```
> restart:
> read "ODE3solve.mpl":
```

Package "Solving third-order holonomic differential equations", Maple 16
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Package "Hypergeometric Summation", Maple V - Maple 17
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Here are the Maple implementations in chapter 4 related just to the Bessel square root functions with the square root of the change of variable parameters in k(x).

```
>                    ####### THE EXPONENT DIFFERENCES #######
```

In chapter 4, section 4.1.1 which is called "Exponent differences", we have the following Maple implementations:

```
> eq:=HolonomicDE(BesselI(nu, x)^2,Y(x));
```

$$eq := -4\,Y(x)\,x + \left(-4\,\nu^2 - 4\,x^2 + 1\right)\left(\frac{d}{dx}\,Y(x)\right) + \left(\frac{d^3}{dx^3}\,Y(x)\right)x^2 + 3\left(\frac{d^2}{dx^2}\,Y(x)\right)x \tag{2}$$

```
> LBB:=de2diffop(eq, Y(x));
```

$$LBB := x^2\,Dx^3 + 3\,x\,Dx^2 + \left(-4\,\nu^2 - 4\,x^2 + 1\right)Dx - 4\,x \tag{3}$$

```
> gen_exp(LBB,t,x=infinity);
```

$$\left[\left[1, t = \frac{1}{x}\right], \left[\frac{2}{t} + 1, t = \frac{1}{x}\right], \left[-\frac{2}{t} + 1, t = \frac{1}{x}\right]\right] \tag{4}$$

```
> gen_exp(LBB,t,x=0);
```

$$\left[[0, t = x], [2\,\nu, t = x], [-2\,\nu, t = x]\right] \tag{5}$$

```
>                    ####### EXAMPLE IN THE THESIS #######
```

In chapter 4, section 4.1.5 which is called "Change of variable parameters are square of rational functions in k(x)", those are the Maple implementations for the example that we have used:

```
> eq:=HolonomicDE(BesselI(nu, x)^2,Y(x));
```

$$eq := -4\,Y(x)\,x + \left(-4\,\nu^2 - 4\,x^2 + 1\right)\left(\frac{d}{dx}\,Y(x)\right) + \left(\frac{d^3}{dx^3}\,Y(x)\right)x^2 + 3\left(\frac{d^2}{dx^2}\,Y(x)\right)x \tag{6}$$

```
> LBB:=de2diffop(eq, Y(x));
```

$$LBB := x^2\,Dx^3 + 3\,x\,Dx^2 + \left(-4\,\nu^2 - 4\,x^2 + 1\right)Dx - 4\,x \tag{7}$$

```
> LBB:=subs(nu=a*RootOf(x^2+2)+1/2,LBB);
```

(8)

$$LBB := x^2\,Dx^3 + 3\,x\,Dx^2 + \left(-4\left(a\,RootOf\left(\_Z^2+2\right) + \frac{1}{2}\right)^2 - 4\,x^2 + 1\right)Dx - 4\,x \tag{8}$$

```
> f:=(x-2)^2*(x-7)/(x-1)^5;
```

$$f := \frac{(x-2)^2\,(x-7)}{(x-1)^5} \tag{9}$$

```
> L:=ChangeOfVariables(LBB,f);
```

$$L := Dx^3\,(x-2)^2\,(x-7)^2\,(x-1)^{13}\,\left(x^2-13\,x+27\right)^2 + 3\,\left(x^4-26\,x^3+188\,x^2-512\,x + 439\right)Dx^2\,(x-2)\,(x-7)\,(x-1)^{12}\,\left(x^2-13\,x+27\right) - \left(6668295207 - 28099018824\,x + 16\,RootOf\left(\_Z^2+2\right)a\,x^{18} - 992\,RootOf\left(\_Z^2+2\right)a\,x^{17} + 26992\,RootOf\left(\_Z^2+2\right)a\,x^{16} - 426880\,RootOf\left(\_Z^2+2\right)a\,x^{15} + 4394096\,RootOf\left(\_Z^2+2\right)a\,x^{14} - 31339552\,RootOf\left(\_Z^2+2\right)a\,x^{13} + 161327472\,RootOf\left(\_Z^2+2\right)a\,x^{12} - 616713600\,RootOf\left(\_Z^2+2\right)a\,x^{11} + 1786029216\,RootOf\left(\_Z^2+2\right)a\,x^{10} - 3971204992\,RootOf\left(\_Z^2+2\right)a\,x^{9} + 6831709856\,RootOf\left(\_Z^2+2\right)a\,x^{8} - 9114573568\,RootOf\left(\_Z^2+2\right)a\,x^{7} + 9399885424\,RootOf\left(\_Z^2+2\right)a\,x^{6} - 7419183392\,RootOf\left(\_Z^2+2\right)a\,x^{5} + 4397270896\,RootOf\left(\_Z^2+2\right)a\,x^{4} - 1893784320\,RootOf\left(\_Z^2+2\right)a\,x^{3} + 559487088\,RootOf\left(\_Z^2+2\right)a\,x^{2} - 101406816\,RootOf\left(\_Z^2+2\right)a\,x - 17006112\,a^2 - 61591453506\,x^3 + 53675365323\,x^2 + 8503056\,a\,RootOf\left(\_Z^2+2\right) + 3\,x^{18} - 186\,x^{17} + 5061\,x^{16} - 80970\,x^{15} + 849004\,x^{14} - 6178910\,x^{13} + 32446722\,x^{12} - 126753214\,x^{11} + 380186954\,x^{10} - 923670030\,x^9 + 2035186004\,x^8 - 4736476918\,x^7 + 11776763148\,x^6 - 26665094450\,x^5 + 47608789582\,x^4 - 1118974176\,a^2\,x^2 + 202813632\,a^2\,x + 3787568640\,a^2\,x^3 + 18229147136\,a^2\,x^7 - 18799770848\,a^2\,x^6 + 14838366784\,a^2\,x^5 - 8794541792\,a^2\,x^4 - 13663419712\,a^2\,x^8 + 7942409984\,a^2\,x^9 + 1233427200\,a^2\,x^{11} - 3572058432\,a^2\,x^{10} - 322654944\,a^2\,x^{12} + 853760\,a^2\,x^{15} - 8788192\,a^2\,x^{14} + 62679104\,a^2\,x^{13} - 32\,a^2\,x^{18} + 1984\,a^2\,x^{17} - 53984\,a^2\,x^{16}\right)(x-1)\,Dx + 32\,(x-2)^3\,\left(x^2-13\,x+27\right)^5\,(x-7) \tag{10}$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \left\{a, RootOf\left(\_Z^2+2\right)\right\} \tag{11}$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```

$$ext := \left\{a, RootOf\left(\_Z^2+2\right)\right\} \tag{12}$$

```
> extppp:={};
```

$$extppp := \varnothing \tag{13}$$

```
> E:= Singular(L,extppp);
```

$$E := \left[[x-2, 2], \left[x^2-13\,x+27, RootOf\left(\_Z^2-13\,\_Z+27\right)\right], [x-1, 1], [x-7, 7], [\infty, \infty]\right] \tag{14}$$

```
> F:=NotAppSing(L,E,ext);
```

$$F := \left[[x-7, 7], [x-1, 1], [x-2, 2], [\infty, \infty]\right] \tag{15}$$

```
> Sirr:=irrsingBesSq(L,t,F,ext);
```

$$Sirr := \left[[[x-1,1]], \left[\left[5, \frac{60}{t^5} - \frac{104}{t^4} + \frac{48}{t^3} - \frac{4}{t^2} + 5, -\frac{60}{t^5} + \frac{104}{t^4} - \frac{48}{t^3} + \frac{4}{t^2} + 5 \right] \right], \right. \quad \mathbf{(16)}$$

$$\left[\left[\frac{60}{t^5} - \frac{104}{t^4} + \frac{48}{t^3} - \frac{4}{t^2}, -\frac{60}{t^5} + \frac{104}{t^4} - \frac{48}{t^3} + \frac{4}{t^2}, -\frac{120}{t^5} + \frac{208}{t^4} - \frac{96}{t^3} + \frac{8}{t^2} \right] \right],$$

$$[5], [1], \left[\left[\left[\frac{60}{t^5} - \frac{104}{t^4} + \frac{48}{t^3} - \frac{4}{t^2} + 5, 5 \right], \left[-\frac{60}{t^5} + \frac{104}{t^4} - \frac{48}{t^3} + \frac{4}{t^2} + 5, 5 \right], \left[\right. \right. \right.$$

$$\left. \left. \left. -\frac{60}{t^5} + \frac{104}{t^4} - \frac{48}{t^3} + \frac{4}{t^2} + 5, \frac{60}{t^5} - \frac{104}{t^4} + \frac{48}{t^3} - \frac{4}{t^2} + 5 \right] \right] \right], [[60\,t^5 - 104\,t^4 + 48\,t^3$$

$$-4\,t^2, -60\,t^5 + 104\,t^4 - 48\,t^3 + 4\,t^2, -120\,t^5 + 208\,t^4 - 96\,t^3 + 8\,t^2]], [[0,0,0]], [[[x$$

$$-7,7], [x-2,2], [\infty,\infty]], [[[0, 2\,a\,RootOf(\_Z^2+2)+1, -2\,a\,RootOf(\_Z^2+2)$$

$$-1], [2\,a\,RootOf(\_Z^2+2)+1, -2\,a\,RootOf(\_Z^2+2)-1, -4\,a\,RootOf(\_Z^2+2)$$

$$-2], [1,1,1], [[2\,a\,RootOf(\_Z^2+2)+1, 0], [-2\,a\,RootOf(\_Z^2+2)-1, 0], [$$

$$-2\,a\,RootOf(\_Z^2+2)-1, 2\,a\,RootOf(\_Z^2+2)+1]], 2], [[0, 4\,a\,RootOf(\_Z^2+2)$$

$$+2, -4\,a\,RootOf(\_Z^2+2)-2], [4\,a\,RootOf(\_Z^2+2)+2, -4\,a\,RootOf(\_Z^2+2)$$

$$-2, -8\,a\,RootOf(\_Z^2+2)-4], [1,1,1], [[4\,a\,RootOf(\_Z^2+2)+2, 0], [$$

$$-4\,a\,RootOf(\_Z^2+2)-2, 0], [-4\,a\,RootOf(\_Z^2+2)-2, 4\,a\,RootOf(\_Z^2+2)+2]],$$

$$2], [[0, 4\,a\,RootOf(\_Z^2+2)+2, -4\,a\,RootOf(\_Z^2+2)-2], [4\,a\,RootOf(\_Z^2+2)$$

$$+2, -4\,a\,RootOf(\_Z^2+2)-2, -8\,a\,RootOf(\_Z^2+2)-4], [1,1,1],$$

$$[[4\,a\,RootOf(\_Z^2+2)+2, 0], [-4\,a\,RootOf(\_Z^2+2)-2, 0], [-4\,a\,RootOf(\_Z^2+2)$$

$$-2, 4\,a\,RootOf(\_Z^2+2)+2]], 2]]]]$$

```
> Sreg:=regsingtrueBesSq(L,t,Sirr[-1],ext);
```

$$Sreg := [[[x-7,7], [x-2,2], [\infty,\infty]], [[0, 2\,a\,RootOf(\_Z^2+2)+1, -2\,a\,RootOf(\_Z^2 \quad \mathbf{(17)}$$

$$+2)-1], [0, 4\,a\,RootOf(\_Z^2+2)+2, -4\,a\,RootOf(\_Z^2+2)-2], [0,$$

$$4\,a\,RootOf(\_Z^2+2)+2, -4\,a\,RootOf(\_Z^2+2)-2]], [[2\,a\,RootOf(\_Z^2+2)+1,$$

$$-2\,a\,RootOf(\_Z^2+2)-1, -4\,a\,RootOf(\_Z^2+2)-2], [4\,a\,RootOf(\_Z^2+2)+2,$$

$$-4\,a\,RootOf(\_Z^2+2)-2, -8\,a\,RootOf(\_Z^2+2)-4], [4\,a\,RootOf(\_Z^2+2)+2,$$

$$-4\,a\,RootOf(\_Z^2+2)-2, -8\,a\,RootOf(\_Z^2+2)-4]], [[[2\,a\,RootOf(\_Z^2+2)+1,$$

$$0], [-2\,a\,RootOf(\_Z^2+2)-1, 0], [-2\,a\,RootOf(\_Z^2+2)-1, 2\,a\,RootOf(\_Z^2+2)$$

$$+1]], [[4\,a\,RootOf(\_Z^2+2)+2, 0], [-4\,a\,RootOf(\_Z^2+2)-2, 0], [$$

$$-4\,a\,RootOf(\_Z^2+2)-2, 4\,a\,RootOf(\_Z^2+2)+2]], [[4\,a\,RootOf(\_Z^2+2)+2, 0],$$

$$[-4\,a\,RootOf(\_Z^2+2)-2, 0], [-4\,a\,RootOf(\_Z^2+2)-2, 4\,a\,RootOf(\_Z^2+2)+2]]$$

$$]]$$

```
> NRemSreg:=SregseptrueBesSq(L,Sreg,ext)[1];
```

$$NRemSreg := [[[x-7,7], [x-2,2], [\infty,\infty]], [[0, 2\,a\,RootOf(\_Z^2+2)+1, \quad \mathbf{(18)}$$

$$-2\,a\,RootOf(\_Z^2+2)-1], [0, 4\,a\,RootOf(\_Z^2+2)+2, -4\,a\,RootOf(\_Z^2+2)-2],$$

$$[0, 4\,a\,RootOf(\_Z^2+2)+2, -4\,a\,RootOf(\_Z^2+2)-2]], [[[2\,a\,RootOf(\_Z^2+2)$$

$$+1, -2\,a\,RootOf(\_Z^2+2)-1, -4\,a\,RootOf(\_Z^2+2)-2], [\,]], [[4\,a\,RootOf(\_Z^2$$

$+2)+2, -4\, a\, RootOf(\_Z^2+2)-2, -8\, a\, RootOf(\_Z^2+2)-4], [\,]],$
$[[4\, a\, RootOf(\_Z^2+2)+2, -4\, a\, RootOf(\_Z^2+2)-2, -8\, a\, RootOf(\_Z^2+2)-4],$
$[\,]]]]$

```
> LogSreg:=SregseptrueBesSq(L,Sreg,ext)[3];
```

$$LogSreg := [\,] \quad \textbf{(19)}$$

```
> RemSreg:=SregseptrueBesSq(L,Sreg,ext)[2];
```

$$RemSreg := [\,] \quad \textbf{(20)}$$

```
> R1:=IrrRegAppsingBesSq(L,t,E,ext);
```

$$R1 := \Bigg[\Bigg[[[x-1,1]], \Bigg[\Bigg[5, \frac{60}{t^5}-\frac{104}{t^4}+\frac{48}{t^3}-\frac{4}{t^2}+5, -\frac{60}{t^5}+\frac{104}{t^4}-\frac{48}{t^3}+\frac{4}{t^2}+5\Bigg]\Bigg], \quad \textbf{(21)}$$
$$\Bigg[\Bigg[\frac{60}{t^5}-\frac{104}{t^4}+\frac{48}{t^3}-\frac{4}{t^2}, -\frac{60}{t^5}+\frac{104}{t^4}-\frac{48}{t^3}+\frac{4}{t^2}, -\frac{120}{t^5}+\frac{208}{t^4}-\frac{96}{t^3}+\frac{8}{t^2}\Bigg]\Bigg],$$
$$[5], [1], \Bigg[\Bigg[\Bigg[\frac{60}{t^5}-\frac{104}{t^4}+\frac{48}{t^3}-\frac{4}{t^2}+5, 5\Bigg], \Bigg[-\frac{60}{t^5}+\frac{104}{t^4}-\frac{48}{t^3}+\frac{4}{t^2}+5, 5\Bigg], \Bigg[$$
$$-\frac{60}{t^5}+\frac{104}{t^4}-\frac{48}{t^3}+\frac{4}{t^2}+5, \frac{60}{t^5}-\frac{104}{t^4}+\frac{48}{t^3}-\frac{4}{t^2}+5\Bigg]\Bigg]\Bigg], [[60\, t^5-104\, t^4+48\, t^3$$
$$-4\, t^2, -60\, t^5+104\, t^4-48\, t^3+4\, t^2, -120\, t^5+208\, t^4-96\, t^3+8\, t^2]], [[0,0,0]]\Bigg], [[[x$$
$$-7,7], [x-2,2], [\infty, \infty]], [[0, 2\, a\, RootOf(\_Z^2+2)+1, -2\, a\, RootOf(\_Z^2+2)$$
$$-1], [0, 4\, a\, RootOf(\_Z^2+2)+2, -4\, a\, RootOf(\_Z^2+2)-2], [0, 4\, a\, RootOf(\_Z^2$$
$$+2)+2, -4\, a\, RootOf(\_Z^2+2)-2]], [[2\, a\, RootOf(\_Z^2+2)+1, -2\, a\, RootOf(\_Z^2$$
$$+2)-1, -4\, a\, RootOf(\_Z^2+2)-2], [4\, a\, RootOf(\_Z^2+2)+2, -4\, a\, RootOf(\_Z^2$$
$$+2)-2, -8\, a\, RootOf(\_Z^2+2)-4], [4\, a\, RootOf(\_Z^2+2)+2, -4\, a\, RootOf(\_Z^2$$
$$+2)-2, -8\, a\, RootOf(\_Z^2+2)-4]], [[[2\, a\, RootOf(\_Z^2+2)+1, 0], [$$
$$-2\, a\, RootOf(\_Z^2+2)-1, 0], [-2\, a\, RootOf(\_Z^2+2)-1, 2\, a\, RootOf(\_Z^2+2)+1]],$$
$$[[4\, a\, RootOf(\_Z^2+2)+2, 0], [-4\, a\, RootOf(\_Z^2+2)-2, 0], [-4\, a\, RootOf(\_Z^2+2)$$
$$-2, 4\, a\, RootOf(\_Z^2+2)+2]], [[4\, a\, RootOf(\_Z^2+2)+2, 0], [-4\, a\, RootOf(\_Z^2$$
$$+2)-2, 0], [-4\, a\, RootOf(\_Z^2+2)-2, 4\, a\, RootOf(\_Z^2+2)+2]]]]], [[[[x-7,7],$$
$$[x-2,2], [\infty, \infty]], [[0, 2\, a\, RootOf(\_Z^2+2)+1, -2\, a\, RootOf(\_Z^2+2)-1], [0,$$

$4\,a\,RootOf(\_Z^2+2)+2,\ -4\,a\,RootOf(\_Z^2+2)-2],\ [0,\ 4\,a\,RootOf(\_Z^2+2)+2,$

$-4\,a\,RootOf(\_Z^2+2)-2]],\ [[[2\,a\,RootOf(\_Z^2+2)+1,\ -2\,a\,RootOf(\_Z^2+2)-1,$

$-4\,a\,RootOf(\_Z^2+2)-2],\ [\ ]],\ [[4\,a\,RootOf(\_Z^2+2)+2,\ -4\,a\,RootOf(\_Z^2+2)$

$-2,\ -8\,a\,RootOf(\_Z^2+2)-4],\ [\ ]],\ [[4\,a\,RootOf(\_Z^2+2)+2,\ -4\,a\,RootOf(\_Z^2$

$+2)-2,\ -8\,a\,RootOf(\_Z^2+2)-4],\ [\ ]]]],\ [\ ],\ [\ ]],\ [[[x^2-13\,x+27,\ RootOf(\_Z^2$

$-13\,\_Z+27)]],\ [[0,2,4]],\ [[2,4,2]],\ [[[2,0],[4,0],[4,2]]]],\ \Big[[[x-7,7],\ [x-1,$

$1],\ [x-2,2],\ [\infty,\infty]],\ \Big[[0,\ 2\,a\,RootOf(\_Z^2+2)+1,\ -2\,a\,RootOf(\_Z^2+2)-1],\ \Big[5,$

$\frac{60}{t^5}-\frac{104}{t^4}+\frac{48}{t^3}-\frac{4}{t^2}+5,\ -\frac{60}{t^5}+\frac{104}{t^4}-\frac{48}{t^3}+\frac{4}{t^2}+5\Big],\ [0,\ 4\,a\,RootOf(\_Z^2+2)$

$+2,\ -4\,a\,RootOf(\_Z^2+2)-2],\ [0,\ 4\,a\,RootOf(\_Z^2+2)+2,\ -4\,a\,RootOf(\_Z^2+2)$

$-2]],\ \Big[[2\,a\,RootOf(\_Z^2+2)+1,\ -2\,a\,RootOf(\_Z^2+2)-1,\ -4\,a\,RootOf(\_Z^2+2)$

$-2],\ \Big[\frac{60}{t^5}-\frac{104}{t^4}+\frac{48}{t^3}-\frac{4}{t^2},\ -\frac{60}{t^5}+\frac{104}{t^4}-\frac{48}{t^3}+\frac{4}{t^2},\ -\frac{120}{t^5}+\frac{208}{t^4}-\frac{96}{t^3}$

$+\frac{8}{t^2}\Big],\ [4\,a\,RootOf(\_Z^2+2)+2,\ -4\,a\,RootOf(\_Z^2+2)-2,\ -8\,a\,RootOf(\_Z^2+2)$

$-4],\ [4\,a\,RootOf(\_Z^2+2)+2,\ -4\,a\,RootOf(\_Z^2+2)-2,\ -8\,a\,RootOf(\_Z^2+2)$

$-4]],\ \Big[[[2\,a\,RootOf(\_Z^2+2)+1,0],\ [-2\,a\,RootOf(\_Z^2+2)-1,0],\ [$

$-2\,a\,RootOf(\_Z^2+2)-1,\ 2\,a\,RootOf(\_Z^2+2)+1]],\ \Big[\Big[\frac{60}{t^5}-\frac{104}{t^4}+\frac{48}{t^3}-\frac{4}{t^2}$

$+5,5\Big],\ \Big[-\frac{60}{t^5}+\frac{104}{t^4}-\frac{48}{t^3}+\frac{4}{t^2}+5,5\Big],\ \Big[-\frac{60}{t^5}+\frac{104}{t^4}-\frac{48}{t^3}+\frac{4}{t^2}+5,\ \frac{60}{t^5}$

$-\frac{104}{t^4}+\frac{48}{t^3}-\frac{4}{t^2}+5\Big]\Big],\ [[4\,a\,RootOf(\_Z^2+2)+2,0],\ [-4\,a\,RootOf(\_Z^2+2)$

$-2,0],\ [-4\,a\,RootOf(\_Z^2+2)-2,\ 4\,a\,RootOf(\_Z^2+2)+2]],\ [[4\,a\,RootOf(\_Z^2$

$+2)+2,0],\ [-4\,a\,RootOf(\_Z^2+2)-2,0],\ [-4\,a\,RootOf(\_Z^2+2)-2,$

$4\,a\,RootOf(\_Z^2+2)+2]]],\ [[1,1,1],[1,1,1],[1,1,1],[1,1,1]]]]$

```
> F1:= BesSqSubst(L,x,t,R1[1],ext);
```

$$F1 := \left[-\frac{x^3-11\,x^2+32\,x-28}{(x-1)^5},\ \frac{x^3-11\,x^2+32\,x-28}{(x-1)^5}\right] \quad \textbf{(22)}$$

```
> CandichangvarBesSq(F1,R1,ext);
```

$$\left\{\left[\frac{(x-2)^2\,(x-7)}{(x-1)^5}, [1,2,2], [[[x-7,7],[x-2,2],[\infty,\infty]],[[0,2\,a\,RootOf(\_Z^2+2)+1,-2\,a\,RootOf(\_Z^2+2)-1],[0,4\,a\,RootOf(\_Z^2+2)+2,-4\,a\,RootOf(\_Z^2+2)-2],[0,4\,a\,RootOf(\_Z^2+2)+2,-4\,a\,RootOf(\_Z^2+2)-2]],[[2\,a\,RootOf(\_Z^2+2)+1,-2\,a\,RootOf(\_Z^2+2)-1,-4\,a\,RootOf(\_Z^2+2)-2],[4\,a\,RootOf(\_Z^2+2)+2,-4\,a\,RootOf(\_Z^2+2)-2,-8\,a\,RootOf(\_Z^2+2)-4],[4\,a\,RootOf(\_Z^2+2)+2,-4\,a\,RootOf(\_Z^2+2)-2,-8\,a\,RootOf(\_Z^2+2)-4]],[[[2\,a\,RootOf(\_Z^2+2)+1,0],[-2\,a\,RootOf(\_Z^2+2)-1,0],[-2\,a\,RootOf(\_Z^2+2)-1,2\,a\,RootOf(\_Z^2+2)+1]],[[4\,a\,RootOf(\_Z^2+2)+2,0],[-4\,a\,RootOf(\_Z^2+2)-2,0],[-4\,a\,RootOf(\_Z^2+2)-2,4\,a\,RootOf(\_Z^2+2)+2]],[[4\,a\,RootOf(\_Z^2+2)+2,0],[-4\,a\,RootOf(\_Z^2+2)-2,0],[-4\,a\,RootOf(\_Z^2+2)-2,4\,a\,RootOf(\_Z^2+2)+2]]]]\right], \left[-\frac{(x-2)^2\,(x-7)}{(x-1)^5}, [1,2,2], [[[x-7,7],[x-2,2],[\infty,\infty]],[[0,2\,a\,RootOf(\_Z^2+2)+1,-2\,a\,RootOf(\_Z^2+2)-1],[0,4\,a\,RootOf(\_Z^2+2)+2,-4\,a\,RootOf(\_Z^2+2)-2],[0,4\,a\,RootOf(\_Z^2+2)+2,-4\,a\,RootOf(\_Z^2+2)-2]],[[2\,a\,RootOf(\_Z^2+2)+1,-2\,a\,RootOf(\_Z^2+2)-1,-4\,a\,RootOf(\_Z^2+2)-2],[4\,a\,RootOf(\_Z^2+2)+2,-4\,a\,RootOf(\_Z^2+2)-2,-8\,a\,RootOf(\_Z^2+2)-4],[4\,a\,RootOf(\_Z^2+2)+2,-4\,a\,RootOf(\_Z^2+2)-2,-8\,a\,RootOf(\_Z^2+2)-4]],[[[2\,a\,RootOf(\_Z^2+2)+1,0],[-2\,a\,RootOf(\_Z^2+2)-1,0],[-2\,a\,RootOf(\_Z^2+2)-1,2\,a\,RootOf(\_Z^2+2)+1]],[[4\,a\,RootOf(\_Z^2+2)+2,0],[-4\,a\,RootOf(\_Z^2+2)-2,0],[-4\,a\,RootOf(\_Z^2+2)-2,4\,a\,RootOf(\_Z^2+2)+2]],[[4\,a\,RootOf(\_Z^2+2)+2,0],[-4\,a\,RootOf(\_Z^2+2)-2,0],[-4\,a\,RootOf(\_Z^2+2)-2,4\,a\,RootOf(\_Z^2+2)+2]]]]\right]\right\} \tag{23}$$

```
> findBesSqIrr(L,R1,F1,x,t,ext);
```

$$\left[\left[\left[a\,RootOf(\_Z^2+2)+\frac{1}{2}, a\,RootOf(\_Z^2+2)+1\right], \frac{(x-2)^2\,(x-7)}{(x-1)^5}\right], \left[\left[a\,RootOf(\_Z^2 \tag{24}$$

$$+2) + \frac{1}{2}, a\,RootOf(\_Z^2+2)+1\Big], -\frac{(x-2)^2(x-7)}{(x-1)^5}\Big]\Big]$$

```
> TIME :=time();
  BesSqSolutions(L);
  time() - TIME;
```

$$TIME := 1.765$$

$$\left\{\left[a\,RootOf(\_Z^2+2)+\frac{1}{2}, [0], [1], \frac{(x-2)^2(x-7)}{(x-1)^5}\right], \left[a\,RootOf(\_Z^2+2)+\frac{1}{2}, [0], [1], -\frac{(x-2)^2(x-7)}{(x-1)^5}\right]\right\}$$

$$0.094 \tag{25}$$

Here are another examples related to the Bessel square type solutions with the square root of the change of variables parameters in k(x). Those examples are not in my PhD thesis.

```
>                    ####### THE INTEGER CASE #######
```

```
> eq:=HolonomicDE(BesselI(nu, x)^2,Y(x));
```

$$eq := -4\,Y(x)\,x + (-4\nu^2-4x^2+1)\left(\frac{d}{dx}Y(x)\right) + \left(\frac{d^3}{dx^3}Y(x)\right)x^2 + 3\left(\frac{d^2}{dx^2}Y(x)\right)x \tag{26}$$

```
> LBB:=de2diffop(eq, Y(x));
```

$$LBB := x^2\,Dx^3 + 3\,x\,Dx^2 + (-4\nu^2-4x^2+1)\,Dx - 4\,x \tag{27}$$

```
> LBB:=subs(nu=1/4,LBB);
```

$$LBB := x^2\,Dx^3 + 3\,x\,Dx^2 + \left(\frac{3}{4}-4x^2\right)Dx - 4\,x \tag{28}$$

```
> f:=(x-1)^2/(x-12);
```

$$f := \frac{(x-1)^2}{x-12} \tag{29}$$

```
> M:=ChangeOfVariables(LBB,f);
```

$$M := 4\,Dx^3\,(x-12)^5\,(x-1)\,(x-23)^2 + 12\,(x^2-46\,x+287)\,Dx^2\,(x-12)^4\,(x-23) - (16\,x^7-1520\,x^6+55245\,x^5-935127\,x^4+6951111\,x^3-15485061\,x^2+11130392\,x + 6015392)\,Dx\,(x-12) - 16\,(x-1)^2\,(x-23)^5 \tag{30}$$

```
> r:=(x-5)*(x-9);
```

$$r := (x-5)\,(x-9) \tag{31}$$

```
> L:=ExpProduct(M,r);
```

$$L := 4\,Dx^3\,(x-12)^5\,(x-1)\,(x-23)^2 - 12\,(x^5-50\,x^4+860\,x^3-6251\,x^2+17905\,x - 12707)\,Dx^2\,(x-12)^4\,(x-23) + (12\,x^{11}-1476\,x^{10}+79692\,x^9-2490516\,x^8 + 49978796\,x^7-674496700\,x^6+6221254983\,x^5-38968032069\,x^4+160783263633\,x^3 \tag{32}$$

$$-408870438747x^2+556733891176x-280722974624\big)\,Dx\,(x-12)-4x^{14}+596x^{13}-40208x^{12}+1627812x^{11}-44170500x^{10}+849303636x^9-11921213943x^8+123901106091x^7-956028831394x^6+5432426961430x^5-22271460509795x^4+63415994348499x^3-117118443948236x^2+122741333264976x-52328387056144$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \varnothing \quad (33)$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```

$$ext := \varnothing \quad (34)$$

```
> extppp:={};
```

$$extppp := \varnothing \quad (35)$$

```
> E:= Singular(L,extppp);
```

$$E := [[x-12, 12], [x-1, 1], [\infty, \infty], [x-23, 23]] \quad (36)$$

```
> F:=NotAppSing(L,E,ext);
```

$$F := [[x-1, 1], [x-12, 12], [\infty, \infty]] \quad (37)$$

```
> Sirr:=irrsingBesSq(L,t,F,ext);
```

$$Sirr := \Bigg[[[x-12, 12], [\infty, \infty]], \Bigg[\Bigg[1, \frac{242}{t}+1, -\frac{242}{t}+1\Bigg], \Bigg[-\frac{43}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}, -\frac{45}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}, -\frac{47}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}\Bigg]\Bigg], \Bigg[\Bigg[\frac{242}{t}, -\frac{242}{t}, -\frac{484}{t}\Bigg], \Bigg[-\frac{2}{t}, -\frac{2}{t}, -\frac{4}{t}\Bigg]\Bigg], [1, 1], [1, 1], \Bigg[\Bigg[\Bigg[\frac{242}{t}+1, 1\Bigg], \Bigg[-\frac{242}{t}+1, 1\Bigg], \Bigg[-\frac{242}{t}+1, \frac{242}{t}+1\Bigg]\Bigg], \Bigg[\Bigg[-\frac{45}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}, -\frac{43}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}\Bigg], \Bigg[-\frac{47}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}, -\frac{45}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}\Bigg], \Bigg[-\frac{47}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}, -\frac{43}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}\Bigg]\Bigg]\Bigg], [[242\,t, -242\,t, -484\,t], [-2\,t, -2\,t, -4\,t]], [[0, 0, 0], [0, 0, 0]], [[[x-1, 1]], [[[-1, 0, 1], [1, 2, 1], [1, 1, 1], [[0, -1], [1, -1], [1, 0]], 4]]]\Bigg] \quad (38)$$

```
> Sreg:=regsingtrueBesSq(L,t,Sirr[-1],ext);
```

$$Sreg := [\;] \quad (39)$$

```
> NRemSreg:=SregseptrueBesSq(L,Sreg,ext)[1];
```

$$NRemSreg := [\;] \quad (40)$$

```
> LogSreg:=SregseptrueBesSq(L,Sreg,ext)[3];
```

$$LogSreg := [\;] \quad (41)$$

```
> RemSreg:=SregseptrueBesSq(L,Sreg,ext)[2];
```

$$RemSreg := [\;] \quad (42)$$

```
> R1:=IrrRegAppsingBesSq(L,t,E,ext);
```

$$R1 := \Bigg[\Bigg[[[x-12, 12], [\infty, \infty]], \Bigg[\Bigg[1, \frac{242}{t}+1, -\frac{242}{t}+1\Bigg], \Bigg[-\frac{43}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}, -\frac{45}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}, -\frac{47}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}\Bigg]\Bigg], \Bigg[\Bigg[\frac{242}{t}, -\frac{242}{t}, -\frac{484}{t}\Bigg], \Bigg[-\frac{2}{t}, -\frac{2}{t}, -\frac{4}{t}\Bigg]\Bigg], [1, 1], [1, 1], \Bigg[\Bigg[\Bigg[\frac{242}{t}+1, 1\Bigg], \Bigg[-\frac{242}{t}+1, 1\Bigg], \Bigg[-\frac{242}{t}+1, \frac{242}{t}+1\Bigg]\Bigg], \Bigg[\Bigg[\quad (43)$$

$$-\frac{45}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}, -\frac{43}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}\Big], \Big[-\frac{47}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}, -\frac{45}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}\Big], \Big[-\frac{47}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}, -\frac{43}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}\Big]\Big]\Big], [[242\,t, -242\,t, -484\,t], [-2\,t, -2\,t, -4\,t]], [[0,0,0],[0,0,0]]\Big], [\,], [[\,],[\,],[\,]], [[[x-1,1],[x-23,23]], [[-1,0,1],[0,2,4]], [[1,2,1],[2,4,2]], [[[0,-1],[1,-1],[1,0]], [[2,0],[4,0],[4,2]]]]], \Big[[[x-1,1],[x-12,12],[\infty,\infty]], \Big[[-1,0,1], \Big[1, \frac{242}{t}+1, -\frac{242}{t}+1\Big], \Big[-\frac{43}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}, -\frac{45}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}, -\frac{47}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}\Big]\Big], \Big[[1,2,1], \Big[\frac{242}{t}, -\frac{242}{t}, -\frac{484}{t}\Big], \Big[-\frac{2}{t}, -\frac{2}{t}, -\frac{4}{t}\Big]\Big], \Big[[[0,-1],[1,-1],[1,0]], \Big[\Big[\frac{242}{t}+1, 1\Big], \Big[-\frac{242}{t}+1, 1\Big], \Big[-\frac{242}{t}+1, \frac{242}{t}+1\Big]\Big], \Big[\Big[-\frac{45}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}, -\frac{43}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}\Big], \Big[-\frac{47}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}, -\frac{45}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}\Big], \Big[-\frac{47}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}, -\frac{43}{t}+1+\frac{14}{t^2}-\frac{1}{t^3}\Big]\Big]\Big], [[1,1,1],[1,1,1],[1,1,1]]\Big]\Big]$$

```
> F1:= BesSqSubst(L,x,t,R1[1],ext);
```

$$F1 := \left[-\frac{x^2-12\,x+121}{x-12}, \frac{x^2-12\,x-121}{x-12}, -\frac{x^2-12\,x-121}{x-12}, \frac{x^2-12\,x+121}{x-12}\right] \quad \textbf{(44)}$$

```
> findBesSqvfInt(L,R1,F1,x,t,ext);
```

$$\left[\left[\left[\frac{1}{4}\right], -\frac{(x-1)^2}{x-12}\right], \left[\left[\frac{1}{4}\right], -\frac{(x-23)^2}{x-12}\right], \left[\left[\frac{1}{4}\right], \frac{(x-23)^2}{x-12}\right], \left[\left[\frac{1}{4}\right], \frac{(x-1)^2}{x-12}\right]\right] \quad \textbf{(45)}$$

```
> TIME :=time();
  BesSqSolutions(L);
  time() - TIME;
```

$$TIME := 8.328$$

$$\left\{\left[\frac{1}{4}, [(x-5)\,(x-9)], [1], \frac{(x-1)^2}{x-12}\right], \left[\frac{1}{4}, [(x-5)\,(x-9)], [1], -\frac{(x-1)^2}{x-12}\right]\right\}$$

$$2.125 \quad \textbf{(46)}$$

```
>                ####### THE LOGARITHMIC CASE #######
```

```
> eq:=HolonomicDE(BesselI(nu, x)^2,Y(x));
```

$$eq := -4\,Y(x)\,x + \left(-4\,\nu^2-4\,x^2+1\right)\left(\frac{d}{dx}\,Y(x)\right) + \left(\frac{d^3}{dx^3}\,Y(x)\right)x^2 + 3\left(\frac{d^2}{dx^2}\,Y(x)\right)x \quad \textbf{(47)}$$

```
> LBB:=de2diffop(eq, Y(x));
```

$$LBB := x^2 Dx^3 + 3\,x\,Dx^2 + \left(-4\,\nu^2 - 4\,x^2 + 1\right) Dx - 4\,x \tag{48}$$

```
> LBB:=subs(nu=3,LBB);
```

$$LBB := x^2 Dx^3 + 3\,x\,Dx^2 + \left(-4\,x^2 - 35\right) Dx - 4\,x \tag{49}$$

```
> f:=(x-1)/((x-7)^2);
```

$$f := \frac{x-1}{(x-7)^2} \tag{50}$$

```
> M:=ChangeOfVariables(LBB,f);
```

$$M := Dx^3 (x-1)^2 (x-7)^7 (x+5)^2 + 3\left(x^2 + 10\,x - 47\right) Dx^2 (x-1)(x-7)^6 (x+5) - \left(35\,x^8 - 280\,x^7 - 4056\,x^6 + 30944\,x^5 + 124286\,x^4 - 154296\,x^3 - 9292672\,x^2 + 21781616\,x + 46300983\right) Dx\,(x-7) + 4\,(x+5)^5 (x-1) \tag{51}$$

```
> r:=x;
```

$$r := x \tag{52}$$

```
> L1:=ExpProduct(M,r);
```

$$L1 := Dx^3 (x-1)^2 (x-7)^7 (x+5)^2 - 3\left(x^4 - 3\,x^3 - 34\,x^2 + 25\,x + 47\right) Dx^2 (x-1)(x-7)^6 (x+5) + \left(3\,x^{12} - 102\,x^{11} + 1206\,x^{10} - 3588\,x^9 - 39428\,x^8 + 329428\,x^7 - 298380\,x^6 - 4611296\,x^5 + 14023435\,x^4 + 3070482\,x^3 - 39730946\,x^2 + 23597284\,x - 55124658\right) Dx\,(x-7) - x^{14} + 41\,x^{13} - 637\,x^{12} + 3900\,x^{11} + 6212\,x^{10} - 206592\,x^9 + 800017\,x^8 + 1580072\,x^7 - 17220581\,x^6 + 22757129\,x^5 + 74892153\,x^4 - 165812620\,x^3 + 44214202\,x^2 - 230777610\,x - 82955045 \tag{53}$$

```
> r0:=0;
```

$$r0 := 0 \tag{54}$$

```
> r1:=1;
```

$$r1 := 1 \tag{55}$$

```
> r2:=0;
```

$$r2 := 0 \tag{56}$$

```
> L:=GaugeTransf(L1,r0,r1,r2);
```

$$L := (x+5)^3 \left(x^{14} - 41\,x^{13} + 637\,x^{12} - 3900\,x^{11} - 6212\,x^{10} + 206592\,x^9 - 800017\,x^8 - 1580072\,x^7 + 17220581\,x^6 - 22757129\,x^5 - 74892153\,x^4 + 165812620\,x^3 - 44214202\,x^2 + 230777610\,x + 82955045\right)^3 (x-1)^5 (x-7)^{14} Dx^3 - \left(3\,x^{18} - 132\,x^{17} + 2181\,x^{16} - 13308\,x^{15} - 49251\,x^{14} + 1101798\,x^{13} - 3875266\,x^{12} - 18477035\,x^{11} + 157255132\,x^{10} - 105772395\,x^9 - 1844470753\,x^8 + 4247460806\,x^7 + 5553299897\,x^6 - 19176601600\,x^5 + 1225062152\,x^4 - 17722577007\,x^3 + 15466614523\,x^2 + 48387738265\,x + 27322786790\right)(x+5)^2 \left(x^{14} - 41\,x^{13} + 637\,x^{12} - 3900\,x^{11} - 6212\,x^{10} + 206592\,x^9 - 800017\,x^8 - 1580072\,x^7 + 17220581\,x^6 - 22757129\,x^5 - 74892153\,x^4 + 165812620\,x^3 - 44214202\,x^2 + 230777610\,x + 82955045\right)^2 (x-1)^4 (x-7)^{13} Dx^2 + \left(3\,x^{25} - 240\,x^{24} + 8505\,x^{23} - 172683\,x^{22} + 2133121\,x^{21} - 14717439\,x^{20} + 20286152\,x^{19} + 609393748\,x^{18} - 5673438350\,x^{17} + 16487326981\,x^{16} + 67717529222\,x^{15} - 721496452810\,x^{14} + 1852980250742\,x^{13} + 3845506071638\,x^{12} \tag{57}$$

$$- 34308160018532\, x^{11} + 61529587429212\, x^{10} + 78743219114627\, x^{9} - 426460816951810\, x^{8} + 438552218254841\, x^{7} + 117785789138701\, x^{6} - 1052894091678791\, x^{5} + 4344110881937929\, x^{4} - 6872500316324124\, x^{3} + 2230897290793880\, x^{2} - 1903678744352168\, x - 146476091913075\big)\, (x+5)^{2} \big(x^{14} - 41\, x^{13} + 637\, x^{12} - 3900\, x^{11} - 6212\, x^{10} + 206592\, x^{9} - 800017\, x^{8} - 1580072\, x^{7} + 17220581\, x^{6} - 22757129\, x^{5} - 74892153\, x^{4} + 165812620\, x^{3} - 44214202\, x^{2} + 230777610\, x + 82955045\big)^{2} (x-1)^{3} (x-7)^{8} Dx - \big(x^{27} - 87\, x^{26} + 3393\, x^{25} - 77245\, x^{24} + 1108262\, x^{23} - 9769502\, x^{22} + 39810222\, x^{21} + 161688184\, x^{20} - 3259100209\, x^{19} + 17614590021\, x^{18} - 7862152289\, x^{17} - 414984671595\, x^{16} + 2128866755468\, x^{15} - 1504507264140\, x^{14} - 24668440418920\, x^{13} + 92582504935372\, x^{12} - 26440940190049\, x^{11} - 554851048194125\, x^{10} + 1153272190379339\, x^{9} - 20171437573975\, x^{8} - 2473847761588770\, x^{7} + 7823057284780298\, x^{6} - 25563018307209542\, x^{5} + 39646128197290764\, x^{4} - 32626448170072447\, x^{3} + 41393065788567423\, x^{2} - 13746539671737003\, x + 22841552409157551\big)\, (x+5)^{2} \big(x^{14} - 41\, x^{13} + 637\, x^{12} - 3900\, x^{11} - 6212\, x^{10} + 206592\, x^{9} - 800017\, x^{8} - 1580072\, x^{7} + 17220581\, x^{6} - 22757129\, x^{5} - 74892153\, x^{4} + 165812620\, x^{3} - 44214202\, x^{2} + 230777610\, x + 82955045\big)^{2} (x-1)^{3} (x-7)^{7}$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \varnothing \qquad (58)$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```

$$ext := \varnothing \qquad (59)$$

```
> extppp:={};
```

$$extppp := \varnothing \qquad (60)$$

```
> E:= Singular(L,extppp);
```

$$E := \big[\big[x^{14} - 41\, x^{13} + 637\, x^{12} - 3900\, x^{11} - 6212\, x^{10} + 206592\, x^{9} - 800017\, x^{8} - 1580072\, x^{7} + 17220581\, x^{6} - 22757129\, x^{5} - 74892153\, x^{4} + 165812620\, x^{3} - 44214202\, x^{2} + 230777610\, x + 82955045, \mathit{RootOf}\big(\_Z^{14} - 41\, \_Z^{13} + 637\, \_Z^{12} - 3900\, \_Z^{11} - 6212\, \_Z^{10} + 206592\, \_Z^{9} - 800017\, \_Z^{8} - 1580072\, \_Z^{7} + 17220581\, \_Z^{6} - 22757129\, \_Z^{5} - 74892153\, \_Z^{4} + 165812620\, \_Z^{3} - 44214202\, \_Z^{2} + 230777610\, \_Z + 82955045\big)\big], [x - 1, 1], [x - 7, 7], [\infty, \infty], [x + 5, -5]\big] \qquad (61)$$

```
> F:=NotAppSing(L,E,ext);
```

$$F := [[x - 1, 1], [x - 7, 7], [\infty, \infty]] \qquad (62)$$

```
> Sirr:=irrsingBesSq(L,t,F,ext);
```

$$Sirr := \Big[[[x - 7, 7]], \Big[\Big[1, \frac{24}{t^2} + \frac{2}{t} - 1, -\frac{24}{t^2} - \frac{2}{t} - 1\Big]\Big], \Big[\Big[\frac{24}{t^2} + \frac{2}{t} - 2, -\frac{24}{t^2} - \frac{2}{t} - 2, -\frac{48}{t^2} - \frac{4}{t}\Big]\Big], [2], [1], \Big[\Big[\Big[\frac{24}{t^2} + \frac{2}{t} - 1, 1\Big], \Big[-\frac{24}{t^2} - \frac{2}{t} - 1, 1\Big], \Big[-\frac{24}{t^2} - \frac{2}{t} \qquad (63)$$

$-1, \frac{24}{t^2}+\frac{2}{t}-1\Big]\Big]\Big]\Big], [[24 t^2+2 t, -24 t^2-2 t, -48 t^2-4 t]], [[-2, -2, 0]], \Big[[[x-1, 1], [\infty, \infty]], \Big[[[-7, 0, 5], [7, 12, 5], [1, 1, 1], [[0, -7], [5, -7], [5, 0]], 4], \Big[\Big[-7-\frac{1}{t^2}, -1-\frac{1}{t^2}, 5-\frac{1}{t^2}\Big], [6, 12, 6], [1, 1, 1], \Big[\Big[-1-\frac{1}{t^2}, -7-\frac{1}{t^2}\Big], \Big[5-\frac{1}{t^2}, -7-\frac{1}{t^2}\Big], \Big[5-\frac{1}{t^2}, -1-\frac{1}{t^2}\Big]\Big], 4\Big]\Big]\Big]\Big]$

```
> Sreg:=regsingtrueBesSq(L,t,Sirr[-1],ext);
```

$$Sreg := \Big[[[x-1, 1], [\infty, \infty]], \Big[[-7, 0, 5], \Big[-7-\frac{1}{t^2}, -1-\frac{1}{t^2}, 5-\frac{1}{t^2}\Big]\Big], [[7, 12, 5], [6, 6, 12]], \Big[[[0, -7], [5, -7], [5, 0]], \Big[\Big[-1-\frac{1}{t^2}, -7-\frac{1}{t^2}\Big], \Big[5-\frac{1}{t^2}, -1-\frac{1}{t^2}\Big], \Big[5-\frac{1}{t^2}, -7-\frac{1}{t^2}\Big]\Big]\Big]\Big] \quad (64)$$

```
> NRemSreg:=SregseptrueBesSq(L,Sreg,ext)[1];
```

$$NRemSreg := [\,] \quad (65)$$

```
> LogSreg:=SregseptrueBesSq(L,Sreg,ext)[3];
```

$$LogSreg := \Big[[[x-1, 1], [\infty, \infty]], \Big[[-7, 0, 5], \Big[-7-\frac{1}{t^2}, -1-\frac{1}{t^2}, 5-\frac{1}{t^2}\Big]\Big], [[[\,], [7, 12, 5]], [[\,], [6, 6, 12]]]\Big] \quad (66)$$

```
> RemSreg:=SregseptrueBesSq(L,Sreg,ext)[2];
```

$$RemSreg := [\,] \quad (67)$$

```
> R1:=IrrRegAppsingBesSq(L,t,E,ext);
```

$$R1 := \Big[\Big[\Big[[[x-7, 7]], \Big[\Big[1, \frac{24}{t^2}+\frac{2}{t}-1, -\frac{24}{t^2}-\frac{2}{t}-1\Big]\Big], \Big[\Big[\frac{24}{t^2}+\frac{2}{t}-2, -\frac{24}{t^2}-\frac{2}{t}-2, -\frac{48}{t^2}-\frac{4}{t}\Big]\Big], [2], [1], \Big[\Big[\Big[\Big[\frac{24}{t^2}+\frac{2}{t}-1, 1\Big], \Big[-\frac{24}{t^2}-\frac{2}{t}-1, 1\Big], \Big[-\frac{24}{t^2}-\frac{2}{t}-1, \frac{24}{t^2}+\frac{2}{t}-1\Big]\Big]\Big]\Big], [[24 t^2+2 t, -24 t^2-2 t, -48 t^2-4 t]], [[-2, -2, 0]]\Big], \Big[[[x-1, 1], [\infty, \infty]], \Big[[-7, 0, 5], \Big[-7-\frac{1}{t^2}, -1-\frac{1}{t^2}, 5-\frac{1}{t^2}\Big]\Big], [[7, 12, 5], [6, 6, 12]], \Big[[[0, -7], [5, -7], [5, 0]], \Big[\Big[-1-\frac{1}{t^2}, -7-\frac{1}{t^2}\Big], \Big[5-\frac{1}{t^2}, -1-\frac{1}{t^2}\Big], \Big[5-\frac{1}{t^2}, -7-\frac{1}{t^2}\Big]\Big]\Big]\Big], \Big[[\,], [\,], \Big[[[x-1, 1], [\infty, \infty]], \Big[[-7, 0, 5], \Big[-7-\frac{1}{t^2}, -1-\frac{1}{t^2}, 5-\frac{1}{t^2}\Big]\Big], [[[\,], [7, 12, 5]], [[\,], [6, 12, 6]]]\Big]\Big], [[[x^{14}-41 x^{13}+637 x^{12}-3900 x^{11}-6212 x^{10}+206592 x^9-800017 x^8-1580072 x^7+17220581 x^6-22757129 x^5-74892153 x^4 \quad (68)$$

$+ 165812620\,x^3 - 44214202\,x^2 + 230777610\,x + 82955045,\ RootOf\big(\_Z^{14} - 41\,\_Z^{13} + 637\,\_Z^{12} - 3900\,\_Z^{11} - 6212\,\_Z^{10} + 206592\,\_Z^9 - 800017\,\_Z^8 - 1580072\,\_Z^7 + 17220581\,\_Z^6 - 22757129\,\_Z^5 - 74892153\,\_Z^4 + 165812620\,\_Z^3 - 44214202\,\_Z^2 + 230777610\,\_Z + 82955045\big)\,],\ [x+5, -5]],\ [[0,1,3],[0,1,3]],\ [[1,3,2],[1,3,2]],\ [[[1,0],[3,0],[3,1]],[[1,0],[3,0],[3,1]]]],\ \Big[[[x-1,1],[x-7,7],[\infty,\infty]],\ \Big[[-7,0,5],\Big[1,\frac{24}{t^2}+\frac{2}{t}-1,-\frac{24}{t^2}-\frac{2}{t}-1\Big],\Big[-7-\frac{1}{t^2},-1-\frac{1}{t^2},5-\frac{1}{t^2}\Big]\Big],\ \Big[[7,12,5],\Big[\frac{24}{t^2}+\frac{2}{t}-2,-\frac{24}{t^2}-\frac{2}{t}-2,-\frac{48}{t^2}-\frac{4}{t}\Big],[6,12,6]\Big],\ \Big[[[0,-7],[5,-7],[5,0]],\Big[\Big[\frac{24}{t^2}+\frac{2}{t}-1,1\Big],\Big[-\frac{24}{t^2}-\frac{2}{t}-1,1\Big],\Big[-\frac{24}{t^2}-\frac{2}{t}-1,\frac{24}{t^2}+\frac{2}{t}-1\Big]\Big],\Big[\Big[-1-\frac{1}{t^2},-7-\frac{1}{t^2}\Big],\Big[5-\frac{1}{t^2},-7-\frac{1}{t^2}\Big],\Big[5-\frac{1}{t^2},-1-\frac{1}{t^2}\Big]\Big]\Big],\ [[1,1,1],[1,1,1],[1,1,1]]\Big]\Big]$

```
> F1:= BesSqSubst(L,x,t,R1[1],ext);
```

$$F1 := \left[-\frac{x-1}{(x-7)^2}, \frac{x-1}{(x-7)^2}\right] \tag{69}$$

```
> CandichangvarBesSq(F1,R1,ext);
```

$$\left\{\left[\left[\frac{x-1}{(x-7)^2},[1,1],\left[[[x-1,1],[\infty,\infty]],\left[[-7,0,5],\left[-7-\frac{1}{t^2},-1-\frac{1}{t^2},5-\frac{1}{t^2}\right]\right],[[7,12,5],[6,6,12]],\left[[[0,-7],[5,-7],[5,0]],\left[\left[-1-\frac{1}{t^2},-7-\frac{1}{t^2}\right],\left[5-\frac{1}{t^2},-1-\frac{1}{t^2}\right],\left[5-\frac{1}{t^2},-7-\frac{1}{t^2}\right]\right]\right]\right]\right],\left[-\frac{x-1}{(x-7)^2},[1,1],\left[[[x-1,1],[\infty,\infty]],\left[[-7,0,5],\left[-7-\frac{1}{t^2},-1-\frac{1}{t^2},5-\frac{1}{t^2}\right]\right],[[7,12,5],[6,6,12]],\left[[[0,-7],[5,-7],[5,0]],\left[\left[-1-\frac{1}{t^2},-7-\frac{1}{t^2}\right],\left[5-\frac{1}{t^2},-1-\frac{1}{t^2}\right],\left[5-\frac{1}{t^2},-7-\frac{1}{t^2}\right]\right]\right]\right]\right]\right]\right\} \tag{70}$$

```
> findBesSqln(L,R1,F1,x,t,ext);
```

$$\left\{\left[[3],\frac{x-1}{(x-7)^2}\right],\left[[3],-\frac{x-1}{(x-7)^2}\right]\right\} \tag{71}$$

```
> TIME :=time();
  BesSqSolutions(L);
  time() - TIME;
```

$$TIME := 41.234$$

$$\left\{\left[3, [x], [Dx], \frac{x-1}{(x-7)^2}\right], \left[3, [x], [Dx], -\frac{x-1}{(x-7)^2}\right]\right\}$$
14.359 **(72)**

```
> ####### THE RATIONAL CASE #######
```

```
> eq:=HolonomicDE(BesselI(nu, x)^2,Y(x));
```

$$eq := -4\,Y(x)\,x + \left(-4\,\nu^2 - 4\,x^2 + 1\right)\left(\frac{d}{dx}\,Y(x)\right) + \left(\frac{d^3}{dx^3}\,Y(x)\right)x^2 + 3\left(\frac{d^2}{dx^2}\,Y(x)\right)x \quad \textbf{(73)}$$

```
> LBB:=de2diffop(eq, Y(x));
```

$$LBB := x^2\,Dx^3 + 3\,x\,Dx^2 + \left(-4\,\nu^2 - 4\,x^2 + 1\right)Dx - 4\,x \quad \textbf{(74)}$$

```
> LBB:=subs(nu=1/3,LBB);
```

$$LBB := x^2\,Dx^3 + 3\,x\,Dx^2 + \left(\frac{5}{9} - 4\,x^2\right)Dx - 4\,x \quad \textbf{(75)}$$

```
> f:=(x-1)/(x-12);
```

$$f := \frac{x-1}{x-12} \quad \textbf{(76)}$$

```
> M:=ChangeOfVariables(LBB,f);
```

$$M := 9\,Dx^3\,(x-1)^2\,(x-12)^5 + 27\,(-13+2\,x)\,Dx^2\,(x-1)\,(x-12)^4 + \left(54\,x^4 - 1998\,x^3 + 21521\,x^2 - 122448\,x + 176076\right)Dx\,(x-12) + 47916\,x - 47916 \quad \textbf{(77)}$$

```
> r:=(x-9);
```

$$r := x - 9 \quad \textbf{(78)}$$

```
> L:=ExpProduct(M,r);
```

$$L := 9\,Dx^3\,(x-1)^2\,(x-12)^5 - 27\left(x^3 - 22\,x^2 + 127\,x - 95\right)Dx^2\,(x-1)\,(x-12)^4 + \left(27\,x^8 - 1836\,x^7 + 52299\,x^6 - 803952\,x^5 + 7135830\,x^4 - 36040032\,x^3 + 93852569\,x^2 - 98170032\,x + 34048332\right)Dx\,(x-12) - 9\,x^{10} + 801\,x^9 - 31221\,x^8 + 697572\,x^7 - 9809397\,x^6 + 89568396\,x^5 - 527135447\,x^4 + 1908325917\,x^3 - 3815933832\,x^2 + 3356793000\,x - 1008917820 \quad \textbf{(79)}$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \varnothing \quad \textbf{(80)}$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```

$$ext := \varnothing \quad \textbf{(81)}$$

```
> extppp:={};
```

$$extppp := \varnothing \quad \textbf{(82)}$$

```
> E:= Singular(L,extppp);
```

$$E := [[x-12, 12], [x-1, 1], [\infty, \infty]] \quad \textbf{(83)}$$

```
> F:=NotAppSing(L,E,ext);
```

$$F := [[x-1, 1], [x-12, 12], [\infty, \infty]] \quad \textbf{(84)}$$

```
> Sirr:=irrsingBesSq(L,t,F,ext);
```

$$Sirr := \left[[[x-12, 12]], \left[\left[1, \frac{22}{t}+1, -\frac{22}{t}+1\right]\right], \left[\left[\frac{22}{t}, -\frac{22}{t}, -\frac{44}{t}\right]\right], [1], [1], \left[\left[\left[\frac{22}{t}\right.\right.\right.\right. \quad \textbf{(85)}$$

$$+1, 1\right], \left[-\frac{22}{t}+1, 1\right], \left[-\frac{22}{t}+1, \frac{22}{t}+1\right]\right]\right], [[22\,t, -22\,t, -44\,t]], [[0, 0, 0]], \left[[[x-1, 1], [\infty, \infty]], \left[\left[\left[0, \frac{2}{3}, -\frac{2}{3}\right], \left[\frac{2}{3}, -\frac{2}{3}, -\frac{4}{3}\right], [1, 1, 1], \left[\left[\frac{2}{3}, 0\right], \left[-\frac{2}{3}, 0\right], \left[-\frac{2}{3}, \frac{2}{3}\right]\right], 2\right], \left[\left[\frac{9}{t}-\frac{1}{t^2}, 1+\frac{9}{t}-\frac{1}{t^2}, 2+\frac{9}{t}-\frac{1}{t^2}\right], [1, 2, 1], [1, 1, 1], \left[\left[1+\frac{9}{t}-\frac{1}{t^2}, \frac{9}{t}-\frac{1}{t^2}\right], \left[2+\frac{9}{t}-\frac{1}{t^2}, \frac{9}{t}-\frac{1}{t^2}\right], \left[2+\frac{9}{t}-\frac{1}{t^2}, 1+\frac{9}{t}-\frac{1}{t^2}\right]\right], 4\right]\right]\right]\right]$$

```
> Sreg:=regsingtrueBesSq(L,t,Sirr[-1],ext);
```

$$Sreg := \left[[[x-1, 1]], \left[\left[0, \frac{2}{3}, -\frac{2}{3}\right]\right], \left[\left[\frac{2}{3}, -\frac{2}{3}, -\frac{4}{3}\right]\right], \left[\left[\left[\frac{2}{3}, 0\right], \left[-\frac{2}{3}, 0\right], \left[-\frac{2}{3}, \frac{2}{3}\right]\right]\right]\right] \quad \textbf{(86)}$$

```
> NRemSreg:=SregseptrueBesSq(L,Sreg,ext)[1];
```

$$NRemSreg := \left[[[x-1, 1]], \left[\left[0, \frac{2}{3}, -\frac{2}{3}\right]\right], \left[\left[\left[\frac{2}{3}, -\frac{2}{3}, -\frac{4}{3}\right], [\,]\right]\right]\right] \quad \textbf{(87)}$$

```
> LogSreg:=SregseptrueBesSq(L,Sreg,ext)[3];
```

$$LogSreg := [\,] \quad \textbf{(88)}$$

```
> RemSreg:=SregseptrueBesSq(L,Sreg,ext)[2];
```

$$RemSreg := [\,] \quad \textbf{(89)}$$

```
> R1:=IrrRegAppsingBesSq(L,t,E,ext);
```

$$R1 := \left[\left[\left[[[x-12, 12]], \left[\left[1, \frac{22}{t}+1, -\frac{22}{t}+1\right]\right], \left[\left[\frac{22}{t}, -\frac{22}{t}, -\frac{44}{t}\right]\right], [1], [1], \left[\left[\left[\frac{22}{t}+1, 1\right], \left[-\frac{22}{t}+1, 1\right], \left[-\frac{22}{t}+1, \frac{22}{t}+1\right]\right]\right], [[22\,t, -22\,t, -44\,t]], [[0, 0, 0]]\right], \left[[[x-1, 1]], \left[\left[0, \frac{2}{3}, -\frac{2}{3}\right]\right], \left[\left[\frac{2}{3}, -\frac{2}{3}, -\frac{4}{3}\right]\right], \left[\left[\left[\frac{2}{3}, 0\right], \left[-\frac{2}{3}, 0\right], \left[-\frac{2}{3}, \frac{2}{3}\right]\right]\right]\right], \left[\left[[[x-1, 1]], \left[\left[0, \frac{2}{3}, -\frac{2}{3}\right]\right], \left[\left[\left[\frac{2}{3}, -\frac{2}{3}, -\frac{4}{3}\right], [\,]\right]\right]\right], [\,], [\,]\right], \left[[[\infty, \infty]], \left[\left[\frac{9}{t}-\frac{1}{t^2}, 1+\frac{9}{t}-\frac{1}{t^2}, 2+\frac{9}{t}-\frac{1}{t^2}\right]\right], [[1, 2, 1]], \left[\left[\left[1+\frac{9}{t}-\frac{1}{t^2}, \frac{9}{t}-\frac{1}{t^2}\right], \left[2+\frac{9}{t}-\frac{1}{t^2}, \frac{9}{t}-\frac{1}{t^2}\right], \left[2+\frac{9}{t}-\frac{1}{t^2}, 1+\frac{9}{t}-\frac{1}{t^2}\right]\right]\right]\right]\right], \left[[[x-1, 1], [x-12, 12], [\infty, \infty]], \left[\left[0, \frac{2}{3}, -\frac{2}{3}\right], \left[1, \frac{22}{t}+1, -\frac{22}{t}+1\right], \left[\frac{9}{t}-\frac{1}{t^2}, 1+\frac{9}{t}-\frac{1}{t^2}, 2+\frac{9}{t}-\frac{1}{t^2}\right]\right], \left[\left[\frac{2}{3}, -\frac{2}{3}, -\frac{4}{3}\right], \left[\frac{22}{t}, -\frac{22}{t}, -\frac{44}{t}\right], [1, 2, 1]\right], \left[\left[\left[\frac{2}{3}, 0\right], \left[-\frac{2}{3}, 0\right], \left[-\frac{2}{3}, \frac{2}{3}\right]\right], \left[\left[\frac{22}{t}+1, 1\right], \left[-\frac{22}{t}+1, 1\right], \left[-\frac{22}{t}+1, \frac{22}{t}+1\right]\right], \left[\left[1+\frac{9}{t}-\frac{1}{t^2}, \frac{9}{t}-\frac{1}{t^2}\right], \left[2+\frac{9}{t}-\frac{1}{t^2}, \frac{9}{t}-\frac{1}{t^2}\right], \left[2+\frac{9}{t}-\frac{1}{t^2}, 1+\frac{9}{t}-\frac{1}{t^2}\right]\right]\right], [[1, 1, 1], [1, 1, 1], [1, 1, 1]]\right]\right] \quad \textbf{(90)}$$

```
> F1:= BesSqSubst(L,x,t,R1[1],ext);
```

$$F1 := \left[-\frac{11}{x-12}, \frac{11}{x-12}\right] \quad \mathbf{(91)}$$

```
> CandichangvarBesSq(F1,R1,ext);
```

$$\left\{\left[\frac{x-1}{x-12}, [1], \left[[[x-1,1]], \left[\left[0, \frac{2}{3}, -\frac{2}{3}\right]\right], \left[\left[\frac{2}{3}, -\frac{2}{3}, -\frac{4}{3}\right]\right], \left[\left[\left[\frac{2}{3}, 0\right], \left[-\frac{2}{3}, 0\right], \left[-\frac{2}{3}, \frac{2}{3}\right]\right]\right]\right]\right], \left[-\frac{x-1}{x-12}, [1], \left[[[x-1,1]], \left[\left[0, \frac{2}{3}, -\frac{2}{3}\right]\right], \left[\left[\frac{2}{3}, -\frac{2}{3}, -\frac{4}{3}\right]\right], \left[\left[\left[\frac{2}{3}, 0\right], \left[-\frac{2}{3}, 0\right], \left[-\frac{2}{3}, \frac{2}{3}\right]\right]\right]\right]\right]\right\} \quad \mathbf{(92)}$$

```
> findBesSqvfRat(L,R1,F1,x,t,ext);
```

$$\left[\left[\left[\frac{1}{3}, \frac{5}{6}\right], \frac{x-1}{x-12}\right], \left[\left[\frac{1}{3}, \frac{5}{6}\right], -\frac{x-1}{x-12}\right]\right] \quad \mathbf{(93)}$$

```
> TIME :=time();
  BesSqSolutions(L);
  time() - TIME;
```

$$TIME := 57.171$$

$$\left\{\left[\frac{1}{3}, [x-9], [1], \frac{x-1}{x-12}\right], \left[\frac{1}{3}, [x-9], [1], -\frac{x-1}{x-12}\right]\right\}$$

$$0.204 \quad \mathbf{(94)}$$

```
>                   ####### THE BASEFIELD CASE #######
```

```
> eq:=HolonomicDE(BesselI(nu, x)^2,Y(x));
```

$$eq := -4\,Y(x)\,x + \left(-4\,\nu^2 - 4\,x^2 + 1\right)\left(\frac{d}{dx}\,Y(x)\right) + \left(\frac{d^3}{dx^3}\,Y(x)\right)x^2 + 3\left(\frac{d^2}{dx^2}\,Y(x)\right)x \quad \mathbf{(95)}$$

```
> LBB:=de2diffop(eq, Y(x));
```

$$LBB := x^2\,Dx^3 + 3\,x\,Dx^2 + \left(-4\,\nu^2 - 4\,x^2 + 1\right)Dx - 4\,x \quad \mathbf{(96)}$$

```
> LBB:=subs(nu=sqrt(2)+1/2,LBB);
```

$$LBB := x^2\,Dx^3 + 3\,x\,Dx^2 + \left(-4\left(\sqrt{2} + \frac{1}{2}\right)^2 - 4\,x^2 + 1\right)Dx - 4\,x \quad \mathbf{(97)}$$

```
> f:=(x-7)/x;
```

$$f := \frac{x-7}{x} \quad \mathbf{(98)}$$

```
> L:=ChangeOfVariables(LBB,f);
```

$$L := Dx^3\,(x-7)^2\,x^5 + 3\,(-7+2\,x)\,Dx^2\,(x-7)\,x^4 - 2\,x\left(-3\,x^4 + 98\sqrt{2}\,x^2 + 21\,x^3 + 294\,x^2 - 1372\,x + 4802\right)Dx - 1372\,x + 9604 \quad \mathbf{(99)}$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \varnothing \quad \mathbf{(100)}$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```

$$ext := \varnothing \quad \mathbf{(101)}$$

```
> extppp:={};
```

$$\mathbf{(102)}$$

$$extppp := \varnothing \tag{102}$$

```
> E:= Singular(L,extppp);
```

$$E := [[x, 0], [x-7, 7]] \tag{103}$$

```
> F:=NotAppSing(L,E,ext);
```

$$F := [[x, 0], [x-7, 7]] \tag{104}$$

```
> Sirr:=irrsingBesSq(L,t,F,ext);
```

$$Sirr := \Big[[[x, 0]], \Big[\Big[1, -\frac{14}{t}+1, \frac{14}{t}+1\Big]\Big], \Big[\Big[-\frac{14}{t}, \frac{14}{t}, \frac{28}{t}\Big]\Big], [1], [1], \Big[\Big[\Big[-\frac{14}{t}+1, 1\Big], \Big[\frac{14}{t}+1, 1\Big], \Big[\frac{14}{t}+1, -\frac{14}{t}+1\Big]\Big]\Big], [[-14\,t, 14\,t, 28\,t]], [[0, 0, 0]], \big[[[x-7, 7]], [[[0, 1+2\sqrt{2}, -1-2\sqrt{2}], [1+2\sqrt{2}, -1-2\sqrt{2}, -2-4\sqrt{2}], [1, 1, 1], [[1+2\sqrt{2}, 0], [-1-2\sqrt{2}, 0], [-1-2\sqrt{2}, 1+2\sqrt{2}]], 2]]]\big]\Big] \tag{105}$$

```
> Sreg:=regsingtrueBesSq(L,t,Sirr[-1],ext);
```

$$Sreg := \big[[[x-7, 7]], [[0, 1+2\sqrt{2}, -1-2\sqrt{2}]], [[1+2\sqrt{2}, -1-2\sqrt{2}, -2-4\sqrt{2}]], [[[1+2\sqrt{2}, 0], [-1-2\sqrt{2}, 0], [-1-2\sqrt{2}, 1+2\sqrt{2}]]]\big] \tag{106}$$

```
> NRemSreg:=SregseptrueBesSq(L,Sreg,ext)[1];
```

$$NRemSreg := \big[[[x-7, 7]], [[0, 1+2\sqrt{2}, -1-2\sqrt{2}]], [[[1+2\sqrt{2}, -1-2\sqrt{2}, -2-4\sqrt{2}], [\,]]]\big] \tag{107}$$

```
> LogSreg:=SregseptrueBesSq(L,Sreg,ext)[3];
```

$$LogSreg := [\,] \tag{108}$$

```
> RemSreg:=SregseptrueBesSq(L,Sreg,ext)[2];
```

$$RemSreg := [\,] \tag{109}$$

```
> R1:=IrrRegAppsingBesSq(L,t,E,ext);
```

$$R1 := \Big[\Big[[[x, 0]], \Big[\Big[1, -\frac{14}{t}+1, \frac{14}{t}+1\Big]\Big], \Big[\Big[-\frac{14}{t}, \frac{14}{t}, \frac{28}{t}\Big]\Big], [1], [1], \Big[\Big[\Big[-\frac{14}{t}+1, 1\Big], \Big[\frac{14}{t}+1, 1\Big], \Big[\frac{14}{t}+1, -\frac{14}{t}+1\Big]\Big]\Big], [[-14\,t, 14\,t, 28\,t]], [[0, 0, 0]]\Big], \big[[[x-7, 7]], [[0, 1+2\sqrt{2}, -1-2\sqrt{2}]], [[1+2\sqrt{2}, -1-2\sqrt{2}, -2-4\sqrt{2}]], [[[1+2\sqrt{2}, 0], [-1-2\sqrt{2}, 0], [-1-2\sqrt{2}, 1+2\sqrt{2}]]]\big], \big[[[[x-7, 7]], [[0, 1+2\sqrt{2}, -1-2\sqrt{2}]], [[[1+2\sqrt{2}, -1-2\sqrt{2}, -2-4\sqrt{2}], [\,]]]], [\,], [\,]\big], [\,], \Big[[[x, 0], [x-7, 7]], \Big[\Big[1, -\frac{14}{t}+1, \frac{14}{t}+1\Big], [0, 1+2\sqrt{2}, -1-2\sqrt{2}]\Big], \Big[\Big[-\frac{14}{t}, \frac{14}{t}, \frac{28}{t}\Big], [1+2\sqrt{2}, -1-2\sqrt{2}, -2-4\sqrt{2}]\Big], \Big[\Big[\Big[-\frac{14}{t}+1, 1\Big], \Big[\frac{14}{t}+1, 1\Big], \Big[\frac{14}{t}+1, -\frac{14}{t}+1\Big]\Big], [[1+2\sqrt{2}, 0], [-1-2\sqrt{2}, 0], [-1-2\sqrt{2}, 1+2\sqrt{2}]]\Big], [[1, 1, 1], [1, 1, 1]]\Big]\Big] \tag{110}$$

```
> F1:= BesSqSubst(L,x,t,R1[1],ext);
```

(111)

$$F1 := \left[-\frac{7}{x}, \frac{7}{x} \right] \quad (111)$$

```
> CandichangvarBesSq(F1,R1,ext);
```

$$\left\{ \left[\frac{x-7}{x}, [1], [[[x-7,7]], [[0, 1+2\sqrt{2}, -1-2\sqrt{2}]], [[1+2\sqrt{2}, -1-2\sqrt{2}, -2 - 4\sqrt{2}]], [[[1+2\sqrt{2}, 0], [-1-2\sqrt{2}, 0], [-1-2\sqrt{2}, 1+2\sqrt{2}]]]] \right], \left[-\frac{x-7}{x}, [1], [[[x-7,7]], [[0, 1+2\sqrt{2}, -1-2\sqrt{2}]], [[1+2\sqrt{2}, -1-2\sqrt{2}, -2 - 4\sqrt{2}]], [[[1+2\sqrt{2}, 0], [-1-2\sqrt{2}, 0], [-1-2\sqrt{2}, 1+2\sqrt{2}]]]] \right] \right\} \quad (112)$$

```
> findBesSqvfBasfield(L,R1,F1,x,t,ext);
```

$$\left[\left[\left[\sqrt{2} + \frac{1}{2}, 1+\sqrt{2} \right], -\frac{x-7}{x} \right], \left[\left[\sqrt{2} + \frac{1}{2}, 1+\sqrt{2} \right], \frac{x-7}{x} \right] \right] \quad (113)$$

```
> TIME :=time();
  BesSqSolutions(L);
  time() - TIME;
```

$$TIME := 57.796$$

$$\left\{ \left[\sqrt{2} + \frac{1}{2}, [0], [1], \frac{x-7}{x} \right], \left[\sqrt{2} + \frac{1}{2}, [0], [1], -\frac{x-7}{x} \right] \right\}$$

$$0.063 \quad (114)$$

```
>                    ####### THE IRRATIONAL CASE #######
```

```
> eq:=HolonomicDE(BesselI(nu, x)^2,Y(x));
```

$$eq := -4\, Y(x)\, x + \left(-4\, \nu^2 - 4\, x^2 + 1 \right) \left(\frac{d}{dx} Y(x) \right) + \left(\frac{d^3}{dx^3} Y(x) \right) x^2 + 3 \left(\frac{d^2}{dx^2} Y(x) \right) x \quad (115)$$

```
> LBB:=de2diffop(eq, Y(x));
```

$$LBB := x^2\, Dx^3 + 3\, x\, Dx^2 + \left(-4\, \nu^2 - 4\, x^2 + 1 \right) Dx - 4\, x \quad (116)$$

```
> LBB:=x^2*Dx^3+3*x*Dx^2+(1-4*x^2-4*nu^2)*Dx-4*x;
```

$$LBB := x^2\, Dx^3 + 3\, x\, Dx^2 + \left(-4\, \nu^2 - 4\, x^2 + 1 \right) Dx - 4\, x \quad (117)$$

```
> LBB:=subs(nu=a*RootOf(x^2+1)+1/2,LBB);
```

$$LBB := x^2\, Dx^3 + 3\, x\, Dx^2 + \left(-4 \left(a\, RootOf\left(\_Z^2 + 1\right) + \frac{1}{2} \right)^2 - 4\, x^2 + 1 \right) Dx - 4\, x \quad (118)$$

```
> f:=(x-2)/(x-1);
```

$$f := \frac{x-2}{x-1} \quad (119)$$

```
> L:=ChangeOfVariables(LBB,f);
```

$$L := Dx^3\, (x-2)^2\, (x-1)^5 + 3\, (-3 + 2\, x)\, Dx^2\, (x-2)\, (x-1)^4 - 2 \left(2\, RootOf\left(\_Z^2 + 1\right) a\, x^2 - 2\, a^2\, x^2 - 3\, x^4 - 4\, RootOf\left(\_Z^2 + 1\right) a\, x + 4\, a^2\, x + 15\, x^3 + 2\, a\, RootOf\left(\_Z^2 + 1\right) - 2\, a^2 - 25\, x^2 + 13\, x + 2 \right) (x-1)\, Dx - 4\, x + 8 \quad (120)$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \left\{ a, RootOf\left(\_Z^2 + 1\right) \right\} \quad (121)$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```
$$ext := \{a, RootOf(\_Z^2+1)\} \tag{122}$$

```
> extppp:={};
```
$$extppp := \emptyset \tag{123}$$

```
> E:= Singular(L,extppp);
```
$$E := [[x-2, 2], [x-1, 1]] \tag{124}$$

```
> F:=NotAppSing(L,E,ext);
```
$$F := [[x-1, 1], [x-2, 2]] \tag{125}$$

```
> Sirr:=irrsingBesSq(L,t,F,ext);
```
$$Sirr := \left[[[x-1, 1]], \left[\left[1, \frac{2}{t}+1, -\frac{2}{t}+1\right]\right], \left[\left[\frac{2}{t}, -\frac{2}{t}, -\frac{4}{t}\right]\right], [1], [1], \left[\left[\left[\frac{2}{t}+1, 1\right], \left[-\frac{2}{t}+1, 1\right], \left[-\frac{2}{t}+1, \frac{2}{t}+1\right]\right]\right], [[2\,t, -2\,t, -4\,t]], [[0, 0, 0]], [[[x-2, 2]], [[[0, 2\,a\,RootOf(\_Z^2+1)+1, -2\,a\,RootOf(\_Z^2+1)-1], [2\,a\,RootOf(\_Z^2+1)+1, -2\,a\,RootOf(\_Z^2+1)-1, -4\,a\,RootOf(\_Z^2+1)-2], [1, 1, 1], [[2\,a\,RootOf(\_Z^2+1)+1, 0], [-2\,a\,RootOf(\_Z^2+1)-1, 0], [-2\,a\,RootOf(\_Z^2+1)-1, 2\,a\,RootOf(\_Z^2+1)+1]], 2]]]\right] \tag{126}$$

```
> Sreg:=regsingtrueBesSq(L,t,Sirr[-1],ext);
```
$$Sreg := [[[x-2, 2]], [[0, 2\,a\,RootOf(\_Z^2+1)+1, -2\,a\,RootOf(\_Z^2+1)-1]], [[2\,a\,RootOf(\_Z^2+1)+1, -2\,a\,RootOf(\_Z^2+1)-1, -4\,a\,RootOf(\_Z^2+1)-2]], [[[2\,a\,RootOf(\_Z^2+1)+1, 0], [-2\,a\,RootOf(\_Z^2+1)-1, 0], [-2\,a\,RootOf(\_Z^2+1)-1, 2\,a\,RootOf(\_Z^2+1)+1]]]] \tag{127}$$

```
> NRemSreg:=SregseptrueBesSq(L,Sreg,ext)[1];
```
$$NRemSreg := [[[x-2, 2]], [[0, 2\,a\,RootOf(\_Z^2+1)+1, -2\,a\,RootOf(\_Z^2+1)-1]], [[[2\,a\,RootOf(\_Z^2+1)+1, -2\,a\,RootOf(\_Z^2+1)-1, -4\,a\,RootOf(\_Z^2+1)-2], [\,]]]] \tag{128}$$

```
> LogSreg:=SregseptrueBesSq(L,Sreg,ext)[3];
```
$$LogSreg := [\,] \tag{129}$$

```
> RemSreg:=SregseptrueBesSq(L,Sreg,ext)[2];
```
$$RemSreg := [\,] \tag{130}$$

```
> R1:=IrrRegAppsingBesSq(L,t,E,ext);
```
$$R1 := \left[\left[[[x-1, 1]], \left[\left[1, \frac{2}{t}+1, -\frac{2}{t}+1\right]\right], \left[\left[\frac{2}{t}, -\frac{2}{t}, -\frac{4}{t}\right]\right], [1], [1], \left[\left[\left[\frac{2}{t}+1, 1\right], \left[-\frac{2}{t}+1, 1\right], \left[-\frac{2}{t}+1, \frac{2}{t}+1\right]\right]\right], [[2\,t, -2\,t, -4\,t]], [[0, 0, 0]]\right], [[[x-2, 2]], [[0, 2\,a\,RootOf(\_Z^2+1)+1, -2\,a\,RootOf(\_Z^2+1)-1]], [[2\,a\,RootOf(\_Z^2+1)+1, -2\,a\,RootOf(\_Z^2+1)-1, -4\,a\,RootOf(\_Z^2+1)-2]], [[[2\,a\,RootOf(\_Z^2+1)+1, 0], [-2\,a\,RootOf(\_Z^2+1)-1, 0], [-2\,a\,RootOf(\_Z^2+1)-1, 2\,a\,RootOf(\_Z^2+1)+1) \tag{131}$$

$$+1]]]], [[[[x-2,2]], [[0, 2\,a\,\mathit{RootOf}(\_Z^2+1)+1, -2\,a\,\mathit{RootOf}(\_Z^2+1)-1]], [[[2\,a\,\mathit{RootOf}(\_Z^2+1)+1, -2\,a\,\mathit{RootOf}(\_Z^2+1)-1, -4\,a\,\mathit{RootOf}(\_Z^2+1)-2], []]]], [\,], [\,]], [\,], \left[[[x-1,1],[x-2,2]], \left[\left[1, \frac{2}{t}+1, -\frac{2}{t}+1\right], [0, 2\,a\,\mathit{RootOf}(\_Z^2+1)+1, -2\,a\,\mathit{RootOf}(\_Z^2+1)-1]\right], \left[\left[\frac{2}{t}, -\frac{2}{t}, -\frac{4}{t}\right], [2\,a\,\mathit{RootOf}(\_Z^2+1)+1, -2\,a\,\mathit{RootOf}(\_Z^2+1)-1, -4\,a\,\mathit{RootOf}(\_Z^2+1)-2]\right], \left[\left[\left[\frac{2}{t}+1, 1\right], \left[-\frac{2}{t}+1, 1\right], \left[-\frac{2}{t}+1, \frac{2}{t}+1\right]\right], [[2\,a\,\mathit{RootOf}(\_Z^2+1)+1, 0], [-2\,a\,\mathit{RootOf}(\_Z^2+1)-1, 0], [-2\,a\,\mathit{RootOf}(\_Z^2+1)-1, 2\,a\,\mathit{RootOf}(\_Z^2+1)+1]]\right], [[1,1,1],[1,1,1]]\right]\right]$$

```
> F1:= BesSqSubst(L,x,t,R1[1],ext);
```

$$F1 := \left[\frac{1}{x-1}, -\frac{1}{x-1}\right] \quad \mathbf{(132)}$$

```
> CandichangvarBesSq(F1,R1,ext);
```

$$\left\{\left[\frac{x-2}{x-1}, [1], [[[x-2,2]], [[0, 2\,a\,\mathit{RootOf}(\_Z^2+1)+1, -2\,a\,\mathit{RootOf}(\_Z^2+1)-1]], [[2\,a\,\mathit{RootOf}(\_Z^2+1)+1, -2\,a\,\mathit{RootOf}(\_Z^2+1)-1, -4\,a\,\mathit{RootOf}(\_Z^2+1)-2]], [[[2\,a\,\mathit{RootOf}(\_Z^2+1)+1, 0], [-2\,a\,\mathit{RootOf}(\_Z^2+1)-1, 0], [-2\,a\,\mathit{RootOf}(\_Z^2+1)-1, 2\,a\,\mathit{RootOf}(\_Z^2+1)+1]]]]\right], \left[-\frac{x-2}{x-1}, [1], [[[x-2,2]], [[0, 2\,a\,\mathit{RootOf}(\_Z^2+1)+1, -2\,a\,\mathit{RootOf}(\_Z^2+1)-1]], [[2\,a\,\mathit{RootOf}(\_Z^2+1)+1, -2\,a\,\mathit{RootOf}(\_Z^2+1)-1, -4\,a\,\mathit{RootOf}(\_Z^2+1)-2]], [[[2\,a\,\mathit{RootOf}(\_Z^2+1)+1, 0], [-2\,a\,\mathit{RootOf}(\_Z^2+1)-1, 0], [-2\,a\,\mathit{RootOf}(\_Z^2+1)-1, 2\,a\,\mathit{RootOf}(\_Z^2+1)+1]]]]\right]\right\} \quad \mathbf{(133)}$$

```
> findBesSqIrr(L,R1,F1,x,t,ext);
```

$$\left[\left[\left[a\,\mathit{RootOf}(\_Z^2+1)+\frac{1}{2}, a\,\mathit{RootOf}(\_Z^2+1)+1\right], \frac{x-2}{x-1}\right], \left[\left[a\,\mathit{RootOf}(\_Z^2+1)+\frac{1}{2}, a\,\mathit{RootOf}(\_Z^2+1)+1\right], -\frac{x-2}{x-1}\right]\right] \quad \mathbf{(134)}$$

```
> TIME :=time();
  BesSqSolutions(L);
  time() - TIME;
```

$$TIME := 58.343$$

$$\left\{\left[a\,\mathit{RootOf}(\_Z^2+1)+\frac{1}{2}, [0], [1], \frac{x-2}{x-1}\right], \left[a\,\mathit{RootOf}(\_Z^2+1)+\frac{1}{2}, [0], [1], -\frac{x-2}{x-1}\right]\right\}$$

$$0.063 \quad \mathbf{(135)}$$