```
> restart:
> read "ODE3solve.mpl":
```

Package "Solving third-order holonomic differential equations", Maple 16
Copyright 2017, Mouafo Wouodjie Merlin, University of Kassel
Package "Hypergeometric Summation", Maple V - Maple 17
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Here are the Maple implementations related just to the differential of the 1F1 square functions.

```
>                  ####### THE EXPONENT DIFFERENCES #######
```

For the "Exponent differences", we have the following Maple implementations:

```
> L11 := x^2*Dx^3+3*x*(-x+b+1)*Dx^2-(-2*x^2+4*x*(a+b+2)-b*(2*b+3)
  -1)*Dx-2*a*(-2*x+2*b+1)*(a+1);
```

$$L11 := x^2 Dx^3 + 3x(-x+b+1)Dx^2 - \left(-2x^2 + 4x(a+b+2) - b(2b+3) - 1\right)Dx - 2a(-2x+2b+1)(a+1) \quad \textbf{(2)}$$

```
> gen_exp(L11,t,x=0);
```

$$[[0, t=x], [-b, t=x], [-2b, t=x]] \quad \textbf{(3)}$$

```
> gen_exp(L11,t,x=infinity);
```

$$\left[\left[2a^2+2a, t=\frac{1}{x}\right], \left[-\frac{1}{t} - 4a^2 + b + 5, t=\frac{1}{x}\right], \left[-\frac{2}{t} + 2a^2 - 2a + 2b - 2, t=\frac{1}{x}\right]\right] \quad \textbf{(4)}$$

```
>                  ####### EXAMPLE NOT IN THE THESIS #######
```

Those are the Maple implementations for examples related to the differential of the 1F1 square type solutions:

```
> LA:=MinOp(diff(hypergeom([a],[b],x),x)^2);
```

$$LA := Dx^3 + \frac{3(-x+b+1)Dx^2}{x} - \frac{\left(4xa - 2b^2 + 4bx - 2x^2 - 3b + 8x - 1\right)Dx}{x^2} - \frac{2(-2x+2b+1)(a+1)}{x^2} \quad \textbf{(5)}$$

```
> L1:=subs(b=1,LA);
```

$$L1 := Dx^3 + \frac{3(-x+2)Dx^2}{x} - \frac{\left(4xa - 2x^2 + 12x - 6\right)Dx}{x^2} - \frac{2(-2x+3)(a+1)}{x^2} \quad \textbf{(6)}$$

```
> f:=(x-7)^5/((x-1)*(x-3));
```

$$f := \frac{(x-7)^5}{(x-1)(x-3)} \quad \textbf{(7)}$$

```
> L:=ChangeOfVariables(L1,f);
```

$$L := Dx^3\,(x-1)^5\,(x-3)^5\,(x-7)^2\,\left(3\,x^2-2\,x-13\right)^2-3\,\left(9\,x^9-327\,x^8+4756\,x^7 - 34120\,x^6+111358\,x^5-21854\,x^4-783388\,x^3+1290992\,x^2+1152385\,x-2839555\right) Dx^2\,(x-1)^3\,(x-3)^3\,(x-7)\,\left(3\,x^2-2\,x-13\right)-2\,\left(-81\,x^{18}+5886\,x^{17}+162\,a\,x^{15} - 192537\,x^{16}-6750\,a\,x^{14}+3724773\,x^{15}+117018\,a\,x^{13}-46946335\,x^{14} - 1065558\,a\,x^{12}+397907849\,x^{13}+5078426\,a\,x^{11}-2224863625\,x^{12}-8018246\,a\,x^{10} + 7277629969\,x^{11}-35168430\,a\,x^9-5629725401\,x^{10}+180214338\,a\,x^8 - 60083935295\,x^9-135944058\,a\,x^7+249394747605\,x^8-786413754\,a\,x^6 - 216111728961\,x^7+1622885550\,a\,x^5-1008500660445\,x^6+833748510\,a\,x^4 + 2706154165243\,x^5-4508274498\,a\,x^3+17916353989\,x^4+1587947998\,a\,x^2 - 7429891265525\,x^3+4125047654\,x\,a+6001354618790\,x^2-2880148362\,a + 6554732399357\,x-8068568860792\right) Dx\,(x-1)\,(x-3)+2\,\left(2\,x^5-70\,x^4+980\,x^3 - 6863\,x^2+24022\,x-33623\right)(a+1)\,(x-7)^4\,\left(3\,x^2-2\,x-13\right)^5 \quad \textbf{(8)}$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \{a\} \quad \textbf{(9)}$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```

$$ext := \{a\} \quad \textbf{(10)}$$

```
> extppp:={};
```

$$extppp := \emptyset \quad \textbf{(11)}$$

```
> E:= Singular(L,extppp);
```

$$E := \left[[x-7,7],\,[\infty,\infty],\,[x-1,1],\,\left[x^2-\frac{2}{3}\,x-\frac{13}{3},\,RootOf\left(3\,\_Z^2-2\,\_Z-13\right)\right],\,[x-3,3]\right] \quad \textbf{(12)}$$

```
> F:= NotAppSing(L,E,ext);
```

$$F := [[x-3,3],\,[x-7,7],\,[\infty,\infty],\,[x-1,1]] \quad \textbf{(13)}$$

```
> Sirr:=irrsingdiff1F1sq(L,t,F,ext);
```

$$Sirr := \Bigg[[[x-3,3],\,[\infty,\infty],\,[x-1,1]],\,\Bigg[\left[2\,a+2,\,\frac{512}{t}+2,\,\frac{1024}{t}-2\,a+2\right],\,\left[6\,a+6, -\frac{6}{t^3}+\frac{124}{t^2}-\frac{726}{t}-6\,a+6,\,-\frac{3}{t^3}+\frac{62}{t^2}-\frac{363}{t}+6\right],\,\left[2\,a+2,\,-\frac{7776}{t}-2\,a+2, -\frac{3888}{t}+2\right]\Bigg],\,\Bigg[\left[\frac{512}{t}-2\,a,\,\frac{512}{t}-2\,a,\,\frac{1024}{t}-4\,a\right],\,\left[-\frac{3}{t^3}+\frac{62}{t^2}-\frac{363}{t}-6\,a,\,\frac{3}{t^3} - \frac{62}{t^2}+\frac{363}{t}+6\,a,\,-\frac{6}{t^3}+\frac{124}{t^2}-\frac{726}{t}-12\,a\right],\,\left[-\frac{3888}{t}-2\,a,\,\frac{3888}{t}+2\,a, -\frac{7776}{t}-4\,a\right]\Bigg],\,[1,3,1],\,[1,1,1],\,\Bigg[\Bigg[\left[\frac{512}{t}+2,\,2\,a+2\right],\,\left[\frac{1024}{t}-2\,a+2,\,\frac{512}{t} +2\right],\,\left[\frac{1024}{t}-2\,a+2,\,2\,a+2\right]\Bigg],\,\Bigg[\left[-\frac{3}{t^3}+\frac{62}{t^2}-\frac{363}{t}+6,\,6\,a+6\right],\,\left[-\frac{3}{t^3}+\frac{62}{t^2} -\frac{363}{t}+6,\,-\frac{6}{t^3}+\frac{124}{t^2}-\frac{726}{t}-6\,a+6\right],\,\left[-\frac{6}{t^3}+\frac{124}{t^2}-\frac{726}{t}-6\,a+6,\,6\,a \quad \textbf{(14)}$$

$+6\Big]\Big]$, $\Big[\Big[-\frac{3888}{t}+2, 2a+2\Big], \Big[-\frac{3888}{t}+2, -\frac{7776}{t}-2a+2\Big], \Big[-\frac{7776}{t}-2a+2,$
$2a+2\Big]\Big]\Big]$, $[[512\,t, 512\,t, 1024\,t], [-3\,t^3+62\,t^2-363\,t, 3\,t^3-62\,t^2+363\,t, -6\,t^3$
$+124\,t^2-726\,t], [-3888\,t, 3888\,t, -7776\,t]], [[-2\,a, -2\,a, -4\,a], [-6\,a, 6\,a, -12\,a], [$
$-2\,a, 2\,a, -4\,a]], [[[x-7, 7]], [[[-10, -5, 0], [5, 10, 5], [1, 1, 1], [[-5, -10], [0,$
$-10], [0, -5]], 4]]]]$

```
> Sreg:=regsingtruediff1F1sq(L,t,Sirr[-1],ext);
```

$$Sreg := [[[x-7, 7]], [[-10, -5, 0]], [[5, 5, 10]], [[[-5, -10], [0, -5], [0, -10]]]] \quad (15)$$

```
> RSreg:=Sregseptruediff1F1sq(L,Sreg,ext);
```

$$RSreg := [[\,], [\,], [[[x-7, 7]], [[-10, -5, 0]], [[[\,], [5, 5, 10]]]]] \quad (16)$$

```
> R1:=IrrRegAppsingdiff1F1sq(L,t,E,ext);
```

$R1 := \Big[\Big[[[x-3, 3], [\infty, \infty], [x-1, 1]], \Big[\Big[2a+2, \frac{512}{t}+2, \frac{1024}{t}-2a+2\Big], \Big[6a+6,$ **(17)**

$-\frac{6}{t^3}+\frac{124}{t^2}-\frac{726}{t}-6a+6, -\frac{3}{t^3}+\frac{62}{t^2}-\frac{363}{t}+6\Big], \Big[2a+2, -\frac{7776}{t}-2a+2,$
$-\frac{3888}{t}+2\Big]\Big], \Big[\Big[\frac{512}{t}-2a, \frac{512}{t}-2a, \frac{1024}{t}-4a\Big], \Big[-\frac{3}{t^3}+\frac{62}{t^2}-\frac{363}{t}-6a, \frac{3}{t^3}$
$-\frac{62}{t^2}+\frac{363}{t}+6a, -\frac{6}{t^3}+\frac{124}{t^2}-\frac{726}{t}-12a\Big], \Big[-\frac{3888}{t}-2a, \frac{3888}{t}+2a,$
$-\frac{7776}{t}-4a\Big]\Big], [1, 3, 1], [1, 1, 1], \Big[\Big[\Big[\frac{512}{t}+2, 2a+2\Big], \Big[\frac{1024}{t}-2a+2, \frac{512}{t}$
$+2\Big], \Big[\frac{1024}{t}-2a+2, 2a+2\Big]\Big], \Big[\Big[-\frac{3}{t^3}+\frac{62}{t^2}-\frac{363}{t}+6, 6a+6\Big], \Big[-\frac{3}{t^3}+\frac{62}{t^2}$
$-\frac{363}{t}+6, -\frac{6}{t^3}+\frac{124}{t^2}-\frac{726}{t}-6a+6\Big], \Big[-\frac{6}{t^3}+\frac{124}{t^2}-\frac{726}{t}-6a+6, 6a$
$+6\Big]\Big], \Big[\Big[-\frac{3888}{t}+2, 2a+2\Big], \Big[-\frac{3888}{t}+2, -\frac{7776}{t}-2a+2\Big], \Big[-\frac{7776}{t}-2a+2,$
$2a+2\Big]\Big]\Big], [[512\,t, 512\,t, 1024\,t], [-3\,t^3+62\,t^2-363\,t, 3\,t^3-62\,t^2+363\,t, -6\,t^3$
$+124\,t^2-726\,t], [-3888\,t, 3888\,t, -7776\,t]], [[-2\,a, -2\,a, -4\,a], [-6\,a, 6\,a, -12\,a], [$
$-2\,a, 2\,a, -4\,a]]], [[[x-7, 7]], [[-10, -5, 0]], [[5, 5, 10]], [[[-5, -10], [0, -5],$
$[0, -10]]]], [[\,], [\,], [[[x-7, 7]], [[-10, -5, 0]], [[[\,], [5, 10, 5]]]]]], \Big[\Big[\Big[\Big[x^2-\frac{2}{3}\,x$
$-\frac{13}{3}, RootOf(3\_Z^2-2\_Z-13)\Big]\Big], [[0, 2, 4]], [[2, 4, 2]], [[[2, 0], [4, 0], [4, 2]]]\Big],$
$\Big[[[x-3, 3], [x-7, 7], [\infty, \infty], [x-1, 1]], \Big[\Big[2a+2, \frac{512}{t}+2, \frac{1024}{t}-2a+2\Big],$

$[-10, -5, 0], \left[6a+6, -\frac{6}{t^3}+\frac{124}{t^2}-\frac{726}{t}-6a+6, -\frac{3}{t^3}+\frac{62}{t^2}-\frac{363}{t}+6\right], \left[2a+2, -\frac{7776}{t}-2a+2, -\frac{3888}{t}+2\right]\Big]\Big], \left[\left[\frac{512}{t}-2a, \frac{512}{t}-2a, \frac{1024}{t}-4a\right], [5, 10, 5], \left[-\frac{3}{t^3}+\frac{62}{t^2}-\frac{363}{t}-6a, \frac{3}{t^3}-\frac{62}{t^2}+\frac{363}{t}+6a, -\frac{6}{t^3}+\frac{124}{t^2}-\frac{726}{t}-12a\right], \left[-\frac{3888}{t}-2a, \frac{3888}{t}+2a, -\frac{7776}{t}-4a\right]\right], \left[\left[\left[\frac{512}{t}+2, 2a+2\right], \left[\frac{1024}{t}-2a+2, \frac{512}{t}+2\right], \left[\frac{1024}{t}-2a+2, 2a+2\right]\right], [[-5, -10], [0, -10], [0, -5]], \left[\left[-\frac{3}{t^3}+\frac{62}{t^2}-\frac{363}{t}+6, 6a+6\right], \left[-\frac{3}{t^3}+\frac{62}{t^2}-\frac{363}{t}+6, -\frac{6}{t^3}+\frac{124}{t^2}-\frac{726}{t}-6a+6\right], \left[-\frac{6}{t^3}+\frac{124}{t^2}-\frac{726}{t}-6a+6, 6a+6\right]\right], \left[\left[-\frac{3888}{t}+2, 2a+2\right], \left[-\frac{3888}{t}+2, -\frac{7776}{t}-2a+2\right], \left[-\frac{7776}{t}-2a+2, 2a+2\right]\right]\right], [[1, 1, 1], [1, 1, 1], [1, 1, 1], [1, 1, 1]]\Big]\Big]$

```
> F1:=Hypdiff1F1sqSubst(L,x,t,R1[1],ext);
```

$$F1 := \left[-\frac{x^5-35x^4+490x^3-1545x^2+5489x-12176}{(x-1)(x-3)}, -\frac{x^5-35x^4+490x^3-1545x^2-2287x+11152}{(x-1)(x-3)}, \frac{x^5-35x^4+490x^3-1545x^2-3311x+12176}{(x-1)(x-3)}, \frac{x^5-35x^4+490x^3-1545x^2+4465x-11152}{(x-1)(x-3)}, -\frac{x^5-35x^4+490x^3-1545x^2+4465x-11152}{(x-1)(x-3)}, -\frac{x^5-35x^4+490x^3-1545x^2-3311x+12176}{(x-1)(x-3)}, \frac{x^5-35x^4+490x^3-1545x^2-2287x+11152}{(x-1)(x-3)}, \frac{x^5-35x^4+490x^3-1545x^2+5489x-12176}{(x-1)(x-3)}\right] \quad \textbf{(18)}$$

```
> finddiff1F1sqln(L,R1,F1,x,t,ext);
```

$$\left[\left[\left[\left\{a, -a, \frac{1}{2}-a, \frac{1}{2}+a\right\}, [1]\right], -\frac{(x-7)^5}{(x-1)(x-3)}\right], \left[\left[\left\{a, -a, \frac{1}{2}-a, \frac{1}{2}+a\right\}, [1]\right], \frac{(x-7)^5}{(x-1)(x-3)}\right], \left[\left[\left\{a, -a, \frac{1}{2}-a, \frac{1}{2}+a\right\}, [1]\right], \right. \right. \quad \textbf{(19)}$$

$$\frac{(x-7)\left(x^4-28x^3+294x^2-332x-1231\right)}{(x-1)(x-3)}\Big], \Big[\Big[\Big[\Big\{a, -a, \frac{1}{2}-a, \frac{1}{2}+a\Big\}, [1]\Big],$$
$$-\frac{(x-7)\left(x^4-28x^3+294x^2-76x-1487\right)}{(x-1)(x-3)}\Big], \Big[\Big[\Big[\Big\{a, -a, \frac{1}{2}-a, \frac{1}{2}+a\Big\}, [1]\Big],$$
$$-\frac{(x-7)\left(x^4-28x^3+294x^2-1628x+2657\right)}{(x-1)(x-3)}\Big], \Big[\Big[\Big[\Big\{a, -a, \frac{1}{2}-a, \frac{1}{2}+a\Big\}, [1]\Big],$$
$$-\frac{(x-7)\left(x^4-28x^3+294x^2-332x-1231\right)}{(x-1)(x-3)}\Big], \Big[\Big[\Big[\Big\{a, -a, \frac{1}{2}-a, \frac{1}{2}+a\Big\}, [1]\Big],$$
$$\frac{(x-7)\left(x^4-28x^3+294x^2-76x-1487\right)}{(x-1)(x-3)}\Big], \Big[\Big[\Big[\Big\{a, -a, \frac{1}{2}-a, \frac{1}{2}+a\Big\}, [1]\Big],$$
$$\frac{(x-7)\left(x^4-28x^3+294x^2-1628x+2657\right)}{(x-1)(x-3)}\Big]\Big]$$

```
> TIME :=time();
  Hypdiff1F1sqSolutions(L);
  time() - TIME;
```

$$TIME := 11.187$$

$$\Big\{\Big[[[[a], 1, [0], [1]]], \frac{(x-7)^5}{(x-1)(x-3)}\Big], \Big[\Big[\Big[[-a], 1,$$
$$\Big[\frac{6x^7-214x^6+3054x^5-21635x^4+73050x^3-59792x^2-244782x+436937}{(x-7)(x-1)^2(x-3)^2}\Big], [x^5$$
$$-35x^4+490x^3-3430x^2+12005x-16807]\Big]\Big], -\frac{(x-7)^5}{(x-1)(x-3)}\Big]\Big\}$$

$$2.641 \tag{20}$$

```
>                    ####### THE INTEGER CASE #######
```

```
> LA:=MinOp(diff(hypergeom([a],[b],x),x)^2);
```

$$LA := Dx^3 + \frac{3(-x+b+1)Dx^2}{x} - \frac{\left(4xa-2b^2+4bx-2x^2-3b+8x-1\right)Dx}{x^2} - \frac{2(-2x+2b+1)(a+1)}{x^2} \tag{21}$$

```
> L1:=subs({a=1/3,b=1/2},LA);
```

$$L1 := Dx^3 + \frac{3\left(-x+\frac{3}{2}\right)Dx^2}{x} - \frac{\left(\frac{34}{3}x-3-2x^2\right)Dx}{x^2} - \frac{8(-2x+2)}{3x^2} \tag{22}$$

```
> f:=(x-1)^2/x;
```

$$f := \frac{(x-1)^2}{x} \tag{23}$$

```
> L:=ChangeOfVariables(L1,f);
```

$$L := 6\,Dx^3 x^5 (x-1)^2 (x+1)^2 - 9\left(2x^4-3x^3-10x^2+x+2\right)Dx^2 x^3 (x-1)(x+1) \tag{24}$$

$+2\left(6x^8-34x^7-83x^6+88x^5+217x^4-20x^3-92x^2-16x+6\right)Dx\,x+32\left(x^2-3x+1\right)(x+1)^5(x-1)$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \varnothing \tag{25}$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```

$$ext := \varnothing \tag{26}$$

```
> extppp:={};
```

$$extppp := \varnothing \tag{27}$$

```
> E:= Singular(L,extppp);
```

$$E := [[x, 0], [x+1, -1], [\infty, \infty], [x-1, 1]] \tag{28}$$

```
> F:= NotAppSing(L,E,ext);
```

$$F := [[x, 0], [\infty, \infty], [x-1, 1]] \tag{29}$$

```
> Sirr:=irrsingdiff1F1sq(L,t,F,ext);
```

$$Sirr := \left[[[x,0],[\infty,\infty]], \left[\left[\frac{8}{3}, -\frac{1}{t}+\frac{3}{2}, -\frac{2}{t}+\frac{1}{3}\right], \left[\frac{8}{3}, -\frac{1}{t}+\frac{3}{2}, -\frac{2}{t}+\frac{1}{3}\right]\right], \left[\left[-\frac{1}{t}-\frac{7}{6}, -\frac{1}{t}-\frac{7}{6}, -\frac{2}{t}-\frac{7}{3}\right], \left[-\frac{1}{t}-\frac{7}{6}, -\frac{1}{t}-\frac{7}{6}, -\frac{2}{t}-\frac{7}{3}\right]\right], [1,1], [1,1], \left[\left[\left[-\frac{1}{t}+\frac{3}{2}, \frac{8}{3}\right], \left[-\frac{2}{t}+\frac{1}{3}, -\frac{1}{t}+\frac{3}{2}\right], \left[-\frac{2}{t}+\frac{1}{3}, \frac{8}{3}\right]\right], \left[\left[-\frac{1}{t}+\frac{3}{2}, \frac{8}{3}\right], \left[-\frac{2}{t}+\frac{1}{3}, -\frac{1}{t}+\frac{3}{2}\right], \left[-\frac{2}{t}+\frac{1}{3}, \frac{8}{3}\right]\right]\right], [[-t,-t,-2t],[-t,-t,-2t]], \left[\left[-\frac{7}{6}, -\frac{7}{6}, -\frac{7}{3}\right], \left[-\frac{7}{6}, -\frac{7}{6}, -\frac{7}{3}\right]\right], [[[x-1,1]], [[[-2,-1,0],[1,2,1],[1,1,1],[[-1,-2],[0,-2],[0,-1]],4]]]\right] \tag{30}$$

```
> Sreg:=regsingtruediff1F1sq(L,t,Sirr[-1],ext);
```

$$Sreg := [\ ] \tag{31}$$

```
> RSreg:=Sregseptruediff1F1sq(L,Sreg,ext);
```

$$RSreg := [[\ ],[\ ],[\ ]] \tag{32}$$

```
> R1:=IrrRegAppsingdiff1F1sq(L,t,E,ext);
```

$$R1 := \left[\left[[[x,0],[\infty,\infty]], \left[\left[\frac{8}{3}, -\frac{1}{t}+\frac{3}{2}, -\frac{2}{t}+\frac{1}{3}\right], \left[\frac{8}{3}, -\frac{1}{t}+\frac{3}{2}, -\frac{2}{t}+\frac{1}{3}\right]\right], \left[\left[-\frac{1}{t}-\frac{7}{6}, -\frac{1}{t}-\frac{7}{6}, -\frac{2}{t}-\frac{7}{3}\right], \left[-\frac{1}{t}-\frac{7}{6}, -\frac{1}{t}-\frac{7}{6}, -\frac{2}{t}-\frac{7}{3}\right]\right], [1,1], [1,1], \left[\left[\left[-\frac{1}{t}+\frac{3}{2}, \frac{8}{3}\right], \left[-\frac{2}{t}+\frac{1}{3}, -\frac{1}{t}+\frac{3}{2}\right], \left[-\frac{2}{t}+\frac{1}{3}, \frac{8}{3}\right]\right], \left[\left[-\frac{1}{t}+\frac{3}{2}, \frac{8}{3}\right], \left[-\frac{2}{t}+\frac{1}{3}, -\frac{1}{t}+\frac{3}{2}\right], \left[-\frac{2}{t}+\frac{1}{3}, \frac{8}{3}\right]\right]\right], [[-t,-t,-2t],[-t,-t,-2t]], \left[\left[-\frac{7}{6}, -\frac{7}{6}, -\frac{7}{3}\right], \left[-\frac{7}{6}, -\frac{7}{6}, -\frac{7}{3}\right]\right]\right], [\ ], [[\ ],[\ ],[\ ]], [[[x+1,-1],[x-1,1]], [[0,2,4],[-2,-1,0]], [[2,4,2],[1,2,1]], [[[2,0],[4,0],[4,2]], [[-1,-2],[0,-2],[0,-1]]]]], \left[[[x,0],[\infty,\infty],[x-1,1]], \left[\left[\frac{8}{3}, -\frac{1}{t}+\frac{3}{2}, -\frac{2}{t}+\frac{1}{3}\right], \left[\frac{8}{3}, -\frac{1}{t}+\frac{3}{2}, -\frac{2}{t}\right.\right.\right. \tag{33}$$

$$+\frac{1}{3}\Big],[-2,-1,0]\Big],\Big[\Big[-\frac{1}{t}-\frac{7}{6},-\frac{1}{t}-\frac{7}{6},-\frac{2}{t}-\frac{7}{3}\Big],\Big[-\frac{1}{t}-\frac{7}{6},-\frac{1}{t}-\frac{7}{6},-\frac{2}{t}$$
$$-\frac{7}{3}\Big],[1,2,1]\Big],\Big[\Big[\Big[-\frac{1}{t}+\frac{3}{2},\frac{8}{3}\Big],\Big[-\frac{2}{t}+\frac{1}{3},-\frac{1}{t}+\frac{3}{2}\Big],\Big[-\frac{2}{t}+\frac{1}{3},\frac{8}{3}\Big]\Big],\Big[\Big[-\frac{1}{t}$$
$$+\frac{3}{2},\frac{8}{3}\Big],\Big[-\frac{2}{t}+\frac{1}{3},-\frac{1}{t}+\frac{3}{2}\Big],\Big[-\frac{2}{t}+\frac{1}{3},\frac{8}{3}\Big]\Big],[[-1,-2],[0,-2],[0,-1]]\Big],$$
$$[[1,1,1],[1,1,1],[1,1,1]]\Big]\Big]$$

```
> F1:=Hypdiff1F1sqSubst(L,x,t,R1[1],ext);
```

$$F1 := \left[-\frac{x^2+1}{x},\frac{x^2-1}{x},-\frac{x^2-1}{x},\frac{x^2+1}{x}\right] \quad \textbf{(34)}$$

```
> finddiff1F1sqInt(L,R1,F1,x,t,ext);
```

$$\Big[\Big[\Big[\Big\{\frac{1}{3},\frac{1}{6},\frac{2}{3},\frac{5}{6}\Big\},\Big[\frac{1}{2}\Big]\Big],\frac{(x-1)^2}{x}\Big],\Big[\Big[\Big\{\frac{1}{3},\frac{1}{6},\frac{2}{3},\frac{5}{6}\Big\},\Big[\frac{1}{2}\Big]\Big],\frac{(x+1)^2}{x}\Big],\Big[\Big[\Big\{\frac{1}{3},$$
$$\frac{1}{6},\frac{2}{3},\frac{5}{6}\Big\},\Big[\frac{1}{2}\Big]\Big],-\frac{(x+1)^2}{x}\Big],\Big[\Big[\Big\{\frac{1}{3},\frac{1}{6},\frac{2}{3},\frac{5}{6}\Big\},\Big[\frac{1}{2}\Big]\Big],-\frac{(x-1)^2}{x}\Big]\Big] \quad \textbf{(35)}$$

```
> TIME :=time();
  Hypdiff1F1sqSolutions(L);
  time() - TIME;
```

$$TIME := 97.046$$

$$\Big\{\Big[\Big[\Big[\Big[\frac{1}{3}\Big],\frac{1}{2},[0],[1]\Big]\Big],\frac{(x-1)^2}{x}\Big],\Big[\Big[\Big[\Big[\frac{1}{6}\Big],\frac{1}{2},\Big[\frac{2x^3-5x^2+2}{(x-1)x^2}\Big],$$
$$\Big[\frac{x^4(x-1)(x^2-2x+1)Dx^2}{(x+1)^2}$$
$$-\frac{x^2(6x^6-61x^5+54x^4+74x^3-90x^2+11x+6)Dx}{6(x+1)^3}$$
$$-\frac{x(66x^6-319x^5-11x^4+555x^3-259x^2-56x+24)}{18(x+1)^3}\Big]\Big]\Big],-\frac{(x-1)^2}{x}\Big]\Big\}$$

$$17.985 \quad \textbf{(36)}$$

```
>                 ####### THE LOGARITHMIC CASE #######
```

```
> LA:=MinOp(diff(hypergeom([a],[b],x),x)^2);
```

$$LA := Dx^3+\frac{3(-x+b+1)Dx^2}{x}-\frac{(4xa-2b^2+4bx-2x^2-3b+8x-1)Dx}{x^2}$$
$$-\frac{2(-2x+2b+1)(a+1)}{x^2} \quad \textbf{(37)}$$

```
> L1:=subs({a=1/7,b=1},LA);
```

(38)

$$L1 := Dx^3 + \frac{3\,(-x+2)\,Dx^2}{x} - \frac{\left(\frac{88}{7}\,x - 6 - 2\,x^2\right)\,Dx}{x^2} - \frac{16\,(-2\,x+3)}{7\,x^2} \quad (38)$$

```
> f:=(x-1)/(x-12);
```

$$f := \frac{x-1}{x-12} \quad (39)$$

```
> L:=ChangeOfVariables(L1,f);
```

$$L := 7\,Dx^3\,(x-12)^5\,(x-1)^2 + 21\,(2\,x^2 - 37\,x + 277)\,Dx^2\,(x-12)^3\,(x-1) + 2\,(21\,x^4 - 777\,x^3 + 10433\,x^2 - 71859\,x + 369643)\,Dx\,(x-12) + 21296\,x - 724064 \quad (40)$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \varnothing \quad (41)$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```

$$ext := \varnothing \quad (42)$$

```
> extppp:={};
```

$$extppp := \varnothing \quad (43)$$

```
> E:= Singular(L,extppp);
```

$$E := [[x-12, 12], [x-1, 1]] \quad (44)$$

```
> F:= NotAppSing(L,E,ext);
```

$$F := [[x-12, 12], [x-1, 1]] \quad (45)$$

```
> Sirr:=irrsingdiff1F1sq(L,t,F,ext);
```

$$Sirr := \left[[[x-12, 12]], \left[\left[\frac{16}{7}, -\frac{22}{t} + \frac{12}{7}, -\frac{11}{t} + 2\right]\right], \left[\left[-\frac{11}{t} - \frac{2}{7}, \frac{11}{t} + \frac{2}{7}, -\frac{22}{t} - \frac{4}{7}\right]\right], [1], [1], \left[\left[\left[-\frac{11}{t} + 2, \frac{16}{7}\right], \left[-\frac{11}{t} + 2, -\frac{22}{t} + \frac{12}{7}\right], \left[-\frac{22}{t} + \frac{12}{7}, \frac{16}{7}\right]\right]\right], [[-11\,t, 11\,t, -22\,t]], \left[\left[-\frac{2}{7}, \frac{2}{7}, -\frac{4}{7}\right]\right], [[[x-1, 1]], [[[-2, -1, 0], [1, 2, 1], [1, 1, 1], [[-1, -2], [0, -2], [0, -1]], 4]]]\right] \quad (46)$$

```
> Sreg:=regsingtruediff1F1sq(L,t,Sirr[-1],ext);
```

$$Sreg := [[[x-1, 1]], [[-2, -1, 0]], [[1, 1, 2]], [[[-1, -2], [0, -1], [0, -2]]]] \quad (47)$$

```
> RSreg:=Sregseptruediff1F1sq(L,Sreg,ext);
```

$$RSreg := [[\ ], [\ ], [[[x-1, 1]], [[-2, -1, 0]], [[[\ ], [1, 1, 2]]]]] \quad (48)$$

```
> R1:=IrrRegAppsingdiff1F1sq(L,t,E,ext);
```

$$R1 := \left[\left[[[x-12, 12]], \left[\left[\frac{16}{7}, -\frac{22}{t} + \frac{12}{7}, -\frac{11}{t} + 2\right]\right], \left[\left[-\frac{11}{t} - \frac{2}{7}, \frac{11}{t} + \frac{2}{7}, -\frac{22}{t} - \frac{4}{7}\right]\right], [1], [1], \left[\left[\left[-\frac{11}{t} + 2, \frac{16}{7}\right], \left[-\frac{11}{t} + 2, -\frac{22}{t} + \frac{12}{7}\right], \left[-\frac{22}{t} + \frac{12}{7}, \frac{16}{7}\right]\right]\right], [[-11\,t, 11\,t, -22\,t]], \left[\left[-\frac{2}{7}, \frac{2}{7}, -\frac{4}{7}\right]\right]\right], [[[x-1, 1]], [[-2, -1, 0]], [[1, 1, 2]], [[[-1, -2], [0, -1], [0, -2]]]], [[\ ], [\ ], [[[x-1, 1]], [[-2, -1, 0]], [[[\ ], [1, 2, 1]]]]], [\ ], \left[[[x-12, 12], [x-1, 1]], \left[\left[\frac{16}{7}, -\frac{22}{t} + \frac{12}{7}, -\frac{11}{t} + 2\right], [-2, -1, \right.\right. \quad (49)$$

$$0]\Big], \Big[\Big[\Big[-\frac{11}{t}-\frac{2}{7}, \frac{11}{t}+\frac{2}{7}, -\frac{22}{t}-\frac{4}{7}\Big], [1,2,1]\Big], \Big[\Big[\Big[-\frac{11}{t}+2, \frac{16}{7}\Big], \Big[-\frac{11}{t}+2, -\frac{22}{t}+\frac{12}{7}\Big], \Big[-\frac{22}{t}+\frac{12}{7}, \frac{16}{7}\Big]\Big], [[-1,-2],[0,-2],[0,-1]]\Big], [[1,1,1],[1,1,1]]\Big]\Big]$$

```
> F1:=Hypdiff1F1sqSubst(L,x,t,R1[1],ext);
```

$$F1 := \left[-\frac{11}{x-12}, \frac{11}{x-12}\right] \tag{50}$$

```
> finddiff1F1sqln(L,R1,F1,x,t,ext);
```

$$\left[\left[\left[\left\{\frac{1}{7}, \frac{5}{14}, \frac{6}{7}, \frac{9}{14}\right\}, [1]\right], -\frac{x-1}{x-12}\right], \left[\left[\left\{\frac{1}{7}, \frac{5}{14}, \frac{6}{7}, \frac{9}{14}\right\}, [1]\right], \frac{x-1}{x-12}\right]\right] \tag{51}$$

```
> TIME :=time();
  Hypdiff1F1sqSolutions(L);
  time() - TIME;
```

$$TIME := 137.234$$

$$\left\{\left[\left[\left[\left[\left[\frac{1}{7}\right], 1, [0], [1]\right]\right], \frac{x-1}{x-12}\right], \left[\left[\left[\left[\frac{6}{7}\right], 1, \left[\frac{x^2-46x+166}{(x-12)^2(x-1)}\right], \left[(x-12)^2(x-1)Dx^2 + \left(\frac{55x}{7}+\frac{187}{7}\right)Dx - \frac{11(35x^2-169x-1318)}{49(x^2-13x+12)}\right]\right]\right], -\frac{x-1}{x-12}\right]\right\}$$

$$1.578 \tag{52}$$

```
>                    ####### THE RATIONAL AND IRRATIONAL CASE #######
```

```
> LA:=MinOp(diff(hypergeom([a],[b],x),x)^2);
```

$$LA := Dx^3 + \frac{3(-x+b+1)Dx^2}{x} - \frac{(4xa-2b^2+4bx-2x^2-3b+8x-1)Dx}{x^2} - \frac{2(-2x+2b+1)(a+1)}{x^2} \tag{53}$$

```
> L1:=subs({a=1/3,b=1/2},LA);
```

$$L1 := Dx^3 + \frac{3\left(-x+\frac{3}{2}\right)Dx^2}{x} - \frac{\left(\frac{34}{3}x-3-2x^2\right)Dx}{x^2} - \frac{8(2-2x)}{3x^2} \tag{54}$$

```
> f:=(x-3)/(x-7);
```

$$f := \frac{x-3}{x-7} \tag{55}$$

```
> L:=ChangeOfVariables(L1,f);
```

$$L := -4096 + 3Dx^3(x-7)^5(x-3)^2 + 18(x^2-11x+36)Dx^2(x-7)^3(x-3) + 2(9x^4 - 198x^3 + 1576x^2 - 5434x + 7887)Dx(x-7) \tag{56}$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \varnothing \tag{57}$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```
$$ext := \varnothing \quad (58)$$

```
> extppp:={};
```
$$extppp := \varnothing \quad (59)$$

```
> E:= Singular(L,extppp);
```
$$E := [[x-7, 7], [x-3, 3]] \quad (60)$$

```
> F:= NotAppSing(L,E,ext);
```
$$F := [[x-3, 3], [x-7, 7]] \quad (61)$$

```
> Sirr:=irrsingdiff1F1sq(L,t,F,ext);
```
$$Sirr := \left[[[x-7,7]], \left[\left[\frac{8}{3}, -\frac{8}{t}+\frac{1}{3}, -\frac{4}{t}+\frac{3}{2}\right]\right], \left[\left[-\frac{4}{t}-\frac{7}{6}, \frac{4}{t}+\frac{7}{6}, -\frac{8}{t}-\frac{7}{3}\right]\right], [1], [1], \left[\left[\left[-\frac{4}{t}+\frac{3}{2}, \frac{8}{3}\right], \left[-\frac{4}{t}+\frac{3}{2}, -\frac{8}{t}+\frac{1}{3}\right], \left[-\frac{8}{t}+\frac{1}{3}, \frac{8}{3}\right]\right]\right], [[-4t, 4t, -8t]], \left[\left[-\frac{7}{6}, \frac{7}{6}, -\frac{7}{3}\right]\right], \left[[[x-3,3]], \left[\left[\left[-1, 0, -\frac{1}{2}\right], \left[1, \frac{1}{2}, -\frac{1}{2}\right], [1,1,1], \left[[0, -1], \left[-\frac{1}{2}, -1\right], \left[-\frac{1}{2}, 0\right]\right], 3\right]\right]\right]\right] \quad (62)$$

```
> Sreg:=regsingtruediff1F1sq(L,t,Sirr[-1],ext);
```
$$Sreg := \left[[[x-3,3]], \left[\left[-1, 0, -\frac{1}{2}\right]\right], \left[\left[\frac{1}{2}, -\frac{1}{2}, 1\right]\right], \left[\left[\left[-\frac{1}{2}, -1\right], \left[-\frac{1}{2}, 0\right], [0, -1]\right]\right]\right] \quad (63)$$

```
> RSreg:=Sregseptruediff1F1sq(L,Sreg,ext);
```
$$RSreg := \left[\left[[[x-3,3]], \left[\left[-1, 0, -\frac{1}{2}\right]\right], \left[\left[\left[\frac{1}{2}, -\frac{1}{2}\right], [1]\right]\right]\right], [\ ], [\ ]\right] \quad (64)$$

```
> R1:=IrrRegAppsingdiff1F1sq(L,t,E,ext);
```
$$R1 := \left[\left[[[x-7,7]], \left[\left[\frac{8}{3}, -\frac{8}{t}+\frac{1}{3}, -\frac{4}{t}+\frac{3}{2}\right]\right], \left[\left[-\frac{4}{t}-\frac{7}{6}, \frac{4}{t}+\frac{7}{6}, -\frac{8}{t}-\frac{7}{3}\right]\right], [1], [1], \left[\left[\left[-\frac{4}{t}+\frac{3}{2}, \frac{8}{3}\right], \left[-\frac{4}{t}+\frac{3}{2}, -\frac{8}{t}+\frac{1}{3}\right], \left[-\frac{8}{t}+\frac{1}{3}, \frac{8}{3}\right]\right]\right], [[-4t, 4t, -8t]], \left[\left[-\frac{7}{6}, \frac{7}{6}, -\frac{7}{3}\right]\right]\right], \left[[[x-3,3]], \left[\left[-1, 0, -\frac{1}{2}\right]\right], \left[\left[\frac{1}{2}, -\frac{1}{2}, 1\right]\right], \left[\left[\left[-\frac{1}{2}, -1\right], \left[-\frac{1}{2}, 0\right], [0, -1]\right]\right]\right], \left[\left[[[x-3,3]], \left[\left[-1, 0, -\frac{1}{2}\right]\right], \left[\left[\left[\frac{1}{2}, -\frac{1}{2}\right], [1]\right]\right]\right], [\ ], [\ ]\right], [\ ], \left[[[x-3,3], [x-7,7]], \left[\left[-1, 0, -\frac{1}{2}\right], \left[\frac{8}{3}, -\frac{8}{t}+\frac{1}{3}, -\frac{4}{t}+\frac{3}{2}\right]\right], \left[\left[1, \frac{1}{2}, -\frac{1}{2}\right], \left[-\frac{4}{t}-\frac{7}{6}, \frac{4}{t}+\frac{7}{6}, -\frac{8}{t}-\frac{7}{3}\right]\right], \left[\left[[0, -1], \left[-\frac{1}{2}, -1\right], \left[-\frac{1}{2}, 0\right]\right], \left[\left[-\frac{4}{t}+\frac{3}{2}, \frac{8}{3}\right], \left[-\frac{4}{t}+\frac{3}{2}, -\frac{8}{t}+\frac{1}{3}\right], \left[-\frac{8}{t}+\frac{1}{3}, \frac{8}{3}\right]\right]\right], [[1,1,1], [1,1,1]]\right]\right] \quad (65)$$

```
> F1:=Hypdiff1F1sqSubst(L,x,t,R1[1],ext);
```
$$F1 := \left[-\frac{4}{x-7}, \frac{4}{x-7}\right] \quad (66)$$

```
> finddiff1F1sqRatIrr(L,R1,F1,x,t,ext);
```
$$\left[\left[\left[\left\{\frac{1}{3}, \frac{1}{6}, \frac{2}{3}, \frac{5}{6}\right\}, \left[\frac{1}{2}\right]\right], \frac{x-3}{x-7}\right], \left[\left[\left\{\frac{1}{3}, \frac{1}{6}, \frac{2}{3}, \frac{5}{6}\right\}, \left[\frac{1}{2}\right]\right], -\frac{x-3}{x-7}\right]\right] \quad (67)$$

```
> TIME :=time();
```

```
Hypdiff1F1sqSolutions(L);
time() - TIME;
```

$$TIME := 142.609$$

$$\left\{\left[\left[\left[\left[\left[\frac{1}{3}\right], \frac{1}{2}, [0], [1]\right]\right], \frac{x-3}{x-7}\right], \left[\left[\left[\left[\frac{1}{6}\right], \frac{1}{2}, \left[\frac{x^2-14x+17}{(x-7)^2(x-3)}\right], \left[(x-3)^3 Dx^2 - \frac{2(x-3)(19x^2-214x+471)Dx}{3(x^2-14x+49)} + \frac{2(57x^3-761x^2+2339x-1707)}{9(x^3-21x^2+147x-343)}\right]\right]\right], -\frac{x-3}{x-7}\right]\right]\right\}$$

$$1.375 \tag{68}$$

```
> LA:=MinOp(diff(hypergeom([a],[b],x),x)^2);
```

$$LA := Dx^3 + \frac{3(-x+b+1)Dx^2}{x} - \frac{(4xa-2b^2+4bx-2x^2-3b+8x-1)Dx}{x^2} - \frac{2(-2x+2b+1)(a+1)}{x^2} \tag{69}$$

```
> L1:=subs({a=1/7,b=RootOf(x^2+2)},LA);
```

$$L1 := Dx^3 + \frac{3(-x+RootOf(\_Z^2+2)+1)Dx^2}{x} - \frac{1}{x^2}\left(\left(\frac{60x}{7} - 2 RootOf(\_Z^2+2)^2 + 4 RootOf(\_Z^2+2)x - 2x^2 - 3 RootOf(\_Z^2+2) - 1\right)Dx\right) - \frac{16(-2x+2RootOf(\_Z^2+2)+1)}{7x^2} \tag{70}$$

```
> f:=(x-1)/x;
```

$$f := \frac{x-1}{x} \tag{71}$$

```
> L:=ChangeOfVariables(L1,f);
```

$$L := 7Dx^3 x^5 (x-1)^2 + 21(RootOf(\_Z^2+2)x + 2x^2 - 2x + 1)(x-1)x^3 Dx^2 + x(42 RootOf(\_Z^2+2)x^3 + 42x^4 - 49 RootOf(\_Z^2+2)x^2 - 84x^3 + 28 RootOf(\_Z^2+2)x + 17x^2 - 10x + 14)Dx + \frac{16(2RootOf(\_Z^2+2)-1)(-9x+4RootOf(\_Z^2+2)+2)}{9} \tag{72}$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \{RootOf(\_Z^2+2)\} \tag{73}$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```

$$ext := \{RootOf(\_Z^2+2)\} \tag{74}$$

```
> extppp:={};
```

$$extppp := \varnothing \tag{75}$$

```
> E:= Singular(L,extppp);
```

(76)

$$E := [[x, 0], [x-1, 1]] \tag{76}$$

```
> F:= NotAppSing(L,E,ext);
```

$$F := [[x-1, 1], [x, 0]] \tag{77}$$

```
> Sirr:=irrsingdiff1F1sq(L,t,F,ext);
```

$$Sirr := \left[[[x, 0]], \left[\left[\frac{16}{7}, \frac{2}{t} - \frac{2}{7} + 2\, RootOf(\_Z^2+2), \frac{1}{t} + 1 + RootOf(\_Z^2+2) \right]\right], \left[\left[\frac{1}{t} - \frac{9}{7} + RootOf(\_Z^2+2), -\frac{1}{t} + \frac{9}{7} - RootOf(\_Z^2+2), \frac{2}{t} - \frac{18}{7} + 2\, RootOf(\_Z^2+2) \right]\right], [1], [1], \left[\left[\left[\frac{1}{t} + 1 + RootOf(\_Z^2+2), \frac{16}{7} \right], \left[\frac{1}{t} + 1 + RootOf(\_Z^2+2), \frac{2}{t} - \frac{2}{7} + 2\, RootOf(\_Z^2+2) \right], \left[\frac{2}{t} - \frac{2}{7} + 2\, RootOf(\_Z^2+2), \frac{16}{7} \right]\right]\right], [[t, -t, 2\, t]], \left[\left[-\frac{9}{7} + RootOf(\_Z^2+2), \frac{9}{7} - RootOf(\_Z^2+2), -\frac{18}{7} + 2\, RootOf(\_Z^2+2) \right]\right], [[[x-1, 1]], [[[0, -RootOf(\_Z^2+2), -2\, RootOf(\_Z^2+2)], [-RootOf(\_Z^2+2), -2\, RootOf(\_Z^2+2), -RootOf(\_Z^2+2)], [1, 1, 1], [[-RootOf(\_Z^2+2), 0], [-2\, RootOf(\_Z^2+2), 0], [-2\, RootOf(\_Z^2+2), -RootOf(\_Z^2+2)]], 2]]] \right] \tag{78}$$

```
> Sreg:=regsingtruediff1F1sq(L,t,Sirr[-1],ext);
```

$$Sreg := [[[x-1, 1]], [[0, -RootOf(\_Z^2+2), -2\, RootOf(\_Z^2+2)]], [[-RootOf(\_Z^2+2), -RootOf(\_Z^2+2), -2\, RootOf(\_Z^2+2)]], [[[-RootOf(\_Z^2+2), 0], [-2\, RootOf(\_Z^2+2), -RootOf(\_Z^2+2)], [-2\, RootOf(\_Z^2+2), 0]]]] \tag{79}$$

```
> RSreg:=Sregseptruediff1F1sq(L,Sreg,ext);
```

$$RSreg := [[[[x-1, 1]], [[0, -RootOf(\_Z^2+2), -2\, RootOf(\_Z^2+2)]], [[[-RootOf(\_Z^2+2), -RootOf(\_Z^2+2), -2\, RootOf(\_Z^2+2)], [\,]]]], [\,], [\,]] \tag{80}$$

```
> R1:=IrrRegAppsingdiff1F1sq(L,t,E,ext);
```

$$R1 := \left[\left[[[x, 0]], \left[\left[\frac{16}{7}, \frac{2}{t} - \frac{2}{7} + 2\, RootOf(\_Z^2+2), \frac{1}{t} + 1 + RootOf(\_Z^2+2) \right]\right], \left[\left[\frac{1}{t} - \frac{9}{7} + RootOf(\_Z^2+2), -\frac{1}{t} + \frac{9}{7} - RootOf(\_Z^2+2), \frac{2}{t} - \frac{18}{7} + 2\, RootOf(\_Z^2+2) \right]\right], [1], [1], \left[\left[\left[\frac{1}{t} + 1 + RootOf(\_Z^2+2), \frac{16}{7} \right], \left[\frac{1}{t} + 1 + RootOf(\_Z^2+2), \frac{2}{t} - \frac{2}{7} + 2\, RootOf(\_Z^2+2) \right], \left[\frac{2}{t} - \frac{2}{7} + 2\, RootOf(\_Z^2+2), \frac{16}{7} \right]\right]\right], [[t, -t, 2\, t]], \left[\left[-\frac{9}{7} + RootOf(\_Z^2+2), \frac{9}{7} - RootOf(\_Z^2+2), -\frac{18}{7} + 2\, RootOf(\_Z^2+2) \right]\right]\right], [[[x-1, 1]], [[0, -RootOf(\_Z^2+2), -2\, RootOf(\_Z^2+2)]], [[-RootOf(\_Z^2+2), -RootOf(\_Z^2+2), -2\, RootOf(\_Z^2+2)]], [[[-RootOf(\_Z^2+2), 0], [-2\, RootOf(\_Z^2+2), -RootOf(\_Z^2+2)], [-2\, RootOf(\_Z^2+2) \tag{81}$$

$+2), 0]]]], [[[[x-1, 1]], [[0, -RootOf(\_Z^2+2), -2\,RootOf(\_Z^2+2)]], [[[$
$-RootOf(\_Z^2+2), -2\,RootOf(\_Z^2+2), -RootOf(\_Z^2+2)], [\,]]]], [\,], [\,]], [\,], \Big[[[x$
$-1, 1], [x, 0]], \Big[[0, -RootOf(\_Z^2+2), -2\,RootOf(\_Z^2+2)], \Big[\frac{16}{7}, \frac{2}{t}-\frac{2}{7}$
$+2\,RootOf(\_Z^2+2), \frac{1}{t}+1+RootOf(\_Z^2+2)\Big]\Big], \Big[[-RootOf(\_Z^2+2),$
$-2\,RootOf(\_Z^2+2), -RootOf(\_Z^2+2)], \Big[\frac{1}{t}-\frac{9}{7}+RootOf(\_Z^2+2), -\frac{1}{t}+\frac{9}{7}$
$-RootOf(\_Z^2+2), \frac{2}{t}-\frac{18}{7}+2\,RootOf(\_Z^2+2)\Big]\Big], \Big[[[-RootOf(\_Z^2+2), 0], [$
$-2\,RootOf(\_Z^2+2), 0], [-2\,RootOf(\_Z^2+2), -RootOf(\_Z^2+2)]], \Big[\Big[\frac{1}{t}+1$
$+RootOf(\_Z^2+2), \frac{16}{7}\Big], \Big[\frac{1}{t}+1+RootOf(\_Z^2+2), \frac{2}{t}-\frac{2}{7}+2\,RootOf(\_Z^2$
$+2)\Big], \Big[\frac{2}{t}-\frac{2}{7}+2\,RootOf(\_Z^2+2), \frac{16}{7}\Big]\Big]\Big], [[1, 1, 1], [1, 1, 1]]\Big]\Big]$

```
> F1:=Hypdiff1F1sqSubst(L,x,t,R1[1],ext);
```

$$F1 := \left[\frac{1}{x}, -\frac{1}{x}\right] \tag{82}$$

```
> finddiff1F1sqRatIrr(L,R1,F1,x,t,ext);
```

$$\Big[\Big[\Big[\Big\{\frac{1}{7}, \frac{9}{14}, RootOf(\_Z^2+2)+\frac{5}{14}, RootOf(\_Z^2+2)+\frac{6}{7}\Big\}, [RootOf(\_Z^2+2)]\Big], \tag{83}$$
$\frac{x-1}{x}\Big], \Big[\Big[\Big\{\frac{1}{7}, \frac{9}{14}, RootOf(\_Z^2+2)+\frac{5}{14}, RootOf(\_Z^2+2)+\frac{6}{7}\Big\}, [RootOf(\_Z^2$
$+2)]\Big], -\frac{x-1}{x}\Big]\Big]$

```
> TIME :=time();
  Hypdiff1F1sqSolutions(L);
  time() - TIME;
```

$$TIME := 154.421$$

$\Big\{\Big[\Big[\Big[\Big[\frac{1}{7}\Big], RootOf(\_Z^2+2), [0], [1]\Big]\Big], \frac{x-1}{x}\Big], \Big[\Big[\Big[\Big[RootOf(\_Z^2+2)+\frac{6}{7}\Big], RootOf(\_Z^2$
$+2), \Big[\frac{2}{x^2}\Big], \Big[\frac{1}{5202}\big((x-1)^2(-4998\,RootOf(\_Z^2+2)\,x+5202\,x^2$
$+686\,RootOf(\_Z^2+2)-1428\,x-2303)\,Dx^2\big)+\frac{1}{36414\,x^2}\big((-67830\,RootOf(\_Z^2$
$+2)\,x^4+72828\,x^5+176806\,RootOf(\_Z^2+2)\,x^3-175440\,x^4-177919\,RootOf(\_Z^2$
$+2)\,x^2+173732\,x^3+73745\,RootOf(\_Z^2+2)\,x-30842\,x^2-4802\,RootOf(\_Z^2+2)$

$$-56399\,x+16121\big)\,Dx\big)+\frac{1}{537792297\,x^3}\big(2\,\big(3061+896\,RootOf\big(\_Z^2+2\big)\big)\,\big($$

$$-141414\,RootOf\big(\_Z^2+2\big)\,x^2+75954\,x^3+175616\,RootOf\big(\_Z^2+2\big)\,x-144102\,x^2$$

$$-33614\,RootOf\big(\_Z^2+2\big)+20335\,x+46991\big)\big)\big]\big]\big],-\frac{x-1}{x}\big]\big\}$$

$$2.188 \tag{84}$$