,RootOf\left(242+\_Z^3,\,index=2\right)}{t}-\frac{15\,RootOf\left(242+\_Z^3,\,index=1\right)}{t},\right.$$

$$\frac{15\,RootOf\left(242+\_Z^3,\,index=3\right)}{t}-\frac{15\,RootOf\left(242+\_Z^3,\,index=1\right)}{t},$$

$$\left.\frac{15\,RootOf\left(242+\_Z^3,\,index=3\right)}{t}-\frac{15\,RootOf\left(242+\_Z^3,\,index=2\right)}{t}\right]\Bigg],\left[\frac{1}{3},\,\frac{2}{3},\,1\right],$$

$$[3,3,1],\left[\left[\left[\frac{1}{\left(\frac{1}{4}\,2^{2/3}-\frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right)t^{1/3}}+\frac{1}{21},\,-\frac{2^{1/3}}{t^{1/3}}+\frac{1}{21}\right],\right.\right.$$

$$\left[\frac{1}{\left(\frac{1}{4}\,2^{2/3}+\frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right)t^{1/3}}+\frac{1}{21},\,-\frac{2^{1/3}}{t^{1/3}}+\frac{1}{21}\right],\left[\frac{1}{\left(\frac{1}{4}\,2^{2/3}+\frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right)t^{1/3}}\right.$$

$$\left.\left.+\frac{1}{21},\,\frac{1}{\left(\frac{1}{4}\,2^{2/3}-\frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right)t^{1/3}}+\frac{1}{21}\right]\right],$$

$$\left[\left[\frac{16384}{9\left(-\frac{8}{3}\,12^{1/3}+\frac{8}{3}\,I\sqrt{3}\,12^{1/3}\right)^2t^{2/3}}+\frac{2}{21},\,\frac{16}{3}\,\frac{12^{1/3}}{t^{2/3}}+\frac{2}{21}\right],\right.$$

$$\left[\frac{16384}{9\left(-\frac{8}{3}\,12^{1/3}-\frac{8}{3}\,I\sqrt{3}\,12^{1/3}\right)^2t^{2/3}}+\frac{2}{21},\,\frac{16}{3}\,\frac{12^{1/3}}{t^{2/3}}+\frac{2}{21}\right],$$

$$\left.\left[\frac{16384}{9\left(-\frac{8}{3}\,12^{1/3}-\frac{8}{3}\,I\sqrt{3}\,12^{1/3}\right)^2t^{2/3}}+\frac{2}{21},\,\frac{16384}{9\left(-\frac{8}{3}\,12^{1/3}+\frac{8}{3}\,I\sqrt{3}\,12^{1/3}\right)^2t^{2/3}}\right.\right.$$

$$+\frac{2}{21}\Big]\Big], \Big[\Big[\frac{15\,RootOf(242+\_Z^3, index=2)}{t}+\frac{1}{7}, \frac{15\,RootOf(242+\_Z^3, index=1)}{t}+\frac{1}{7}\Big], \Big[\frac{15\,RootOf(242+\_Z^3, index=3)}{t}+\frac{1}{7}, \frac{15\,RootOf(242+\_Z^3, index=1)}{t}+\frac{1}{7}\Big], \Big[\frac{15\,RootOf(242+\_Z^3, index=3)}{t}+\frac{1}{7}, \frac{15\,RootOf(242+\_Z^3, index=2)}{t}+\frac{1}{7}\Big]\Big]\Big], \Big[\Big[\frac{1}{2}\,(3\,2^{1/3}+I\,2^{1/3}\sqrt{3})\,t, -\frac{1}{2}\,(-3\,2^{1/3}+I\,2^{1/3}\sqrt{3})\,t, -I\sqrt{3}\,t\,2^{1/3}\Big], \Big[\frac{8}{3}\,(-3\,12^{1/3}+I\,12^{1/3}\sqrt{3})\,t^2, -\frac{8}{3}\,(3\,12^{1/3}+I\,12^{1/3}\sqrt{3})\,t^2, -\frac{16}{3}\,I\sqrt{3}\,t^2\,12^{1/3}\Big],$$

$$[15\,(RootOf(242+\_Z^3, index=2)-RootOf(242+\_Z^3, index=1))\,t, 15\,(RootOf(242+\_Z^3, index=3)-RootOf(242+\_Z^3, index=1))\,t, 15\,(RootOf(242+\_Z^3, index=3)-RootOf(242+\_Z^3, index=2))\,t]], [[0,0,0],[0,0,0],[0,0,0]], \Big[[[x-1,1],[x-7,7],[x-3,3]], \Big[\Big[\Big[0,0,\frac{12}{7}\Big], \Big[0,\frac{12}{7},\frac{12}{7}\Big], [1,1,1], \Big[[0,0], \Big[\frac{12}{7},0\Big], \Big[\frac{12}{7},0\Big]\Big], 3\Big], \Big[\Big[0,0,\frac{18}{7}\Big], \Big[0,\frac{18}{7},\frac{18}{7}\Big], [1,1,1], \Big[[0,0], \Big[\frac{18}{7},0\Big], \Big[\frac{18}{7},0\Big]\Big], 3\Big], \Big[\Big[0,0,\frac{6}{7}\Big], \Big[0,\frac{6}{7},\frac{6}{7}\Big], [1,1,1], \Big[[0,0], \Big[\frac{6}{7},0\Big], \Big[\frac{6}{7},0\Big]\Big], 3\Big]\Big]\Big]$$

```
> Sreg:=regsingtrue0F2(L,t,Sirr[-1],ext);
```

$$Sreg := \Big[[[x-1,1],[x-7,7],[x-3,3]], \Big[\Big[0,0,\frac{12}{7}\Big], \Big[0,0,\frac{18}{7}\Big], \Big[0,0,\frac{6}{7}\Big]\Big], \Big[\Big[0,\frac{12}{7},\frac{12}{7}\Big], \Big[0,\frac{18}{7},\frac{18}{7}\Big], \Big[0,\frac{6}{7},\frac{6}{7}\Big]\Big], \Big[\Big[[0,0], \Big[\frac{12}{7},0\Big], \Big[\frac{12}{7},0\Big]\Big], \Big[[0,0], \Big[\frac{18}{7},0\Big], \Big[\frac{18}{7},0\Big]\Big], \Big[[0,0], \Big[\frac{6}{7},0\Big], \Big[\frac{6}{7},0\Big]\Big]\Big]\Big] \quad (87)$$

```
> RSreg:=Sregseptrue0F2(L,Sreg,ext);
```

$$RSreg := \Big[[\,], [\,], \Big[[[x-1,1],[x-7,7],[x-3,3]], \Big[\Big[0,0,\frac{12}{7}\Big], \Big[0,0,\frac{18}{7}\Big], \Big[0,0,\frac{6}{7}\Big]\Big], \Big[\Big[\Big[\frac{12}{7},\frac{12}{7}\Big],[0]\Big], \Big[\Big[\frac{18}{7},\frac{18}{7}\Big],[0]\Big], \Big[\Big[\frac{6}{7},\frac{6}{7}\Big],[0]\Big]\Big]\Big] \quad (88)$$

```
> R1:=IrrRegAppsing0F2(L,t,E,ext);
> l:=1;
```

$$l := 1 \quad (89)$$

```
> F1:= Sirr0F2info1(L,R1[1],R1[2],l,x,t,ext);
```

$$F1 := \Big[\Big[\Big[\infty, \frac{1}{x}, [2\,x], 1, \{\,\}, \{\,\}\Big], \Big[9, x-9, \Big[\frac{2048}{9\,(x-9)^2}\Big], 2, \{\,\}, \{\,\}\Big], \Big[12, x-12, \Big[-\frac{30250}{(x-12)^3}\Big], 3, \{RootOf(242+\_Z^3, index=1), RootOf(242+\_Z^3, index=2)\},$$

(90)

$$\left\{RootOf\left(242+\_Z^3, index=1\right), RootOf\left(242+\_Z^3, index=2\right)\right\}\Big]\Big], 6, 6, (x-9)^2\,(x-12)^3, (x-9)\,(x-12)^2\Big]$$

```
> find0F2ln(L,R1,F1,x,t,ext);
```

$$\left[\left[\left[\left[1, \frac{1}{7}\right]\right], \frac{2\,(x-1)^2\,(x-3)\,(x-7)^3}{(x-9)^2\,(x-12)^3}\right], \left[\left[\left[1, \frac{1}{7}\right]\right], -\frac{2\,(x-1)^2\,(x-3)\,(x-7)^3}{(x-9)^2\,(x-12)^3}\right]\right] \quad (91)$$

```
> TIME := time();
  Hyp0F2Solutions(L);
  time()-TIME;
```

$$TIME := 70.375$$

$$\left\{\left[\left\{\left[\left[1, \frac{1}{7}\right], [0], [1]\right]\right\}, \frac{2\,(x-1)^2\,(x-3)\,(x-7)^3}{(x-9)^2\,(x-12)^3}\right]\right\}$$

$$1.437 \quad (92)$$

```
> F:=sumdiffeq(hyperterm([],[b1,b2],x,k),k,J(x));
```

$$F := \left(\frac{d^3}{dx^3} J(x)\right) x^2 + (b2+1+b1)\left(\frac{d^2}{dx^2} J(x)\right) x - J(x) + b2\,b1\left(\frac{d}{dx} J(x)\right) = 0 \quad (93)$$

```
> LA:=de2diffop(F,J(x));
```

$$LA := x^2\,Dx^3 + (x\,b2 + x + x\,b1)\,Dx^2 + b2\,b1\,Dx - 1 \quad (94)$$

```
> L12:=subs({b1=1,b2=1/7},LA);
```

$$L12 := x^2\,Dx^3 + \frac{15}{7}\,x\,Dx^2 + \frac{1}{7}\,Dx - 1 \quad (95)$$

```
> f:=(2*(x-1)*(x-3)*(x-7)^3)/((x-9)*(x-12)^3);
```

$$f := \frac{2\,(x-1)\,(x-3)\,(x-7)^3}{(x-9)\,(x-12)^3} \quad (96)$$

```
> L:=ChangeOfVariables(L12,f);
```

$$L := 7\,Dx^3\,(x-1)^2\,(x-3)^2\,(x-7)^2\,(x-9)^4\,(x-12)^6\,(3177-2724\,x+654\,x^2-52\,x^3+x^4)^2 + 3\,(20830689-27346680\,x+17177688\,x^2-6348216\,x^3+1444242\,x^4+5\,x^8-196296\,x^5+14768\,x^6-520\,x^7)\,Dx^2\,(x-1)\,(x-3)\,(x-7)\,(x-9)^3\,(x-12)^5\,(3177-2724\,x+654\,x^2-52\,x^3+x^4) + (564228922154400\,x-777883985371440\,x^2+624680810940240\,x^3-328646471988996\,x^4-208\,x^{15}+x^{16}+51096393696\,x^9-1231064592\,x^{10}-172695696\,x^{11}+20647516\,x^{12}-1008736\,x^{13}+23376\,x^{14}-681599791818\,x^8+119460916666080\,x^5-30650023473552\,x^6+5538208328208\,x^7-174852961459983)\,Dx\,(x-9)^2\,(x-12)^4 - 14\,(3177-2724\,x+654\,x^2-52\,x^3+x^4)^5\,(x-7)^2 \quad (97)$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \{\} \quad (98)$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```

$$ext := \{\} \tag{99}$$

```
> extppp:={};
```

$$extppp := \{\} \tag{100}$$

```
> E:= Singular(L,extppp);
```

$$E := \left[[x-3, 3], [\infty, \infty], [x-1, 1], [x-9, 9], [x-12, 12], \left[3177 - 2724\,x + 654\,x^2 - 52\,x^3 + x^4, RootOf\left(3177 - 2724\,\_Z + 654\,\_Z^2 - 52\,\_Z^3 + \_Z^4\right)\right], [x-7, 7] \right] \tag{101}$$

```
> F:=NotAppSing(L,E,ext);
```

$$F := [[x-1, 1], [x-7, 7], [x-3, 3], [\infty, \infty], [x-9, 9], [x-12, 12]] \tag{102}$$

```
> Sirr:=irrsing0F2(L,t,F,ext);
```

$$Sirr := \left[[[\infty, \infty], [x-9, 9], [x-12, 12]], \left[\left[-\frac{2^{1/3}}{t^{1/3}} + \frac{1}{21}, \frac{1}{\left(\frac{1}{4}\,2^{2/3} - \frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right) t^{1/3}} + \frac{1}{21}, \frac{1}{\left(\frac{1}{4}\,2^{2/3} + \frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right) t^{1/3}} + \frac{1}{21} \right], \left[\frac{4}{9}\,\frac{18^{2/3}}{t^{1/3}} + \frac{1}{21}, \frac{1}{\left(-\frac{1}{16}\,18^{1/3} + \frac{1}{16}\,I\sqrt{3}\,18^{1/3}\right) t^{1/3}} + \frac{1}{21}, \frac{1}{\left(-\frac{1}{16}\,18^{1/3} - \frac{1}{16}\,I\sqrt{3}\,18^{1/3}\right) t^{1/3}} + \frac{1}{21} \right], \left[\frac{15\,RootOf\left(66 + \_Z^3, index = 1\right)}{t} + \frac{1}{7}, \frac{15\,RootOf\left(66 + \_Z^3, index = 2\right)}{t} + \frac{1}{7}, \frac{15\,RootOf\left(66 + \_Z^3, index = 3\right)}{t} + \frac{1}{7} \right] \right], \left[\left[\frac{1}{\left(\frac{1}{4}\,2^{2/3} - \frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right) t^{1/3}} + \frac{2^{1/3}}{t^{1/3}}, \frac{1}{\left(\frac{1}{4}\,2^{2/3} + \frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right) t^{1/3}} + \frac{2^{1/3}}{t^{1/3}}, \frac{1}{\left(\frac{1}{4}\,2^{2/3} + \frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right) t^{1/3}} - \frac{1}{\left(\frac{1}{4}\,2^{2/3} - \frac{1}{4}\,I\sqrt{3}\,2^{2/3}\right) t^{1/3}} \right], \left[\frac{1}{\left(-\frac{1}{16}\,18^{1/3} + \frac{1}{16}\,I\sqrt{3}\,18^{1/3}\right) t^{1/3}} - \frac{4}{9}\,\frac{18^{2/3}}{t^{1/3}}, \frac{1}{\left(-\frac{1}{16}\,18^{1/3} - \frac{1}{16}\,I\sqrt{3}\,18^{1/3}\right) t^{1/3}} - \frac{4}{9}\,\frac{18^{2/3}}{t^{1/3}}, \frac{1}{\left(-\frac{1}{16}\,18^{1/3} - \frac{1}{16}\,I\sqrt{3}\,18^{1/3}\right) t^{1/3}} - \frac{1}{\left(-\frac{1}{16}\,18^{1/3} + \frac{1}{16}\,I\sqrt{3}\,18^{1/3}\right) t^{1/3}} \right], \left[\frac{15\,RootOf\left(66 + \_Z^3, index = 2\right)}{t} - \frac{15\,RootOf\left(66 + \_Z^3, index = 1\right)}{t}, \frac{15\,RootOf\left(66 + \_Z^3, index = 3\right)}{t} - \frac{15\,RootOf\left(66 + \_Z^3, index = 1\right)}{t}, \frac{15\,RootOf\left(66 + \_Z^3, index = 3\right)}{t} \right] \right] \right] \tag{103}$$

$$- \frac{15\ RootOf\left(66 + \_Z^3,\ index = 2\right)}{t} \Bigg]\Bigg], \left[\frac{1}{3}, \frac{1}{3}, 1\right], [3, 3, 1],$$

$$\Bigg[\Bigg[\Bigg[\left[\frac{1}{\left(\frac{1}{4}\ 2^{2/3} - \frac{1}{4}\ I\sqrt{3}\ 2^{2/3}\right) t^{1/3}} + \frac{1}{21}, -\frac{2^{1/3}}{t^{1/3}} + \frac{1}{21}\right],$$

$$\left[\frac{1}{\left(\frac{1}{4}\ 2^{2/3} + \frac{1}{4}\ I\sqrt{3}\ 2^{2/3}\right) t^{1/3}} + \frac{1}{21}, -\frac{2^{1/3}}{t^{1/3}} + \frac{1}{21}\right], \left[\frac{1}{\left(\frac{1}{4}\ 2^{2/3} + \frac{1}{4}\ I\sqrt{3}\ 2^{2/3}\right) t^{1/3}}\right.$$

$$\left. + \frac{1}{21}, \frac{1}{\left(\frac{1}{4}\ 2^{2/3} - \frac{1}{4}\ I\sqrt{3}\ 2^{2/3}\right) t^{1/3}} + \frac{1}{21}\right]\Bigg], \Bigg[\Bigg[\frac{1}{\left(-\frac{1}{16}\ 18^{1/3} + \frac{1}{16}\ I\sqrt{3}\ 18^{1/3}\right) t^{1/3}}$$

$$+ \frac{1}{21}, \frac{4}{9}\ \frac{18^{2/3}}{t^{1/3}} + \frac{1}{21}\Bigg], \left[\frac{1}{\left(-\frac{1}{16}\ 18^{1/3} - \frac{1}{16}\ I\sqrt{3}\ 18^{1/3}\right) t^{1/3}} + \frac{1}{21}, \frac{4}{9}\ \frac{18^{2/3}}{t^{1/3}}\right.$$

$$\left. + \frac{1}{21}\right], \left[\frac{1}{\left(-\frac{1}{16}\ 18^{1/3} - \frac{1}{16}\ I\sqrt{3}\ 18^{1/3}\right) t^{1/3}} + \frac{1}{21},\right.$$

$$\left.\frac{1}{\left(-\frac{1}{16}\ 18^{1/3} + \frac{1}{16}\ I\sqrt{3}\ 18^{1/3}\right) t^{1/3}} + \frac{1}{21}\right]\Bigg], \Bigg[\Bigg[\frac{15\ RootOf\left(66 + \_Z^3,\ index = 2\right)}{t}$$

$$+ \frac{1}{7}, \frac{15\ RootOf\left(66 + \_Z^3,\ index = 1\right)}{t} + \frac{1}{7}\Bigg], \left[\frac{15\ RootOf\left(66 + \_Z^3,\ index = 3\right)}{t}\right.$$

$$\left. + \frac{1}{7}, \frac{15\ RootOf\left(66 + \_Z^3,\ index = 1\right)}{t} + \frac{1}{7}\right], \left[\frac{15\ RootOf\left(66 + \_Z^3,\ index = 3\right)}{t}\right.$$

$$\left. + \frac{1}{7}, \frac{15\ RootOf\left(66 + \_Z^3,\ index = 2\right)}{t} + \frac{1}{7}\right]\Bigg]\Bigg], \Bigg[\Bigg[\frac{1}{2}\ \left(3\ 2^{1/3} + I\ 2^{1/3}\sqrt{3}\right) t, -\frac{1}{2}\ \left(\right.\right.$$

$$\left.\left.-3\ 2^{1/3} + I\ 2^{1/3}\sqrt{3}\right) t, -I\sqrt{3}\ t\ 2^{1/3}\right], \left[-\frac{2}{9}\ \left(3\ 18^{2/3} + I\ 18^{2/3}\sqrt{3}\right) t, \frac{2}{9}\ \left(-3\ 18^{2/3}\right.\right.$$

$$\left.\left. + I\ 18^{2/3}\sqrt{3}\right) t, \frac{4}{9}\ I\sqrt{3}\ t\ 18^{2/3}\right], [15\ \left(RootOf\left(66 + \_Z^3,\ index = 2\right) - RootOf\left(66\right.\right.$$

$$\left.\left. + \_Z^3,\ index = 1\right)\right)\ t, 15\ \left(RootOf\left(66 + \_Z^3,\ index = 3\right) - RootOf\left(66 + \_Z^3,\ index\right.\right.$$

$$\left.\left. = 1\right)\right)\ t, 15\ \left(RootOf\left(66 + \_Z^3,\ index = 3\right) - RootOf\left(66 + \_Z^3,\ index = 2\right)\right)\ t]], [[0, 0,$$

$$0], [0, 0, 0], [0, 0, 0]], \Bigg[[[x - 1, 1], [x - 7, 7], [x - 3, 3]], \Bigg[\Bigg[\Bigg[0, 0, \frac{6}{7}\Bigg], \left[0, \frac{6}{7}, \frac{6}{7}\right],$$

$$[1, 1, 1], \left[[0, 0], \left[\frac{6}{7}, 0\right], \left[\frac{6}{7}, 0\right]\right], 3\Bigg], \Bigg[\Bigg[0, 0, \frac{18}{7}\Bigg], \left[0, \frac{18}{7}, \frac{18}{7}\right], [1, 1, 1], \left[[0, 0],\right.$$

$$\left.\left[\frac{18}{7}, 0\right], \left[\frac{18}{7}, 0\right]\right], 3\Bigg], \Bigg[\Bigg[0, 0, \frac{6}{7}\Bigg], \left[0, \frac{6}{7}, \frac{6}{7}\right], [1, 1, 1], \left[[0, 0], \left[\frac{6}{7}, 0\right], \left[\frac{6}{7}, 0\right]\right], 3\Bigg]\Bigg]\Bigg]$$

```
]
> Sreg:=regsingtrue0F2(L,t,Sirr[-1],ext);
```

$$Sreg := \left[[[x-1,1],[x-7,7],[x-3,3]], \left[\left[0,0,\frac{6}{7}\right],\left[0,0,\frac{18}{7}\right],\left[0,0,\frac{6}{7}\right]\right], \left[\left[0,\frac{6}{7},\frac{6}{7}\right],\left[0,\frac{18}{7},\frac{18}{7}\right],\left[0,\frac{6}{7},\frac{6}{7}\right]\right], \left[\left[[0,0],\left[\frac{6}{7},0\right],\left[\frac{6}{7},0\right]\right],\left[[0,0],\left[\frac{18}{7},0\right],\left[\frac{18}{7},0\right]\right],\left[[0,0],\left[\frac{6}{7},0\right],\left[\frac{6}{7},0\right]\right]\right]\right] \quad (104)$$

```
> RSreg:=Sregseptrue0F2(L,Sreg,ext);
```

$$RSreg := \left[[\,], [\,], \left[[[x-1,1],[x-7,7],[x-3,3]], \left[\left[0,0,\frac{6}{7}\right],\left[0,0,\frac{18}{7}\right],\left[0,0,\frac{6}{7}\right]\right], \left[\left[\left[\frac{6}{7},\frac{6}{7}\right],[0]\right],\left[\left[\frac{18}{7},\frac{18}{7}\right],[0]\right],\left[\left[\frac{6}{7},\frac{6}{7}\right],[0]\right]\right]\right]\right] \quad (105)$$

```
> R1:=IrrRegAppsing0F2(L,t,E,ext);
```

$$R1 := \left[\left[[[x-9,9],[x-12,12],[\infty,\infty]], \left[\left[\frac{8\,2^{1/3}}{t^{1/3}} + \frac{1}{3} + \frac{RootOf(\_Z^2+1)}{3}, \frac{16\,2^{1/3}}{(I\sqrt{3}-1)\,t^{1/3}} + \frac{1}{3} + \frac{RootOf(\_Z^2+1)}{3}, -\frac{16\,2^{1/3}}{(I\sqrt{3}+1)\,t^{1/3}} + \frac{1}{3} + \frac{RootOf(\_Z^2+1)}{3} \right], \left[-\frac{1980^{2/3}}{6\,t^{1/3}} + \frac{1}{3} + \frac{RootOf(\_Z^2+1)}{3}, -\frac{1980^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}} + \frac{1}{3} + \frac{RootOf(\_Z^2+1)}{3}, \frac{1980^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}} + \frac{1}{3} + \frac{RootOf(\_Z^2+1)}{3} \right], \left[\frac{3\,RootOf(\_Z^3+2,index=1)}{t} + 1 + RootOf(\_Z^2+1), \frac{3\,RootOf(\_Z^3+2,index=2)}{t} + 1 + RootOf(\_Z^2+1), \frac{3\,RootOf(\_Z^3+2,index=3)}{t} + 1 + RootOf(\_Z^2+1) \right] \right], \left[\left[\frac{16\,2^{1/3}}{(I\sqrt{3}-1)\,t^{1/3}} - \frac{8\,2^{1/3}}{t^{1/3}}, -\frac{16\,2^{1/3}}{(I\sqrt{3}+1)\,t^{1/3}} - \frac{8\,2^{1/3}}{t^{1/3}}, -\frac{16\,2^{1/3}}{(I\sqrt{3}+1)\,t^{1/3}} - \frac{16\,2^{1/3}}{(I\sqrt{3}-1)\,t^{1/3}} \right], \left[-\frac{1980^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}} + \frac{1980^{2/3}}{6\,t^{1/3}}, \frac{1980^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}} + \frac{1980^{2/3}}{6\,t^{1/3}}, \frac{1980^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}} + \frac{1980^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}} \right], \left[\frac{3\,RootOf(\_Z^3+2,index=2)}{t} - \frac{3\,RootOf(\_Z^3+2,index=1)}{t}, \frac{3\,RootOf(\_Z^3+2,index=3)}{t} - \frac{3\,RootOf(\_Z^3+2,index=1)}{t}, \right. \right. \quad (106)$$

$$\frac{3\,RootOf\left(\_Z^3+2,\,index=3\right)}{t}\;\_\;\frac{3\,RootOf\left(\_Z^3+2,\,index=2\right)}{t}\;\Bigg]\Bigg],\left[\frac{1}{3},\frac{1}{3},1\right],[3,3,$$

$$1],\left[\left[\left[\frac{16\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3},\frac{8\,2^{1/3}}{t^{1/3}}+\frac{1}{3}\right.\right.\right.$$

$$\left.+\frac{RootOf\left(\_Z^2+1\right)}{3}\right],\left[-\frac{16\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3},\frac{8\,2^{1/3}}{t^{1/3}}+\frac{1}{3}\right.$$

$$\left.+\frac{RootOf\left(\_Z^2+1\right)}{3}\right],\left[-\frac{16\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3},\right.$$

$$\left.\left.\frac{16\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3}\right]\right],\left[\left[-\frac{1980^{2/3}}{3\left(I\sqrt{3}-1\right)t^{1/3}}+\frac{1}{3}\right.\right.$$

$$\left.+\frac{RootOf\left(\_Z^2+1\right)}{3},-\frac{1980^{2/3}}{6\,t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3}\right],\left[\frac{1980^{2/3}}{3\left(I\sqrt{3}+1\right)t^{1/3}}\right.$$

$$\left.+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3},-\frac{1980^{2/3}}{6\,t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3}\right],$$

$$\left[\frac{1980^{2/3}}{3\left(I\sqrt{3}+1\right)t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3},-\frac{1980^{2/3}}{3\left(I\sqrt{3}-1\right)t^{1/3}}+\frac{1}{3}\right.$$

$$\left.+\frac{RootOf\left(\_Z^2+1\right)}{3}\right]\Bigg],\left[\left[\frac{3\,RootOf\left(\_Z^3+2,\,index=2\right)}{t}+1+RootOf\left(\_Z^2+1\right),\right.\right.$$

$$\left.\frac{3\,RootOf\left(\_Z^3+2,\,index=1\right)}{t}+1+RootOf\left(\_Z^2+1\right)\right],\left[\frac{3\,RootOf\left(\_Z^3+2,\,index=3\right)}{t}\right.$$

$$\left.+1+RootOf\left(\_Z^2+1\right),\frac{3\,RootOf\left(\_Z^3+2,\,index=1\right)}{t}+1+RootOf\left(\_Z^2+1\right)\right],$$

$$\left[\frac{3\,RootOf\left(\_Z^3+2,\,index=3\right)}{t}+1+RootOf\left(\_Z^2+1\right),\frac{3\,RootOf\left(\_Z^3+2,\,index=2\right)}{t}\right.$$

$$\left.\left.+1+RootOf\left(\_Z^2+1\right)\right]\right]\Bigg],\left[\left[-4\left(I\sqrt{3}\;2^{1/3}+3\,2^{1/3}\right)t,4\left(I\sqrt{3}\;2^{1/3}-3\,2^{1/3}\right)t,\right.\right.$$

$$\left.8\,I\,2^{1/3}\sqrt{3}\;t\right],\left[\frac{\left(3\,1980^{2/3}+I\sqrt{3}\;1980^{2/3}\right)t}{12},-\frac{\left(I\sqrt{3}\;1980^{2/3}-3\,1980^{2/3}\right)t}{12},\right.$$

$$\left.-\frac{I}{6}\;1980^{2/3}\sqrt{3}\;t\right],[3\left(RootOf\left(\_Z^3+2,\,index=2\right)-RootOf\left(\_Z^3+2,\,index=1\right)\right)t,$$

$$3\left(RootOf\left(\_Z^3+2,\,index=3\right)-RootOf\left(\_Z^3+2,\,index=1\right)\right)t,3\left(RootOf\left(\_Z^3+2,\right.\right.$$

$$\left.\left.index=3\right)-RootOf\left(\_Z^3+2,\,index=2\right)\right)t]],[[0,0,0],[0,0,0],[0,0,0]]],[[[x-1,$$

$$1], [x-3, 3], [x-7, 7]], [[0, 0, -2\,RootOf(\_Z^2+1)], [0, 0, -RootOf(\_Z^2+1)], [0, 0, -2\,RootOf(\_Z^2+1)]], [[0, -2\,RootOf(\_Z^2+1), -2\,RootOf(\_Z^2+1)], [0, -RootOf(\_Z^2+1), -RootOf(\_Z^2+1)], [0, -2\,RootOf(\_Z^2+1), -2\,RootOf(\_Z^2+1)]], [[[0, 0], [-2\,RootOf(\_Z^2+1), 0], [-2\,RootOf(\_Z^2+1), 0]], [[0, 0], [-RootOf(\_Z^2+1), 0], [-RootOf(\_Z^2+1), 0]], [[0, 0], [-2\,RootOf(\_Z^2+1), 0], [-2\,RootOf(\_Z^2+1), 0]]]], [[], []], [[[x-1, 1], [x-3, 3], [x-7, 7]], [[0, 0, -2\,RootOf(\_Z^2+1)], [0, 0, -RootOf(\_Z^2+1)], [0, 0, -2\,RootOf(\_Z^2+1)]], [[[-2\,RootOf(\_Z^2+1), -2\,RootOf(\_Z^2+1)], [0]], [[-RootOf(\_Z^2+1), -RootOf(\_Z^2+1)], [0]], [[-2\,RootOf(\_Z^2+1), -2\,RootOf(\_Z^2+1)], [0]]]], \left[\left[\left[x^4-\frac{98}{3}x^3+\frac{1058}{3}x^2-1450\,x+1833, RootOf(3\,\_Z^4-98\,\_Z^3+1058\,\_Z^2-4350\,\_Z+5499)\right]\right], [[0, 2, 4]], [[2, 4, 2]], [[[2, 0], [4, 0], [4, 2]]]\right], \left[[[x-1, 1], [x-3, 3], [x-9, 9], [x-12, 12], [\infty, \infty], [x-7, 7]], \left[[0, 0, -2\,RootOf(\_Z^2+1)], [0, 0, -RootOf(\_Z^2+1)], \left[\frac{8\,2^{1/3}}{t^{1/3}}+\frac{1}{3}+\frac{RootOf(\_Z^2+1)}{3}, \frac{16\,2^{1/3}}{(I\sqrt{3}-1)\,t^{1/3}}+\frac{1}{3}+\frac{RootOf(\_Z^2+1)}{3}, -\frac{16\,2^{1/3}}{(I\sqrt{3}+1)\,t^{1/3}}+\frac{1}{3}+\frac{RootOf(\_Z^2+1)}{3}\right], \left[-\frac{1980^{2/3}}{6\,t^{1/3}}+\frac{1}{3}+\frac{RootOf(\_Z^2+1)}{3}, -\frac{1980^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}}+\frac{1}{3}+\frac{RootOf(\_Z^2+1)}{3}, \frac{1980^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}}+\frac{1}{3}+\frac{RootOf(\_Z^2+1)}{3}\right], \left[\frac{3\,RootOf(\_Z^3+2, index=1)}{t}+1+RootOf(\_Z^2+1), \frac{3\,RootOf(\_Z^3+2, index=2)}{t}+1+RootOf(\_Z^2+1), \frac{3\,RootOf(\_Z^3+2, index=3)}{t}+1+RootOf(\_Z^2+1)\right], [0, 0, -2\,RootOf(\_Z^2+1)]\right],$$

$$\left[[0, -2\,\mathrm{RootOf}(\_Z^2+1), -2\,\mathrm{RootOf}(\_Z^2+1)], [0, -\mathrm{RootOf}(\_Z^2+1), -\mathrm{RootOf}(\_Z^2+1)], \left[\frac{16\,2^{1/3}}{(I\sqrt{3}-1)\,t^{1/3}} - \frac{8\,2^{1/3}}{t^{1/3}}, -\frac{16\,2^{1/3}}{(I\sqrt{3}+1)\,t^{1/3}} - \frac{8\,2^{1/3}}{t^{1/3}}, -\frac{16\,2^{1/3}}{(I\sqrt{3}+1)\,t^{1/3}} - \frac{16\,2^{1/3}}{(I\sqrt{3}-1)\,t^{1/3}}\right], \left[-\frac{1980^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}} + \frac{1980^{2/3}}{6\,t^{1/3}}, \frac{1980^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}} + \frac{1980^{2/3}}{6\,t^{1/3}}, \frac{1980^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}} + \frac{1980^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}}\right], \left[\frac{3\,\mathrm{RootOf}(\_Z^3+2, index=2)}{t} - \frac{3\,\mathrm{RootOf}(\_Z^3+2, index=1)}{t}, \frac{3\,\mathrm{RootOf}(\_Z^3+2, index=3)}{t} - \frac{3\,\mathrm{RootOf}(\_Z^3+2, index=1)}{t}, \frac{3\,\mathrm{RootOf}(\_Z^3+2, index=3)}{t} - \frac{3\,\mathrm{RootOf}(\_Z^3+2, index=2)}{t}\right], [0, -2\,\mathrm{RootOf}(\_Z^2+1), -2\,\mathrm{RootOf}(\_Z^2+1)]\right], \left[[[0,0],[-2\,\mathrm{RootOf}(\_Z^2+1),0],[-2\,\mathrm{RootOf}(\_Z^2+1),0]], [[0,0],[-\mathrm{RootOf}(\_Z^2+1),0],[-\mathrm{RootOf}(\_Z^2+1),0]], \left[\left[\frac{16\,2^{1/3}}{(I\sqrt{3}-1)\,t^{1/3}} + \frac{1}{3} + \frac{\mathrm{RootOf}(\_Z^2+1)}{3}, \frac{8\,2^{1/3}}{t^{1/3}} + \frac{1}{3} + \frac{\mathrm{RootOf}(\_Z^2+1)}{3}\right], \left[-\frac{16\,2^{1/3}}{(I\sqrt{3}+1)\,t^{1/3}} + \frac{1}{3} + \frac{\mathrm{RootOf}(\_Z^2+1)}{3}, \frac{8\,2^{1/3}}{t^{1/3}} + \frac{1}{3} + \frac{\mathrm{RootOf}(\_Z^2+1)}{3}\right], \left[-\frac{16\,2^{1/3}}{(I\sqrt{3}+1)\,t^{1/3}} + \frac{1}{3} + \frac{\mathrm{RootOf}(\_Z^2+1)}{3}, \frac{16\,2^{1/3}}{(I\sqrt{3}-1)\,t^{1/3}} + \frac{1}{3} + \frac{\mathrm{RootOf}(\_Z^2+1)}{3}\right]\right], \left[\left[-\frac{1980^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}} + \frac{1}{3} + \frac{\mathrm{RootOf}(\_Z^2+1)}{3}, -\frac{1980^{2/3}}{6\,t^{1/3}} + \frac{1}{3} + \frac{\mathrm{RootOf}(\_Z^2+1)}{3}\right], \left[\frac{1980^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}} + \frac{1}{3} + \frac{\mathrm{RootOf}(\_Z^2+1)}{3}, -\frac{1980^{2/3}}{6\,t^{1/3}} + \frac{1}{3} + \frac{\mathrm{RootOf}(\_Z^2+1)}{3}\right], \left[\frac{1980^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}} + \frac{1}{3} + \frac{\mathrm{RootOf}(\_Z^2+1)}{3}, -\frac{1980^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}} + \frac{1}{3} + \frac{\mathrm{RootOf}(\_Z^2+1)}{3}\right]\right], \left[\left[\frac{3\,\mathrm{RootOf}(\_Z^3+2, index=2)}{t} + 1 + \mathrm{RootOf}(\_Z^2+1), \frac{3\,\mathrm{RootOf}(\_Z^3+2, index=1)}{t} + 1 + \mathrm{RootOf}(\_Z^2+1)\right], \left[\frac{3\,\mathrm{RootOf}(\_Z^3+2, index=3)}{t}\right.$$

$$+1+RootOf\left(\_Z^2+1\right),\ \frac{3\ RootOf\left(\_Z^3+2,\ index=1\right)}{t}+1+RootOf\left(\_Z^2+1\right)\Bigg],$$

$$\left[\frac{3\ RootOf\left(\_Z^3+2,\ index=3\right)}{t}+1+RootOf\left(\_Z^2+1\right),\ \frac{3\ RootOf\left(\_Z^3+2,\ index=2\right)}{t}\right.$$

$$\left.+1+RootOf\left(\_Z^2+1\right)\right]\Bigg],\ [[0,0],\ [-2\ RootOf\left(\_Z^2+1\right),0],\ [-2\ RootOf\left(\_Z^2+1\right),$$

$$0]]\Bigg],\ [[1,1,1],[1,1,1],[3,3,3],[3,3,3],[1,1,1],[1,1,1]]\Bigg]$$

```
> F1:= Sirr0F2info1(L,R1[1],R1[2],x,t,ext);
```

$$F1 := \left[\left[\left[9,\ x-9,\ \left[-\frac{1024}{x-9}\right],\ 1,\ \varnothing,\ \{RootOf\left(\_Z^2+1\right)\}\right],\left[12,\ x-12,\ \left[\frac{18150}{x-12}\right],\ 1,\ \varnothing,\right.\right.\right. \tag{107}$$

$$\left.\left.\left.\{RootOf\left(\_Z^2+1\right)\}\right],\left[\infty,\ \frac{1}{x},\ [-2\ x^3],\ 3,\ \{RootOf\left(\_Z^3+2,\ index=1\right),\ RootOf\left(\_Z^3+2,\right.\right.\right.$$

$$\left.\left.\left.index=2\right)\},\ \{RootOf\left(\_Z^2+1\right),\ RootOf\left(\_Z^3+2,\ index=1\right),\ RootOf\left(\_Z^3+2,\ index\right.\right.\right.$$

$$\left.\left.\left.=2\right)\}\right]\right],\ 5,\ 6,\ (x-9)\ (x-12),\ 1\Bigg]$$

```
> find0F2ln(L,R1,F1,x,t,ext);
```

$$\left[\left[[[1,\ RootOf\left(\_Z^2+1\right)]],\ \frac{2\ (x-1)^2\ (x-3)\ (x-7)^2}{(x-9)\ (x-12)}\right],\left[[[1,\ RootOf\left(\_Z^2+1\right)]],\right.\right. \tag{108}$$

$$\left.\left.-\frac{2\ (x-1)^2\ (x-3)\ (x-7)^2}{(x-9)\ (x-12)}\right]\right]$$

```
> TIME := time();
  Hyp0F2Solutions(L);
  time()-TIME;
```

$$TIME := 40.046$$

$$\left\{\left[\left\{\left[\left[\left[1,\ RootOf\left(\_Z^2+1\right)\right],\left[-\frac{2}{x-7}\right],\left[\frac{9\ (x-3)\ (x-7)^2\ (x-9)^3\ (x-12)^3\ Dx^2}{\left(3\ x^4-98\ x^3+1058\ x^2-4350\ x+5499\right)^2}\right.\right.\right.\right.$$

$$+\left(9\ \left(15\ x^{13}-1623\ x^{12}+79349\ x^{11}-2318073\ x^{10}+45075498\ x^9-614907602\ x^8+6040382598\ x^7-43\right.\right.$$

$$+223351175871\ x^5-824867387271\ x^4+2104961242149\ x^3-3505189872969\ x^2$$

$$\left.\left.+3413344871016\ x-1477552421520\right)\ Dx\right)\Big/\left(\left(3\ x^4-98\ x^3+1058\ x^2-4350\ x\right.\right.$$

$$\left.\left.+5499\right)^2\ \left(3\ x^5-101\ x^4+1156\ x^3-5408\ x^2+9849\ x-5499\right)\right)+\left(18\ \left(6\ x^{12}\right.\right.$$

$$-621\ x^{11}+28733\ x^{10}-785302\ x^9+14104058\ x^8-175093272\ x^7+1537782840\ x^6$$

$$-9607222674\ x^5+42285234816\ x^4-127655542107\ x^3+250793603163\ x^2$$

$$\left.\left.-288655394040\ x+148734510768\right)\right)\Big/\left(\left(3\ x^4-98\ x^3+1058\ x^2-4350\ x\right.\right.$$

$$+5499)^2\,(3x^5-101x^4+1156x^3-5408x^2+9849x-5499))]]\},$$

$$\left.\left.\frac{2\,(x-1)^2\,(x-3)\,(x-7)^2}{(x-9)\,(x-12)}\right]\right\}$$

14.750 **(109)**

```
> F:=sumdiffeq(hyperterm([],[b1,b2],x,k),k,J(x));
```

$$F := \left(\frac{d^3}{dx^3}J(x)\right)x^2+(b1+b2+1)\left(\frac{d^2}{dx^2}J(x)\right)x+b1\,b2\left(\frac{d}{dx}J(x)\right)-J(x)=0 \quad \textbf{(110)}$$

```
> LA:=de2diffop(F,J(x));
```

$$LA := x^2\,Dx^3+(x\,b1+x\,b2+x)\,Dx^2+b2\,b1\,Dx-1 \quad \textbf{(111)}$$

```
> L12:=subs({b1=1,b2=1+RootOf(x^2+1)},LA);
```

$$L12 := x^2\,Dx^3+(2\,x+x\,(1+RootOf(\_Z^2+1)))\,Dx^2+(1+RootOf(\_Z^2+1))\,Dx-1 \quad \textbf{(112)}$$

```
> f:=(2*(x-1)^2*(x-3)*(x-7)^2)/((x-9)*(x-12));
```

$$f := \frac{2\,(x-1)^2\,(x-3)\,(x-7)^2}{(x-9)\,(x-12)} \quad \textbf{(113)}$$

```
> L:=ChangeOfVariables(L12,f);
```

$$L := Dx^3\,(x-1)^2\,(x-3)^2\,(x-7)^2\,(x-9)^4\,(x-12)^4\,(3x^4-98x^3+1058x^2-4350x \quad \textbf{(114)}$$
$$+5499)^2+(1+RootOf(\_Z^2+1))\,(9x^8+176\,RootOf(\_Z^2+1)\,x^6-588x^7$$
$$-5806\,RootOf(\_Z^2+1)\,x^5+15776x^6+72218\,RootOf(\_Z^2+1)\,x^4-227662x^5$$
$$-411684\,RootOf(\_Z^2+1)\,x^3+1932740x^4+1000188\,RootOf(\_Z^2+1)\,x^2$$
$$-9870720x^3-434862\,RootOf(\_Z^2+1)\,x+29558196x^2-1335366\,RootOf(\_Z^2+1)$$
$$-47406438x+31574367)\,(3x^4-98x^3+1058x^2-4350x+5499)\,(x-7)\,(x$$
$$-3)\,(x-1)\,(x-9)^3\,(x-12)^3\,Dx^2-\frac{1}{5}((1+3\,RootOf(\_Z^2+1))\,($$
$$-547157401526799+1640315296209240x+1877136788751552x^3-350016x^{14}$$
$$+12594984x^{13}-306602642x^{12}+5353573328x^{11}-69354920712x^{10}$$
$$+679945841960x^9-45x^{16}+5880x^{15}-5097833895956x^8+29323504806456x^7$$
$$-128971487877984x^6-2253196066142232x^2-17123543914158\,RootOf(\_Z^2+1)$$
$$-1059559724735838x^4+429181878089880x^5+528\,RootOf(\_Z^2+1)\,x^{14}$$
$$-34132\,RootOf(\_Z^2+1)\,x^{13}+856206\,RootOf(\_Z^2+1)\,x^{12}-7848464\,RootOf(\_Z^2$$
$$+1)\,x^{11}-89708864\,RootOf(\_Z^2+1)\,x^{10}+3652659780\,RootOf(\_Z^2+1)\,x^9$$
$$-53915229562\,RootOf(\_Z^2+1)\,x^8+488313786912\,RootOf(\_Z^2+1)\,x^7$$
$$-3003412628448\,RootOf(\_Z^2+1)\,x^6+12913918449300\,RootOf(\_Z^2+1)\,x^5$$
$$-38725548708006\,RootOf(\_Z^2+1)\,x^4+78674056953264\,RootOf(\_Z^2+1)\,x^3$$
$$-101103717757104\,RootOf(\_Z^2+1)\,x^2+70417349719740\,RootOf(\_Z^2+1)\,x)\,(x$$
$$-9)^2\,(x-12)^2\,Dx)-2\,(3x^4-98x^3+1058x^2-4350x+5499)^5\,(x-1)\,(x-7)$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \{RootOf(\_Z^2+1)\} \tag{115}$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```

$$ext := \{RootOf(\_Z^2+1)\} \tag{116}$$

```
> extppp:={};
```

$$extppp := \varnothing \tag{117}$$

```
> E:= Singular(L,extppp);
```

$$E := \left[\left[x^4 - \frac{98}{3}x^3 + \frac{1058}{3}x^2 - 1450\,x + 1833,\ RootOf(3\,\_Z^4 - 98\,\_Z^3 + 1058\,\_Z^2 - 4350\,\_Z + 5499)\right], [x-1,1], [x-12,12], [\infty,\infty], [x-3,3], [x-9,9], [x-7,7]\right] \tag{118}$$

```
> F:=NotAppSing(L,E,ext);
```

$$F := [[x-1,1], [x-3,3], [x-9,9], [x-12,12], [\infty,\infty], [x-7,7]] \tag{119}$$

```
> Sirr:=irrsing0F2(L,t,F,ext);
```

$$Sirr := \left[[[x-9,9], [x-12,12], [\infty,\infty]], \left[\left[\frac{8\,2^{1/3}}{t^{1/3}} + \frac{1}{3} + \frac{RootOf(\_Z^2+1)}{3},\ \frac{16\,2^{1/3}}{(I\sqrt{3}-1)\,t^{1/3}} + \frac{1}{3} + \frac{RootOf(\_Z^2+1)}{3},\ -\frac{16\,2^{1/3}}{(I\sqrt{3}+1)\,t^{1/3}} + \frac{1}{3} + \frac{RootOf(\_Z^2+1)}{3}\right], \left[-\frac{1980^{2/3}}{6\,t^{1/3}} + \frac{1}{3} + \frac{RootOf(\_Z^2+1)}{3},\ -\frac{1980^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}} + \frac{1}{3} + \frac{RootOf(\_Z^2+1)}{3},\ \frac{1980^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}} + \frac{1}{3} + \frac{RootOf(\_Z^2+1)}{3}\right], \left[\frac{3\,RootOf(\_Z^3+2, index=1)}{t} + 1 + RootOf(\_Z^2+1),\ \frac{3\,RootOf(\_Z^3+2, index=2)}{t} + 1 + RootOf(\_Z^2+1),\ \frac{3\,RootOf(\_Z^3+2, index=3)}{t} + 1 + RootOf(\_Z^2+1)\right]\right], \left[\left[\frac{16\,2^{1/3}}{(I\sqrt{3}-1)\,t^{1/3}} - \frac{8\,2^{1/3}}{t^{1/3}},\ -\frac{16\,2^{1/3}}{(I\sqrt{3}+1)\,t^{1/3}} - \frac{8\,2^{1/3}}{t^{1/3}},\ -\frac{16\,2^{1/3}}{(I\sqrt{3}+1)\,t^{1/3}} - \frac{16\,2^{1/3}}{(I\sqrt{3}-1)\,t^{1/3}}\right], \left[-\frac{1980^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}} + \frac{1980^{2/3}}{6\,t^{1/3}},\ \frac{1980^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}} + \frac{1980^{2/3}}{6\,t^{1/3}},\ \frac{1980^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}} + \frac{1980^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}}\right], \left[\frac{3\,RootOf(\_Z^3+2, index=2)}{t} - \frac{3\,RootOf(\_Z^3+2, index=1)}{t},\ \frac{3\,RootOf(\_Z^3+2, index=3)}{t} - \frac{3\,RootOf(\_Z^3+2, index=1)}{t},\ \frac{3\,RootOf(\_Z^3+2, index=3)}{t} - \frac{3\,RootOf(\_Z^3+2, index=2)}{t}\right]\right], \left[\frac{1}{3}, \frac{1}{3}, 1\right], [3, 3,\right.$$

$$1], \left[\left[\left[\frac{16\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3},\ \frac{8\,2^{1/3}}{t^{1/3}}+\frac{1}{3}\right.\right.\right.$$
$$\left.+\frac{RootOf\left(\_Z^2+1\right)}{3}\right],\left[-\frac{16\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3},\ \frac{8\,2^{1/3}}{t^{1/3}}+\frac{1}{3}\right.$$
$$\left.+\frac{RootOf\left(\_Z^2+1\right)}{3}\right],\left[-\frac{16\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3},\right.$$
$$\left.\left.\frac{16\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3}\right]\right],\left[\left[-\frac{1980^{2/3}}{3\left(I\sqrt{3}-1\right)t^{1/3}}+\frac{1}{3}\right.\right.$$
$$\left.+\frac{RootOf\left(\_Z^2+1\right)}{3},-\frac{1980^{2/3}}{6\,t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3}\right],\left[\frac{1980^{2/3}}{3\left(I\sqrt{3}+1\right)t^{1/3}}\right.$$
$$\left.+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3},-\frac{1980^{2/3}}{6\,t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3}\right],$$
$$\left[\frac{1980^{2/3}}{3\left(I\sqrt{3}+1\right)t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3},-\frac{1980^{2/3}}{3\left(I\sqrt{3}-1\right)t^{1/3}}+\frac{1}{3}\right.$$
$$\left.\left.+\frac{RootOf\left(\_Z^2+1\right)}{3}\right]\right],\left[\left[\frac{3\,RootOf\left(\_Z^3+2,\,index=2\right)}{t}+1+RootOf\left(\_Z^2+1\right),\right.\right.$$
$$\left.\frac{3\,RootOf\left(\_Z^3+2,\,index=1\right)}{t}+1+RootOf\left(\_Z^2+1\right)\right],\left[\frac{3\,RootOf\left(\_Z^3+2,\,index=3\right)}{t}\right.$$
$$\left.+1+RootOf\left(\_Z^2+1\right),\frac{3\,RootOf\left(\_Z^3+2,\,index=1\right)}{t}+1+RootOf\left(\_Z^2+1\right)\right],$$
$$\left[\frac{3\,RootOf\left(\_Z^3+2,\,index=3\right)}{t}+1+RootOf\left(\_Z^2+1\right),\frac{3\,RootOf\left(\_Z^3+2,\,index=2\right)}{t}\right.$$
$$\left.\left.+1+RootOf\left(\_Z^2+1\right)\right]\right]\right],\left[\left[-4\left(I\sqrt{3}\;2^{1/3}+3\,2^{1/3}\right)t,\,4\left(I\sqrt{3}\;2^{1/3}-3\,2^{1/3}\right)t,\right.\right.$$
$$\left.8\,I\,2^{1/3}\sqrt{3}\;t\right],\left[\frac{\left(3\,1980^{2/3}+I\sqrt{3}\;1980^{2/3}\right)t}{12},-\frac{\left(I\sqrt{3}\;1980^{2/3}-3\,1980^{2/3}\right)t}{12},\right.$$
$$\left.-\frac{I}{6}\;1980^{2/3}\sqrt{3}\;t\right],\left[3\left(RootOf\left(\_Z^3+2,\,index=2\right)-RootOf\left(\_Z^3+2,\,index=1\right)\right)t,\right.$$
$$3\left(RootOf\left(\_Z^3+2,\,index=3\right)-RootOf\left(\_Z^3+2,\,index=1\right)\right)t,3\left(RootOf\left(\_Z^3+2,\right.\right.$$
$$\left.\left.index=3\right)-RootOf\left(\_Z^3+2,\,index=2\right)\right)t\left]\right],\left[\left[0,0,0\right],\left[0,0,0\right],\left[0,0,0\right]\right],\left[\left[x-1,\right.\right.$$
$$\left.1\right],\left[x-3,3\right],\left[x-7,7\right]\left]\right],\left[\left[\left[0,0,-2\,RootOf\left(\_Z^2+1\right)\right],\left[0,-2\,RootOf\left(\_Z^2+1\right),\right.\right.\right.$$
$$\left.-2\,RootOf\left(\_Z^2+1\right)\right],\left[1,1,1\right],\left[\left[0,0\right],\left[-2\,RootOf\left(\_Z^2+1\right),0\right],\left[-2\,RootOf\left(\_Z^2\right.\right.\right.$$
$$\left.\left.+1\right),0\right]\left]\right],3\left]\right],\left[\left[0,0,-RootOf\left(\_Z^2+1\right)\right],\left[0,-RootOf\left(\_Z^2+1\right),-RootOf\left(\_Z^2+1\right)\right],\right.$$
$$\left[1,1,1\right],\left[\left[0,0\right],\left[-RootOf\left(\_Z^2+1\right),0\right],\left[-RootOf\left(\_Z^2+1\right),0\right]\right],3\left]\right],\left[\left[0,0,\right.\right.$$
$$\left.-2\,RootOf\left(\_Z^2+1\right)\right],\left[0,-2\,RootOf\left(\_Z^2+1\right),-2\,RootOf\left(\_Z^2+1\right)\right],\left[1,1,1\right],\left[\left[0,\right.\right.$$
$$\left.\left.0\right],\left[-2\,RootOf\left(\_Z^2+1\right),0\right],\left[-2\,RootOf\left(\_Z^2+1\right),0\right]\left]\right],3\left]\right]\right]\right]$$

```
> Sreg:=regsingtrue0F2(L,t,Sirr[-1],ext);
```

$$Sreg := [[[x-1,1],[x-3,3],[x-7,7]],[[0,0,-2\,RootOf(\_Z^2+1)],[0,0,-RootOf(\_Z^2+1)],[0,0,-2\,RootOf(\_Z^2+1)]],[[0,-2\,RootOf(\_Z^2+1),-2\,RootOf(\_Z^2+1)],[0,-RootOf(\_Z^2+1),-RootOf(\_Z^2+1)],[0,-2\,RootOf(\_Z^2+1),-2\,RootOf(\_Z^2+1)]],[[[0,0],[-2\,RootOf(\_Z^2+1),0],[-2\,RootOf(\_Z^2+1),0]],[[0,0],[-RootOf(\_Z^2+1),0],[-RootOf(\_Z^2+1),0]],[[0,0],[-2\,RootOf(\_Z^2+1),0],[-2\,RootOf(\_Z^2+1),0]]]] \quad (121)$$

```
> RSreg:=Sregseptrue0F2(L,Sreg,ext);
```

$$RSreg := [[\,],[\,],[[[x-1,1],[x-3,3],[x-7,7]],[[0,0,-2\,RootOf(\_Z^2+1)],[0,0,-RootOf(\_Z^2+1)],[0,0,-2\,RootOf(\_Z^2+1)]],[[[-2\,RootOf(\_Z^2+1),-2\,RootOf(\_Z^2+1)],[0]],[[-RootOf(\_Z^2+1),-RootOf(\_Z^2+1)],[0]],[[-2\,RootOf(\_Z^2+1),-2\,RootOf(\_Z^2+1)],[0]]]] \quad (122)$$

```
> R1:=IrrRegAppsing0F2(L,t,E,ext);
```

$$R1 := \left[\left[[[x-9,9],[x-12,12],[\infty,\infty]],\left[\left[\frac{8\,2^{1/3}}{t^{1/3}}+\frac{1}{3}+\frac{RootOf(\_Z^2+1)}{3},\frac{16\,2^{1/3}}{(I\sqrt{3}-1)\,t^{1/3}}+\frac{1}{3}+\frac{RootOf(\_Z^2+1)}{3},-\frac{16\,2^{1/3}}{(I\sqrt{3}+1)\,t^{1/3}}+\frac{1}{3}+\frac{RootOf(\_Z^2+1)}{3}\right],\left[-\frac{1980^{2/3}}{6\,t^{1/3}}+\frac{1}{3}+\frac{RootOf(\_Z^2+1)}{3},-\frac{1980^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}}+\frac{1}{3}+\frac{RootOf(\_Z^2+1)}{3},\frac{1980^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}}+\frac{1}{3}+\frac{RootOf(\_Z^2+1)}{3}\right],\left[\frac{3\,RootOf(\_Z^3+2,index=1)}{t}+1+RootOf(\_Z^2+1),\frac{3\,RootOf(\_Z^3+2,index=2)}{t}+1+RootOf(\_Z^2+1),\frac{3\,RootOf(\_Z^3+2,index=3)}{t}+1+RootOf(\_Z^2+1)\right]\right],\left[\left[\frac{16\,2^{1/3}}{(I\sqrt{3}-1)\,t^{1/3}}-\frac{8\,2^{1/3}}{t^{1/3}},-\frac{16\,2^{1/3}}{(I\sqrt{3}+1)\,t^{1/3}}-\frac{8\,2^{1/3}}{t^{1/3}},-\frac{16\,2^{1/3}}{(I\sqrt{3}+1)\,t^{1/3}}-\frac{16\,2^{1/3}}{(I\sqrt{3}-1)\,t^{1/3}}\right],\left[-\frac{1980^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}}+\frac{1980^{2/3}}{6\,t^{1/3}},\frac{1980^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}}+\frac{1980^{2/3}}{6\,t^{1/3}},\frac{1980^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}}+\frac{1980^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}}\right],\left[\frac{3\,RootOf(\_Z^3+2,index=2)}{t}-\frac{3\,RootOf(\_Z^3+2,index=1)}{t},\frac{3\,RootOf(\_Z^3+2,index=3)}{t}-\frac{3\,RootOf(\_Z^3+2,index=1)}{t},\right. \quad (123)$$

$$\frac{3\,RootOf\left(\_Z^3+2,\,index=3\right)}{t}\;\_\;\frac{3\,RootOf\left(\_Z^3+2,\,index=2\right)}{t}\;\Bigg]\Bigg],\left[\frac{1}{3},\frac{1}{3},1\right],[3,3,$$

$$1],\left[\left[\left[\frac{16\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3},\frac{8\,2^{1/3}}{t^{1/3}}+\frac{1}{3}\right.\right.\right.$$

$$\left.+\frac{RootOf\left(\_Z^2+1\right)}{3}\right],\left[-\frac{16\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3},\frac{8\,2^{1/3}}{t^{1/3}}+\frac{1}{3}\right.$$

$$\left.+\frac{RootOf\left(\_Z^2+1\right)}{3}\right],\left[-\frac{16\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3},\right.$$

$$\left.\left.\frac{16\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3}\right]\right],\left[\left[-\frac{1980^{2/3}}{3\left(I\sqrt{3}-1\right)t^{1/3}}+\frac{1}{3}\right.\right.$$

$$\left.+\frac{RootOf\left(\_Z^2+1\right)}{3},-\frac{1980^{2/3}}{6\,t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3}\right],\left[\frac{1980^{2/3}}{3\left(I\sqrt{3}+1\right)t^{1/3}}\right.$$

$$\left.+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3},-\frac{1980^{2/3}}{6\,t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3}\right],$$

$$\left[\frac{1980^{2/3}}{3\left(I\sqrt{3}+1\right)t^{1/3}}+\frac{1}{3}+\frac{RootOf\left(\_Z^2+1\right)}{3},-\frac{1980^{2/3}}{3\left(I\sqrt{3}-1\right)t^{1/3}}+\frac{1}{3}\right.$$

$$\left.+\frac{RootOf\left(\_Z^2+1\right)}{3}\right]\Bigg],\left[\left[\frac{3\,RootOf\left(\_Z^3+2,\,index=2\right)}{t}+1+RootOf\left(\_Z^2+1\right),\right.\right.$$

$$\left.\frac{3\,RootOf\left(\_Z^3+2,\,index=1\right)}{t}+1+RootOf\left(\_Z^2+1\right)\right],\left[\frac{3\,RootOf\left(\_Z^3+2,\,index=3\right)}{t}\right.$$

$$\left.+1+RootOf\left(\_Z^2+1\right),\frac{3\,RootOf\left(\_Z^3+2,\,index=1\right)}{t}+1+RootOf\left(\_Z^2+1\right)\right],$$

$$\left[\frac{3\,RootOf\left(\_Z^3+2,\,index=3\right)}{t}+1+RootOf\left(\_Z^2+1\right),\frac{3\,RootOf\left(\_Z^3+2,\,index=2\right)}{t}\right.$$

$$\left.\left.+1+RootOf\left(\_Z^2+1\right)\right]\right]\Bigg],\left[\left[-4\left(I\sqrt{3}\;2^{1/3}+3\,2^{1/3}\right)t,4\left(I\sqrt{3}\;2^{1/3}-3\,2^{1/3}\right)t,\right.\right.$$

$$\left.8\,I\,2^{1/3}\sqrt{3}\;t\right],\left[\frac{\left(3\,1980^{2/3}+I\sqrt{3}\;1980^{2/3}\right)t}{12},-\frac{\left(I\sqrt{3}\;1980^{2/3}-3\,1980^{2/3}\right)t}{12},\right.$$

$$\left.-\frac{I}{6}\;1980^{2/3}\sqrt{3}\;t\right],[3\left(RootOf\left(\_Z^3+2,\,index=2\right)-RootOf\left(\_Z^3+2,\,index=1\right)\right)t,$$

$$3\left(RootOf\left(\_Z^3+2,\,index=3\right)-RootOf\left(\_Z^3+2,\,index=1\right)\right)t,3\left(RootOf\left(\_Z^3+2,\right.\right.$$

$$\left.\left.index=3\right)-RootOf\left(\_Z^3+2,\,index=2\right)\right)t]],[[0,0,0],[0,0,0],[0,0,0]]],[[[x-1,$$

$$1], [x-3, 3], [x-7, 7]], [[0, 0, -2\,RootOf(\_Z^2+1)], [0, 0, -RootOf(\_Z^2+1)], [0, 0, -2\,RootOf(\_Z^2+1)]], [[0, -2\,RootOf(\_Z^2+1), -2\,RootOf(\_Z^2+1)], [0, -RootOf(\_Z^2+1), -RootOf(\_Z^2+1)], [0, -2\,RootOf(\_Z^2+1), -2\,RootOf(\_Z^2+1)]], [[[0, 0], [-2\,RootOf(\_Z^2+1), 0], [-2\,RootOf(\_Z^2+1), 0]], [[0, 0], [-RootOf(\_Z^2+1), 0], [-RootOf(\_Z^2+1), 0]], [[0, 0], [-2\,RootOf(\_Z^2+1), 0], [-2\,RootOf(\_Z^2+1), 0]]]], [[], []], [[[x-1, 1], [x-3, 3], [x-7, 7]], [[0, 0, -2\,RootOf(\_Z^2+1)], [0, 0, -RootOf(\_Z^2+1)], [0, 0, -2\,RootOf(\_Z^2+1)]], [[[-2\,RootOf(\_Z^2+1), -2\,RootOf(\_Z^2+1)], [0]], [[-RootOf(\_Z^2+1), -RootOf(\_Z^2+1)], [0]], [[-2\,RootOf(\_Z^2+1), -2\,RootOf(\_Z^2+1)], [0]]]], \left[\left[\left[x^4-\frac{98}{3}x^3+\frac{1058}{3}x^2-1450\,x+1833, RootOf(3\,\_Z^4-98\,\_Z^3+1058\,\_Z^2-4350\,\_Z+5499)\right]\right], [[0, 2, 4]], [[2, 4, 2]], [[[2, 0], [4, 0], [4, 2]]]\right], \left[[[x-1, 1], [x-3, 3], [x-9, 9], [x-12, 12], [\infty, \infty], [x-7, 7]], \left[[0, 0, -2\,RootOf(\_Z^2+1)], [0, 0, -RootOf(\_Z^2+1)], \left[\frac{8\,2^{1/3}}{t^{1/3}}+\frac{1}{3}+\frac{RootOf(\_Z^2+1)}{3}, \frac{16\,2^{1/3}}{(I\sqrt{3}-1)\,t^{1/3}}+\frac{1}{3}+\frac{RootOf(\_Z^2+1)}{3}, -\frac{16\,2^{1/3}}{(I\sqrt{3}+1)\,t^{1/3}}+\frac{1}{3}+\frac{RootOf(\_Z^2+1)}{3}\right], \left[-\frac{1980^{2/3}}{6\,t^{1/3}}+\frac{1}{3}+\frac{RootOf(\_Z^2+1)}{3}, -\frac{1980^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}}+\frac{1}{3}+\frac{RootOf(\_Z^2+1)}{3}, \frac{1980^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}}+\frac{1}{3}+\frac{RootOf(\_Z^2+1)}{3}\right], \left[\frac{3\,RootOf(\_Z^3+2, index=1)}{t}+1+RootOf(\_Z^2+1), \frac{3\,RootOf(\_Z^3+2, index=2)}{t}+1+RootOf(\_Z^2+1), \frac{3\,RootOf(\_Z^3+2, index=3)}{t}+1+RootOf(\_Z^2+1)\right], [0, 0, -2\,RootOf(\_Z^2+1)]\right],$$

$$\left[[0, -2\,\mathrm{RootOf}(\_Z^2+1), -2\,\mathrm{RootOf}(\_Z^2+1)], [0, -\mathrm{RootOf}(\_Z^2+1), -\mathrm{RootOf}(\_Z^2+1)], \left[\frac{16\,2^{1/3}}{(I\sqrt{3}-1)\,t^{1/3}} - \frac{8\,2^{1/3}}{t^{1/3}}, -\frac{16\,2^{1/3}}{(I\sqrt{3}+1)\,t^{1/3}} - \frac{8\,2^{1/3}}{t^{1/3}}, -\frac{16\,2^{1/3}}{(I\sqrt{3}+1)\,t^{1/3}} - \frac{16\,2^{1/3}}{(I\sqrt{3}-1)\,t^{1/3}}\right], \left[-\frac{1980^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}} + \frac{1980^{2/3}}{6\,t^{1/3}}, \frac{1980^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}} + \frac{1980^{2/3}}{6\,t^{1/3}}, \frac{1980^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}} + \frac{1980^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}}\right], \left[\frac{3\,\mathrm{RootOf}(\_Z^3+2, index=2)}{t} - \frac{3\,\mathrm{RootOf}(\_Z^3+2, index=1)}{t}, \frac{3\,\mathrm{RootOf}(\_Z^3+2, index=3)}{t} - \frac{3\,\mathrm{RootOf}(\_Z^3+2, index=1)}{t}, \frac{3\,\mathrm{RootOf}(\_Z^3+2, index=3)}{t} - \frac{3\,\mathrm{RootOf}(\_Z^3+2, index=2)}{t}\right], [0, -2\,\mathrm{RootOf}(\_Z^2+1), -2\,\mathrm{RootOf}(\_Z^2+1)]\right], \left[[[0,0],[-2\,\mathrm{RootOf}(\_Z^2+1),0],[-2\,\mathrm{RootOf}(\_Z^2+1),0]], [[0,0],[-\mathrm{RootOf}(\_Z^2+1),0],[-\mathrm{RootOf}(\_Z^2+1),0]], \left[\left[\frac{16\,2^{1/3}}{(I\sqrt{3}-1)\,t^{1/3}} + \frac{1}{3} + \frac{\mathrm{RootOf}(\_Z^2+1)}{3}, \frac{8\,2^{1/3}}{t^{1/3}} + \frac{1}{3} + \frac{\mathrm{RootOf}(\_Z^2+1)}{3}\right], \left[-\frac{16\,2^{1/3}}{(I\sqrt{3}+1)\,t^{1/3}} + \frac{1}{3} + \frac{\mathrm{RootOf}(\_Z^2+1)}{3}, \frac{8\,2^{1/3}}{t^{1/3}} + \frac{1}{3} + \frac{\mathrm{RootOf}(\_Z^2+1)}{3}\right], \left[-\frac{16\,2^{1/3}}{(I\sqrt{3}+1)\,t^{1/3}} + \frac{1}{3} + \frac{\mathrm{RootOf}(\_Z^2+1)}{3}, \frac{16\,2^{1/3}}{(I\sqrt{3}-1)\,t^{1/3}} + \frac{1}{3} + \frac{\mathrm{RootOf}(\_Z^2+1)}{3}\right]\right], \left[\left[-\frac{1980^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}} + \frac{1}{3} + \frac{\mathrm{RootOf}(\_Z^2+1)}{3}, -\frac{1980^{2/3}}{6\,t^{1/3}} + \frac{1}{3} + \frac{\mathrm{RootOf}(\_Z^2+1)}{3}\right], \left[\frac{1980^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}} + \frac{1}{3} + \frac{\mathrm{RootOf}(\_Z^2+1)}{3}, -\frac{1980^{2/3}}{6\,t^{1/3}} + \frac{1}{3} + \frac{\mathrm{RootOf}(\_Z^2+1)}{3}\right], \left[\frac{1980^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}} + \frac{1}{3} + \frac{\mathrm{RootOf}(\_Z^2+1)}{3}, -\frac{1980^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}} + \frac{1}{3} + \frac{\mathrm{RootOf}(\_Z^2+1)}{3}\right]\right], \left[\left[\frac{3\,\mathrm{RootOf}(\_Z^3+2, index=2)}{t} + 1 + \mathrm{RootOf}(\_Z^2+1), \frac{3\,\mathrm{RootOf}(\_Z^3+2, index=1)}{t} + 1 + \mathrm{RootOf}(\_Z^2+1)\right], \left[\frac{3\,\mathrm{RootOf}(\_Z^3+2, index=3)}{t}\right.$$

$$+1+RootOf\left(\_Z^2+1\right),\ \frac{3\ RootOf\left(\_Z^3+2,\ index=1\right)}{t}+1+RootOf\left(\_Z^2+1\right)\Bigg],$$

$$\left[\frac{3\ RootOf\left(\_Z^3+2,\ index=3\right)}{t}+1+RootOf\left(\_Z^2+1\right),\ \frac{3\ RootOf\left(\_Z^3+2,\ index=2\right)}{t}\right.$$

$$\left.+1+RootOf\left(\_Z^2+1\right)\right]\Bigg],\ [[0,0],[-2\ RootOf\left(\_Z^2+1\right),0],[-2\ RootOf\left(\_Z^2+1\right),$$

$$0]]\Bigg],\ [[1,1,1],[1,1,1],[3,3,3],[3,3,3],[1,1,1],[1,1,1]]\Bigg]\Bigg]$$

```
> F1:= Sirr0F2info1(L,R1[1],R1[2],x,t,ext);
```

$$F1 := \left[\left[\left[9,\ x-9,\ \left[-\frac{1024}{x-9}\right],\ 1,\ \varnothing,\ \{RootOf\left(\_Z^2+1\right)\}\right],\left[12,\ x-12,\ \left[\frac{18150}{x-12}\right],\ 1,\ \varnothing,\right.\right.\right. \tag{124}$$

$$\left.\left.\left.\{RootOf\left(\_Z^2+1\right)\}\right],\left[\infty,\ \frac{1}{x},\ [-2\ x^3],\ 3,\ \{RootOf\left(\_Z^3+2,\ index=1\right),\ RootOf\left(\_Z^3+2,\right.\right.\right.$$

$$\left.\left.\left.index=2\right)\},\ \{RootOf\left(\_Z^2+1\right),\ RootOf\left(\_Z^3+2,\ index=1\right),\ RootOf\left(\_Z^3+2,\ index\right.\right.\right.$$

$$\left.\left.\left.=2\right)\}\right]\right],\ 5,\ 6,\ (x-9)\ (x-12),\ 1\Bigg]$$

```
> find0F2ln(L,R1,F1,x,t,ext);
```

$$\left[\left[[[1,\ RootOf\left(\_Z^2+1\right)]],\ -\frac{2\ (x-1)^2\ (x-3)\ (x-7)^2}{(x-9)\ (x-12)}\right],\left[[[1,\ RootOf\left(\_Z^2+1\right)]],\right.\right. \tag{125}$$

$$\left.\left.\frac{2\ (x-1)^2\ (x-3)\ (x-7)^2}{(x-9)\ (x-12)}\right]\right]$$

```
> TIME := time();
  Hyp0F2Solutions(L);
  time()-TIME;
```

$$TIME := 84.390$$

$$\left\{\left[\left\{\left[\left[\left[1,\ RootOf\left(\_Z^2+1\right)\right],\left[-\frac{2}{x-7}\right],\left[\frac{9\ (x-3)\ (x-7)^2\ (x-9)^3\ (x-12)^3\ Dx^2}{\left(3\ x^4-98\ x^3+1058\ x^2-4350\ x+5499\right)^2}\right.\right.\right.\right.$$

$$+\left(9\ \left(15\ x^{13}-1623\ x^{12}+79349\ x^{11}-2318073\ x^{10}+45075498\ x^9-614907602\ x^8+6040382598\ x^7-43\right.\right.$$

$$+223351175871\ x^5-824867387271\ x^4+2104961242149\ x^3-3505189872969\ x^2$$

$$\left.\left.+3413344871016\ x-1477552421520\right)\ Dx\right)\Big/\left(\left(3\ x^4-98\ x^3+1058\ x^2-4350\ x\right.\right.$$

$$\left.\left.+5499\right)^2\ \left(3\ x^5-101\ x^4+1156\ x^3-5408\ x^2+9849\ x-5499\right)\right)+\left(18\ \left(6\ x^{12}\right.\right.$$

$$-621\ x^{11}+28733\ x^{10}-785302\ x^9+14104058\ x^8-175093272\ x^7+1537782840\ x^6$$

$$-9607222674\ x^5+42285234816\ x^4-127655542107\ x^3+250793603163\ x^2$$

$$\left.\left.-288655394040\ x+148734510768\right)\right)\Big/\left(\left(3\ x^4-98\ x^3+1058\ x^2-4350\ x\right.\right.$$

$$+5499)^2\,(3x^5-101x^4+1156x^3-5408x^2+9849x-5499))]]\},$$

$$\left.\left.\frac{2\,(x-1)^2\,(x-3)\,(x-7)^2}{(x-9)\,(x-12)}\right]\right\}$$

14.250 **(126)**

```
> ####### THE IRRATIONAL CASE #######
```

```
> F:=sumdiffeq(hyperterm([],[b1,b2],x,k),k,J(x));
```

$$F := \left(\frac{d^3}{dx^3}J(x)\right)x^2+(b1+b2+1)\left(\frac{d^2}{dx^2}J(x)\right)x+b1\,b2\left(\frac{d}{dx}J(x)\right)-J(x)=0 \quad \textbf{(127)}$$

```
> LA:=de2diffop(F,J(x));
```

$$LA := x^2\,Dx^3+(x\,b1+x\,b2+x)\,Dx^2+b2\,b1\,Dx-1 \quad \textbf{(128)}$$

```
> L12:=subs({b1=1/7,b2=1+RootOf(x^2-2)},LA);
```

$$L12 := x^2\,Dx^3+\left(\frac{8x}{7}+x\,(1+RootOf(\_Z^2-2))\right)Dx^2+\frac{(1+RootOf(\_Z^2-2))\,Dx}{7}-1 \quad \textbf{(129)}$$

```
> f:=(2*(x-1)*(x-3)^2*(x-7))/((x-9)*(x-12)^3);
```

$$f := \frac{2\,(x-1)\,(x-3)^2\,(x-7)}{(x-9)\,(x-12)^3} \quad \textbf{(130)}$$

```
> L:=ChangeOfVariables(L12,f);
```

$$\begin{aligned}L :=\ & 7\,Dx^3\,(x-7)^2\,(x-1)^2\,(x-3)^2\,(x-9)^4\,(x-12)^6\,(31x^3-547x^2+2745x-3285)^2\\ & -(6727\,RootOf(\_Z^2-2)\,x^6-1302x^7-237398\,RootOf(\_Z^2-2)\,x^5+49527x^6\\ & +3285793\,RootOf(\_Z^2-2)\,x^4-762264x^5-22446900\,RootOf(\_Z^2-2)\,x^3\\ & +6101781x^4+77901705\,RootOf(\_Z^2-2)\,x^2-27313902x^3-126242550\,RootOf(\_Z^2\\ & -2)\,x+69066513x^2+75538575\,RootOf(\_Z^2-2)-96311268x+65897955)\,(31x^3\\ & -547x^2+2745x-3285)\,(x-7)\,(x-3)\,(x-1)\,(x-9)^3\,(x-12)^5\,Dx^2-(\\ & -478680988091490x-342243011108160x^3-40362x^{14}+3070674x^{13}-104557477x^{12}\\ & +2097443056x^{11}-27418943328x^{10}+243293866666x^9-1470299830879x^8\\ & +5794560788112x^7-12387190731426x^6+557038208557260x^2\\ & +106972196481405x^4-2687997918042x^5+417074\,RootOf(\_Z^2-2)\,x^{13}\\ & -26889741\,RootOf(\_Z^2-2)\,x^{12}+747053128\,RootOf(\_Z^2-2)\,x^{11}\\ & -11503383074\,RootOf(\_Z^2-2)\,x^{10}+102759431586\,RootOf(\_Z^2-2)\,x^9\\ & -455042593291\,RootOf(\_Z^2-2)\,x^8-462164433984\,RootOf(\_Z^2-2)\,x^7\\ & +19390488741252\,RootOf(\_Z^2-2)\,x^6-125759039823306\,RootOf(\_Z^2-2)\,x^5\\ & +442964053072845\,RootOf(\_Z^2-2)\,x^4-941306001487560\,RootOf(\_Z^2-2)\,x^3\\ & +1189869961924350\,RootOf(\_Z^2-2)\,x^2-814340021848650\,RootOf(\_Z^2-2)\,x\end{aligned} \quad \textbf{(131)}$$

$$+ 228762261520875\ RootOf\left(\_Z^2-2\right) + 166203122715495\Big)\ (x-9)^2\ (x-12)^4\ Dx$$
$$+ 14\ \left(31\ x^3 - 547\ x^2 + 2745\ x - 3285\right)^5\ (x-3)$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \{RootOf\left(\_Z^2-2\right)\} \tag{132}$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```

$$ext := \{RootOf\left(\_Z^2-2\right)\} \tag{133}$$

```
> extppp:={};
```

$$extppp := \varnothing \tag{134}$$

```
> E:= Singular(L,extppp);
```

$$E := \left[[x-1,1],\left[x^3 - \frac{547}{31}\,x^2 + \frac{2745}{31}\,x - \frac{3285}{31}, RootOf\left(31\ \_Z^3 - 547\ \_Z^2 + 2745\ \_Z - 3285\right)\right], [x-12,12], [x-3,3], [x-9,9], [x-7,7]\right] \tag{135}$$

```
> F:=NotAppSing(L,E,ext);
```

$$F := [[x-1,1], [x-3,3], [x-9,9], [x-12,12], [x-7,7]] \tag{136}$$

```
> Sirr:=irrsing0F2(L,t,F,ext);
```

$$Sirr := \left[[[x-9,9], [x-12,12]], \left[\left[\frac{2\ 12^{2/3}}{3\ t^{1/3}} + \frac{1}{21} + \frac{RootOf\left(\_Z^2-2\right)}{3}, \frac{4\ 12^{2/3}}{3\left(I\sqrt{3}-1\right)t^{1/3}} + \frac{1}{21} + \frac{RootOf\left(\_Z^2-2\right)}{3}, -\frac{4\ 12^{2/3}}{3\left(I\sqrt{3}+1\right)t^{1/3}} + \frac{1}{21} + \frac{RootOf\left(\_Z^2-2\right)}{3}\right], \left[\frac{9\ RootOf\left(\_Z^3+110, index=1\right)}{t} + \frac{1}{7} + RootOf\left(\_Z^2-2\right), \frac{9\ RootOf\left(\_Z^3+110, index=2\right)}{t} + \frac{1}{7} + RootOf\left(\_Z^2-2\right), \frac{9\ RootOf\left(\_Z^3+110, index=3\right)}{t} + \frac{1}{7} + RootOf\left(\_Z^2-2\right)\right]\right], \left[\left[\frac{4\ 12^{2/3}}{3\left(I\sqrt{3}-1\right)t^{1/3}} - \frac{2\ 12^{2/3}}{3\ t^{1/3}}, -\frac{4\ 12^{2/3}}{3\left(I\sqrt{3}+1\right)t^{1/3}} - \frac{2\ 12^{2/3}}{3\ t^{1/3}}, -\frac{4\ 12^{2/3}}{3\left(I\sqrt{3}+1\right)t^{1/3}} - \frac{4\ 12^{2/3}}{3\left(I\sqrt{3}-1\right)t^{1/3}}\right], \left[\frac{9\ RootOf\left(\_Z^3+110, index=2\right)}{t} - \frac{9\ RootOf\left(\_Z^3+110, index=1\right)}{t}, \frac{9\ RootOf\left(\_Z^3+110, index=3\right)}{t} - \frac{9\ RootOf\left(\_Z^3+110, index=1\right)}{t}, \frac{9\ RootOf\left(\_Z^3+110, index=3\right)}{t} - \frac{9\ RootOf\left(\_Z^3+110, index=2\right)}{t}\right]\right], \left[\frac{1}{3}, 1\right], [3,1], \left[\left[\left[\frac{4\ 12^{2/3}}{3\left(I\sqrt{3}-1\right)t^{1/3}} + \frac{1}{21} + \frac{RootOf\left(\_Z^2-2\right)}{3}, \frac{2\ 12^{2/3}}{3\ t^{1/3}} + \frac{1}{21} + \frac{RootOf\left(\_Z^2-2\right)}{3}\right], \left[-\frac{4\ 12^{2/3}}{3\left(I\sqrt{3}+1\right)t^{1/3}}\right.\right.\right. \tag{137}$$

$$+\frac{1}{21}+\frac{RootOf\left(\_Z^2-2\right)}{3},\frac{2\,12^{2/3}}{3\,t^{1/3}}+\frac{1}{21}+\frac{RootOf\left(\_Z^2-2\right)}{3}\Bigg],\Bigg[$$

$$-\frac{4\,12^{2/3}}{3\left(I\sqrt{3}+1\right)t^{1/3}}+\frac{1}{21}+\frac{RootOf\left(\_Z^2-2\right)}{3},\frac{4\,12^{2/3}}{3\left(I\sqrt{3}-1\right)t^{1/3}}+\frac{1}{21}$$

$$+\frac{RootOf\left(\_Z^2-2\right)}{3}\Bigg]\Bigg],\Bigg[\Bigg[\frac{9\,RootOf\left(\_Z^3+110,\,index=2\right)}{t}+\frac{1}{7}+RootOf\left(\_Z^2-2\right),$$

$$\frac{9\,RootOf\left(\_Z^3+110,\,index=1\right)}{t}+\frac{1}{7}+RootOf\left(\_Z^2-2\right)\Bigg],$$

$$\Bigg[\frac{9\,RootOf\left(\_Z^3+110,\,index=3\right)}{t}+\frac{1}{7}+RootOf\left(\_Z^2-2\right),$$

$$\frac{9\,RootOf\left(\_Z^3+110,\,index=1\right)}{t}+\frac{1}{7}+RootOf\left(\_Z^2-2\right)\Bigg],$$

$$\Bigg[\frac{9\,RootOf\left(\_Z^3+110,\,index=3\right)}{t}+\frac{1}{7}+RootOf\left(\_Z^2-2\right),$$

$$\frac{9\,RootOf\left(\_Z^3+110,\,index=2\right)}{t}+\frac{1}{7}+RootOf\left(\_Z^2-2\right)\Bigg]\Bigg],\Bigg[\Bigg[$$

$$-\frac{\left(3\,12^{2/3}+I\sqrt{3}\,12^{2/3}\right)t}{3},\frac{\left(I\sqrt{3}\,12^{2/3}-3\,12^{2/3}\right)t}{3},\frac{2\,I}{3}\,12^{2/3}\sqrt{3}\,t\Bigg],$$

$$\left[9\left(RootOf\left(\_Z^3+110,\,index=2\right)-RootOf\left(\_Z^3+110,\,index=1\right)\right)t,\,9\left(RootOf\left(\_Z^3\right.\right.\right.$$

$$\left.\left.\left.+110,\,index=3\right)-RootOf\left(\_Z^3+110,\,index=1\right)\right)t,\,9\left(RootOf\left(\_Z^3+110,\,index=3\right)\right.\right.$$

$$\left.\left.-RootOf\left(\_Z^3+110,\,index=2\right)\right)t\right]],\left[[0,0,0],[0,0,0]\right],\Bigg[[x-1,1],[x-3,3],[x$$

$$-7,7]],\Bigg[\Bigg[\Bigg[0,\frac{6}{7},-RootOf\left(\_Z^2-2\right)\Bigg],\Bigg[\frac{6}{7},-RootOf\left(\_Z^2-2\right),-RootOf\left(\_Z^2-2\right)$$

$$-\frac{6}{7}\Bigg],[1,1,1],\Bigg[\Bigg[\frac{6}{7},0\Bigg],\left[-RootOf\left(\_Z^2-2\right),0\right],\Bigg[-RootOf\left(\_Z^2-2\right),\frac{6}{7}\Bigg]\Bigg],2\Bigg],\Bigg[\Bigg[0,$$

$$\frac{12}{7},-2\,RootOf\left(\_Z^2-2\right)\Bigg],\Bigg[\frac{12}{7},-2\,RootOf\left(\_Z^2-2\right),-\frac{12}{7}-2\,RootOf\left(\_Z^2-2\right)\Bigg],$$

$$[1,1,1],\Bigg[\Bigg[\frac{12}{7},0\Bigg],\left[-2\,RootOf\left(\_Z^2-2\right),0\right],\Bigg[-2\,RootOf\left(\_Z^2-2\right),\frac{12}{7}\Bigg]\Bigg],2\Bigg],\Bigg[\Bigg[0,$$

$$\frac{6}{7},-RootOf\left(\_Z^2-2\right)\Bigg],\Bigg[\frac{6}{7},-RootOf\left(\_Z^2-2\right),-RootOf\left(\_Z^2-2\right)-\frac{6}{7}\Bigg],[1,1,1],$$

$$\Bigg[\Bigg[\frac{6}{7},0\Bigg],\left[-RootOf\left(\_Z^2-2\right),0\right],\Bigg[-RootOf\left(\_Z^2-2\right),\frac{6}{7}\Bigg]\Bigg],2\Bigg]\Bigg]\Bigg]$$

```
> Sreg:=regsingtrue0F2(L,t,Sirr[-1],ext);
```

$$Sreg:=\Bigg[[x-1,1],[x-3,3],[x-7,7]],\Bigg[\Bigg[0,\frac{6}{7},-RootOf\left(\_Z^2-2\right)\Bigg],\Bigg[0,\frac{12}{7},$$

$$-2\,RootOf\left(\_Z^2-2\right)\Bigg],\Bigg[0,\frac{6}{7},-RootOf\left(\_Z^2-2\right)\Bigg]\Bigg],\Bigg[\Bigg[\frac{6}{7},-RootOf\left(\_Z^2-2\right),\qquad(138)$$

$$-RootOf(\_Z^2-2)-\frac{6}{7}\Big],\Big[\frac{12}{7},-2\,RootOf(\_Z^2-2),-\frac{12}{7}-2\,RootOf(\_Z^2-2)\Big],$$
$$\Big[\frac{6}{7},-RootOf(\_Z^2-2),-RootOf(\_Z^2-2)-\frac{6}{7}\Big]\Big],\Big[\Big[\Big[\frac{6}{7},0\Big],[-RootOf(\_Z^2-2),$$
$$0],\Big[-RootOf(\_Z^2-2),\frac{6}{7}\Big]\Big],\Big[\Big[\frac{12}{7},0\Big],[-2\,RootOf(\_Z^2-2),0],\Big[-2\,RootOf(\_Z^2$$
$$-2),\frac{12}{7}\Big]\Big],\Big[\Big[\frac{6}{7},0\Big],[-RootOf(\_Z^2-2),0],\Big[-RootOf(\_Z^2-2),\frac{6}{7}\Big]\Big]\Big]$$

```
> RSreg:=Sregseptrue0F2(L,Sreg,ext);
```

$$RSreg := \Big[\Big[[[x-1,1],[x-3,3],[x-7,7]],\Big[\Big[0,\frac{6}{7},-RootOf(\_Z^2-2)\Big],\Big[0,\frac{12}{7},$$
$$-2\,RootOf(\_Z^2-2)\Big],\Big[0,\frac{6}{7},-RootOf(\_Z^2-2)\Big]\Big],\Big[\Big[\Big[\frac{6}{7},-RootOf(\_Z^2-2),$$
$$-RootOf(\_Z^2-2)-\frac{6}{7}\Big],[\,]\Big],\Big[\Big[\frac{12}{7},-2\,RootOf(\_Z^2-2),-\frac{12}{7}-2\,RootOf(\_Z^2$$
$$-2)\Big],[\,]\Big],\Big[\Big[\frac{6}{7},-RootOf(\_Z^2-2),-RootOf(\_Z^2-2)-\frac{6}{7}\Big],[\,]\Big]\Big],[\,],[\,]\Big] \quad (139)$$

```
> R1:=IrrRegAppsing0F2(L,t,E,ext);
```

$$R1 := \Big[\Big[[[x-9,9],[x-12,12]],\Big[\Big[\frac{2\,12^{2/3}}{3\,t^{1/3}}+\frac{1}{21}+\frac{RootOf(\_Z^2-2)}{3},$$
$$\frac{4\,12^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}}+\frac{1}{21}+\frac{RootOf(\_Z^2-2)}{3},-\frac{4\,12^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}}+\frac{1}{21}$$
$$+\frac{RootOf(\_Z^2-2)}{3}\Big],\Big[\frac{9\,RootOf(\_Z^3+110,index=1)}{t}+\frac{1}{7}+RootOf(\_Z^2-2),$$
$$\frac{9\,RootOf(\_Z^3+110,index=2)}{t}+\frac{1}{7}+RootOf(\_Z^2-2),$$
$$\frac{9\,RootOf(\_Z^3+110,index=3)}{t}+\frac{1}{7}+RootOf(\_Z^2-2)\Big]\Big],\Big[\Big[\frac{4\,12^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}}$$
$$-\frac{2\,12^{2/3}}{3\,t^{1/3}},-\frac{4\,12^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}}-\frac{2\,12^{2/3}}{3\,t^{1/3}},-\frac{4\,12^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}}$$
$$-\frac{4\,12^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}}\Big],\Big[\frac{9\,RootOf(\_Z^3+110,index=2)}{t}$$
$$-\frac{9\,RootOf(\_Z^3+110,index=1)}{t},\frac{9\,RootOf(\_Z^3+110,index=3)}{t}$$
$$-\frac{9\,RootOf(\_Z^3+110,index=1)}{t},\frac{9\,RootOf(\_Z^3+110,index=3)}{t}$$
$$-\frac{9\,RootOf(\_Z^3+110,index=2)}{t}\Big]\Big],\Big[\frac{1}{3},1\Big],[3,1],\Big[\Big[\Big[\frac{4\,12^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}}+\frac{1}{21} \quad (140)$$

$$+\frac{RootOf\left(\_Z^2-2\right)}{3},\ \frac{2\,12^{2/3}}{3\,t^{1/3}}+\frac{1}{21}+\frac{RootOf\left(\_Z^2-2\right)}{3}\Bigg],\Bigg[-\frac{4\,12^{2/3}}{3\left(I\sqrt{3}+1\right)t^{1/3}}$$

$$+\frac{1}{21}+\frac{RootOf\left(\_Z^2-2\right)}{3},\ \frac{2\,12^{2/3}}{3\,t^{1/3}}+\frac{1}{21}+\frac{RootOf\left(\_Z^2-2\right)}{3}\Bigg],\Bigg[$$

$$-\frac{4\,12^{2/3}}{3\left(I\sqrt{3}+1\right)t^{1/3}}+\frac{1}{21}+\frac{RootOf\left(\_Z^2-2\right)}{3},\ \frac{4\,12^{2/3}}{3\left(I\sqrt{3}-1\right)t^{1/3}}+\frac{1}{21}$$

$$+\frac{RootOf\left(\_Z^2-2\right)}{3}\Bigg]\Bigg],\Bigg[\Bigg[\frac{9\,RootOf\left(\_Z^3+110,\,index=2\right)}{t}+\frac{1}{7}+RootOf\left(\_Z^2-2\right),$$

$$\frac{9\,RootOf\left(\_Z^3+110,\,index=1\right)}{t}+\frac{1}{7}+RootOf\left(\_Z^2-2\right)\Bigg],$$

$$\Bigg[\frac{9\,RootOf\left(\_Z^3+110,\,index=3\right)}{t}+\frac{1}{7}+RootOf\left(\_Z^2-2\right),$$

$$\frac{9\,RootOf\left(\_Z^3+110,\,index=1\right)}{t}+\frac{1}{7}+RootOf\left(\_Z^2-2\right)\Bigg],$$

$$\Bigg[\frac{9\,RootOf\left(\_Z^3+110,\,index=3\right)}{t}+\frac{1}{7}+RootOf\left(\_Z^2-2\right),$$

$$\frac{9\,RootOf\left(\_Z^3+110,\,index=2\right)}{t}+\frac{1}{7}+RootOf\left(\_Z^2-2\right)\Bigg]\Bigg]\Bigg],\Bigg[\Bigg[$$

$$-\frac{\left(3\,12^{2/3}+I\sqrt{3}\,12^{2/3}\right)t}{3},\ \frac{\left(I\sqrt{3}\,12^{2/3}-3\,12^{2/3}\right)t}{3},\ \frac{2\,I}{3}\,12^{2/3}\sqrt{3}\,t\Bigg],$$

$$\left[9\left(RootOf\left(\_Z^3+110,\,index=2\right)-RootOf\left(\_Z^3+110,\,index=1\right)\right)t,\ 9\left(RootOf\left(\_Z^3\right.\right.\right.$$

$$\left.\left.\left.+110,\,index=3\right)-RootOf\left(\_Z^3+110,\,index=1\right)\right)t,\ 9\left(RootOf\left(\_Z^3+110,\,index=3\right)\right.\right.$$

$$\left.\left.-RootOf\left(\_Z^3+110,\,index=2\right)\right)t\right]\Big],\ [[0,0,0],[0,0,0]]\Big],\Big[[[x-1,1],[x-3,3],[x$$

$$-7,7]]\Big],\Bigg[\Bigg[0,\frac{6}{7},-RootOf\left(\_Z^2-2\right)\Bigg],\Bigg[0,\frac{12}{7},-2\,RootOf\left(\_Z^2-2\right)\Bigg],\Bigg[0,\frac{6}{7},$$

$$-RootOf\left(\_Z^2-2\right)\Bigg]\Bigg],\Bigg[\Bigg[\frac{6}{7},-RootOf\left(\_Z^2-2\right),-RootOf\left(\_Z^2-2\right)-\frac{6}{7}\Bigg],\Bigg[\frac{12}{7},$$

$$-2\,RootOf\left(\_Z^2-2\right),-\frac{12}{7}-2\,RootOf\left(\_Z^2-2\right)\Bigg],\Bigg[\frac{6}{7},-RootOf\left(\_Z^2-2\right),$$

$$-RootOf\left(\_Z^2-2\right)-\frac{6}{7}\Bigg]\Bigg],\Bigg[\Bigg[\Bigg[\frac{6}{7},0\Bigg],\left[-RootOf\left(\_Z^2-2\right),0\right],\Bigg[-RootOf\left(\_Z^2-2\right),$$

$$\frac{6}{7}\Big]\Big], \Big[\Big[\frac{12}{7}, 0\Big], [-2\,RootOf(\_Z^2-2), 0], \Big[-2\,RootOf(\_Z^2-2), \frac{12}{7}\Big]\Big], \Big[\Big[\frac{6}{7}, 0\Big], [$$

$$-RootOf(\_Z^2-2), 0], \Big[-RootOf(\_Z^2-2), \frac{6}{7}\Big]\Big]\Big], \Big[\Big[[[x-1, 1], [x-3, 3], [x-7,$$

$$7]], \Big[\Big[0, \frac{6}{7}, -RootOf(\_Z^2-2)\Big], \Big[0, \frac{12}{7}, -2\,RootOf(\_Z^2-2)\Big], \Big[0, \frac{6}{7}, -RootOf(\_Z^2$$

$$-2)\Big]\Big], \Big[\Big[\Big[\frac{6}{7}, -RootOf(\_Z^2-2), -RootOf(\_Z^2-2)-\frac{6}{7}\Big], [\,]\Big], \Big[\Big[\frac{12}{7},$$

$$-2\,RootOf(\_Z^2-2), -\frac{12}{7}-2\,RootOf(\_Z^2-2)\Big], [\,]\Big], \Big[\Big[\frac{6}{7}, -RootOf(\_Z^2-2),$$

$$-RootOf(\_Z^2-2)-\frac{6}{7}\Big], [\,]\Big]\Big], [\,], [\,]\Big], \Big[\Big[\Big[x^3-\frac{547}{31}x^2+\frac{2745}{31}x-\frac{3285}{31},$$

$$RootOf(31\,\_Z^3-547\,\_Z^2+2745\,\_Z-3285)\Big]\Big], [[0, 2, 4]], [[2, 4, 2]], [[[2, 0], [4, 0],$$

$$[4, 2]]]\Big], \Big[[[x-1, 1], [x-3, 3], [x-9, 9], [x-12, 12], [x-7, 7]], \Big[\Big[0, \frac{6}{7},$$

$$-RootOf(\_Z^2-2)\Big], \Big[0, \frac{12}{7}, -2\,RootOf(\_Z^2-2)\Big], \Big[\frac{2\,12^{2/3}}{3\,t^{1/3}}+\frac{1}{21}$$

$$+\frac{RootOf(\_Z^2-2)}{3}, \frac{4\,12^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}}+\frac{1}{21}+\frac{RootOf(\_Z^2-2)}{3},$$

$$-\frac{4\,12^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}}+\frac{1}{21}+\frac{RootOf(\_Z^2-2)}{3}\Big], \Big[\frac{9\,RootOf(\_Z^3+110, index=1)}{t}$$

$$+\frac{1}{7}+RootOf(\_Z^2-2), \frac{9\,RootOf(\_Z^3+110, index=2)}{t}+\frac{1}{7}+RootOf(\_Z^2-2),$$

$$\frac{9\,RootOf(\_Z^3+110, index=3)}{t}+\frac{1}{7}+RootOf(\_Z^2-2)\Big], \Big[0, \frac{6}{7}, -RootOf(\_Z^2$$

$$-2)\Big]\Big], \Big[\Big[\frac{6}{7}, -RootOf(\_Z^2-2), -RootOf(\_Z^2-2)-\frac{6}{7}\Big], \Big[\frac{12}{7}, -2\,RootOf(\_Z^2$$

$$-2), -\frac{12}{7}-2\,RootOf(\_Z^2-2)\Big], \Big[\frac{4\,12^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}}-\frac{2\,12^{2/3}}{3\,t^{1/3}},$$

$$-\frac{4\,12^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}}-\frac{2\,12^{2/3}}{3\,t^{1/3}}, -\frac{4\,12^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}}-\frac{4\,12^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}}\Big],$$

$$\Big[\frac{9\,RootOf(\_Z^3+110, index=2)}{t}-\frac{9\,RootOf(\_Z^3+110, index=1)}{t},$$

$$\frac{9\,RootOf(\_Z^3+110, index=3)}{t}-\frac{9\,RootOf(\_Z^3+110, index=1)}{t},$$

$$\frac{9\,RootOf(\_Z^3+110,\,index=3)}{t}\;-\;\frac{9\,RootOf(\_Z^3+110,\,index=2)}{t}\Big],\Big[\frac{6}{7},$$

$$-RootOf(\_Z^2-2),\,-RootOf(\_Z^2-2)-\frac{6}{7}\Big]\Big],\Big[\Big[\Big[\frac{6}{7},0\Big],[-RootOf(\_Z^2-2),0],\Big[$$

$$-RootOf(\_Z^2-2),\,\frac{6}{7}\Big]\Big],\Big[\Big[\frac{12}{7},0\Big],[-2\,RootOf(\_Z^2-2),0],\Big[-2\,RootOf(\_Z^2-2),$$

$$\frac{12}{7}\Big]\Big],\Big[\Big[\frac{4\,12^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}}+\frac{1}{21}+\frac{RootOf(\_Z^2-2)}{3},\frac{2\,12^{2/3}}{3\,t^{1/3}}+\frac{1}{21}$$

$$+\frac{RootOf(\_Z^2-2)}{3}\Big],\Big[-\frac{4\,12^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}}+\frac{1}{21}+\frac{RootOf(\_Z^2-2)}{3},\frac{2\,12^{2/3}}{3\,t^{1/3}}$$

$$+\frac{1}{21}+\frac{RootOf(\_Z^2-2)}{3}\Big],\Big[-\frac{4\,12^{2/3}}{3\,(I\sqrt{3}+1)\,t^{1/3}}+\frac{1}{21}+\frac{RootOf(\_Z^2-2)}{3},$$

$$\frac{4\,12^{2/3}}{3\,(I\sqrt{3}-1)\,t^{1/3}}+\frac{1}{21}+\frac{RootOf(\_Z^2-2)}{3}\Big]\Big],\Big[\Big[\frac{9\,RootOf(\_Z^3+110,\,index=2)}{t}$$

$$+\frac{1}{7}+RootOf(\_Z^2-2),\frac{9\,RootOf(\_Z^3+110,\,index=1)}{t}+\frac{1}{7}+RootOf(\_Z^2-2)\Big],$$

$$\Big[\frac{9\,RootOf(\_Z^3+110,\,index=3)}{t}+\frac{1}{7}+RootOf(\_Z^2-2),$$

$$\frac{9\,RootOf(\_Z^3+110,\,index=1)}{t}+\frac{1}{7}+RootOf(\_Z^2-2)\Big],$$

$$\Big[\frac{9\,RootOf(\_Z^3+110,\,index=3)}{t}+\frac{1}{7}+RootOf(\_Z^2-2),$$

$$\frac{9\,RootOf(\_Z^3+110,\,index=2)}{t}+\frac{1}{7}+RootOf(\_Z^2-2)\Big]\Big],\Big[\Big[\frac{6}{7},0\Big],[-RootOf(\_Z^2$$

$$-2),0],\Big[-RootOf(\_Z^2-2),\,\frac{6}{7}\Big]\Big]\Big],[[1,1,1],[1,1,1],[3,3,3],[1,1,1],[1,1,1]]\Big]$$

```
> F1:= Sirr0F2info1(L,R1[1],R1[2],x,t,ext);
```

$$F1 := \Big[\Big[\Big[9,\,x-9,\Big[-\frac{128}{3\,(x-9)}\Big],1,\varnothing,\{RootOf(\_Z^2-2)\}\Big],\Big[12,\,x-12,\Big[-\frac{2970}{(x-12)^3}\Big],\quad (141)$$

$$3,\{RootOf(\_Z^3+110,\,index=1),RootOf(\_Z^3+110,\,index=2)\},\{RootOf(\_Z^2-2),$$

$$RootOf(\_Z^3+110,\,index=1),RootOf(\_Z^3+110,\,index=2)\}\Big]\Big],4,5,(x-9)\,(x-12)^3,$$

$$(x-12)^2\Big]$$

```
> find0F2Irr(L,R1,F1,x,t,ext);
```

$$\Big[\Big[\Big[\Big[\frac{1}{7},RootOf(\_Z^2-2)\Big]\Big],\frac{2\,(x-1)\,(x-3)^2\,(x-7)}{(x-9)\,(x-12)^3}\Big],\Big[\Big[\Big[\frac{1}{7},RootOf(\_Z^2-2)\Big]\Big],\quad (142)$$

$$-\frac{2\,(x-1)\,(x-3)^2\,(x-7)}{(x-9)\,(x-12)^3}\Big]\Big]$$

```
> TIME := time();
```

```
Hyp0F2Solutions(L);
time()-TIME;
```

$$TIME := 154.359$$

$$\left\{\left[\left\{\left[\left[\left[\frac{1}{7}, RootOf\left(\_Z^2-2\right)\right]\right], \left[\left[\frac{1}{x-12}\right]\right], \left[\frac{961\,(x-1)\,(x-7)\,(x-9)^3\,(x-12)^4\,Dx^2}{\left(31\,x^3-547\,x^2+2745\,x-3285\right)^2}\right.\right.\right.\right.$$

$$+\left(961\left(3673\,x^{11}-327920\,x^{10}+13045564\,x^9-304411956\,x^8+4612734126\,x^7-47433328812\,x^6+335\right.\right.$$

$$\left.\left.-1618909282188\,x^4+5149002111057\,x^3-10070835984612\,x^2+10527038104752\,x\right.\right.$$

$$\left.\left.-4183786987200\right)Dx\right)\Big/\left(7\left(31\,x^3-547\,x^2+2745\,x-3285\right)^2\left(31\,x^4-640\,x^3\right.\right.$$

$$\left.\left.+4386\,x^2-11520\,x+9855\right)\right)-\left(961\left(3673\,x^{10}-283844\,x^9+9639436\,x^8\right.\right.$$

$$\left.\left.-188738724\,x^7+2347869438\,x^6-19258895556\,x^5+104489321868\,x^4\right.\right.$$

$$\left.\left.-365037419772\,x^3+768553073793\,x^2-848199099096\,x+348648915600\right)\right)\Big/$$

$$\left.\left.\left(7\left(31\,x^3-547\,x^2+2745\,x-3285\right)^2\left(31\,x^4-640\,x^3+4386\,x^2-11520\,x+9855\right)\right)\right]\right]\right\}$$

$$, \left.\left.\left[\frac{2\,(x-1)\,(x-3)^2\,(x-7)}{(x-9)\,(x-12)^3}\right]\right]\right\}$$

13.062 **(143)**

```
> ####### THE RATIONAL CASE #######
```

```
> F:=sumdiffeq(hyperterm([],[b1,b2],x,k),k,J(x));
```

$$F := \left(\frac{d^3}{dx^3}\,J(x)\right)x^2+(b1+b2+1)\left(\frac{d^2}{dx^2}\,J(x)\right)x+b1\,b2\left(\frac{d}{dx}\,J(x)\right)-J(x)=0 \quad \textbf{(144)}$$

```
> LA:=de2diffop(F,J(x));
```

$$LA := x^2\,Dx^3+(x\,b1+x\,b2+x)\,Dx^2+b2\,b1\,Dx-1 \quad \textbf{(145)}$$

```
> L12:=subs({b1=1/7,b2=1/5},LA);
```

$$L12 := x^2\,Dx^3+\frac{47}{35}\,Dx^2\,x+\frac{1}{35}\,Dx-1 \quad \textbf{(146)}$$

```
> f:=(2*(x-1)^2*(x-3)*(x-7)^3)/((x-9)^2*(x-12)^3);
```

$$f := \frac{2\,(x-1)^2\,(x-3)\,(x-7)^3}{(x-9)^2\,(x-12)^3} \quad \textbf{(147)}$$

```
> L:=ChangeOfVariables(L12,f);
```

$$L := 35\,Dx^3\,(x-1)^2\,(x-3)^2\,(x-7)^2\,(x-9)^5\,(x-12)^6\left(x^4-60\,x^3+830\,x^2-3852\,x\right. \quad \textbf{(148)}$$

$$+5193)^2+(47x^8-5640x^7+119120x^6-581208x^5-6480058x^4+86613480x^3-376117272x^2+687325176x-415406637)\,Dx^2\,(x-7)\,(x-1)\,(x-3)\,(x-9)^4\,(x-12)^5\,(x^4-60x^3+830x^2-3852x+5193)+(x^{16}-240x^{15}+95680x^{14}-6405136x^{13}+236830540x^{12}-6159920464x^{11}+118819343328x^{10}-1691620476208x^9+17614025125542x^8-133551397637136x^7+734375296519488x^6-2903314010368752x^5+8107678940829516x^4-15485044941209520x^3+19095958145015712x^2-13609866239187408x+4252340609354817)\,Dx\,(x-9)^3\,(x-12)^4-70\,(x^4-60x^3+830x^2-3852x+5193)^5\,(x-1)\,(x-7)^2$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \varnothing \tag{149}$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```

$$ext := \varnothing \tag{150}$$

```
> extppp:={};
```

$$extppp := \varnothing \tag{151}$$

```
> E:= Singular(L,extppp);
```

$$E := [[x-1,1],[x-12,12],[\infty,\infty],[x-3,3],[x^4-60x^3+830x^2-3852x+5193,\ RootOf(\_Z^4-60\_Z^3+830\_Z^2-3852\_Z+5193)],[x-9,9],[x-7,7]] \tag{152}$$

```
> F:=NotAppSing(L,E,ext);
```

$$F := [[x-1,1],[x-3,3],[x-9,9],[x-12,12],[\infty,\infty],[x-7,7]] \tag{153}$$

```
> Sirr:=irrsing0F2(L,t,F,ext);
```

$$Sirr := \left[[[x-9,9],[x-12,12],[\infty,\infty]],\left[\left[\frac{16\,12^{1/3}}{3t^{2/3}}-\frac{46}{105},\frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2t^{2/3}}-\frac{46}{105},\frac{64\,12^{1/3}}{3\left(I\sqrt{3}+1\right)^2t^{2/3}}-\frac{46}{105}\right],\left[\frac{15\,RootOf(\_Z^3+242,index=1)}{t}-\frac{23}{35},\frac{15\,RootOf(\_Z^3+242,index=2)}{t}-\frac{23}{35},\frac{15\,RootOf(\_Z^3+242,index=3)}{t}-\frac{23}{35}\right],\left[-\frac{2^{1/3}}{t^{1/3}}-\frac{23}{105},-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}-\frac{23}{105},\frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}-\frac{23}{105}\right]\right],\left[\left[\frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2t^{2/3}}-\frac{16\,12^{1/3}}{3t^{2/3}},\frac{64\,12^{1/3}}{3\left(I\sqrt{3}+1\right)^2t^{2/3}}-\frac{16\,12^{1/3}}{3t^{2/3}},\frac{64\,12^{1/3}}{3\left(I\sqrt{3}+1\right)^2t^{2/3}}-\frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2t^{2/3}}\right],\left[\frac{15\,RootOf(\_Z^3+242,index=2)}{t}-\frac{15\,RootOf(\_Z^3+242,index=1)}{t},\frac{15\,RootOf(\_Z^3+242,index=3)}{t}\right]\right] \tag{154}$$

$$-\frac{15\,RootOf\left(\_Z^3+242,\,index=1\right)}{t},\ \frac{15\,RootOf\left(\_Z^3+242,\,index=3\right)}{t}$$

$$-\frac{15\,RootOf\left(\_Z^3+242,\,index=2\right)}{t}\Bigg],\left[-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}+\frac{2^{1/3}}{t^{1/3}},\ \frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}\right.$$

$$\left.+\frac{2^{1/3}}{t^{1/3}},\ \frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}+\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}\right]\Bigg],\left[\frac{2}{3},\,1,\,\frac{1}{3}\right],\,[3,\,1,\,3],$$

$$\left[\left[\left[\frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2t^{2/3}}-\frac{46}{105},\ \frac{16\,12^{1/3}}{3\,t^{2/3}}-\frac{46}{105}\right],\left[\frac{64\,12^{1/3}}{3\left(I\sqrt{3}+1\right)^2t^{2/3}}-\frac{46}{105},\right.\right.\right.$$

$$\left.\left.\left.\frac{16\,12^{1/3}}{3\,t^{2/3}}-\frac{46}{105}\right],\left[\frac{64\,12^{1/3}}{3\left(I\sqrt{3}+1\right)^2t^{2/3}}-\frac{46}{105},\ \frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2t^{2/3}}-\frac{46}{105}\right]\right],$$

$$\left[\left[\frac{15\,RootOf\left(\_Z^3+242,\,index=2\right)}{t}-\frac{23}{35},\ \frac{15\,RootOf\left(\_Z^3+242,\,index=1\right)}{t}-\frac{23}{35}\right],\right.$$

$$\left[\frac{15\,RootOf\left(\_Z^3+242,\,index=3\right)}{t}-\frac{23}{35},\ \frac{15\,RootOf\left(\_Z^3+242,\,index=1\right)}{t}-\frac{23}{35}\right],$$

$$\left.\left[\frac{15\,RootOf\left(\_Z^3+242,\,index=3\right)}{t}-\frac{23}{35},\ \frac{15\,RootOf\left(\_Z^3+242,\,index=2\right)}{t}-\frac{23}{35}\right]\right],$$

$$\left[\left[-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}-\frac{23}{105},\ -\frac{2^{1/3}}{t^{1/3}}-\frac{23}{105}\right],\left[\frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}-\frac{23}{105},\ -\frac{2^{1/3}}{t^{1/3}}\right.\right.$$

$$\left.\left.-\frac{23}{105}\right],\left[\frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}-\frac{23}{105},\ -\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}-\frac{23}{105}\right]\right]\Bigg],$$

$$\left[\left[\frac{8\left(I\sqrt{3}\,12^{1/3}-3\,12^{1/3}\right)t^2}{3},\ -\frac{8\left(I\sqrt{3}\,12^{1/3}+3\,12^{1/3}\right)t^2}{3},\ -\frac{16\,I}{3}\sqrt{3}\,12^{1/3}t^2\right],\right.$$

$$\left[15\left(RootOf\left(\_Z^3+242,\,index=2\right)-RootOf\left(\_Z^3+242,\,index=1\right)\right)t,\,15\left(RootOf\left(\_Z^3\right.\right.\right.$$

$$\left.\left.+242,\,index=3\right)-RootOf\left(\_Z^3+242,\,index=1\right)\right)t,\,15\left(RootOf\left(\_Z^3+242,\,index=3\right)$$

$$\left.\left.-RootOf\left(\_Z^3+242,\,index=2\right)\right)t\right],\left[\frac{\left(I\sqrt{3}\,2^{1/3}+3\,2^{1/3}\right)t}{2},\right.$$

$$\left.\left.-\frac{\left(I\sqrt{3}\,2^{1/3}-3\,2^{1/3}\right)t}{2},\ -I\,2^{1/3}\sqrt{3}\,t\right]\right],\,[[0,0,0],[0,0,0],[0,0,0]],\left[[x-1,1],\right.$$

$$\left.[x-3,3],[x-7,7]\right],\left[\left[\left[0,\frac{12}{7},\frac{8}{5}\right],\left[\frac{12}{7},\frac{8}{5},-\frac{4}{35}\right],[1,1,1],\left[\left[\frac{12}{7},0\right],\left[\frac{8}{5},0\right],\right.\right.\right.$$

$$\left.\left.\left.\left[\frac{8}{5},\frac{12}{7}\right]\right],2\right],\left[\left[0,\frac{6}{7},\frac{4}{5}\right],\left[\frac{6}{7},\frac{4}{5},-\frac{2}{35}\right],[1,1,1],\left[\left[\frac{6}{7},0\right],\left[\frac{4}{5},0\right],\left[\frac{4}{5},\frac{6}{7}\right]\right],2\right],$$

$$\left.\left[\left[0,\frac{18}{7},\frac{12}{5}\right],\left[\frac{18}{7},\frac{12}{5},-\frac{6}{35}\right],[1,1,1],\left[\left[\frac{18}{7},0\right],\left[\frac{12}{5},0\right],\left[\frac{12}{5},\frac{18}{7}\right]\right],2\right]\right]\right]$$

```
> Sreg:=regsingtrue0F2(L,t,Sirr[-1],ext);
```

$$Sreg := \left[[[x-1,1],[x-3,3],[x-7,7]], \left[\left[0, \frac{12}{7}, \frac{8}{5}\right], \left[0, \frac{6}{7}, \frac{4}{5}\right], \left[0, \frac{18}{7}, \frac{12}{5}\right] \right], \right. \tag{155}$$
$$\left[\left[\frac{12}{7}, \frac{8}{5}, -\frac{4}{35}\right], \left[\frac{6}{7}, \frac{4}{5}, -\frac{2}{35}\right], \left[\frac{18}{7}, \frac{12}{5}, -\frac{6}{35}\right] \right], \left[\left[\left[\frac{12}{7}, 0\right], \left[\frac{8}{5}, 0\right], \left[\frac{8}{5}, \frac{12}{7}\right] \right], \left[\left[\frac{6}{7}, 0\right], \left[\frac{4}{5}, 0\right], \left[\frac{4}{5}, \frac{6}{7}\right] \right], \left[\left[\frac{18}{7}, 0\right], \left[\frac{12}{5}, 0\right], \left[\frac{12}{5}, \frac{18}{7}\right] \right] \right] \right]$$

```
> RSreg:=Sregseptrue0F2(L,Sreg,ext);
```

$$RSreg := \left[\left[[[x-1,1],[x-3,3],[x-7,7]], \left[\left[0, \frac{12}{7}, \frac{8}{5}\right], \left[0, \frac{6}{7}, \frac{4}{5}\right], \left[0, \frac{18}{7}, \frac{12}{5}\right] \right], \right. \right. \tag{156}$$
$$\left. \left. \left[\left[\left[\frac{12}{7}, \frac{8}{5}, -\frac{4}{35}\right], [\,]\right], \left[\left[\frac{6}{7}, \frac{4}{5}, -\frac{2}{35}\right], [\,]\right], \left[\left[\frac{18}{7}, \frac{12}{5}, -\frac{6}{35}\right], [\,]\right] \right], [\,], [\,] \right]$$

```
> R1:=IrrRegAppsing0F2(L,t,E,ext);
```

$$R1 := \left[\left[[[x-9,9],[x-12,12],[\infty,\infty]], \left[\left[\frac{16\,12^{1/3}}{3\,t^{2/3}} - \frac{46}{105}, \frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2 t^{2/3}} \right. \right. \right. \tag{157}$$
$$\left. - \frac{46}{105}, \frac{64\,12^{1/3}}{3\left(I\sqrt{3}+1\right)^2 t^{2/3}} - \frac{46}{105} \right], \left[\frac{15\,RootOf\left(\_Z^3+242, index=1\right)}{t} - \frac{23}{35}, \right.$$
$$\left. \frac{15\,RootOf\left(\_Z^3+242, index=2\right)}{t} - \frac{23}{35}, \frac{15\,RootOf\left(\_Z^3+242, index=3\right)}{t} - \frac{23}{35} \right], \left[\right.$$
$$\left. - \frac{2^{1/3}}{t^{1/3}} - \frac{23}{105}, -\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}} - \frac{23}{105}, \frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}} - \frac{23}{105} \right] \right],$$
$$\left[\left[\frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2 t^{2/3}} - \frac{16\,12^{1/3}}{3\,t^{2/3}}, \frac{64\,12^{1/3}}{3\left(I\sqrt{3}+1\right)^2 t^{2/3}} - \frac{16\,12^{1/3}}{3\,t^{2/3}}, \right. \right.$$
$$\left. \left. \frac{64\,12^{1/3}}{3\left(I\sqrt{3}+1\right)^2 t^{2/3}} - \frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2 t^{2/3}} \right], \left[\frac{15\,RootOf\left(\_Z^3+242, index=2\right)}{t} \right.$$
$$\left. - \frac{15\,RootOf\left(\_Z^3+242, index=1\right)}{t}, \frac{15\,RootOf\left(\_Z^3+242, index=3\right)}{t} \right.$$
$$\left. - \frac{15\,RootOf\left(\_Z^3+242, index=1\right)}{t}, \frac{15\,RootOf\left(\_Z^3+242, index=3\right)}{t} \right.$$
$$\left. - \frac{15\,RootOf\left(\_Z^3+242, index=2\right)}{t} \right], \left[-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}} + \frac{2^{1/3}}{t^{1/3}}, \frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}} \right.$$
$$\left. + \frac{2^{1/3}}{t^{1/3}}, \frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}} + \frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}} \right] \right], \left[\frac{2}{3}, 1, \frac{1}{3} \right], [3,1,3],$$
$$\left[\left[\left[\frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2 t^{2/3}} - \frac{46}{105}, \frac{16\,12^{1/3}}{3\,t^{2/3}} - \frac{46}{105} \right], \left[\frac{64\,12^{1/3}}{3\left(I\sqrt{3}+1\right)^2 t^{2/3}} - \frac{46}{105}, \right. \right.$$

$$\frac{16\,12^{1/3}}{3\,t^{2/3}}-\frac{46}{105}\Big],\Big[\frac{64\,12^{1/3}}{3\left(I\sqrt{3}+1\right)^2 t^{2/3}}-\frac{46}{105},\frac{64\,12^{1/3}}{3\left(I\sqrt{3}-1\right)^2 t^{2/3}}-\frac{46}{105}\Big]\Big],$$

$$\Big[\Big[\frac{15\,RootOf\left(\_Z^3+242,\,index=2\right)}{t}-\frac{23}{35},\frac{15\,RootOf\left(\_Z^3+242,\,index=1\right)}{t}-\frac{23}{35}\Big],$$

$$\Big[\frac{15\,RootOf\left(\_Z^3+242,\,index=3\right)}{t}-\frac{23}{35},\frac{15\,RootOf\left(\_Z^3+242,\,index=1\right)}{t}-\frac{23}{35}\Big],$$

$$\Big[\frac{15\,RootOf\left(\_Z^3+242,\,index=3\right)}{t}-\frac{23}{35},\frac{15\,RootOf\left(\_Z^3+242,\,index=2\right)}{t}-\frac{23}{35}\Big]\Big],$$

$$\Big[\Big[-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}-\frac{23}{105},-\frac{2^{1/3}}{t^{1/3}}-\frac{23}{105}\Big],\Big[\frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}-\frac{23}{105},-\frac{2^{1/3}}{t^{1/3}}$$

$$-\frac{23}{105}\Big],\Big[\frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}-\frac{23}{105},-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}-\frac{23}{105}\Big]\Big]\Big],$$

$$\Big[\Big[\frac{8\left(I\sqrt{3}\;12^{1/3}-3\,12^{1/3}\right)t^2}{3},-\frac{8\left(I\sqrt{3}\;12^{1/3}+3\,12^{1/3}\right)t^2}{3},-\frac{16\,I}{3}\sqrt{3}\;12^{1/3}t^2\Big],$$

$$\Big[15\left(RootOf\left(\_Z^3+242,\,index=2\right)-RootOf\left(\_Z^3+242,\,index=1\right)\right)t,\,15\left(RootOf\left(\_Z^3$$

$$+242,\,index=3\right)-RootOf\left(\_Z^3+242,\,index=1\right)\right)t,\,15\left(RootOf\left(\_Z^3+242,\,index=3\right)$$

$$-RootOf\left(\_Z^3+242,\,index=2\right)\right)t\Big],\Big[\frac{\left(I\sqrt{3}\;2^{1/3}+3\,2^{1/3}\right)t}{2},$$

$$-\frac{\left(I\sqrt{3}\;2^{1/3}-3\,2^{1/3}\right)t}{2},-I\,2^{1/3}\sqrt{3}\;t\Big]\Big],[[0,0,0],[0,0,0],[0,0,0]]\Big],\Big[[[x-1,$$

$$1],[x-3,3],[x-7,7]],\Big[\Big[0,\frac{12}{7},\frac{8}{5}\Big],\Big[0,\frac{6}{7},\frac{4}{5}\Big],\Big[0,\frac{18}{7},\frac{12}{5}\Big]\Big],\Big[\Big[\frac{12}{7},\frac{8}{5},$$

$$-\frac{4}{35}\Big],\Big[\frac{6}{7},\frac{4}{5},-\frac{2}{35}\Big],\Big[\frac{18}{7},\frac{12}{5},-\frac{6}{35}\Big]\Big],\Big[\Big[\Big[\frac{12}{7},0\Big],\Big[\frac{8}{5},0\Big],\Big[\frac{8}{5},\frac{12}{7}\Big]\Big],\Big[\Big[\frac{6}{7},0\Big],$$

$$\Big[\frac{4}{5},0\Big],\Big[\frac{4}{5},\frac{6}{7}\Big]\Big],\Big[\Big[\frac{18}{7},0\Big],\Big[\frac{12}{5},0\Big],\Big[\frac{12}{5},\frac{18}{7}\Big]\Big]\Big]\Big],\Big[\Big[[[x-1,1],[x-3,3],[x$$

$$-7,7]],\Big[\Big[0,\frac{12}{7},\frac{8}{5}\Big],\Big[0,\frac{6}{7},\frac{4}{5}\Big],\Big[0,\frac{18}{7},\frac{12}{5}\Big]\Big],\Big[\Big[\Big[\frac{12}{7},\frac{8}{5},-\frac{4}{35}\Big],[\,]\Big],\Big[\Big[\frac{6}{7},$$

$$\frac{4}{5},-\frac{2}{35}\Big],[\,]\Big],\Big[\Big[\frac{18}{7},\frac{12}{5},-\frac{6}{35}\Big],[\,]\Big]\Big],[\,],[\,]\Big],[[[x^4-60\,x^3+830\,x^2-3852\,x$$

$$+5193, \mathit{RootOf}\left(\_Z^4-60\,\_Z^3+830\,\_Z^2-3852\,\_Z+5193\right)\,]],\,[\,[0,2,4\,]],\,[\,[2,4,2\,]],$$

$$[\,[\,[2,0\,],\,[4,0\,],\,[4,2\,]\,]\,],\,\left[\,[\,[x-1,1\,],\,[x-3,3\,],\,[x-9,9\,],\,[x-12,12\,],\,[\,\infty,\,\infty\,],\,[x\right.$$

$$\left.-7,7\,]\,],\,\left[\left[0,\,\frac{12}{7},\,\frac{8}{5}\right],\left[0,\,\frac{6}{7},\,\frac{4}{5}\right],\left[\frac{16\,12^{1/3}}{3\,t^{2/3}}-\frac{46}{105},\,\frac{64\,12^{1/3}}{3\left(\mathrm{I}\sqrt{3}-1\right)^2 t^{2/3}}-\frac{46}{105},\right.\right.$$

$$\left.\left.\frac{64\,12^{1/3}}{3\left(\mathrm{I}\sqrt{3}+1\right)^2 t^{2/3}}-\frac{46}{105}\right],\left[\frac{15\,\mathit{RootOf}\left(\_Z^3+242,\mathit{index}=1\right)}{t}-\frac{23}{35},\right.\right.$$

$$\left.\left.\frac{15\,\mathit{RootOf}\left(\_Z^3+242,\mathit{index}=2\right)}{t}-\frac{23}{35},\,\frac{15\,\mathit{RootOf}\left(\_Z^3+242,\mathit{index}=3\right)}{t}-\frac{23}{35}\right],\left[\right.\right.$$

$$\left.\left.-\frac{2^{1/3}}{t^{1/3}}-\frac{23}{105},\,-\frac{2\,2^{1/3}}{\left(\mathrm{I}\sqrt{3}-1\right)t^{1/3}}-\frac{23}{105},\,\frac{2\,2^{1/3}}{\left(\mathrm{I}\sqrt{3}+1\right)t^{1/3}}-\frac{23}{105}\right],\left[0,\,\frac{18}{7},\right.\right.$$

$$\left.\left.\frac{12}{5}\right]\right],\left[\left[\left[\frac{12}{7},\,\frac{8}{5},\,-\frac{4}{35}\right],\left[\frac{6}{7},\,\frac{4}{5},\,-\frac{2}{35}\right],\left[\frac{64\,12^{1/3}}{3\left(\mathrm{I}\sqrt{3}-1\right)^2 t^{2/3}}-\frac{16\,12^{1/3}}{3\,t^{2/3}},\right.\right.\right.$$

$$\left.\left.\left.\frac{64\,12^{1/3}}{3\left(\mathrm{I}\sqrt{3}+1\right)^2 t^{2/3}}-\frac{16\,12^{1/3}}{3\,t^{2/3}},\,\frac{64\,12^{1/3}}{3\left(\mathrm{I}\sqrt{3}+1\right)^2 t^{2/3}}-\frac{64\,12^{1/3}}{3\left(\mathrm{I}\sqrt{3}-1\right)^2 t^{2/3}}\right],\right.$$

$$\left[\frac{15\,\mathit{RootOf}\left(\_Z^3+242,\mathit{index}=2\right)}{t}-\frac{15\,\mathit{RootOf}\left(\_Z^3+242,\mathit{index}=1\right)}{t},\right.$$

$$\frac{15\,\mathit{RootOf}\left(\_Z^3+242,\mathit{index}=3\right)}{t}-\frac{15\,\mathit{RootOf}\left(\_Z^3+242,\mathit{index}=1\right)}{t},$$

$$\left.\frac{15\,\mathit{RootOf}\left(\_Z^3+242,\mathit{index}=3\right)}{t}-\frac{15\,\mathit{RootOf}\left(\_Z^3+242,\mathit{index}=2\right)}{t}\right],\left[\right.$$

$$-\frac{2\,2^{1/3}}{\left(\mathrm{I}\sqrt{3}-1\right)t^{1/3}}+\frac{2^{1/3}}{t^{1/3}},\,\frac{2\,2^{1/3}}{\left(\mathrm{I}\sqrt{3}+1\right)t^{1/3}}+\frac{2^{1/3}}{t^{1/3}},\,\frac{2\,2^{1/3}}{\left(\mathrm{I}\sqrt{3}+1\right)t^{1/3}}$$

$$\left.+\frac{2\,2^{1/3}}{\left(\mathrm{I}\sqrt{3}-1\right)t^{1/3}}\right],\left[\frac{18}{7},\,\frac{12}{5},\,-\frac{6}{35}\right]\Bigg],\left[\left[\left[\frac{12}{7},0\right],\left[\frac{8}{5},0\right],\left[\frac{8}{5},\,\frac{12}{7}\right]\right],\left[\left[\frac{6}{7},0\right],\right.\right.$$

$$\left.\left.\left[\frac{4}{5},0\right],\left[\frac{4}{5},\,\frac{6}{7}\right]\right],\left[\left[\frac{64\,12^{1/3}}{3\left(\mathrm{I}\sqrt{3}-1\right)^2 t^{2/3}}-\frac{46}{105},\,\frac{16\,12^{1/3}}{3\,t^{2/3}}-\frac{46}{105}\right],\right.\right.$$

$$\left.\left.\left[\frac{64\,12^{1/3}}{3\left(\mathrm{I}\sqrt{3}+1\right)^2 t^{2/3}}-\frac{46}{105},\,\frac{16\,12^{1/3}}{3\,t^{2/3}}-\frac{46}{105}\right],\left[\frac{64\,12^{1/3}}{3\left(\mathrm{I}\sqrt{3}+1\right)^2 t^{2/3}}-\frac{46}{105},\right.\right.\right.$$

$$\left.\left.\left.\frac{64\,12^{1/3}}{3\left(\mathrm{I}\sqrt{3}-1\right)^2 t^{2/3}}-\frac{46}{105}\right]\right],\left[\left[\frac{15\,\mathit{RootOf}\left(\_Z^3+242,\mathit{index}=2\right)}{t}-\frac{23}{35},\right.\right.$$

$$\frac{15\,RootOf\left(\_Z^3+242,\,index=1\right)}{t}-\frac{23}{35}\Bigg],\left[\frac{15\,RootOf\left(\_Z^3+242,\,index=3\right)}{t}-\frac{23}{35},$$

$$\frac{15\,RootOf\left(\_Z^3+242,\,index=1\right)}{t}-\frac{23}{35}\Bigg],\left[\frac{15\,RootOf\left(\_Z^3+242,\,index=3\right)}{t}-\frac{23}{35},$$

$$\frac{15\,RootOf\left(\_Z^3+242,\,index=2\right)}{t}-\frac{23}{35}\Bigg]\Bigg],\left[\left[-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}-\frac{23}{105},-\frac{2^{1/3}}{t^{1/3}}\right.\right.$$

$$\left.-\frac{23}{105}\right],\left[\frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}-\frac{23}{105},-\frac{2^{1/3}}{t^{1/3}}-\frac{23}{105}\right],\left[\frac{2\,2^{1/3}}{\left(I\sqrt{3}+1\right)t^{1/3}}-\frac{23}{105},\right.$$

$$\left.\left.-\frac{2\,2^{1/3}}{\left(I\sqrt{3}-1\right)t^{1/3}}-\frac{23}{105}\right]\right],\left[\left[\frac{18}{7},0\right],\left[\frac{12}{5},0\right],\left[\frac{12}{5},\frac{18}{7}\right]\right]\Bigg],[[1,1,1],[1,1,1],$$

$$[3,3,3],[1,1,1],[3,3,3],[1,1,1]]\Bigg]\Bigg]$$

```
> F1:= Sirr0F2info1(L,R1[1],R1[2],x,t,ext);
```

$$F1:=\left[\left[\left[9,x-9,\left[\frac{2048}{9\,(x-9)^2}\right],2,\varnothing,\varnothing\right],\left[12,x-12,\left[-\frac{30250}{(x-12)^3}\right],3,\{RootOf\left(\_Z^3\right.\right.\right.\right. \tag{158}$$

$$\left.+242,\,index=1\right),RootOf\left(\_Z^3+242,\,index=2\right)\},\{RootOf\left(\_Z^3+242,\,index=1\right),$$

$$RootOf\left(\_Z^3+242,\,index=2\right)\}],\left[\infty,\frac{1}{x},[2\,x],1,\varnothing,\varnothing\right]\Bigg],6,6,(x-9)^2\,(x-12)^3,(x$$

$$\left.-9)\,(x-12)^2\right]$$

```
> find0F2Rat(L,R1,F1,x,t,T,ext);
```

$$\left[\left[\left[\left[\frac{1}{5},\frac{1}{7}\right]\right],-\frac{2\,(x-1)^2\,(x-3)\,(x-7)^3}{(x-9)^2\,(x-12)^3}\right]\right] \tag{159}$$

```
> TIME := time();
  Hyp0F2Solutions(L);
  time()-TIME;
```

$$TIME:=170.968$$

$$\left\{\left[\left\{\left[\left[\frac{1}{5},\frac{1}{7}\right],[0],[1]\right]\right\},\frac{2\,(x-1)^2\,(x-3)\,(x-7)^3}{(x-9)^2\,(x-12)^3}\right]\right\}$$

$$0.703 \tag{160}$$