```
> restart:
> read "ODE3solve.mpl":
```

Here are the Maple implementations in chapter 4 related just to the Bessel square root functions with the square root of the change of variable parameters not in k(x).

```
>                    ####### THE EXPONENT DIFFERENCES #######
```

In chapter 4, section 4.2.1 which is called "Exponent differences", we have the following Maple implementations:

```
> eq:=HolonomicDE(BesselI(nu, sqrt(x))^2,Y(x));
```

$$eq := -Y(x) + \left(-2\,v^2 - 2\,x + 2\right)\left(\frac{d}{dx}\,Y(x)\right) + 2\left(\frac{d^3}{dx^3}\,Y(x)\right)x^2 + 6\left(\frac{d^2}{dx^2}\,Y(x)\right)x \qquad \textbf{(2)}$$

```
> LBB:=de2diffop(eq, Y(x));
```

$$LBB := 2\,x^2\,Dx^3 + 6\,x\,Dx^2 + \left(-2\,v^2 - 2\,x + 2\right)Dx - 1 \qquad \textbf{(3)}$$

```
> gen_exp(LBB,t,x=infinity);
```

$$\left[\left[\frac{1}{2}, t = \frac{1}{x}\right], \left[\frac{1}{t} + \frac{1}{2}, t^2 = \frac{1}{x}\right]\right] \qquad \textbf{(4)}$$

```
> gen_exp(LBB,t,x=0);
```

$$\left[\,[0, t = x], [v, t = x], [-v, t = x]\,\right] \qquad \textbf{(5)}$$

```
>                    ####### EXAMPLE IN THE THESIS #######
```

In chapter 4, section 4.2.6 which is called "Change of variable parameters are not square of rational functions in k(x)", those are the Maple implementations for the example that we have used:

```
> eq:=HolonomicDE(BesselI(nu, sqrt(x))^2,Y(x));
```

$$eq := -Y(x) + \left(-2\,v^2 - 2\,x + 2\right)\left(\frac{d}{dx}\,Y(x)\right) + 2\left(\frac{d^3}{dx^3}\,Y(x)\right)x^2 + 6\left(\frac{d^2}{dx^2}\,Y(x)\right)x \qquad \textbf{(6)}$$

```
> LBB:=de2diffop(eq, Y(x));
```

$$LBB := 2\,x^2\,Dx^3 + 6\,x\,Dx^2 + \left(-2\,v^2 - 2\,x + 2\right)Dx - 1 \qquad \textbf{(7)}$$

```
> LBB:=subs(nu=5/12,LBB);
```

$$LBB := 2\,x^2\,Dx^3 + 6\,x\,Dx^2 + \left(\frac{119}{72} - 2\,x\right)Dx - 1 \qquad \textbf{(8)}$$

```
> f:=(x-1)^5*(x-7)^4/((x-12)*(x-14)*(x-3));
```

$$f := \frac{(x-1)^5 (x-7)^4}{(x-12)(x-14)(x-3)} \tag{9}$$

```
> L:=ChangeOfVariables(LBB,f);
```

$$L := 36\, Dx^3 (x-1)^2 (x-7)^2 (x-12)^4 (x-14)^4 (x-3)^4 \left(3\, x^4 - 109\, x^3 + 1307\, x^2 - 5878\, x \right. \tag{10}$$
$$\left. + 8967\right)^2 + 108\, \left(3\, x^8 - 218\, x^7 + 6499\, x^6 - 104180\, x^5 + 987827\, x^4 - 5732068\, x^3 \right.$$
$$+ 20096501\, x^2 - 38751258\, x + 30858534\right) Dx^2 (x-1)(x-7)(x-12)^3 (x$$
$$- 14)^3 (x-3)^3 \left(3\, x^4 - 109\, x^3 + 1307\, x^2 - 5878\, x + 8967\right) - \left(11664\, x^{25} - 2080080\, x^{24}\right.$$
$$+ 173832480\, x^{23} - 9053528544\, x^{22} + 329691581136\, x^{21} - 8928362729232\, x^{20}$$
$$+ 186723184797765\, x^{19} - 3091611815345853\, x^{18} + 41222620813286268\, x^{17}$$
$$- 447919384137869370\, x^{16} + 3998651202636212263\, x^{15} - 29483091110607109603\, x^{14}$$
$$+ 180075462606323412536\, x^{13} - 911870515880074445602\, x^{12}$$
$$+ 3824107057746349765223\, x^{11} - 13240379544983023993231\, x^{10}$$
$$+ 37650005677084427654216\, x^9 - 87252176473291126851370\, x^8$$
$$+ 163041223796184338489462\, x^7 - 242129495391633879835802\, x^6$$
$$+ 280228836976025631929820\, x^5 - 245979402650313330482820\, x^4$$
$$+ 157429488642064200020025\, x^3 - 69036545299869074699541\, x^2$$
$$+ 18520665120908031123126\, x - 2300502610610535723336\right) Dx (x-12)(x-14)(x$$
$$- 3) - 144 (x-1)^4 (x-7)^3 \left(3\, x^4 - 109\, x^3 + 1307\, x^2 - 5878\, x + 8967\right)^5$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \varnothing \tag{11}$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```

$$ext := \varnothing \tag{12}$$

```
> extppp:={};
```

$$extppp := \varnothing \tag{13}$$

```
> E:= Singular(L,extppp);
```

$$E := \left[[x-1, 1], \left[x^4 - \frac{109}{3}\, x^3 + \frac{1307}{3}\, x^2 - \frac{5878}{3}\, x + 2989, RootOf\left(3\, \_Z^4 - 109\, \_Z^3\right.\right.\right. \tag{14}$$
$$\left.\left. + 1307\, \_Z^2 - 5878\, \_Z + 8967\right) \right], [x-3, 3], [\infty, \infty], [x-7, 7], [x-14, 14], [x-12,$$
$$\left. 12] \right]$$

```
> F:=NotAppSing(L,E,ext);
```

$$F := [[\infty, \infty], [x-1, 1], [x-14, 14], [x-12, 12], [x-7, 7], [x-3, 3]] \tag{15}$$

```
> Sirr:= irrsingBesSqRoot(L,t,F,ext);
```

$$Sirr := \left[[\infty, \infty], [x-14, 14], [x-12, 12], [x-3, 3]], \left[\left[3, \frac{6}{t^3} - \frac{8}{t^2} + \frac{78}{t} + 3, -\frac{6}{t^3}\right.\right.\right. \tag{16}$$
$$\left. + \frac{8}{t^2} - \frac{78}{t} + 3\right], \left[\frac{1}{2}, \frac{8281\sqrt{286}}{22\sqrt{t}} + \frac{1}{2}, -\frac{8281\sqrt{286}}{22\sqrt{t}} + \frac{1}{2}\right], \left[\frac{1}{2}, -\frac{3025\, I\sqrt{22}}{6\sqrt{t}}\right.$$

$$+ \frac{1}{2}, \frac{3025\, I\sqrt{22}}{6\sqrt{t}} + \frac{1}{2}\Big], \Big[\frac{1}{2}, \frac{64\sqrt{22}}{33\sqrt{t}} + \frac{1}{2}, -\frac{64\sqrt{22}}{33\sqrt{t}} + \frac{1}{2}\Big]\Big], \Big[\Big[\frac{6}{t^3} - \frac{8}{t^2}$$

$$+ \frac{78}{t}, -\frac{6}{t^3} + \frac{8}{t^2} - \frac{78}{t}, -\frac{12}{t^3} + \frac{16}{t^2} - \frac{156}{t}\Big], \Big[\frac{8281\sqrt{286}}{22\sqrt{t}}, -\frac{8281\sqrt{286}}{22\sqrt{t}},$$

$$-\frac{8281\sqrt{286}}{11\sqrt{t}}\Big], \Big[\frac{-\frac{3025\,I}{6}\sqrt{22}}{\sqrt{t}}, \frac{\frac{3025\,I}{6}\sqrt{22}}{\sqrt{t}}, \frac{\frac{3025\,I}{3}\sqrt{22}}{\sqrt{t}}\Big], \Big[\frac{64\sqrt{22}}{33\sqrt{t}},$$

$$-\frac{64\sqrt{22}}{33\sqrt{t}}, -\frac{128\sqrt{22}}{33\sqrt{t}}\Big]\Big], \Big[3, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\Big], [1,2,2,2], \Big[\Big[\Big[\frac{6}{t^3} - \frac{8}{t^2} + \frac{78}{t} + 3, 3\Big], \Big[$$

$$-\frac{6}{t^3} + \frac{8}{t^2} - \frac{78}{t} + 3, 3\Big], \Big[-\frac{6}{t^3} + \frac{8}{t^2} - \frac{78}{t} + 3, \frac{6}{t^3} - \frac{8}{t^2} + \frac{78}{t} + 3\Big]\Big],$$

$$\Big[\Big[\frac{8281\sqrt{286}}{22\sqrt{t}} + \frac{1}{2}, \frac{1}{2}\Big], \Big[-\frac{8281\sqrt{286}}{22\sqrt{t}} + \frac{1}{2}, \frac{1}{2}\Big], \Big[-\frac{8281\sqrt{286}}{22\sqrt{t}} + \frac{1}{2},$$

$$\frac{8281\sqrt{286}}{22\sqrt{t}} + \frac{1}{2}\Big]\Big], \Big[\Big[-\frac{3025\,I\sqrt{22}}{6\sqrt{t}} + \frac{1}{2}, \frac{1}{2}\Big], \Big[\frac{3025\,I\sqrt{22}}{6\sqrt{t}} + \frac{1}{2}, \frac{1}{2}\Big],$$

$$\Big[\frac{3025\,I\sqrt{22}}{6\sqrt{t}} + \frac{1}{2}, -\frac{3025\,I\sqrt{22}}{6\sqrt{t}} + \frac{1}{2}\Big]\Big], \Big[\Big[\frac{64\sqrt{22}}{33\sqrt{t}} + \frac{1}{2}, \frac{1}{2}\Big], \Big[-\frac{64\sqrt{22}}{33\sqrt{t}} + \frac{1}{2},$$

$$\frac{1}{2}\Big], \Big[-\frac{64\sqrt{22}}{33\sqrt{t}} + \frac{1}{2}, \frac{64\sqrt{22}}{33\sqrt{t}} + \frac{1}{2}\Big]\Big]\Big], \Big[6\,t^3 - 8\,t^2 + 78\,t, -6\,t^3 + 8\,t^2 - 78\,t, -12\,t^3$$

$$+ 16\,t^2 - 156\,t\Big], \Big[\frac{8281\sqrt{286}\,t}{22}, -\frac{8281\sqrt{286}\,t}{22}, -\frac{8281\sqrt{286}\,t}{11}\Big], \Big[-\frac{3025\,I}{6}\sqrt{22}\,t,$$

$$\frac{3025\,I}{6}\sqrt{22}\,t, \frac{3025\,I}{3}\sqrt{22}\,t\Big], \Big[\frac{64\sqrt{22}\,t}{33}, -\frac{64\sqrt{22}\,t}{33}, -\frac{128\sqrt{22}\,t}{33}\Big]\Big], [[0,0,0],$$

$$[0,0,0], [0,0,0], [0,0,0]], \Big[[[x-1,1], [x-7,7]], \Big[\Big[0, \frac{25}{12}, -\frac{25}{12}\Big], \Big[\frac{25}{12}, -\frac{25}{12},$$

$$-\frac{25}{6}\Big], [1,1,1], \Big[\Big[\frac{25}{12}, 0\Big], \Big[-\frac{25}{12}, 0\Big], \Big[-\frac{25}{12}, \frac{25}{12}\Big]\Big], 2\Big], \Big[\Big[0, \frac{5}{3}, -\frac{5}{3}\Big], \Big[\frac{5}{3}, -\frac{5}{3},$$

$$-\frac{10}{3}\Big], [1,1,1], \Big[\Big[\frac{5}{3}, 0\Big], \Big[-\frac{5}{3}, 0\Big], \Big[-\frac{5}{3}, \frac{5}{3}\Big]\Big], 2\Big]\Big]\Big]\Big]$$

```
> Sreg:=regsingtrueBesSqRoot(L,t,Sirr[-1],ext);
```

$$Sreg := \Big[[[x-1,1], [x-7,7]], \Big[\Big[0, \frac{25}{12}, -\frac{25}{12}\Big], \Big[0, \frac{5}{3}, -\frac{5}{3}\Big]\Big], \Big[\Big[\frac{25}{12}, -\frac{25}{12}, -\frac{25}{6}\Big], \tag{17}$$

$$\Big[\frac{5}{3}, -\frac{5}{3}, -\frac{10}{3}\Big]\Big], \Big[\Big[\Big[\frac{25}{12}, 0\Big], \Big[-\frac{25}{12}, 0\Big], \Big[-\frac{25}{12}, \frac{25}{12}\Big]\Big], \Big[\Big[\frac{5}{3}, 0\Big], \Big[-\frac{5}{3}, 0\Big], \Big[-\frac{5}{3},$$

$$\frac{5}{3}\Big]\Big]\Big]\Big]$$

```
> NRemSreg:=SregseptrueBesSqRoot(L,Sreg,ext)[1];
```

$$NRemSreg := \left[[[x-1, 1], [x-7, 7]], \left[\left[0, \frac{25}{12}, -\frac{25}{12} \right], \left[0, \frac{5}{3}, -\frac{5}{3} \right] \right], \left[\left[\left[\frac{25}{12}, -\frac{25}{12}, \right. \right. \right. \right.$$

$$\left. \left. \left. \left. -\frac{25}{6} \right], [\ ] \right], \left[\left[\frac{5}{3}, -\frac{5}{3}, -\frac{10}{3} \right], [\ ] \right] \right] \right]$$

(18)

```
> LogSreg:=SregseptrueBesSqRoot(L,Sreg,ext)[3];
```

$$LogSreg := [\ ]$$

(19)

```
> RemSreg:=SregseptrueBesSqRoot(L,Sreg,ext)[2];
```

$$RemSreg := [\ ]$$

(20)

```
> R1:=IrrRegAppsingBesSqRoot(L,t,E,ext);
```

$$R1 := \left[\left[[[\infty, \infty], [x-14, 14], [x-12, 12], [x-3, 3]], \left[\left[3, \frac{6}{t^3} - \frac{8}{t^2} + \frac{78}{t} + 3, -\frac{6}{t^3} \right. \right. \right. \right.$$

$$+ \frac{8}{t^2} - \frac{78}{t} + 3 \right], \left[\frac{1}{2}, \frac{8281\sqrt{286}}{22\sqrt{t}} + \frac{1}{2}, -\frac{8281\sqrt{286}}{22\sqrt{t}} + \frac{1}{2} \right], \left[\frac{1}{2}, -\frac{3025\,I\sqrt{22}}{6\sqrt{t}} \right.$$

$$+ \frac{1}{2}, \frac{3025\,I\sqrt{22}}{6\sqrt{t}} + \frac{1}{2} \right], \left[\frac{1}{2}, \frac{64\sqrt{22}}{33\sqrt{t}} + \frac{1}{2}, -\frac{64\sqrt{22}}{33\sqrt{t}} + \frac{1}{2} \right] \right], \left[\left[\frac{6}{t^3} - \frac{8}{t^2} \right. \right.$$

$$+ \frac{78}{t}, -\frac{6}{t^3} + \frac{8}{t^2} - \frac{78}{t}, -\frac{12}{t^3} + \frac{16}{t^2} - \frac{156}{t} \right], \left[\frac{8281\sqrt{286}}{22\sqrt{t}}, -\frac{8281\sqrt{286}}{22\sqrt{t}}, \right.$$

$$\left. -\frac{8281\sqrt{286}}{11\sqrt{t}} \right], \left[-\frac{3025\,I}{6}\sqrt{22}, \frac{3025\,I}{6}\sqrt{22}, \frac{3025\,I}{3}\sqrt{22} \right], \left[\frac{64\sqrt{22}}{33\sqrt{t}}, \right.$$

$$\left. -\frac{64\sqrt{22}}{33\sqrt{t}}, -\frac{128\sqrt{22}}{33\sqrt{t}} \right] \right], \left[3, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right], [1, 2, 2, 2], \left[\left[\left[\frac{6}{t^3} - \frac{8}{t^2} + \frac{78}{t} + 3, 3 \right], \right. \right.$$

$$\left. -\frac{6}{t^3} + \frac{8}{t^2} - \frac{78}{t} + 3, 3 \right], \left[-\frac{6}{t^3} + \frac{8}{t^2} - \frac{78}{t} + 3, \frac{6}{t^3} - \frac{8}{t^2} + \frac{78}{t} + 3 \right] \right],$$

$$\left[\left[\frac{8281\sqrt{286}}{22\sqrt{t}} + \frac{1}{2}, \frac{1}{2} \right], \left[-\frac{8281\sqrt{286}}{22\sqrt{t}} + \frac{1}{2}, \frac{1}{2} \right], \left[-\frac{8281\sqrt{286}}{22\sqrt{t}} + \frac{1}{2}, \right. \right.$$

$$\left. \frac{8281\sqrt{286}}{22\sqrt{t}} + \frac{1}{2} \right], \left[\left[-\frac{3025\,I\sqrt{22}}{6\sqrt{t}} + \frac{1}{2}, \frac{1}{2} \right], \left[\frac{3025\,I\sqrt{22}}{6\sqrt{t}} + \frac{1}{2}, \frac{1}{2} \right], \right.$$

$$\left[\frac{3025\,I\sqrt{22}}{6\sqrt{t}} + \frac{1}{2}, -\frac{3025\,I\sqrt{22}}{6\sqrt{t}} + \frac{1}{2} \right] \right], \left[\left[\frac{64\sqrt{22}}{33\sqrt{t}} + \frac{1}{2}, \frac{1}{2} \right], \left[-\frac{64\sqrt{22}}{33\sqrt{t}} + \frac{1}{2}, \right. \right.$$

(21)

$$\frac{1}{2}\bigg], \bigg[-\frac{64\sqrt{22}}{33\sqrt{t}}+\frac{1}{2}, \frac{64\sqrt{22}}{33\sqrt{t}}+\frac{1}{2}\bigg]\bigg]\bigg], \bigg[[6\,t^3-8\,t^2+78\,t, -6\,t^3+8\,t^2-78\,t, -12\,t^3$$

$$+16\,t^2-156\,t], \bigg[\frac{8281\sqrt{286}\,t}{22}, -\frac{8281\sqrt{286}\,t}{22}, -\frac{8281\sqrt{286}\,t}{11}\bigg], \bigg[-\frac{3025\,\mathrm{I}}{6}\sqrt{22}\,t,$$

$$\frac{3025\,\mathrm{I}}{6}\sqrt{22}\,t, \frac{3025\,\mathrm{I}}{3}\sqrt{22}\,t\bigg], \bigg[\frac{64\sqrt{22}\,t}{33}, -\frac{64\sqrt{22}\,t}{33}, -\frac{128\sqrt{22}\,t}{33}\bigg]\bigg], [[0,0,0],$$

$$[0,0,0], [0,0,0], [0,0,0]]\bigg], \bigg[[[x-1,1],[x-7,7]], \bigg[\bigg[0,\frac{25}{12},-\frac{25}{12}\bigg], \bigg[0,\frac{5}{3},-\frac{5}{3}\bigg]\bigg],$$

$$\bigg[\bigg[\frac{25}{12},-\frac{25}{12},-\frac{25}{6}\bigg], \bigg[\frac{5}{3},-\frac{5}{3},-\frac{10}{3}\bigg]\bigg], \bigg[\bigg[\frac{25}{12},0\bigg], \bigg[-\frac{25}{12},0\bigg], \bigg[-\frac{25}{12},\frac{25}{12}\bigg]\bigg], \bigg[\bigg[\frac{5}{3},$$

$$0\bigg], \bigg[-\frac{5}{3},0\bigg], \bigg[-\frac{5}{3},\frac{5}{3}\bigg]\bigg]\bigg]\bigg], \bigg[[[x-1,1],[x-7,7]], \bigg[\bigg[0,\frac{25}{12},-\frac{25}{12}\bigg], \bigg[0,\frac{5}{3},-\frac{5}{3}\bigg]\bigg],$$

$$\bigg[\bigg[\bigg[\frac{25}{12},-\frac{25}{12},-\frac{25}{6}\bigg],[\,]\bigg], \bigg[\bigg[\frac{5}{3},-\frac{5}{3},-\frac{10}{3}\bigg],[\,]\bigg]\bigg], [\,],[\,]\bigg], \bigg[\bigg[\bigg[x^4-\frac{109}{3}x^3$$

$$+\frac{1307}{3}x^2-\frac{5878}{3}x+2989, RootOf\,(3\,\_Z^4-109\,\_Z^3+1307\,\_Z^2-5878\,\_Z+8967)\bigg]$$

$$\bigg], [[0,2,4]], [[2,4,2]], [[[2,0],[4,0],[4,2]]]\bigg], \bigg[[[\infty,\infty],[x-1,1],[x-14,14],$$

$$[x-12,12],[x-7,7],[x-3,3]], \bigg[\bigg[3,\frac{6}{t^3}-\frac{8}{t^2}+\frac{78}{t}+3, -\frac{6}{t^3}+\frac{8}{t^2}-\frac{78}{t}+3\bigg],$$

$$\bigg[0,\frac{25}{12},-\frac{25}{12}\bigg], \bigg[\frac{1}{2},\frac{8281\sqrt{286}}{22\sqrt{t}}+\frac{1}{2}, -\frac{8281\sqrt{286}}{22\sqrt{t}}+\frac{1}{2}\bigg], \bigg[\frac{1}{2},-\frac{3025\,\mathrm{I}\sqrt{22}}{6\sqrt{t}}$$

$$+\frac{1}{2},\frac{3025\,\mathrm{I}\sqrt{22}}{6\sqrt{t}}+\frac{1}{2}\bigg], \bigg[0,\frac{5}{3},-\frac{5}{3}\bigg], \bigg[\frac{1}{2},\frac{64\sqrt{22}}{33\sqrt{t}}+\frac{1}{2}, -\frac{64\sqrt{22}}{33\sqrt{t}}+\frac{1}{2}\bigg]\bigg],$$

$$\bigg[\bigg[\frac{6}{t^3}-\frac{8}{t^2}+\frac{78}{t}, -\frac{6}{t^3}+\frac{8}{t^2}-\frac{78}{t}, -\frac{12}{t^3}+\frac{16}{t^2}-\frac{156}{t}\bigg], \bigg[\frac{25}{12},-\frac{25}{12},-\frac{25}{6}\bigg],$$

$$\left[\frac{8281\sqrt{286}}{22\sqrt{t}}, -\frac{8281\sqrt{286}}{22\sqrt{t}}, -\frac{8281\sqrt{286}}{11\sqrt{t}}\right], \left[\frac{-\dfrac{3025\,I}{6}\sqrt{22}}{\sqrt{t}}, \frac{\dfrac{3025\,I}{6}\sqrt{22}}{\sqrt{t}},\right.$$

$$\left.\frac{\dfrac{3025\,I}{3}\sqrt{22}}{\sqrt{t}}\right], \left[\frac{5}{3}, -\frac{5}{3}, -\frac{10}{3}\right], \left[\frac{64\sqrt{22}}{33\sqrt{t}}, -\frac{64\sqrt{22}}{33\sqrt{t}}, -\frac{128\sqrt{22}}{33\sqrt{t}}\right]\right], \left[\left[\left[\frac{6}{t^3} - \frac{8}{t^2}\right.\right.\right.$$

$$\left. + \frac{78}{t} + 3, 3\right], \left[-\frac{6}{t^3} + \frac{8}{t^2} - \frac{78}{t} + 3, 3\right], \left[-\frac{6}{t^3} + \frac{8}{t^2} - \frac{78}{t} + 3, \frac{6}{t^3} - \frac{8}{t^2} + \frac{78}{t}\right.$$

$$\left.\left. + 3\right]\right], \left[\left[\frac{25}{12}, 0\right], \left[-\frac{25}{12}, 0\right], \left[-\frac{25}{12}, \frac{25}{12}\right]\right], \left[\left[\frac{8281\sqrt{286}}{22\sqrt{t}} + \frac{1}{2}, \frac{1}{2}\right], \left[-\frac{8281\sqrt{286}}{22\sqrt{t}}\right.\right.$$

$$\left. + \frac{1}{2}, \frac{1}{2}\right], \left[-\frac{8281\sqrt{286}}{22\sqrt{t}} + \frac{1}{2}, \frac{8281\sqrt{286}}{22\sqrt{t}} + \frac{1}{2}\right]\right], \left[\left[-\frac{3025\,I\sqrt{22}}{6\sqrt{t}} + \frac{1}{2}, \frac{1}{2}\right],\right.$$

$$\left[\frac{3025\,I\sqrt{22}}{6\sqrt{t}} + \frac{1}{2}, \frac{1}{2}\right], \left[\frac{3025\,I\sqrt{22}}{6\sqrt{t}} + \frac{1}{2}, -\frac{3025\,I\sqrt{22}}{6\sqrt{t}} + \frac{1}{2}\right]\right], \left[\left[\frac{5}{3}, 0\right], \left[-\frac{5}{3},\right.\right.$$

$$\left.0\right], \left[-\frac{5}{3}, \frac{5}{3}\right]\right], \left[\left[\frac{64\sqrt{22}}{33\sqrt{t}} + \frac{1}{2}, \frac{1}{2}\right], \left[-\frac{64\sqrt{22}}{33\sqrt{t}} + \frac{1}{2}, \frac{1}{2}\right], \left[-\frac{64\sqrt{22}}{33\sqrt{t}} + \frac{1}{2},\right.$$

$$\left.\left.\left.\frac{64\sqrt{22}}{33\sqrt{t}} + \frac{1}{2}\right]\right]\right], [[1,1,1],[1,1,1],[1,2,2],[1,2,2],[1,1,1],[1,2,2]]\right]\right]$$

> `info1:=SirrBesSqRootinfo1(L,R1[1],R1[2],x,t,ext);`

$$info1 := \left[\left[\left[\infty, \frac{1}{x}, \left[x^2\left(x^2 - 2x + 39\right)^2\right]\right], 6, \varnothing, \varnothing\right], \left[14, x - 14, \left[\frac{891474493}{22\,(x-14)}\right], 1, \varnothing, \varnothing\right],\right.\quad\textbf{(22)}$$

$$\left[12, x - 12, \left[-\frac{100656875}{18\,(x-12)}\right], 1, \varnothing, \varnothing\right], \left[3, x - 3, \left[\frac{8192}{99\,(x-3)}\right], 1, \varnothing, \varnothing\right], 9, 8, (x$$

$$\left. - 12)\,(x-14)\,(x-3), 1\right]$$

> `findBesSqRootRat(L,R1,info1,x,t,T,ext);`

$$\left\{\left[\left[\frac{1}{12}\right], \frac{(x-1)\,(x-7)^8}{(x-12)\,(x-14)\,(x-3)}\right], \left[\left[\frac{5}{12}\right], \frac{(x-1)^5\,(x-7)^4}{(x-12)\,(x-14)\,(x-3)}\right]\right\}\quad\textbf{(23)}$$

> `TIME :=time();`
> `BesSqRootSolutions(L);`
> `time() - TIME;`

$$TIME := 3.218$$

$$\left\{\left[\frac{5}{12}, [0], [1], \frac{(x-1)^5\,(x-7)^4}{(x-12)\,(x-14)\,(x-3)}\right]\right\}$$

$$0.797\quad\textbf{(24)}$$

Here are another examples related to the Bessel square type solutions with the square root of the change
of variable parameters not in
k(x). Those examples are not in my PhD thesis.

```
>                    ####### THE EASY CASE #######
```

```
> eq:=HolonomicDE(BesselI(nu, sqrt(x))^2,Y(x));
```

$$eq := -Y(x) + \left(-2\,\nu^2 - 2\,x + 2\right)\left(\frac{d}{dx}\,Y(x)\right) + 2\left(\frac{d^3}{dx^3}\,Y(x)\right)x^2 + 6\left(\frac{d^2}{dx^2}\,Y(x)\right)x \qquad \textbf{(25)}$$

```
> LBB:=de2diffop(eq, Y(x));
```

$$LBB := 2\,x^2\,Dx^3 + 6\,x\,Dx^2 + \left(-2\,\nu^2 - 2\,x + 2\right)Dx - 1 \qquad \textbf{(26)}$$

```
> LBB:=subs(nu=3,LBB);
```

$$LBB := 2\,x^2\,Dx^3 + 6\,x\,Dx^2 + \left(-16 - 2\,x\right)Dx - 1 \qquad \textbf{(27)}$$

```
> f:=(x-1)*(x-7)/((x-12)^3);
```

$$f := \frac{(x-1)\,(x-7)}{(x-12)^3} \qquad \textbf{(28)}$$

```
> L:=ChangeOfVariables(LBB,f);
```

$$L := 2\,Dx^3\,(x-12)^6\,(x-1)^2\,(x-7)^2\,\left(x^2 + 8\,x - 75\right)^2 + 6\,\big(x^4 + 16\,x^3 - 488\,x^2 + 3168\,x$$
$$- 7053\big)\,Dx^2\,(x-12)^5\,(x-1)\,(x-7)\,\left(x^2 + 8\,x - 75\right) - 2\,\big(8\,x^{11} - 31\,x^{10} - 5064\,x^9$$
$$+ 64507\,x^8 - 797280\,x^7 + 38574690\,x^6 - 764079336\,x^5 + 6663080398\,x^4$$
$$- 25163543184\,x^3 + 8829647325\,x^2 + 196140552408\,x - 387786483369\big)\,Dx\,(x-12)$$
$$+ \left(x^2 + 8\,x - 75\right)^5 \qquad \textbf{(29)}$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \varnothing \qquad \textbf{(30)}$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```

$$ext := \varnothing \qquad \textbf{(31)}$$

```
> extppp:={};
```

$$extppp := \varnothing \qquad \textbf{(32)}$$

```
> E:= Singular(L,extppp);
```

$$E := \big[\,[x-1,\,1],\,[x^2 + 8\,x - 75,\,RootOf\left(\_Z^2 + 8\,\_Z - 75\right)],\,[\infty,\,\infty],\,[x-7,\,7],\,[x-12, \qquad \textbf{(33)}$$
$$12]\,\big]$$

```
> F:=NotAppSing(L,E,ext);
```

$$F := [\,[\infty,\,\infty],\,[x-1,\,1],\,[x-12,\,12],\,[x-7,\,7]\,] \qquad \textbf{(34)}$$

```
> Sirr:= irrsingBesSqRoot(L,t,F,ext);
```

$$Sirr := \Bigg[\,[\,[x-12,\,12]\,],\,\Bigg[\Bigg[\frac{3}{2},\,\frac{3\sqrt{55}}{t^{3/2}} + \frac{8\sqrt{55}}{55\sqrt{t}} + \frac{3}{2},\,-\frac{3\sqrt{55}}{t^{3/2}} - \frac{8\sqrt{55}}{55\sqrt{t}} + \frac{3}{2}\Bigg]\Bigg], \qquad \textbf{(35)}$$
$$\Bigg[\Bigg[\frac{3\sqrt{55}}{t^{3/2}} + \frac{8\sqrt{55}}{55\sqrt{t}},\,-\frac{3\sqrt{55}}{t^{3/2}} - \frac{8\sqrt{55}}{55\sqrt{t}},\,-\frac{6\sqrt{55}}{t^{3/2}} - \frac{16\sqrt{55}}{55\sqrt{t}}\Bigg]\Bigg],\,\Bigg[\frac{3}{2}\Bigg],\,[2],$$
$$\Bigg[\Bigg[\Bigg[\frac{3\sqrt{55}}{t^{3/2}} + \frac{8\sqrt{55}}{55\sqrt{t}} + \frac{3}{2},\,\frac{3}{2}\Bigg],\,\Bigg[-\frac{3\sqrt{55}}{t^{3/2}} - \frac{8\sqrt{55}}{55\sqrt{t}} + \frac{3}{2},\,\frac{3}{2}\Bigg],\,\Bigg[-\frac{3\sqrt{55}}{t^{3/2}}$$
$$- \frac{8\sqrt{55}}{55\sqrt{t}} + \frac{3}{2},\,\frac{3\sqrt{55}}{t^{3/2}} + \frac{8\sqrt{55}}{55\sqrt{t}} + \frac{3}{2}\Bigg]\Bigg]\Bigg],\,\Bigg[\Bigg[\frac{(165\,t^3 + 8\,t)\sqrt{55}}{55},$$

$$-\frac{(165\,t^3+8\,t)\,\sqrt{55}}{55},\ -\frac{2\,(165\,t^3+8\,t)\,\sqrt{55}}{55}\Bigg]\Bigg],\ [[0,0,0]],\ [[[\infty,\infty],[x-1,1],[x$$

$$-7,7]],\ [[[-3,0,3],[3,6,3],[1,1,1],[[0,-3],[3,-3],[3,0]],4],[[-3,0,3],$$

$$[3,6,3],[1,1,1],[[0,-3],[3,-3],[3,0]],4],[[-3,0,3],[3,6,3],[1,1,1],[[0,$$

$$-3],[3,-3],[3,0]],4]]]]$$

```
> Sreg:=regsingtrueBesSqRoot(L,t,Sirr[-1],ext);
```

$$Sreg := [[[\infty,\infty],[x-1,1],[x-7,7]],[[-3,0,3],[-3,0,3],[-3,0,3]],[[3,6,3],\tag{36}$$
$$[3,6,3],[3,6,3]],[[[0,-3],[3,-3],[3,0]],[[0,-3],[3,-3],[3,0]],[[0,-3],$$
$$[3,-3],[3,0]]]]$$

```
> NRemSreg:=SregseptrueBesSqRoot(L,Sreg,ext)[1];
```

$$NRemSreg := [\ ]\tag{37}$$

```
> LogSreg:=SregseptrueBesSqRoot(L,Sreg,ext)[3];
```

$$LogSreg := [[[\infty,\infty],[x-1,1],[x-7,7]],[[-3,0,3],[-3,0,3],[-3,0,3]],[[[\ ],\tag{38}$$
$$[3,6,3]],[[\ ],[3,6,3]],[[\ ],[3,6,3]]]]$$

```
> RemSreg:=SregseptrueBesSqRoot(L,Sreg,ext)[2];
```

$$RemSreg := [\ ]\tag{39}$$

```
> R1:=IrrRegAppsingBesSqRoot(L,t,E,ext);
```

$$R1 := \Bigg[\Bigg[[[x-12,12]],\Bigg[\Bigg[\frac{3}{2},\frac{3\sqrt{55}}{t^{3/2}}+\frac{8\sqrt{55}}{55\sqrt{t}}+\frac{3}{2},\ -\frac{3\sqrt{55}}{t^{3/2}}-\frac{8\sqrt{55}}{55\sqrt{t}}+\frac{3}{2}\Bigg]\Bigg],\tag{40}$$

$$\Bigg[\Bigg[\frac{3\sqrt{55}}{t^{3/2}}+\frac{8\sqrt{55}}{55\sqrt{t}},\ -\frac{3\sqrt{55}}{t^{3/2}}-\frac{8\sqrt{55}}{55\sqrt{t}},\ -\frac{6\sqrt{55}}{t^{3/2}}-\frac{16\sqrt{55}}{55\sqrt{t}}\Bigg]\Bigg],\Bigg[\frac{3}{2}\Bigg],[2],$$

$$\Bigg[\Bigg[\Bigg[\frac{3\sqrt{55}}{t^{3/2}}+\frac{8\sqrt{55}}{55\sqrt{t}}+\frac{3}{2},\frac{3}{2}\Bigg],\Bigg[-\frac{3\sqrt{55}}{t^{3/2}}-\frac{8\sqrt{55}}{55\sqrt{t}}+\frac{3}{2},\frac{3}{2}\Bigg],\Bigg[-\frac{3\sqrt{55}}{t^{3/2}}$$

$$-\frac{8\sqrt{55}}{55\sqrt{t}}+\frac{3}{2},\frac{3\sqrt{55}}{t^{3/2}}+\frac{8\sqrt{55}}{55\sqrt{t}}+\frac{3}{2}\Bigg]\Bigg]\Bigg],\Bigg[\Bigg[\frac{(165\,t^3+8\,t)\,\sqrt{55}}{55},$$

$$-\frac{(165\,t^3+8\,t)\,\sqrt{55}}{55},\ -\frac{2\,(165\,t^3+8\,t)\,\sqrt{55}}{55}\Bigg]\Bigg],[[0,0,0]]\Bigg],[[[\infty,\infty],[x-1,1],[x$$

$$-7,7]],[[-3,0,3],[-3,0,3],[-3,0,3]],[[3,6,3],[3,6,3],[3,6,3]],[[[0,-3],$$

$$[3,-3],[3,0]],[[0,-3],[3,-3],[3,0]],[[0,-3],[3,-3],[3,0]]]],[[\ ],[\ ],$$

$$[[[\infty,\infty],[x-1,1],[x-7,7]],[[-3,0,3],[-3,0,3],[-3,0,3]],[[[\ ],[3,6,3]],$$

$$[[\ ],[3,6,3]],[[\ ],[3,6,3]]]]],[[[x^2+8\,x-75,RootOf(\_Z^2+8\,\_Z-75)]],[[0,2,$$

$$4]],[[2,4,2]],[[[2,0],[4,0],[4,2]]]],\Bigg[[[\infty,\infty],[x-1,1],[x-12,12],[x-7,$$

$$7]], \left[[-3, 0, 3], [-3, 0, 3], \left[\frac{3}{2}, \frac{3\sqrt{55}}{t^{3/2}} + \frac{8\sqrt{55}}{55\sqrt{t}} + \frac{3}{2}, -\frac{3\sqrt{55}}{t^{3/2}} - \frac{8\sqrt{55}}{55\sqrt{t}} \right. \right.$$

$$\left. + \frac{3}{2} \right], [-3, 0, 3] \left], \left[[3, 6, 3], [3, 6, 3], \left[\frac{3\sqrt{55}}{t^{3/2}} + \frac{8\sqrt{55}}{55\sqrt{t}}, -\frac{3\sqrt{55}}{t^{3/2}} - \frac{8\sqrt{55}}{55\sqrt{t}}, \right. \right. \right.$$

$$\left. -\frac{6\sqrt{55}}{t^{3/2}} - \frac{16\sqrt{55}}{55\sqrt{t}} \right], [3, 6, 3] \left], \left[[[0, -3], [3, -3], [3, 0]], [[0, -3], [3, -3], [3, \right.$$

$$\left. 0]], \left[\left[\frac{3\sqrt{55}}{t^{3/2}} + \frac{8\sqrt{55}}{55\sqrt{t}} + \frac{3}{2}, \frac{3}{2} \right], \left[-\frac{3\sqrt{55}}{t^{3/2}} - \frac{8\sqrt{55}}{55\sqrt{t}} + \frac{3}{2}, \frac{3}{2} \right], \left[-\frac{3\sqrt{55}}{t^{3/2}} \right. \right. \right.$$

$$\left. -\frac{8\sqrt{55}}{55\sqrt{t}} + \frac{3}{2}, \frac{3\sqrt{55}}{t^{3/2}} + \frac{8\sqrt{55}}{55\sqrt{t}} + \frac{3}{2} \right] \left], [[0, -3], [3, -3], [3, 0]] \right], [[1, 1, 1],$$

$$[1, 1, 1], [1, 2, 2], [1, 1, 1]]]$$

```
> info1:=SirrBesSqRootinfo1(L,R1[1],R1[2],x,t,ext);
```

$$info1 := \left[\left[\left[12, x - 12, \left[\frac{(8x - 41)^2}{55\ (x - 12)^3} \right], 3, \varnothing, \varnothing \right], 3, 5, (x - 12)^3, x - 12 \right] \right. \tag{41}$$

```
> easyBesSqRoot(L,R1,info1,x,t,ext);
```

$$\left\{ \left[[3], \frac{(x - 1)\ (x - 7)}{(x - 12)^3} \right] \right\}, \left\{ \left[[3], -\frac{(x - 1)\ (x - 7)}{(x - 12)^3} \right] \right\} \tag{42}$$

```
> findBesSqRootln(L,R1,info1,x,t,T,ext);
```

$$\left\{ \left[[3], -\frac{(x - 1)\ (x - 7)}{(x - 12)^3} \right] \right\} \tag{43}$$

```
> TIME :=time();
  BesSqRootSolutions(L);
  time() - TIME;
```

$$TIME := 5.171$$

$$\left\{ \left[3, [0], [1], \frac{(x - 1)\ (x - 7)}{(x - 12)^3} \right] \right\}$$

$$0.485 \tag{44}$$

```
> eq:=HolonomicDE(BesselI(nu, sqrt(x))^2,Y(x));
```

$$eq := -Y(x) + \left(-2\nu^2 - 2x + 2 \right) \left(\frac{d}{dx} Y(x) \right) + 2 \left(\frac{d^3}{dx^3} Y(x) \right) x^2 + 6 \left(\frac{d^2}{dx^2} Y(x) \right) x \tag{45}$$

```
> LBB:=de2diffop(eq, Y(x));
```

$$LBB := 2x^2 Dx^3 + 6x Dx^2 + \left(-2\nu^2 - 2x + 2 \right) Dx - 1 \tag{46}$$

```
> LBB:=subs(nu=1/3,LBB);
```

$$LBB := 2x^2 Dx^3 + 6x Dx^2 + \left(\frac{16}{9} - 2x \right) Dx - 1 \tag{47}$$

```
> f:=(x-1)/((x-12)*(x-3)*(x-7));
```

$$f := \frac{x-1}{(x-12)(x-3)(x-7)} \tag{48}$$

```
> L:=ChangeOfVariables(LBB,f);
```

$$L := 18\,Dx^3\,(x-1)^2\,(x-12)^4\,(x-3)^4\,(x-7)^4\,\left(2\,x^3-25\,x^2+44\,x+111\right)^2+54\,\big(2\,x^6 \tag{49}$$
$$-50\,x^5+381\,x^4-8\,x^3-11824\,x^2+48786\,x-54711\big)\,Dx^2\,(x-1)\,(x-12)^3\,(x$$
$$-3)^3\,(x-7)^3\,\left(2\,x^3-25\,x^2+44\,x+111\right)+2\,\big(20\,x^{15}-1440\,x^{14}+45186\,x^{13}$$
$$-821982\,x^{12}+10115082\,x^{11}-103132794\,x^{10}+1098691371\,x^9-11931780267\,x^8$$
$$+107876305716\,x^7-724214858768\,x^6+3478340232684\,x^5-11723995687896\,x^4$$
$$+26953287258582\,x^3-39994279884990\,x^2+34189191422271\,x-12595875042231\big)$$
$$Dx\,(x-12)\,(x-3)\,(x-7)+9\,\left(2\,x^3-25\,x^2+44\,x+111\right)^5$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```

$$ext := \varnothing \tag{50}$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```

$$ext := \varnothing \tag{51}$$

```
> extppp:={};
```

$$extppp := \varnothing \tag{52}$$

```
> E:= Singular(L,extppp);
```

$$E := \left[\left[x^3-\frac{25}{2}\,x^2+22\,x+\frac{111}{2},\,RootOf\left(2\,\_Z^3-25\,\_Z^2+44\,\_Z+111\right)\right],\,[x-1,1],\,[x \tag{53}$$
$$-3,3],\,[\infty,\infty],\,[x-7,7],\,[x-12,12]\right]$$

```
> F:=NotAppSing(L,E,ext);
```

$$F := [[\infty,\infty],\,[x-1,1],\,[x-12,12],\,[x-3,3],\,[x-7,7]] \tag{54}$$

```
> Sirr:= irrsingBesSqRoot(L,t,F,ext);
```

$$Sirr := \Bigg[[[x-12,12],\,[x-3,3],\,[x-7,7]],\,\left[\left[\frac{1}{2},\,\frac{\sqrt{55}}{15\sqrt{t}}+\frac{1}{2},\,-\frac{\sqrt{55}}{15\sqrt{t}}+\frac{1}{2}\right],\,\left[\frac{1}{2},\right.\right. \tag{55}$$
$$\frac{\sqrt{2}}{6\sqrt{t}}+\frac{1}{2},\,-\frac{\sqrt{2}}{6\sqrt{t}}+\frac{1}{2}\right],\,\left[\frac{1}{2},\,-\frac{I\sqrt{30}}{10\sqrt{t}}+\frac{1}{2},\,\frac{I\sqrt{30}}{10\sqrt{t}}+\frac{1}{2}\right]\Bigg],\,\left[\left[\frac{\sqrt{55}}{15\sqrt{t}},\right.\right.$$
$$-\frac{\sqrt{55}}{15\sqrt{t}},\,-\frac{2\sqrt{55}}{15\sqrt{t}}\right],\,\left[\frac{\sqrt{2}}{6\sqrt{t}},\,-\frac{\sqrt{2}}{6\sqrt{t}},\,-\frac{\sqrt{2}}{3\sqrt{t}}\right],\,\left[\frac{-\frac{I}{10}\sqrt{30}}{\sqrt{t}},\,\frac{\frac{I}{10}\sqrt{30}}{\sqrt{t}},\right.$$
$$\frac{\frac{I}{5}\sqrt{30}}{\sqrt{t}}\right]\Bigg],\,\left[\frac{1}{2},\,\frac{1}{2},\,\frac{1}{2}\right],\,[2,2,2],\,\Bigg[\left[\left[\frac{\sqrt{55}}{15\sqrt{t}}+\frac{1}{2},\,\frac{1}{2}\right],\,\left[-\frac{\sqrt{55}}{15\sqrt{t}}+\frac{1}{2},\,\frac{1}{2}\right],\left[\right.$$
$$-\frac{\sqrt{55}}{15\sqrt{t}}+\frac{1}{2},\,\frac{\sqrt{55}}{15\sqrt{t}}+\frac{1}{2}\right]\Bigg],\,\Bigg[\left[\frac{\sqrt{2}}{6\sqrt{t}}+\frac{1}{2},\,\frac{1}{2}\right],\,\left[-\frac{\sqrt{2}}{6\sqrt{t}}+\frac{1}{2},\,\frac{1}{2}\right],\,\left[-\frac{\sqrt{2}}{6\sqrt{t}}\right.$$
$$+\frac{1}{2},\,\frac{\sqrt{2}}{6\sqrt{t}}+\frac{1}{2}\right]\Bigg],\,\Bigg[\left[-\frac{I\sqrt{30}}{10\sqrt{t}}+\frac{1}{2},\,\frac{1}{2}\right],\,\left[\frac{I\sqrt{30}}{10\sqrt{t}}+\frac{1}{2},\,\frac{1}{2}\right],\,\left[\frac{I\sqrt{30}}{10\sqrt{t}}+\frac{1}{2},\right.$$

$$-\frac{I\sqrt{30}}{10\sqrt{t}}+\frac{1}{2}\Bigg]\Bigg]\Bigg],\Bigg[\Bigg[\frac{\sqrt{55}\,t}{15},-\frac{\sqrt{55}\,t}{15},-\frac{2\sqrt{55}\,t}{15}\Bigg],\Bigg[\frac{\sqrt{2}\,t}{6},-\frac{\sqrt{2}\,t}{6},-\frac{\sqrt{2}\,t}{3}\Bigg],\Bigg[$$

$$-\frac{I}{10}\sqrt{30}\,t,\frac{I}{10}\sqrt{30}\,t,\frac{I}{5}\sqrt{30}\,t\Bigg],[[0,0,0],[0,0,0],[0,0,0]],\Bigg[[[\infty,\infty],[x-1,$$

$$1]],\Bigg[\Bigg[0,\frac{2}{3},-\frac{2}{3}\Bigg],\Bigg[\frac{2}{3},-\frac{2}{3},-\frac{4}{3}\Bigg],[1,1,1],\Bigg[\Bigg[\frac{2}{3},0\Bigg],\Bigg[-\frac{2}{3},0\Bigg],\Bigg[-\frac{2}{3},\frac{2}{3}\Bigg]\Bigg],2\Bigg],\Bigg[\Bigg[0,$$

$$\frac{1}{3},-\frac{1}{3}\Bigg],\Bigg[\frac{1}{3},-\frac{1}{3},-\frac{2}{3}\Bigg],[1,1,1],\Bigg[\Bigg[\frac{1}{3},0\Bigg],\Bigg[-\frac{1}{3},0\Bigg],\Bigg[-\frac{1}{3},\frac{1}{3}\Bigg]\Bigg],2\Bigg]\Bigg]\Bigg]\Bigg]$$

```
> Sreg:=regsingtrueBesSqRoot(L,t,Sirr[-1],ext);
```

$$Sreg:=\Bigg[[[\infty,\infty],[x-1,1]],\Bigg[\Bigg[0,\frac{2}{3},-\frac{2}{3}\Bigg],\Bigg[0,\frac{1}{3},-\frac{1}{3}\Bigg]\Bigg],\Bigg[\Bigg[\frac{2}{3},-\frac{2}{3},-\frac{4}{3}\Bigg],\Bigg[\frac{1}{3},-\frac{1}{3},\tag{56}$$

$$-\frac{2}{3}\Bigg]\Bigg],\Bigg[\Bigg[\frac{2}{3},0\Bigg],\Bigg[-\frac{2}{3},0\Bigg],\Bigg[-\frac{2}{3},\frac{2}{3}\Bigg]\Bigg],\Bigg[\Bigg[\frac{1}{3},0\Bigg],\Bigg[-\frac{1}{3},0\Bigg],\Bigg[-\frac{1}{3},\frac{1}{3}\Bigg]\Bigg]\Bigg]\Bigg]$$

```
> NRemSreg:=SregseptrueBesSqRoot(L,Sreg,ext)[1];
```

$$NRemSreg:=\Bigg[[[\infty,\infty],[x-1,1]],\Bigg[\Bigg[0,\frac{2}{3},-\frac{2}{3}\Bigg],\Bigg[0,\frac{1}{3},-\frac{1}{3}\Bigg]\Bigg],\Bigg[\Bigg[\frac{2}{3},-\frac{2}{3},-\frac{4}{3}\Bigg],[\,]\Bigg],\tag{57}$$

$$\Bigg[\Bigg[\frac{1}{3},-\frac{1}{3},-\frac{2}{3}\Bigg],[\,]\Bigg]\Bigg]\Bigg]$$

```
> LogSreg:=SregseptrueBesSqRoot(L,Sreg,ext)[3];
```

$$LogSreg:=[\,]\tag{58}$$

```
> RemSreg:=SregseptrueBesSqRoot(L,Sreg,ext)[2];
```

$$RemSreg:=[\,]\tag{59}$$

```
> R1:=IrrRegAppsingBesSqRoot(L,t,E,ext);
```

$$R1:=\Bigg[\Bigg[[[x-12,12],[x-3,3],[x-7,7]],\Bigg[\Bigg[\frac{1}{2},\frac{\sqrt{55}}{15\sqrt{t}}+\frac{1}{2},-\frac{\sqrt{55}}{15\sqrt{t}}+\frac{1}{2}\Bigg],\Bigg[\frac{1}{2},\tag{60}$$

$$\frac{\sqrt{2}}{6\sqrt{t}}+\frac{1}{2},-\frac{\sqrt{2}}{6\sqrt{t}}+\frac{1}{2}\Bigg],\Bigg[\frac{1}{2},-\frac{I\sqrt{30}}{10\sqrt{t}}+\frac{1}{2},\frac{I\sqrt{30}}{10\sqrt{t}}+\frac{1}{2}\Bigg]\Bigg],\Bigg[\Bigg[\frac{\sqrt{55}}{15\sqrt{t}},$$

$$-\frac{\sqrt{55}}{15\sqrt{t}},-\frac{2\sqrt{55}}{15\sqrt{t}}\Bigg],\Bigg[\frac{\sqrt{2}}{6\sqrt{t}},-\frac{\sqrt{2}}{6\sqrt{t}},-\frac{\sqrt{2}}{3\sqrt{t}}\Bigg],\Bigg[\frac{-\frac{I}{10}\sqrt{30}}{\sqrt{t}},\frac{\frac{I}{10}\sqrt{30}}{\sqrt{t}},$$

$$\frac{\frac{I}{5}\sqrt{30}}{\sqrt{t}}\Bigg]\Bigg],\Bigg[\frac{1}{2},\frac{1}{2},\frac{1}{2}\Bigg],[2,2,2],\Bigg[\Bigg[\Bigg[\frac{\sqrt{55}}{15\sqrt{t}}+\frac{1}{2},\frac{1}{2}\Bigg],\Bigg[-\frac{\sqrt{55}}{15\sqrt{t}}+\frac{1}{2},\frac{1}{2}\Bigg],\Bigg[$$

$$-\frac{\sqrt{55}}{15\sqrt{t}}+\frac{1}{2},\frac{\sqrt{55}}{15\sqrt{t}}+\frac{1}{2}\Bigg]\Bigg],\Bigg[\Bigg[\frac{\sqrt{2}}{6\sqrt{t}}+\frac{1}{2},\frac{1}{2}\Bigg],\Bigg[-\frac{\sqrt{2}}{6\sqrt{t}}+\frac{1}{2},\frac{1}{2}\Bigg],\Bigg[-\frac{\sqrt{2}}{6\sqrt{t}}$$

$$+\frac{1}{2},\frac{\sqrt{2}}{6\sqrt{t}}+\frac{1}{2}\Bigg]\Bigg],\Bigg[\Bigg[-\frac{I\sqrt{30}}{10\sqrt{t}}+\frac{1}{2},\frac{1}{2}\Bigg],\Bigg[\frac{I\sqrt{30}}{10\sqrt{t}}+\frac{1}{2},\frac{1}{2}\Bigg],\Bigg[\frac{I\sqrt{30}}{10\sqrt{t}}+\frac{1}{2},$$

$$-\frac{I\sqrt{30}}{10\sqrt{t}}+\frac{1}{2}\Bigg]\Bigg]\Bigg],\Bigg[\Bigg[\frac{\sqrt{55}\,t}{15},-\frac{\sqrt{55}\,t}{15},-\frac{2\sqrt{55}\,t}{15}\Bigg],\Bigg[\frac{\sqrt{2}\,t}{6},-\frac{\sqrt{2}\,t}{6},-\frac{\sqrt{2}\,t}{3}\Bigg],\Bigg[$$

$$-\frac{I}{10}\sqrt{30}\,t,\frac{I}{10}\sqrt{30}\,t,\frac{I}{5}\sqrt{30}\,t\Bigg]\Bigg],[[0,0,0],[0,0,0],[0,0,0]]\Bigg],\Bigg[[[\infty,\infty],[x-1,$$

$$1]],\Bigg[\Bigg[0,\frac{2}{3},-\frac{2}{3}\Bigg],\Bigg[0,\frac{1}{3},-\frac{1}{3}\Bigg]\Bigg],\Bigg[\Bigg[\frac{2}{3},-\frac{2}{3},-\frac{4}{3}\Bigg],\Bigg[\frac{1}{3},-\frac{1}{3},-\frac{2}{3}\Bigg]\Bigg],\Bigg[\Bigg[\Bigg[\frac{2}{3},0\Bigg],\Bigg[$$

$$-\frac{2}{3},0\Bigg],\Bigg[-\frac{2}{3},\frac{2}{3}\Bigg]\Bigg],\Bigg[\Bigg[\frac{1}{3},0\Bigg],\Bigg[-\frac{1}{3},0\Bigg],\Bigg[-\frac{1}{3},\frac{1}{3}\Bigg]\Bigg]\Bigg]\Bigg],\Bigg[[[\infty,\infty],[x-1,1]],\Bigg[\Bigg[0,$$

$$\frac{2}{3},-\frac{2}{3}\Bigg],\Bigg[0,\frac{1}{3},-\frac{1}{3}\Bigg]\Bigg],\Bigg[\Bigg[\frac{2}{3},-\frac{2}{3},-\frac{4}{3}\Bigg],[\ ]\Bigg],\Bigg[\Bigg[\frac{1}{3},-\frac{1}{3},-\frac{2}{3}\Bigg],[\ ]\Bigg]\Bigg],[\ ],[\ ],$$

$$\Bigg[\Bigg[\Bigg[x^3-\frac{25}{2}\,x^2+22\,x+\frac{111}{2},RootOf(2\_Z^3-25\_Z^2+44\_Z+111)\Bigg]\Bigg],[[0,2,4]],$$

$$[[2,4,2]],[[[2,0],[4,0],[4,2]]]\Bigg],\Bigg[[[\infty,\infty],[x-1,1],[x-12,12],[x-3,3],[x$$

$$-7,7]],\Bigg[\Bigg[0,\frac{2}{3},-\frac{2}{3}\Bigg],\Bigg[0,\frac{1}{3},-\frac{1}{3}\Bigg],\Bigg[\frac{1}{2},\frac{\sqrt{55}}{15\sqrt{t}}+\frac{1}{2},-\frac{\sqrt{55}}{15\sqrt{t}}+\frac{1}{2}\Bigg],\Bigg[\frac{1}{2},\frac{\sqrt{2}}{6\sqrt{t}}$$

$$+\frac{1}{2},-\frac{\sqrt{2}}{6\sqrt{t}}+\frac{1}{2}\Bigg],\Bigg[\frac{1}{2},-\frac{I\sqrt{30}}{10\sqrt{t}}+\frac{1}{2},\frac{I\sqrt{30}}{10\sqrt{t}}+\frac{1}{2}\Bigg]\Bigg],\Bigg[\Bigg[\frac{2}{3},-\frac{2}{3},-\frac{4}{3}\Bigg],\Bigg[\frac{1}{3},$$

$$-\frac{1}{3},-\frac{2}{3}\Bigg],\Bigg[\frac{\sqrt{55}}{15\sqrt{t}},-\frac{\sqrt{55}}{15\sqrt{t}},-\frac{2\sqrt{55}}{15\sqrt{t}}\Bigg],\Bigg[\frac{\sqrt{2}}{6\sqrt{t}},-\frac{\sqrt{2}}{6\sqrt{t}},-\frac{\sqrt{2}}{3\sqrt{t}}\Bigg],$$

$$\Bigg[\frac{-\frac{I}{10}\sqrt{30}}{\sqrt{t}},\frac{\frac{I}{10}\sqrt{30}}{\sqrt{t}},\frac{\frac{I}{5}\sqrt{30}}{\sqrt{t}}\Bigg]\Bigg],\Bigg[\Bigg[\Bigg[\frac{2}{3},0\Bigg],\Bigg[-\frac{2}{3},0\Bigg],\Bigg[-\frac{2}{3},\frac{2}{3}\Bigg]\Bigg],\Bigg[\Bigg[\frac{1}{3},0\Bigg],\Bigg[$$

$$-\frac{1}{3},0\Bigg],\Bigg[-\frac{1}{3},\frac{1}{3}\Bigg]\Bigg],\Bigg[\Bigg[\frac{\sqrt{55}}{15\sqrt{t}}+\frac{1}{2},\frac{1}{2}\Bigg],\Bigg[-\frac{\sqrt{55}}{15\sqrt{t}}+\frac{1}{2},\frac{1}{2}\Bigg],\Bigg[-\frac{\sqrt{55}}{15\sqrt{t}}+\frac{1}{2},$$

$$\frac{\sqrt{55}}{15\sqrt{t}}+\frac{1}{2}\Bigg]\Bigg],\Bigg[\Bigg[\frac{\sqrt{2}}{6\sqrt{t}}+\frac{1}{2},\frac{1}{2}\Bigg],\Bigg[-\frac{\sqrt{2}}{6\sqrt{t}}+\frac{1}{2},\frac{1}{2}\Bigg],\Bigg[-\frac{\sqrt{2}}{6\sqrt{t}}+\frac{1}{2},\frac{\sqrt{2}}{6\sqrt{t}}$$

$$+\frac{1}{2}\Bigg]\Bigg],\Bigg[\Bigg[-\frac{I\sqrt{30}}{10\sqrt{t}}+\frac{1}{2},\frac{1}{2}\Bigg],\Bigg[\frac{I\sqrt{30}}{10\sqrt{t}}+\frac{1}{2},\frac{1}{2}\Bigg],\Bigg[\frac{I\sqrt{30}}{10\sqrt{t}}+\frac{1}{2},-\frac{I\sqrt{30}}{10\sqrt{t}}+\frac{1}{2}\Bigg]\Bigg]\Bigg]\Bigg]$$

$$, [[1, 1, 1], [1, 1, 1], [1, 2, 2], [1, 2, 2], [1, 2, 2]]]$$

```
> info1:=SirrBesSqRootinfo1(L,R1[1],R1[2],x,t,ext);
```

$$info1 := \left[\left[\left[12, x-12, \left[\frac{11}{45\,(x-12)}\right], 1, \varnothing, \varnothing\right], \left[3, x-3, \left[\frac{1}{18\,(x-3)}\right], 1, \varnothing, \varnothing\right], \left[7, x \right.\right.\right. \tag{61}$$

$$\left. -7, \left[-\frac{3}{10\,(x-7)}\right], 1, \varnothing, \varnothing\right]\right], 3, 5, (x-12)\,(x-3)\,(x-7), 1\right]$$

```
> easyBesSqRoot(L,R1,info1,x,t,ext);
```

$$\left\{\left[\left[\frac{1}{3}\right], \frac{x-1}{(x-12)\,(x-3)\,(x-7)}\right], \left[\left[\frac{1}{3}\right], -\frac{x-1}{(x-12)\,(x-3)\,(x-7)}\right]\right\} \tag{62}$$

```
> findBesSqRootRat(L,R1,info1,x,t,T,ext);
```

$$\left\{\left[\left[\frac{1}{3}\right], \frac{x-1}{(x-12)\,(x-3)\,(x-7)}\right], \left[\left[\frac{1}{3}\right], \frac{(x-1)^2}{2\,(x-12)\,(x-3)\,(x-7)}\right]\right\} \tag{63}$$

```
> TIME :=time();
  BesSqRootSolutions(L);
  time() - TIME;
```

$$TIME := 8.046$$

$$\left\{\left[\frac{1}{3}, [0], [1], \frac{x-1}{(x-12)\,(x-3)\,(x-7)}\right]\right\}$$

$$0.579 \tag{64}$$

```
> eq:=HolonomicDE(BesselI(nu, sqrt(x))^2,Y(x));
```

$$eq := -Y(x) + \left(-2\,v^2 - 2\,x + 2\right)\left(\frac{d}{dx}\,Y(x)\right) + 2\left(\frac{d^3}{dx^3}\,Y(x)\right)x^2 + 6\left(\frac{d^2}{dx^2}\,Y(x)\right)x \tag{65}$$

```
> LBB:=de2diffop(eq, Y(x));
```

$$LBB := 2\,x^2\,Dx^3 + 6\,x\,Dx^2 + \left(-2\,v^2 - 2\,x + 2\right)Dx - 1 \tag{66}$$

```
> LBB:=subs(nu=a1*RootOf(x^2+1)+1/2,LBB);
```

$$LBB := 2\,x^2\,Dx^3 + 6\,x\,Dx^2 + \left(-2\left(a1\,RootOf\left(\_Z^2+1\right) + \frac{1}{2}\right)^2 - 2\,x + 2\right)Dx - 1 \tag{67}$$

```
> f:=(x-7)^3/((x-12)*(x-14)*(x-1));
```

$$f := \frac{(x-7)^3}{(x-12)\,(x-14)\,(x-1)} \tag{68}$$

```
> L:=ChangeOfVariables(LBB,f);
```

$$L := Dx^3\,(x-7)^2\left(3\,x^2 - 5\,x - 427\right)^2(x-12)^4\,(x-14)^4\,(x-1)^4 + 3\left(6\,x^5 - 117\,x^4\right. \tag{69}$$

$$\left. -1368\,x^3 + 46217\,x^2 - 334138\,x + 657482\right)Dx^2\,(x-7)\left(3\,x^2 - 5\,x - 427\right)(x$$

$$-12)^3\,(x-14)^3\,(x-1)^3 - \left(-216080762819978\,x - 66624\,x^{11} - 600774\,x^{10}\right.$$

$$+ 292394091662\,x^6 + 35081598\,x^9 - 324\,a1^2\,x^{11} + 10908\,a1^2\,x^{10} + 57888\,a1^2\,x^9$$

$$- 5277576\,a1^2\,x^8 + 21267812\,a1^2\,x^7 + 944479876\,a1^2\,x^6 - 7082440832\,a1^2\,x^5$$

$$- 72513474936\,a1^2\,x^4 + 717037032572\,a1^2\,x^3 + 1772595974156\,a1^2\,x^2$$

$$- 24750874399496\,a1^2\,x - 3360661254552\,x^5 + 22671293966104\,x^4$$

$$-91715812262095\,x^3 + 208025784279285\,x^2 + 22339876769952\,a1^2 - 54\,x^{13} + 3564\,x^{12}$$
$$-22339876769952\,a1\,RootOf\left(\_Z^2+1\right) - 151320372\,x^8 - 12032099168\,x^7$$
$$+324\,RootOf\left(\_Z^2+1\right)a1\,x^{11} - 10908\,RootOf\left(\_Z^2+1\right)a1\,x^{10} - 57888\,RootOf\left(\_Z^2\right.$$
$$\left.+1\right)a1\,x^9 + 5277576\,RootOf\left(\_Z^2+1\right)a1\,x^8 - 21267812\,RootOf\left(\_Z^2+1\right)a1\,x^7$$
$$-944479876\,RootOf\left(\_Z^2+1\right)a1\,x^6 + 7082440832\,RootOf\left(\_Z^2+1\right)a1\,x^5$$
$$+72513474936\,RootOf\left(\_Z^2+1\right)a1\,x^4 - 717037032572\,RootOf\left(\_Z^2+1\right)a1\,x^3$$
$$-1772595974156\,RootOf\left(\_Z^2+1\right)a1\,x^2 + 24750874399496\,RootOf\left(\_Z^2+1\right)a1\,x$$
$$+50915291517020\big)\,(x-12)\,(x-14)\,(x-1)\,Dx + 4\left(3\,x^2 - 5\,x - 427\right)^5 (x-7)^2$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```
$$ext := \left\{a1, RootOf\left(\_Z^2+1\right)\right\} \tag{70}$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```
$$ext := \left\{a1, RootOf\left(\_Z^2+1\right)\right\} \tag{71}$$

```
> extppp:={};
```
$$extppp := \varnothing \tag{72}$$

```
> E:= Singular(L,extppp);
```
$$E := \left[[x-1,1], \left[x^2 - \frac{5}{3}x - \frac{427}{3}, RootOf\left(3\_Z^2 - 5\_Z - 427\right)\right], [x-7,7], [x-14,14],\right. \tag{73}$$
$$\left.[x-12,12]\right]$$

```
> F:=NotAppSing(L,E,ext);
```
$$F := [[x-1,1], [x-12,12], [x-14,14], [x-7,7]] \tag{74}$$

```
> Sirr:= irrsingBesSqRoot(L,t,F,ext);
```
$$Sirr := \left[[[x-1,1], [x-12,12], [x-14,14]], \left[\left[\frac{1}{2}, -\frac{6\,I\sqrt{858}}{143\sqrt{t}} + \frac{1}{2}, \frac{6\,I\sqrt{858}}{143\sqrt{t}}\right.\right.\right. \tag{75}$$

$$+\frac{1}{2}\bigg], \left[\frac{1}{2}, -\frac{5\,I\sqrt{110}}{22\sqrt{t}} + \frac{1}{2}, \frac{5\,I\sqrt{110}}{22\sqrt{t}} + \frac{1}{2}\right], \left[\frac{1}{2}, \frac{7\sqrt{182}}{26\sqrt{t}} + \frac{1}{2}, -\frac{7\sqrt{182}}{26\sqrt{t}}\right.$$

$$+\frac{1}{2}\bigg]\bigg], \left[\left[\frac{-\frac{6\,I}{143}\sqrt{858}}{\sqrt{t}}, \frac{\frac{6\,I}{143}\sqrt{858}}{\sqrt{t}}, \frac{\frac{12\,I}{143}\sqrt{858}}{\sqrt{t}}\right], \left[\frac{-\frac{5\,I}{22}\sqrt{110}}{\sqrt{t}}, \frac{\frac{5\,I}{22}\sqrt{110}}{\sqrt{t}},\right.\right.$$

$$\frac{\frac{5\,I}{11}\sqrt{110}}{\sqrt{t}}\bigg], \left[\frac{7\sqrt{182}}{26\sqrt{t}}, -\frac{7\sqrt{182}}{26\sqrt{t}}, -\frac{7\sqrt{182}}{13\sqrt{t}}\right]\bigg], \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right], [2,2,2], \bigg[\bigg[\bigg[$$

$$-\frac{6\,I\sqrt{858}}{143\sqrt{t}} + \frac{1}{2}, \frac{1}{2}\bigg], \left[\frac{6\,I\sqrt{858}}{143\sqrt{t}} + \frac{1}{2}, \frac{1}{2}\right], \left[\frac{6\,I\sqrt{858}}{143\sqrt{t}} + \frac{1}{2}, -\frac{6\,I\sqrt{858}}{143\sqrt{t}} + \frac{1}{2}\right]\bigg],$$

$$\bigg[\bigg[-\frac{5\,I\sqrt{110}}{22\sqrt{t}} + \frac{1}{2}, \frac{1}{2}\bigg], \left[\frac{5\,I\sqrt{110}}{22\sqrt{t}} + \frac{1}{2}, \frac{1}{2}\right], \left[\frac{5\,I\sqrt{110}}{22\sqrt{t}} + \frac{1}{2}, -\frac{5\,I\sqrt{110}}{22\sqrt{t}}\right.$$

$$+ \frac{1}{2} \Bigg] \Bigg], \Bigg[\Bigg[\frac{7\sqrt{182}}{26\sqrt{t}} + \frac{1}{2}, \frac{1}{2} \Bigg], \Bigg[-\frac{7\sqrt{182}}{26\sqrt{t}} + \frac{1}{2}, \frac{1}{2} \Bigg], \Bigg[-\frac{7\sqrt{182}}{26\sqrt{t}} + \frac{1}{2}, \frac{7\sqrt{182}}{26\sqrt{t}}$$

$$+ \frac{1}{2} \Bigg] \Bigg] \Bigg], \Bigg[\Bigg[-\frac{6\,I}{143}\sqrt{858}\,t, \frac{6\,I}{143}\sqrt{858}\,t, \frac{12\,I}{143}\sqrt{858}\,t \Bigg], \Bigg[-\frac{5\,I}{22}\sqrt{110}\,t, \frac{5\,I}{22}\sqrt{110}\,t,$$

$$\frac{5\,I}{11}\sqrt{110}\,t \Bigg], \Bigg[\frac{7\sqrt{182}\,t}{26}, -\frac{7\sqrt{182}\,t}{26}, -\frac{7\sqrt{182}\,t}{13} \Bigg] \Bigg], [[0,0,0],[0,0,0],[0,0,0]],$$

$$\Bigg[[[x-7,7]], \Bigg[\Bigg[0, 3\,a1\,RootOf(\_Z^2+1) + \frac{3}{2}, -\frac{3}{2} - 3\,a1\,RootOf(\_Z^2+1) \Bigg],$$

$$\Bigg[3\,a1\,RootOf(\_Z^2+1) + \frac{3}{2}, -\frac{3}{2} - 3\,a1\,RootOf(\_Z^2+1), -3 - 6\,a1\,RootOf(\_Z^2$$

$$+ 1) \Bigg], [1,1,1], \Bigg[\Bigg[3\,a1\,RootOf(\_Z^2+1) + \frac{3}{2}, 0 \Bigg], \Bigg[-\frac{3}{2} - 3\,a1\,RootOf(\_Z^2+1), 0 \Bigg], \Bigg[$$

$$-\frac{3}{2} - 3\,a1\,RootOf(\_Z^2+1), 3\,a1\,RootOf(\_Z^2+1) + \frac{3}{2} \Bigg] \Bigg], 2 \Bigg] \Bigg] \Bigg] \Bigg]$$

```
> Sreg:=regsingtrueBesSqRoot(L,t,Sirr[-1],ext);
```

$$Sreg := \Bigg[[[x-7,7]], \Bigg[\Bigg[0, 3\,a1\,RootOf(\_Z^2+1) + \frac{3}{2}, -\frac{3}{2} - 3\,a1\,RootOf(\_Z^2+1) \Bigg] \Bigg], \tag{76}$$

$$\Bigg[\Bigg[3\,a1\,RootOf(\_Z^2+1) + \frac{3}{2}, -\frac{3}{2} - 3\,a1\,RootOf(\_Z^2+1), -3 - 6\,a1\,RootOf(\_Z^2$$

$$+ 1) \Bigg] \Bigg], \Bigg[\Bigg[3\,a1\,RootOf(\_Z^2+1) + \frac{3}{2}, 0 \Bigg], \Bigg[-\frac{3}{2} - 3\,a1\,RootOf(\_Z^2+1), 0 \Bigg], \Bigg[-\frac{3}{2}$$

$$- 3\,a1\,RootOf(\_Z^2+1), 3\,a1\,RootOf(\_Z^2+1) + \frac{3}{2} \Bigg] \Bigg] \Bigg] \Bigg]$$

```
> NRemSreg:=SregseptrueBesSqRoot(L,Sreg,ext)[1];
```

$$NRemSreg := \Bigg[[[x-7,7]], \Bigg[\Bigg[0, 3\,a1\,RootOf(\_Z^2+1) + \frac{3}{2}, -\frac{3}{2} - 3\,a1\,RootOf(\_Z^2 \tag{77}$$

$$+ 1) \Bigg] \Bigg], \Bigg[\Bigg[3\,a1\,RootOf(\_Z^2+1) + \frac{3}{2}, -\frac{3}{2} - 3\,a1\,RootOf(\_Z^2+1), -3$$

$$- 6\,a1\,RootOf(\_Z^2+1) \Bigg], [\;] \Bigg] \Bigg] \Bigg]$$

```
> LogSreg:=SregseptrueBesSqRoot(L,Sreg,ext)[3];
```

$$LogSreg := [\;] \tag{78}$$

```
> RemSreg:=SregseptrueBesSqRoot(L,Sreg,ext)[2];
```

$$RemSreg := [\;] \tag{79}$$

```
> R1:=IrrRegAppsingBesSqRoot(L,t,E,ext);
```

$$R1 := \Bigg[\Bigg[[[x-1,1],[x-12,12],[x-14,14]], \Bigg[\Bigg[\frac{1}{2}, -\frac{6\,I\sqrt{858}}{143\sqrt{t}} + \frac{1}{2}, \frac{6\,I\sqrt{858}}{143\sqrt{t}} \tag{80}$$

$$+ \frac{1}{2} \Bigg], \Bigg[\frac{1}{2}, -\frac{5\,I\sqrt{110}}{22\sqrt{t}} + \frac{1}{2}, \frac{5\,I\sqrt{110}}{22\sqrt{t}} + \frac{1}{2} \Bigg], \Bigg[\frac{1}{2}, \frac{7\sqrt{182}}{26\sqrt{t}} + \frac{1}{2}, -\frac{7\sqrt{182}}{26\sqrt{t}}$$

$$+ \frac{1}{2} \Bigg]\Bigg], \Bigg[\Bigg[\frac{-\frac{6\,I}{143}\sqrt{858}}{\sqrt{t}}, \frac{\frac{6\,I}{143}\sqrt{858}}{\sqrt{t}}, \frac{\frac{12\,I}{143}\sqrt{858}}{\sqrt{t}}\Bigg], \Bigg[\frac{-\frac{5\,I}{22}\sqrt{110}}{\sqrt{t}}, \frac{\frac{5\,I}{22}\sqrt{110}}{\sqrt{t}},$$

$$\frac{\frac{5\,I}{11}\sqrt{110}}{\sqrt{t}}\Bigg], \Bigg[\frac{7\sqrt{182}}{26\sqrt{t}}, -\frac{7\sqrt{182}}{26\sqrt{t}}, -\frac{7\sqrt{182}}{13\sqrt{t}}\Bigg]\Bigg], \Bigg[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\Bigg], [2, 2, 2], \Bigg[\Bigg[\Bigg[$$

$$-\frac{6\,I\sqrt{858}}{143\sqrt{t}} + \frac{1}{2}, \frac{1}{2}\Bigg], \Bigg[\frac{6\,I\sqrt{858}}{143\sqrt{t}} + \frac{1}{2}, \frac{1}{2}\Bigg], \Bigg[\frac{6\,I\sqrt{858}}{143\sqrt{t}} + \frac{1}{2}, -\frac{6\,I\sqrt{858}}{143\sqrt{t}} + \frac{1}{2}\Bigg]\Bigg],$$

$$\Bigg[\Bigg[-\frac{5\,I\sqrt{110}}{22\sqrt{t}} + \frac{1}{2}, \frac{1}{2}\Bigg], \Bigg[\frac{5\,I\sqrt{110}}{22\sqrt{t}} + \frac{1}{2}, \frac{1}{2}\Bigg], \Bigg[\frac{5\,I\sqrt{110}}{22\sqrt{t}} + \frac{1}{2}, -\frac{5\,I\sqrt{110}}{22\sqrt{t}}$$

$$+ \frac{1}{2}\Bigg]\Bigg], \Bigg[\Bigg[\frac{7\sqrt{182}}{26\sqrt{t}} + \frac{1}{2}, \frac{1}{2}\Bigg], \Bigg[-\frac{7\sqrt{182}}{26\sqrt{t}} + \frac{1}{2}, \frac{1}{2}\Bigg], \Bigg[-\frac{7\sqrt{182}}{26\sqrt{t}} + \frac{1}{2}, \frac{7\sqrt{182}}{26\sqrt{t}}$$

$$+ \frac{1}{2}\Bigg]\Bigg]\Bigg], \Bigg[\Bigg[-\frac{6\,I}{143}\sqrt{858}\,t, \frac{6\,I}{143}\sqrt{858}\,t, \frac{12\,I}{143}\sqrt{858}\,t\Bigg], \Bigg[-\frac{5\,I}{22}\sqrt{110}\,t, \frac{5\,I}{22}\sqrt{110}\,t,$$

$$\frac{5\,I}{11}\sqrt{110}\,t\Bigg], \Bigg[\frac{7\sqrt{182}\,t}{26}, -\frac{7\sqrt{182}\,t}{26}, -\frac{7\sqrt{182}\,t}{13}\Bigg]\Bigg], [[0, 0, 0], [0, 0, 0], [0, 0, 0]]\Bigg],$$

$$\Bigg[[[x - 7, 7]], \Bigg[\Bigg[0, 3\,a1\,RootOf(\_Z^2 + 1) + \frac{3}{2}, -\frac{3}{2} - 3\,a1\,RootOf(\_Z^2 + 1)\Bigg]\Bigg],$$

$$\Bigg[\Bigg[3\,a1\,RootOf(\_Z^2 + 1) + \frac{3}{2}, -\frac{3}{2} - 3\,a1\,RootOf(\_Z^2 + 1), -3 - 6\,a1\,RootOf(\_Z^2$$

$$+ 1)\Bigg]\Bigg], \Bigg[\Bigg[\Bigg[3\,a1\,RootOf(\_Z^2 + 1) + \frac{3}{2}, 0\Bigg], \Bigg[-\frac{3}{2} - 3\,a1\,RootOf(\_Z^2 + 1), 0\Bigg], \Bigg[-\frac{3}{2}$$

$$- 3\,a1\,RootOf(\_Z^2 + 1), 3\,a1\,RootOf(\_Z^2 + 1) + \frac{3}{2}\Bigg]\Bigg]\Bigg]\Bigg], \Bigg[[[x - 7, 7]], \Bigg[\Bigg[0,$$

$$3\,a1\,RootOf(\_Z^2 + 1) + \frac{3}{2}, -\frac{3}{2} - 3\,a1\,RootOf(\_Z^2 + 1)\Bigg]\Bigg], \Bigg[\Bigg[\Bigg[3\,a1\,RootOf(\_Z^2 + 1)$$

$$+ \frac{3}{2}, -\frac{3}{2} - 3\,a1\,RootOf(\_Z^2 + 1), -3 - 6\,a1\,RootOf(\_Z^2 + 1)\Bigg], [\ ]\Bigg]\Bigg], [\ ], [\ ], \Bigg[\Bigg[\Bigg[x^2$$

$$-\frac{5}{3}x-\frac{427}{3}, RootOf\left(3\_Z^2-5\_Z-427\right)\bigg]\bigg], [[0,2,4]], [[2,4,2]], [[[2,0],[4,0],$$

$$[4,2]]]\bigg], \bigg[[[x-1,1],[x-12,12],[x-14,14],[x-7,7]], \bigg[\bigg[\frac{1}{2}, -\frac{6\,I\sqrt{858}}{143\sqrt{t}}+\frac{1}{2},$$

$$\frac{6\,I\sqrt{858}}{143\sqrt{t}}+\frac{1}{2}\bigg], \bigg[\frac{1}{2}, -\frac{5\,I\sqrt{110}}{22\sqrt{t}}+\frac{1}{2}, \frac{5\,I\sqrt{110}}{22\sqrt{t}}+\frac{1}{2}\bigg], \bigg[\frac{1}{2}, \frac{7\sqrt{182}}{26\sqrt{t}}+\frac{1}{2},$$

$$-\frac{7\sqrt{182}}{26\sqrt{t}}+\frac{1}{2}\bigg], \bigg[0, 3\,a1\,RootOf\left(\_Z^2+1\right)+\frac{3}{2}, -\frac{3}{2}-3\,a1\,RootOf\left(\_Z^2+1\right)\bigg]\bigg],$$

$$\bigg[\bigg[\frac{-\frac{6\,I}{143}\sqrt{858}}{\sqrt{t}}, \frac{\frac{6\,I}{143}\sqrt{858}}{\sqrt{t}}, \frac{\frac{12\,I}{143}\sqrt{858}}{\sqrt{t}}\bigg], \bigg[\frac{-\frac{5\,I}{22}\sqrt{110}}{\sqrt{t}}, \frac{\frac{5\,I}{22}\sqrt{110}}{\sqrt{t}},$$

$$\frac{\frac{5\,I}{11}\sqrt{110}}{\sqrt{t}}\bigg], \bigg[\frac{7\sqrt{182}}{26\sqrt{t}}, -\frac{7\sqrt{182}}{26\sqrt{t}}, -\frac{7\sqrt{182}}{13\sqrt{t}}\bigg], \bigg[3\,a1\,RootOf\left(\_Z^2+1\right)+\frac{3}{2}, -\frac{3}{2}$$

$$-3\,a1\,RootOf\left(\_Z^2+1\right), -3-6\,a1\,RootOf\left(\_Z^2+1\right)\bigg]\bigg], \bigg[\bigg[\bigg[-\frac{6\,I\sqrt{858}}{143\sqrt{t}}+\frac{1}{2}, \frac{1}{2}\bigg],$$

$$\bigg[\frac{6\,I\sqrt{858}}{143\sqrt{t}}+\frac{1}{2}, \frac{1}{2}\bigg], \bigg[\frac{6\,I\sqrt{858}}{143\sqrt{t}}+\frac{1}{2}, -\frac{6\,I\sqrt{858}}{143\sqrt{t}}+\frac{1}{2}\bigg]\bigg], \bigg[\bigg[-\frac{5\,I\sqrt{110}}{22\sqrt{t}}+\frac{1}{2},$$

$$\frac{1}{2}\bigg], \bigg[\frac{5\,I\sqrt{110}}{22\sqrt{t}}+\frac{1}{2}, \frac{1}{2}\bigg], \bigg[\frac{5\,I\sqrt{110}}{22\sqrt{t}}+\frac{1}{2}, -\frac{5\,I\sqrt{110}}{22\sqrt{t}}+\frac{1}{2}\bigg]\bigg], \bigg[\bigg[\frac{7\sqrt{182}}{26\sqrt{t}}+\frac{1}{2},$$

$$\frac{1}{2}\bigg], \bigg[-\frac{7\sqrt{182}}{26\sqrt{t}}+\frac{1}{2}, \frac{1}{2}\bigg], \bigg[-\frac{7\sqrt{182}}{26\sqrt{t}}+\frac{1}{2}, \frac{7\sqrt{182}}{26\sqrt{t}}+\frac{1}{2}\bigg]\bigg], \bigg[\bigg[3\,a1\,RootOf\left(\_Z^2\right.$$

$$+1\right)+\frac{3}{2}, 0\bigg], \bigg[-\frac{3}{2}-3\,a1\,RootOf\left(\_Z^2+1\right), 0\bigg], \bigg[-\frac{3}{2}-3\,a1\,RootOf\left(\_Z^2+1\right),$$

$$3\,a1\,RootOf\left(\_Z^2+1\right)+\frac{3}{2}\bigg]\bigg]\bigg], [[1,2,2],[1,2,2],[1,2,2],[1,1,1]]\bigg]\bigg]$$

```
> info1:=SirrBesSqRootinfo1(L,R1[1],R1[2],x,t,ext);
```

$$info1 := \bigg[\bigg[\bigg[1, x-1, \bigg[-\frac{216}{143\,(x-1)}\bigg], 1, \varnothing, \{a1, RootOf\left(\_Z^2+1\right)\}\bigg], \bigg[12, x-12, \bigg[\tag{81}$$

$$-\frac{125}{22\,(x-12)}\bigg], 1, \varnothing, \{a1, RootOf\left(\_Z^2+1\right)\}\bigg], \bigg[14, x-14, \bigg[\frac{343}{26\,(x-14)}\bigg], 1, \varnothing, \{a1,$$

$$RootOf\left(\_Z^2+1\right)\}\bigg]\bigg], 3, 4, (x-12)\,(x-14)\,(x-1), 1\bigg]$$

```
> easyBesSqRoot(L,R1,info1,x,t,ext);
```

$$\bigg\{\bigg[\bigg[a1\,RootOf\left(\_Z^2+1\right)+\frac{1}{2}, a1\,RootOf\left(\_Z^2+1\right)+\frac{5}{2}, a1\,RootOf\left(\_Z^2+1\right)+\frac{7}{6}\bigg], \tag{82}$$

$$\frac{(x-7)^3}{(x-12)\,(x-14)\,(x-1)}\Bigg],\left[\left[\mathit{al}\,RootOf\big(\_Z^2+1\big)+\frac{1}{2},\,\mathit{al}\,RootOf\big(\_Z^2+1\big)+\frac{5}{6},\right.\right.$$

$$\left.\left.\mathit{al}\,RootOf\big(\_Z^2+1\big)+\frac{7}{6}\right],\,-\frac{(x-7)^3}{(x-12)\,(x-14)\,(x-1)}\right]\Bigg\}$$

```
> findBesSqRootIrr(L,R1,info1,x,t,T,ext);
```

$$\Bigg\{\left[\left[\mathit{al}\,RootOf\big(\_Z^2+1\big)+\frac{1}{2},\,\mathit{al}\,RootOf\big(\_Z^2+1\big)+\frac{5}{6},\,\mathit{al}\,RootOf\big(\_Z^2+1\big)+\frac{7}{6}\right],\right. \tag{83}$$

$$\left.\frac{(x-7)^3}{(x-12)\,(x-14)\,(x-1)}\right]\Bigg\}$$

```
> TIME :=time();
  BesSqRootSolutions(L);
  time() - TIME;
```

$$TIME := 11.468$$

$$\left\{\left[\mathit{al}\,RootOf\big(\_Z^2+1\big)+\frac{1}{2},\,[0],\,[1],\,\frac{(x-7)^3}{(x-12)\,(x-14)\,(x-1)}\right]\right\}$$

$$0.563 \tag{84}$$


```
>                ####### THE LOGARITHMIC CASE #######
```

```
> eq:=HolonomicDE(BesselI(nu, sqrt(x))^2,Y(x));
```

$$eq := -Y(x) + \big(-2\,\nu^2 - 2\,x + 2\big)\left(\frac{\mathrm{d}}{\mathrm{d}x}\,Y(x)\right) + 2\left(\frac{\mathrm{d}^3}{\mathrm{d}x^3}\,Y(x)\right)x^2 + 6\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2}\,Y(x)\right)x \tag{85}$$

```
> LBB:=de2diffop(eq, Y(x));
```

$$LBB := 2\,x^2\,Dx^3 + 6\,x\,Dx^2 + \big(-2\,\nu^2 - 2\,x + 2\big)Dx - 1 \tag{86}$$

```
> LBB:=subs(nu=3,LBB);
```

$$LBB := 2\,x^2\,Dx^3 + 6\,x\,Dx^2 + (-16 - 2\,x)\,Dx - 1 \tag{87}$$

```
> f:=(x-1)^7/((x-12)*(x-3)^2*(x-7));
```

$$f := \frac{(x-1)^7}{(x-12)\,(x-3)^2\,(x-7)} \tag{88}$$

```
> L:=ChangeOfVariables(LBB,f);
```

$$L := 2\,Dx^3\,(x-1)^2\,(x-12)^4\,(x-3)^5\,(x-7)^4\,\big(x^3 - 29\,x^2 + 233\,x - 513\big)^2 + 6\,\big(x^6 - 58\,x^5 \tag{89}$$

$$+\, 1203\,x^4 - 11984\,x^3 + 61223\,x^2 - 152622\,x + 142893\big)\,Dx^2\,(x-1)\,(x-12)^3\,(x$$

$$-\,3)^4\,(x-7)^3\,\big(x^3 - 29\,x^2 + 233\,x - 513\big) - 2\,\big(9\,x^{19} - 1107\,x^{18} + 61299\,x^{17}$$

$$-\,2025001\,x^{16} + 44605788\,x^{15} - 693923188\,x^{14} + 7876831712\,x^{13} - 66378663720\,x^{12}$$

$$+\,416436089802\,x^{11} - 1900487697278\,x^{10} + 5661386139906\,x^9 - 4400844388430\,x^8$$

$$-\,64684397957276\,x^7 + 466749664160724\,x^6 - 1902141771420960\,x^5$$

$$+\,5356589050131912\,x^4 - 10606735563355539\,x^3 + 14151678477110001\,x^2$$

$$-\,11431958924090421\,x + 4222788193021527\big)\,Dx\,(x-12)\,(x-3)\,(x-7)$$

$$-\,27\,(x-1)^6\,\big(x^3 - 29\,x^2 + 233\,x - 513\big)^5$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```
$$ext := \varnothing \tag{90}$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```
$$ext := \varnothing \tag{91}$$

```
> extppp:={};
```
$$extppp := \varnothing \tag{92}$$

```
> E:= Singular(L,extppp);
```
$$E := \big[\, [x-1,1],\, [x-3,3],\, [\infty,\infty],\, [x^3 - 29\,x^2 + 233\,x - 513,\, RootOf(\_Z^3 - 29\,\_Z^2 \tag{93}$$
$$+ 233\,\_Z - 513)\,],\, [x-7,7],\, [x-12,12]\,\big]$$

```
> F:=NotAppSing(L,E,ext);
```
$$F := [\, [\infty,\infty],\, [x-1,1],\, [x-12,12],\, [x-3,3],\, [x-7,7]\,] \tag{94}$$

```
> Sirr:= irrsingBesSqRoot(L,t,F,ext);
```

$$Sirr := \Bigg[\, [[\infty,\infty],\, [x-12,12],\, [x-3,3],\, [x-7,7]],\, \Bigg[\Bigg[\frac{3}{2},\, \frac{3}{t^{3/2}} + \frac{9}{\sqrt{t}} + \frac{3}{2},\, -\frac{3}{t^{3/2}} \tag{95}$$

$$-\frac{9}{\sqrt{t}} + \frac{3}{2}\Bigg],\, \Bigg[\frac{1}{2},\, \frac{1331\sqrt{55}}{45\sqrt{t}} + \frac{1}{2},\, -\frac{1331\sqrt{55}}{45\sqrt{t}} + \frac{1}{2}\Bigg],\, \Bigg[1,$$

$$\frac{8\,RootOf(\_Z^2 - 2,\, index=1)}{3\,t} + 1,\, \frac{8\,RootOf(\_Z^2 - 2,\, index=2)}{3\,t} + 1\Bigg],\, \Bigg[\frac{1}{2},$$

$$-\frac{54\,I\sqrt{30}}{5\sqrt{t}} + \frac{1}{2},\, \frac{54\,I\sqrt{30}}{5\sqrt{t}} + \frac{1}{2}\Bigg]\Bigg],\, \Bigg[\Bigg[\frac{3}{t^{3/2}} + \frac{9}{\sqrt{t}},\, -\frac{3}{t^{3/2}} - \frac{9}{\sqrt{t}},\, -\frac{6}{t^{3/2}}$$

$$-\frac{18}{\sqrt{t}}\Bigg],\, \Bigg[\frac{1331\sqrt{55}}{45\sqrt{t}},\, -\frac{1331\sqrt{55}}{45\sqrt{t}},\, -\frac{2662\sqrt{55}}{45\sqrt{t}}\Bigg],\, \Bigg[\frac{8\,RootOf(\_Z^2 - 2,\, index=1)}{3\,t},$$

$$\frac{8\,RootOf(\_Z^2 - 2,\, index=2)}{3\,t},\, \frac{8\,RootOf(\_Z^2 - 2,\, index=2)}{3\,t}$$

$$-\frac{8\,RootOf(\_Z^2 - 2,\, index=1)}{3\,t}\Bigg],\, \Bigg[\frac{-\dfrac{54\,I}{5}\sqrt{30}}{\sqrt{t}},\, \frac{\dfrac{54\,I}{5}\sqrt{30}}{\sqrt{t}},\, \frac{\dfrac{108\,I}{5}\sqrt{30}}{\sqrt{t}}\Bigg]\Bigg],\, \Bigg[\frac{3}{2},$$

$$\frac{1}{2},\, 1,\, \frac{1}{2}\Bigg],\, [2,2,1,2],\, \Bigg[\Bigg[\Bigg[\frac{3}{t^{3/2}} + \frac{9}{\sqrt{t}} + \frac{3}{2},\, \frac{3}{2}\Bigg],\, \Bigg[-\frac{3}{t^{3/2}} - \frac{9}{\sqrt{t}} + \frac{3}{2},\, \frac{3}{2}\Bigg],\, \Bigg[-\frac{3}{t^{3/2}}$$

$$-\frac{9}{\sqrt{t}} + \frac{3}{2},\, \frac{3}{t^{3/2}} + \frac{9}{\sqrt{t}} + \frac{3}{2}\Bigg]\Bigg],\, \Bigg[\Bigg[\frac{1331\sqrt{55}}{45\sqrt{t}} + \frac{1}{2},\, \frac{1}{2}\Bigg],\, \Bigg[-\frac{1331\sqrt{55}}{45\sqrt{t}} + \frac{1}{2},$$

$$\frac{1}{2}\Bigg],\, \Bigg[-\frac{1331\sqrt{55}}{45\sqrt{t}} + \frac{1}{2},\, \frac{1331\sqrt{55}}{45\sqrt{t}} + \frac{1}{2}\Bigg]\Bigg],\, \Bigg[\Bigg[\frac{8\,RootOf(\_Z^2 - 2,\, index=1)}{3\,t} + 1,$$

$$1\Big],\ \left[\frac{8\,RootOf\left(\_Z^2-2,\,index=2\right)}{3\,t}+1,\,1\right],\ \left[\frac{8\,RootOf\left(\_Z^2-2,\,index=2\right)}{3\,t}+1,\right.$$

$$\left.\frac{8\,RootOf\left(\_Z^2-2,\,index=1\right)}{3\,t}+1\right]\Big],\ \left[\left[-\frac{54\,I\sqrt{30}}{5\sqrt{t}}+\frac{1}{2},\,\frac{1}{2}\right],\ \left[\frac{54\,I\sqrt{30}}{5\sqrt{t}}+\frac{1}{2},\,\frac{1}{2}\right],\right.$$

$$\left.\left[\frac{54\,I\sqrt{30}}{5\sqrt{t}}+\frac{1}{2},\,-\frac{54\,I\sqrt{30}}{5\sqrt{t}}+\frac{1}{2}\right]\right]\Big],\ \left[\,[3\,t^3+9\,t,\,-3\,t^3-9\,t,\,-6\,t^3-18\,t],\right.$$

$$\left[\frac{1331\sqrt{55}\,t}{45},\,-\frac{1331\sqrt{55}\,t}{45},\,-\frac{2662\sqrt{55}\,t}{45}\right],\ \left[\frac{8\,RootOf\left(\_Z^2-2,\,index=1\right)t}{3},\right.$$

$$\left.\frac{8\,RootOf\left(\_Z^2-2,\,index=2\right)t}{3},\,-\frac{16\,RootOf\left(\_Z^2-2,\,index=1\right)t}{3}\right],\ \left[-\frac{54\,I}{5}\sqrt{30}\,t,\right.$$

$$\left.\frac{54\,I}{5}\sqrt{30}\,t,\,\frac{108\,I}{5}\sqrt{30}\,t\right]\Big],\ [[0,0,0],[0,0,0],[0,0,0],[0,0,0]],\ [[[x-1,1]],$$

$$[[[-21,0,21],[21,42,21],[1,1,1],[[0,-21],[21,-21],[21,0]],4]]]\Bigg]$$

```
> Sreg:=regsingtrueBesSqRoot(L,t,Sirr[-1],ext);
```

$$Sreg := [[[x-1,1]],[[-21,0,21]],[[21,42,21]],[[[0,-21],[21,-21],[21,0]]]] \tag{96}$$

```
> NRemSreg:=SregseptrueBesSqRoot(L,Sreg,ext)[1];
```

$$NRemSreg := [\ ] \tag{97}$$

```
> LogSreg:=SregseptrueBesSqRoot(L,Sreg,ext)[3];
```

$$LogSreg := [[[x-1,1]],[[-21,0,21]],[[[\ ],[21,42,21]]]] \tag{98}$$

```
> RemSreg:=SregseptrueBesSqRoot(L,Sreg,ext)[2];
```

$$RemSreg := [\ ] \tag{99}$$

```
> R1:=IrrRegAppsingBesSqRoot(L,t,E,ext);
```

$$R1 := \Bigg[\Bigg[[[\infty,\infty],[x-12,12],[x-3,3],[x-7,7]],\left[\left[\frac{3}{2},\,\frac{3}{t^{3/2}}+\frac{9}{\sqrt{t}}+\frac{3}{2},\,-\frac{3}{t^{3/2}}\right.\right.\tag{100}$$

$$\left.-\frac{9}{\sqrt{t}}+\frac{3}{2}\right],\ \left[\frac{1}{2},\,\frac{1331\sqrt{55}}{45\sqrt{t}}+\frac{1}{2},\,-\frac{1331\sqrt{55}}{45\sqrt{t}}+\frac{1}{2}\right],\ \Big[1,$$

$$\frac{8\,RootOf\left(\_Z^2-2,\,index=1\right)}{3\,t}+1,\,\frac{8\,RootOf\left(\_Z^2-2,\,index=2\right)}{3\,t}+1\Big],\ \Big[\frac{1}{2},$$

$$-\frac{54\,I\sqrt{30}}{5\sqrt{t}}+\frac{1}{2},\,\frac{54\,I\sqrt{30}}{5\sqrt{t}}+\frac{1}{2}\Big]\Big],\ \left[\left[\frac{3}{t^{3/2}}+\frac{9}{\sqrt{t}},\,-\frac{3}{t^{3/2}}-\frac{9}{\sqrt{t}},\,-\frac{6}{t^{3/2}}\right.\right.$$

$$\left.-\frac{18}{\sqrt{t}}\right],\ \left[\frac{1331\sqrt{55}}{45\sqrt{t}},\,-\frac{1331\sqrt{55}}{45\sqrt{t}},\,-\frac{2662\sqrt{55}}{45\sqrt{t}}\right],\ \Big[\frac{8\,RootOf\left(\_Z^2-2,\,index=1\right)}{3\,t},$$

$$\frac{8\,RootOf\left(\_Z^2-2,\,index=2\right)}{3\,t},\,\frac{8\,RootOf\left(\_Z^2-2,\,index=2\right)}{3\,t}$$

$$-\frac{8\,RootOf\left(\_Z^2-2,\,index=1\right)}{3\,t}\Bigg],\,\left[\frac{-\dfrac{54\,I}{5}\sqrt{30}}{\sqrt{t}},\,\frac{\dfrac{54\,I}{5}\sqrt{30}}{\sqrt{t}},\,\frac{\dfrac{108\,I}{5}\sqrt{30}}{\sqrt{t}}\right]\Bigg],\,\Bigg[\frac{3}{2},$$

$$\frac{1}{2},\,1,\,\frac{1}{2}\Bigg],\,[2,2,1,2],\,\Bigg[\Bigg[\Bigg[\frac{3}{t^{3/2}}+\frac{9}{\sqrt{t}}+\frac{3}{2},\,\frac{3}{2}\Bigg],\,\Bigg[-\frac{3}{t^{3/2}}-\frac{9}{\sqrt{t}}+\frac{3}{2},\,\frac{3}{2}\Bigg],\,\Bigg[-\frac{3}{t^{3/2}}$$

$$-\frac{9}{\sqrt{t}}+\frac{3}{2},\,\frac{3}{t^{3/2}}+\frac{9}{\sqrt{t}}+\frac{3}{2}\Bigg]\Bigg],\,\Bigg[\Bigg[\frac{1331\sqrt{55}}{45\sqrt{t}}+\frac{1}{2},\,\frac{1}{2}\Bigg],\,\Bigg[-\frac{1331\sqrt{55}}{45\sqrt{t}}+\frac{1}{2},$$

$$\frac{1}{2}\Bigg],\,\Bigg[-\frac{1331\sqrt{55}}{45\sqrt{t}}+\frac{1}{2},\,\frac{1331\sqrt{55}}{45\sqrt{t}}+\frac{1}{2}\Bigg]\Bigg],\,\Bigg[\Bigg[\frac{8\,RootOf\left(\_Z^2-2,\,index=1\right)}{3\,t}+1,$$

$$1\Bigg],\,\Bigg[\frac{8\,RootOf\left(\_Z^2-2,\,index=2\right)}{3\,t}+1,\,1\Bigg],\,\Bigg[\frac{8\,RootOf\left(\_Z^2-2,\,index=2\right)}{3\,t}+1,$$

$$\frac{8\,RootOf\left(\_Z^2-2,\,index=1\right)}{3\,t}+1\Bigg]\Bigg],\,\Bigg[\Bigg[-\frac{54\,I\sqrt{30}}{5\sqrt{t}}+\frac{1}{2},\,\frac{1}{2}\Bigg],\,\Bigg[\frac{54\,I\sqrt{30}}{5\sqrt{t}}+\frac{1}{2},\,\frac{1}{2}\Bigg],$$

$$\Bigg[\frac{54\,I\sqrt{30}}{5\sqrt{t}}+\frac{1}{2},\,-\frac{54\,I\sqrt{30}}{5\sqrt{t}}+\frac{1}{2}\Bigg]\Bigg],\,\Bigg[\left[3\,t^3+9\,t,\,-3\,t^3-9\,t,\,-6\,t^3-18\,t\right],$$

$$\Bigg[\frac{1331\sqrt{55}\,t}{45},\,-\frac{1331\sqrt{55}\,t}{45},\,-\frac{2662\sqrt{55}\,t}{45}\Bigg],\,\Bigg[\frac{8\,RootOf\left(\_Z^2-2,\,index=1\right)t}{3},$$

$$\frac{8\,RootOf\left(\_Z^2-2,\,index=2\right)t}{3},\,-\frac{16\,RootOf\left(\_Z^2-2,\,index=1\right)t}{3}\Bigg],\,\Bigg[-\frac{54\,I}{5}\sqrt{30}\,t,$$

$$\frac{54\,I}{5}\sqrt{30}\,t,\,\frac{108\,I}{5}\sqrt{30}\,t\Bigg]\Bigg],\,[[0,0,0],\,[0,0,0],\,[0,0,0],\,[0,0,0]]\Bigg],\,[[[x-1,1]],$$

$$[[-21,0,21]],\,[[21,42,21]],\,[[[0,-21],[21,-21],[21,0]]]],\,[[\,],[\,],[[[x-1,$$

$$1]],\,[[-21,0,21]],\,[[[\,],[21,42,21]]]]],\,[[[x^3-29\,x^2+233\,x-513,\,RootOf\left(\_Z^3\right.$$

$-29\_Z^2 + 233\_Z - 513)\,]],\,[[0, 2, 4]],\,[[2, 4, 2]],\,[[[2, 0], [4, 0], [4, 2]]]],\,[[\infty,$

$\infty],\,[x - 1, 1],\,[x - 12, 12],\,[x - 3, 3],\,[x - 7, 7]],\,\left[\left[\dfrac{3}{2}, \dfrac{3}{t^{3/2}} + \dfrac{9}{\sqrt{t}} + \dfrac{3}{2}, -\dfrac{3}{t^{3/2}}\right.\right.$

$\left.\left.-\dfrac{9}{\sqrt{t}} + \dfrac{3}{2}\right],\,[-21, 0, 21],\,\left[\dfrac{1}{2}, \dfrac{1331\sqrt{55}}{45\sqrt{t}} + \dfrac{1}{2}, -\dfrac{1331\sqrt{55}}{45\sqrt{t}} + \dfrac{1}{2}\right],\,\left[1,\right.\right.$

$\left.\dfrac{8\,RootOf(\_Z^2 - 2,\,index=1)}{3\,t} + 1,\,\dfrac{8\,RootOf(\_Z^2 - 2,\,index=2)}{3\,t} + 1\right],\,\left[\dfrac{1}{2},\right.$

$\left.\left.-\dfrac{54\,I\sqrt{30}}{5\sqrt{t}} + \dfrac{1}{2}, \dfrac{54\,I\sqrt{30}}{5\sqrt{t}} + \dfrac{1}{2}\right]\right],\,\left[\left[\dfrac{3}{t^{3/2}} + \dfrac{9}{\sqrt{t}}, -\dfrac{3}{t^{3/2}} - \dfrac{9}{\sqrt{t}}, -\dfrac{6}{t^{3/2}}\right.\right.$

$\left.\left.-\dfrac{18}{\sqrt{t}}\right],\,[21, 42, 21],\,\left[\dfrac{1331\sqrt{55}}{45\sqrt{t}}, -\dfrac{1331\sqrt{55}}{45\sqrt{t}}, -\dfrac{2662\sqrt{55}}{45\sqrt{t}}\right],\right.$

$\left[\dfrac{8\,RootOf(\_Z^2 - 2,\,index=1)}{3\,t}, \dfrac{8\,RootOf(\_Z^2 - 2,\,index=2)}{3\,t},\right.$

$\left.\dfrac{8\,RootOf(\_Z^2 - 2,\,index=2)}{3\,t} - \dfrac{8\,RootOf(\_Z^2 - 2,\,index=1)}{3\,t}\right],\,\left[\dfrac{-\dfrac{54\,I}{5}\sqrt{30}}{\sqrt{t}},\right.$

$\left.\left.\dfrac{\dfrac{54\,I}{5}\sqrt{30}}{\sqrt{t}}, \dfrac{\dfrac{108\,I}{5}\sqrt{30}}{\sqrt{t}}\right]\right],\,\left[\left[\dfrac{3}{t^{3/2}} + \dfrac{9}{\sqrt{t}} + \dfrac{3}{2}, \dfrac{3}{2}\right], \left[-\dfrac{3}{t^{3/2}} - \dfrac{9}{\sqrt{t}} + \dfrac{3}{2}, \dfrac{3}{2}\right], \right.$

$\left.\left[-\dfrac{3}{t^{3/2}} - \dfrac{9}{\sqrt{t}} + \dfrac{3}{2}, \dfrac{3}{t^{3/2}} + \dfrac{9}{\sqrt{t}} + \dfrac{3}{2}\right]\right],\,[[0, -21], [21, -21], [21, 0]],$

$\left[\left[\dfrac{1331\sqrt{55}}{45\sqrt{t}} + \dfrac{1}{2}, \dfrac{1}{2}\right], \left[-\dfrac{1331\sqrt{55}}{45\sqrt{t}} + \dfrac{1}{2}, \dfrac{1}{2}\right], \left[-\dfrac{1331\sqrt{55}}{45\sqrt{t}} + \dfrac{1}{2}, \dfrac{1331\sqrt{55}}{45\sqrt{t}}\right.\right.$

$\left.\left.+ \dfrac{1}{2}\right]\right],\,\left[\left[\dfrac{8\,RootOf(\_Z^2 - 2,\,index=1)}{3\,t} + 1, 1\right], \left[\dfrac{8\,RootOf(\_Z^2 - 2,\,index=2)}{3\,t} + 1,\right.\right.$

$\left.\left.1\right], \left[\dfrac{8\,RootOf(\_Z^2 - 2,\,index=2)}{3\,t} + 1, \dfrac{8\,RootOf(\_Z^2 - 2,\,index=1)}{3\,t} + 1\right]\right],\,\left[\left[\right.\right.$

$\left.\left.-\dfrac{54\,I\sqrt{30}}{5\sqrt{t}} + \dfrac{1}{2}, \dfrac{1}{2}\right], \left[\dfrac{54\,I\sqrt{30}}{5\sqrt{t}} + \dfrac{1}{2}, \dfrac{1}{2}\right], \left[\dfrac{54\,I\sqrt{30}}{5\sqrt{t}} + \dfrac{1}{2}, -\dfrac{54\,I\sqrt{30}}{5\sqrt{t}} + \dfrac{1}{2}\right]\right]\right]$

$,\,[[1, 2, 2], [1, 1, 1], [1, 2, 2], [1, 1, 1], [1, 2, 2]]]]$

```
> info1:=SirrBesSqRootinfo1(L,R1[1],R1[2],x,t,ext);
```

$$info1 := \left[\left[\left[\infty, \frac{1}{x}, [x(x+9)^2]\right], 3, \varnothing, \varnothing\right], \left[12, x-12, \left[\frac{19487171}{405(x-12)}\right]\right], 1, \varnothing, \varnothing\right], \left[3, x \quad \textbf{(101)}$$

$$-3, \left[\frac{32}{9(x-3)^2}\right], 2, \{RootOf(\_Z^2-2, index=1)\}, \{RootOf(\_Z^2-2, index=1)\}\right],$$

$$\left[7, x-7, \left[-\frac{17496}{5(x-7)}\right], 1, \varnothing, \varnothing\right], 7, 6, (x-12)(x-3)^2(x-7), x-3\right]$$

```
> findBesSqRootln(L,R1,info1,x,t,T,ext);
```

$$\left\{\left[[3], -\frac{(x-1)^7}{(x-12)(x-3)^2(x-7)}\right]\right\} \quad \textbf{(102)}$$

```
> TIME :=time();
  BesSqRootSolutions(L);
  time() - TIME;
```

$$TIME := 14.859$$

$$\left\{\left[3, [0], [1], \frac{(x-1)^7}{(x-12)(x-3)^2(x-7)}\right]\right\}$$

$$0.531 \quad \textbf{(103)}$$

```
>          ####### THE IRRATIONAL CASE #######
```

```
> eq:=HolonomicDE(BesselI(nu, sqrt(x))^2,Y(x));
```

$$eq := -Y(x) + \left(-2v^2 - 2x + 2\right)\left(\frac{d}{dx}Y(x)\right) + 2\left(\frac{d^3}{dx^3}Y(x)\right)x^2 + 6\left(\frac{d^2}{dx^2}Y(x)\right)x \quad \textbf{(104)}$$

```
> LBB:=de2diffop(eq, Y(x));
```

$$LBB := 2x^2 Dx^3 + 6x Dx^2 + \left(-2v^2 - 2x + 2\right)Dx - 1 \quad \textbf{(105)}$$

```
> LBB:=subs(nu=RootOf(x^2-2)+1/2,LBB);
```

$$LBB := 2x^2 Dx^3 + 6x Dx^2 + \left(-2\left(RootOf(\_Z^2-2) + \frac{1}{2}\right)^2 - 2x + 2\right)Dx - 1 \quad \textbf{(106)}$$

```
> f:=(x^2-14)/((x^2-12)*(x^2-3));
```

$$f := \frac{x^2 - 14}{(x^2 - 12)(x^2 - 3)} \quad \textbf{(107)}$$

```
> L:=ChangeOfVariables(LBB,f);
```

$$L := x^2 (x^2-14)^2 (x^4 - 28x^2 + 174)^2 (x^2-12)^4 (x^2-3)^4 Dx^3 + 3x(x^2-14)(x^4 - 28x^2 \quad \textbf{(108)}$$

$$+174)(x^{10} - 55x^8 + 944x^6 - 5730x^4 + 468x^2 + 87696)(x^2-12)^3(x^2-3)^3 Dx^2$$

$$+\frac{1}{2}\left((2 + RootOf(\_Z^2-2))(24592765560x^{10} - 35874048x^{16} + 15622194816x^6\right.$$

$$+618949963008x^4 - 55386x^{20} - 1761409939968x^2 + 485671800x^{14} - 4355417960x^{12}$$

$$+1008x^{22} - 830583548928RootOf(\_Z^2-2) + 1761190x^{18} - 8x^{24} + 4RootOf(\_Z^2$$

$$-2)x^{22} - 771RootOf(\_Z^2-2)x^{20} + 44621RootOf(\_Z^2-2)x^{18}$$

$$-1350112\,RootOf\left(\_Z^2-2\right)x^{16}+26030820\,RootOf\left(\_Z^2-2\right)x^{14}$$
$$-353728076\,RootOf\left(\_Z^2-2\right)x^{12}+3538107204\,RootOf\left(\_Z^2-2\right)x^{10}$$
$$-25866374544\,RootOf\left(\_Z^2-2\right)x^{8}+132149764224\,RootOf\left(\_Z^2-2\right)x^{6}$$
$$-441470590848\,RootOf\left(\_Z^2-2\right)x^{4}+880704969984\,RootOf\left(\_Z^2-2\right)x^{2}$$
$$-73487818080\,x^{8}+1661167097856\big)\left(x^2-3\right)\left(x^2-12\right)Dx\big)+4\,x^5\left(x^4-28\,x^2\right.$$
$$\left.+174\right)^5$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```
$$ext := \left\{RootOf\left(\_Z^2-2\right)\right\} \tag{109}$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```
$$ext := \left\{RootOf\left(\_Z^2-2\right)\right\} \tag{110}$$

```
> extppp:={};
```
$$extppp := \varnothing \tag{111}$$

```
> E:= Singular(L,extppp);
```
$$E := \big[\,[x^4-28\,x^2+174, RootOf\left(\_Z^4-28\,\_Z^2+174\right)], [x,0], [x^2-12, 2\,RootOf\left(\_Z^2 \right. \tag{112}$$
$$\left.-3\right)], [x^2-3, RootOf\left(\_Z^2-3\right)], [\infty,\infty], [x^2-14, RootOf\left(\_Z^2-14\right)]\big]$$

```
> F:=NotAppSing(L,E,ext);
```
$$F := \big[\,[\infty,\infty], [x^2-12, 2\,RootOf\left(\_Z^2-3\right)], [x^2-3, RootOf\left(\_Z^2-3\right)], [x^2-14, \tag{113}$$
$$RootOf\left(\_Z^2-14\right)]\big]$$

```
> Sirr:= irrsingBesSqRoot(L,t,F,ext);
```
$$Sirr := \Bigg[\Big[\,[x^2-12, 2\,RootOf\left(\_Z^2-3\right)], [x^2-3, RootOf\left(\_Z^2-3\right)]\Big],\Bigg[\bigg[\frac{1}{2}, \tag{114}$$

$$\frac{\sqrt{2}}{6\sqrt{-RootOf\left(\_Z^2-3\right)}\,\sqrt{t}}+\frac{1}{2},\ -\frac{\sqrt{2}}{6\sqrt{-RootOf\left(\_Z^2-3\right)}\,\sqrt{t}}+\frac{1}{2}\bigg],\bigg[\frac{1}{2},$$

$$\frac{\sqrt{22}}{6\sqrt{RootOf\left(\_Z^2-3\right)}\,\sqrt{t}}+\frac{1}{2},\ -\frac{\sqrt{22}}{6\sqrt{RootOf\left(\_Z^2-3\right)}\,\sqrt{t}}+\frac{1}{2}\bigg]\bigg],$$

$$\bigg[\bigg[\frac{\sqrt{2}}{6\sqrt{-RootOf\left(\_Z^2-3\right)}\,\sqrt{t}},\ -\frac{\sqrt{2}}{6\sqrt{-RootOf\left(\_Z^2-3\right)}\,\sqrt{t}},$$

$$-\frac{\sqrt{2}}{3\sqrt{-RootOf\left(\_Z^2-3\right)}\,\sqrt{t}}\bigg],\bigg[\frac{\sqrt{22}}{6\sqrt{RootOf\left(\_Z^2-3\right)}\,\sqrt{t}},$$

$$-\frac{\sqrt{22}}{6\sqrt{RootOf\left(\_Z^2-3\right)}\,\sqrt{t}},\ -\frac{\sqrt{22}}{3\sqrt{RootOf\left(\_Z^2-3\right)}\,\sqrt{t}}\bigg]\bigg],\bigg[\frac{1}{2},\frac{1}{2}\bigg], [2,2],$$

$$\bigg[\bigg[\bigg[\frac{\sqrt{2}}{6\sqrt{-RootOf\left(\_Z^2-3\right)}\,\sqrt{t}}+\frac{1}{2},\frac{1}{2}\bigg],\bigg[-\frac{\sqrt{2}}{6\sqrt{-RootOf\left(\_Z^2-3\right)}\,\sqrt{t}}+\frac{1}{2},\frac{1}{2}\bigg],$$

$$\left[-\frac{\sqrt{2}}{6\sqrt{-RootOf(\_Z^2-3)}\,\sqrt{t}}+\frac{1}{2},\;\frac{\sqrt{2}}{6\sqrt{-RootOf(\_Z^2-3)}\,\sqrt{t}}+\frac{1}{2}\right],$$

$$\left[\left[\frac{\sqrt{22}}{6\sqrt{RootOf(\_Z^2-3)}\,\sqrt{t}}+\frac{1}{2},\;\frac{1}{2}\right],\left[-\frac{\sqrt{22}}{6\sqrt{RootOf(\_Z^2-3)}\,\sqrt{t}}+\frac{1}{2},\;\frac{1}{2}\right],\left[\right.\right.$$

$$\left.-\frac{\sqrt{22}}{6\sqrt{RootOf(\_Z^2-3)}\,\sqrt{t}}+\frac{1}{2},\;\frac{\sqrt{22}}{6\sqrt{RootOf(\_Z^2-3)}\,\sqrt{t}}+\frac{1}{2}\right]\right],\left[\left[\right.\right.$$

$$-\frac{\sqrt{2}\,RootOf(\_Z^2-3)\sqrt{-RootOf(\_Z^2-3)}\,t}{18},$$

$$\frac{\sqrt{2}\,RootOf(\_Z^2-3)\sqrt{-RootOf(\_Z^2-3)}\,t}{18},$$

$$\left.\frac{\sqrt{2}\,RootOf(\_Z^2-3)\sqrt{-RootOf(\_Z^2-3)}\,t}{9}\right],\left[\frac{\sqrt{22}\,RootOf(\_Z^2-3)^{3/2}\,t}{18},\right.$$

$$\left.\left.-\frac{\sqrt{22}\,RootOf(\_Z^2-3)^{3/2}\,t}{18},\;-\frac{\sqrt{22}\,RootOf(\_Z^2-3)^{3/2}\,t}{9}\right]\right],[[0,0,0],[0,0,0]],$$

$$\left[\left[[\infty,\infty],[x^2-14,RootOf(\_Z^2-14)]\right],\left[[0,1+2\,RootOf(\_Z^2-2),-1\right.\right.$$

$$-2\,RootOf(\_Z^2-2)],[1+2\,RootOf(\_Z^2-2),-1-2\,RootOf(\_Z^2-2),-2$$

$$-4\,RootOf(\_Z^2-2)],[1,1,1],\left[[1+2\,RootOf(\_Z^2-2),0],[-1-2\,RootOf(\_Z^2\right.$$

$$-2),0],[-1-2\,RootOf(\_Z^2-2),1+2\,RootOf(\_Z^2-2)]],2],\left[\left[0,RootOf(\_Z^2\right.\right.$$

$$-2)+\frac{1}{2},-\frac{1}{2}-RootOf(\_Z^2-2)\right],\left[RootOf(\_Z^2-2)+\frac{1}{2},-\frac{1}{2}-RootOf(\_Z^2\right.$$

$$-2),-1-2\,RootOf(\_Z^2-2)\right],[1,1,1],\left[\left[RootOf(\_Z^2-2)+\frac{1}{2},0\right],\left[-\frac{1}{2}\right.\right.$$

$$\left.\left.\left.\left.\left.-RootOf(\_Z^2-2),0\right],\left[-\frac{1}{2}-RootOf(\_Z^2-2),RootOf(\_Z^2-2)+\frac{1}{2}\right]\right],2\right]\right]\right]\right]$$

```
> Sreg:=regsingtrueBesSqRoot(L,t,Sirr[-1],ext);
```

$$Sreg:=\left[\left[[\infty,\infty],[x^2-14,RootOf(\_Z^2-14)]\right],\left[[0,1+2\,RootOf(\_Z^2-2),-1\right.\right.\qquad\textbf{(115)}$$

$$-2\,RootOf(\_Z^2-2)],\left[0,RootOf(\_Z^2-2)+\frac{1}{2},-\frac{1}{2}-RootOf(\_Z^2-2)\right]\right],\left[[1\right.$$

$$+2\,RootOf(\_Z^2-2),-1-2\,RootOf(\_Z^2-2),-2-4\,RootOf(\_Z^2-2)],$$

$$\left[RootOf(\_Z^2-2)+\frac{1}{2},-\frac{1}{2}-RootOf(\_Z^2-2),-1-2\,RootOf(\_Z^2-2)\right]\right],\left[[[1\right.$$

$$+2\,RootOf(\_Z^2-2),0],[-1-2\,RootOf(\_Z^2-2),0],[-1-2\,RootOf(\_Z^2-2),1$$

$$+\ 2\ RootOf\left(\_Z^2 - 2\right)\,]\,],\ \left[\left[RootOf\left(\_Z^2 - 2\right) + \frac{1}{2},\, 0\right],\ \left[-\frac{1}{2} - RootOf\left(\_Z^2 - 2\right),\, 0\right],\ \left[\right.$$

$$\left.-\frac{1}{2} - RootOf\left(\_Z^2 - 2\right),\, RootOf\left(\_Z^2 - 2\right) + \frac{1}{2}\right]\,]\,]\,]$$

```
> NRemSreg:=SregseptrueBesSqRoot(L,Sreg,ext)[1];
```

$$NRemSreg := \left[\,[\,[\,\infty,\,\infty\,],\ [x^2 - 14,\, RootOf\left(\_Z^2 - 14\right)\,]\,],\ \left[0, 1 + 2\, RootOf\left(\_Z^2 - 2\right),\, -1\right.\right. \tag{116}$$

$$\left.-\,2\, RootOf\left(\_Z^2 - 2\right)\,],\ \left[0,\, RootOf\left(\_Z^2 - 2\right) + \frac{1}{2},\, -\frac{1}{2} - RootOf\left(\_Z^2 - 2\right)\right]\right],\ \left[\,[\,[\,1\right.$$

$$\left.+\,2\, RootOf\left(\_Z^2 - 2\right),\, -1 - 2\, RootOf\left(\_Z^2 - 2\right),\, -2 - 4\, RootOf\left(\_Z^2 - 2\right)\,],\, [\,]\,],\right.$$

$$\left.\left[\left[RootOf\left(\_Z^2 - 2\right) + \frac{1}{2},\, -\frac{1}{2} - RootOf\left(\_Z^2 - 2\right),\, -1 - 2\, RootOf\left(\_Z^2 - 2\right)\right],\, [\,]\,]\,\right]\right]$$

```
> LogSreg:=SregseptrueBesSqRoot(L,Sreg,ext)[3];
```

$$LogSreg := [\,] \tag{117}$$

```
> RemSreg:=SregseptrueBesSqRoot(L,Sreg,ext)[2];
```

$$RemSreg := [\,] \tag{118}$$

```
> R1:=IrrRegAppsingBesSqRoot(L,t,E,ext);
```

$$R1 := \left[\left[\,[\,[x^2 - 12,\, 2\, RootOf\left(\_Z^2 - 3\right)\,],\ [x^2 - 3,\, RootOf\left(\_Z^2 - 3\right)\,]\,],\ \left[\left[\frac{1}{2},\right.\right.\right.\right. \tag{119}$$

$$\left.\frac{\sqrt{2}}{6\,\sqrt{-RootOf\left(\_Z^2 - 3\right)}\,\sqrt{t}} + \frac{1}{2},\, -\frac{\sqrt{2}}{6\,\sqrt{-RootOf\left(\_Z^2 - 3\right)}\,\sqrt{t}} + \frac{1}{2}\right],\ \left[\frac{1}{2},\right.$$

$$\left.\frac{\sqrt{22}}{6\,\sqrt{RootOf\left(\_Z^2 - 3\right)}\,\sqrt{t}} + \frac{1}{2},\, -\frac{\sqrt{22}}{6\,\sqrt{RootOf\left(\_Z^2 - 3\right)}\,\sqrt{t}} + \frac{1}{2}\right]\right],$$

$$\left[\left[\frac{\sqrt{2}}{6\,\sqrt{-RootOf\left(\_Z^2 - 3\right)}\,\sqrt{t}},\, -\frac{\sqrt{2}}{6\,\sqrt{-RootOf\left(\_Z^2 - 3\right)}\,\sqrt{t}},\right.\right.$$

$$\left.-\frac{\sqrt{2}}{3\,\sqrt{-RootOf\left(\_Z^2 - 3\right)}\,\sqrt{t}}\right],\ \left[\frac{\sqrt{22}}{6\,\sqrt{RootOf\left(\_Z^2 - 3\right)}\,\sqrt{t}},\right.$$

$$\left.-\frac{\sqrt{22}}{6\,\sqrt{RootOf\left(\_Z^2 - 3\right)}\,\sqrt{t}},\, -\frac{\sqrt{22}}{3\,\sqrt{RootOf\left(\_Z^2 - 3\right)}\,\sqrt{t}}\right]\right],\ \left[\frac{1}{2},\, \frac{1}{2}\right],\ [2, 2],$$

$$\left[\left[\left[\frac{\sqrt{2}}{6\,\sqrt{-RootOf\left(\_Z^2 - 3\right)}\,\sqrt{t}} + \frac{1}{2},\, \frac{1}{2}\right],\ \left[-\frac{\sqrt{2}}{6\,\sqrt{-RootOf\left(\_Z^2 - 3\right)}\,\sqrt{t}} + \frac{1}{2},\, \frac{1}{2}\right],\right.\right.$$

$$\left.\left[-\frac{\sqrt{2}}{6\,\sqrt{-RootOf\left(\_Z^2 - 3\right)}\,\sqrt{t}} + \frac{1}{2},\, \frac{\sqrt{2}}{6\,\sqrt{-RootOf\left(\_Z^2 - 3\right)}\,\sqrt{t}} + \frac{1}{2}\right]\right],$$

$$\left[\left[\frac{\sqrt{22}}{6\,\sqrt{RootOf\left(\_Z^2 - 3\right)}\,\sqrt{t}} + \frac{1}{2},\, \frac{1}{2}\right],\ \left[-\frac{\sqrt{22}}{6\,\sqrt{RootOf\left(\_Z^2 - 3\right)}\,\sqrt{t}} + \frac{1}{2},\, \frac{1}{2}\right],\ \left[\right.\right.$$

$$-\frac{\sqrt{22}}{6\sqrt{RootOf(\_Z^2-3)}\sqrt{t}}+\frac{1}{2}, \frac{\sqrt{22}}{6\sqrt{RootOf(\_Z^2-3)}\sqrt{t}}+\frac{1}{2}\bigg]\bigg]\bigg], \bigg[\bigg[$$

$$-\frac{\sqrt{2}\,RootOf(\_Z^2-3)\sqrt{-RootOf(\_Z^2-3)}\,t}{18},$$

$$\frac{\sqrt{2}\,RootOf(\_Z^2-3)\sqrt{-RootOf(\_Z^2-3)}\,t}{18},$$

$$\frac{\sqrt{2}\,RootOf(\_Z^2-3)\sqrt{-RootOf(\_Z^2-3)}\,t}{9}\bigg], \bigg[\frac{\sqrt{22}\,RootOf(\_Z^2-3)^{3/2}\,t}{18},$$

$$-\frac{\sqrt{22}\,RootOf(\_Z^2-3)^{3/2}\,t}{18}, -\frac{\sqrt{22}\,RootOf(\_Z^2-3)^{3/2}\,t}{9}\bigg]\bigg], [[0,0,0],[0,0,0]]\bigg],$$

$$\bigg[[[\infty,\infty],[x^2-14, RootOf(\_Z^2-14)]], \bigg[[0,1+2\,RootOf(\_Z^2-2), -1$$

$$-2\,RootOf(\_Z^2-2)], \bigg[0, RootOf(\_Z^2-2)+\frac{1}{2}, -\frac{1}{2}-RootOf(\_Z^2-2)\bigg]\bigg], \bigg[[1$$

$$+2\,RootOf(\_Z^2-2), -1-2\,RootOf(\_Z^2-2), -2-4\,RootOf(\_Z^2-2)],$$

$$\bigg[RootOf(\_Z^2-2)+\frac{1}{2}, -\frac{1}{2}-RootOf(\_Z^2-2), -1-2\,RootOf(\_Z^2-2)\bigg]\bigg], \bigg[[[1$$

$$+2\,RootOf(\_Z^2-2),0],[-1-2\,RootOf(\_Z^2-2),0],[-1-2\,RootOf(\_Z^2-2),1$$

$$+2\,RootOf(\_Z^2-2)]], \bigg[\bigg[RootOf(\_Z^2-2)+\frac{1}{2},0\bigg], \bigg[-\frac{1}{2}-RootOf(\_Z^2-2),0\bigg], \bigg[$$

$$-\frac{1}{2}-RootOf(\_Z^2-2), RootOf(\_Z^2-2)+\frac{1}{2}\bigg]\bigg]\bigg], \bigg[[[\infty,\infty],[x^2-14, RootOf(\_Z^2$$

$$-14)]], \bigg[[0,1+2\,RootOf(\_Z^2-2), -1-2\,RootOf(\_Z^2-2)], \bigg[0, RootOf(\_Z^2-2)$$

$$+\frac{1}{2}, -\frac{1}{2}-RootOf(\_Z^2-2)\bigg]\bigg], \bigg[[[1+2\,RootOf(\_Z^2-2), -1-2\,RootOf(\_Z^2$$

$$-2), -2-4\,RootOf(\_Z^2-2)], [\,]], \bigg[\bigg[RootOf(\_Z^2-2)+\frac{1}{2}, -\frac{1}{2}-RootOf(\_Z^2$$

$$-2), -1-2\,RootOf(\_Z^2-2)\bigg], [\,]\bigg]\bigg]\bigg], [\,], [\,]\bigg], [[[x^4-28\,x^2+174, RootOf(\_Z^4$$

$-28\,\_Z^2 + 174)\,], [x, 0]], [[0, 2, 4], [0, 2, 4]], [[2, 4, 2], [2, 4, 2]], [[[2, 0], [4, 0],$

$[4, 2]], [[2, 0], [4, 0], [4, 2]]]], \left[\,[[\infty, \infty], [x^2 - 12, 2\,RootOf(\_Z^2 - 3)\,], [x^2 - 3,\right.$

$RootOf(\_Z^2 - 3)\,], [x^2 - 14, RootOf(\_Z^2 - 14)\,]], \left[\,[0, 1 + 2\,RootOf(\_Z^2 - 2), -1\right.$

$-2\,RootOf(\_Z^2 - 2)\,], \left[\dfrac{1}{2}, \dfrac{\sqrt{2}}{6\sqrt{-RootOf(\_Z^2 - 3)}\,\sqrt{t}} + \dfrac{1}{2},\right.$

$-\dfrac{\sqrt{2}}{6\sqrt{-RootOf(\_Z^2 - 3)}\,\sqrt{t}} + \dfrac{1}{2}\right], \left[\dfrac{1}{2}, \dfrac{\sqrt{22}}{6\sqrt{RootOf(\_Z^2 - 3)}\,\sqrt{t}} + \dfrac{1}{2},\right.$

$-\dfrac{\sqrt{22}}{6\sqrt{RootOf(\_Z^2 - 3)}\,\sqrt{t}} + \dfrac{1}{2}\right], \left[0, RootOf(\_Z^2 - 2) + \dfrac{1}{2}, -\dfrac{1}{2} - RootOf(\_Z^2\right.$

$-2)\,]\,], \left[\,[1 + 2\,RootOf(\_Z^2 - 2), -1 - 2\,RootOf(\_Z^2 - 2), -2 - 4\,RootOf(\_Z^2\right.$

$-2)\,], \left[\dfrac{\sqrt{2}}{6\sqrt{-RootOf(\_Z^2 - 3)}\,\sqrt{t}}, -\dfrac{\sqrt{2}}{6\sqrt{-RootOf(\_Z^2 - 3)}\,\sqrt{t}},\right.$

$-\dfrac{\sqrt{2}}{3\sqrt{-RootOf(\_Z^2 - 3)}\,\sqrt{t}}\right], \left[\dfrac{\sqrt{22}}{6\sqrt{RootOf(\_Z^2 - 3)}\,\sqrt{t}},\right.$

$-\dfrac{\sqrt{22}}{6\sqrt{RootOf(\_Z^2 - 3)}\,\sqrt{t}}, -\dfrac{\sqrt{22}}{3\sqrt{RootOf(\_Z^2 - 3)}\,\sqrt{t}}\right], \left[RootOf(\_Z^2 - 2) + \dfrac{1}{2},\right.$

$-\dfrac{1}{2} - RootOf(\_Z^2 - 2), -1 - 2\,RootOf(\_Z^2 - 2)\,]\,], \left[\,[[1 + 2\,RootOf(\_Z^2 - 2), 0], [\right.$

$-1 - 2\,RootOf(\_Z^2 - 2), 0], [-1 - 2\,RootOf(\_Z^2 - 2), 1 + 2\,RootOf(\_Z^2 - 2)\,]],$

$\left[\left[\dfrac{\sqrt{2}}{6\sqrt{-RootOf(\_Z^2 - 3)}\,\sqrt{t}} + \dfrac{1}{2}, \dfrac{1}{2}\right], \left[-\dfrac{\sqrt{2}}{6\sqrt{-RootOf(\_Z^2 - 3)}\,\sqrt{t}} + \dfrac{1}{2}, \dfrac{1}{2}\right], \left[\right.\right.$

$-\dfrac{\sqrt{2}}{6\sqrt{-RootOf(\_Z^2 - 3)}\,\sqrt{t}} + \dfrac{1}{2}, \dfrac{\sqrt{2}}{6\sqrt{-RootOf(\_Z^2 - 3)}\,\sqrt{t}} + \dfrac{1}{2}\right]\,],$

$\left[\left[\dfrac{\sqrt{22}}{6\sqrt{RootOf(\_Z^2 - 3)}\,\sqrt{t}} + \dfrac{1}{2}, \dfrac{1}{2}\right], \left[-\dfrac{\sqrt{22}}{6\sqrt{RootOf(\_Z^2 - 3)}\,\sqrt{t}} + \dfrac{1}{2}, \dfrac{1}{2}\right], \left[\right.\right.$

$-\dfrac{\sqrt{22}}{6\sqrt{RootOf(\_Z^2 - 3)}\,\sqrt{t}} + \dfrac{1}{2}, \dfrac{\sqrt{22}}{6\sqrt{RootOf(\_Z^2 - 3)}\,\sqrt{t}} + \dfrac{1}{2}\right]\,], \left[\,[RootOf(\_Z^2\right.$

$$-2) + \frac{1}{2}, 0\Big], \Big[-\frac{1}{2} - RootOf(\_Z^2 - 2), 0\Big], \Big[-\frac{1}{2} - RootOf(\_Z^2 - 2), RootOf(\_Z^2$$

$$-2) + \frac{1}{2}\Big]\Big]\Big], [[1, 1, 1], [1, 2, 2], [1, 2, 2], [1, 1, 1]]\Big]\Big]$$

```
> info1:=SirrBesSqRootinfo1(L,R1[1],R1[2],x,t,ext);
```

$$info1 := \Bigg[\Bigg[\Bigg[2\,RootOf(\_Z^2 - 3), x^2 - 12, \Bigg[-\frac{RootOf(\_Z^2 - 3)}{54\,(x - 2\,RootOf(\_Z^2 - 3))}\Bigg], 1, \varnothing, \tag{120}$$

$$\{RootOf(\_Z^2 - 2)\}\Bigg], \Bigg[RootOf(\_Z^2 - 3), x^2 - 3, \Bigg[\frac{11\,RootOf(\_Z^2 - 3)}{54\,(x - RootOf(\_Z^2 - 3))}\Bigg], 1, \varnothing,$$

$$\{RootOf(\_Z^2 - 2)\}\Bigg]\Bigg], 4, 7, (x^2 - 12)\,(x^2 - 3), 1\Bigg]$$

```
> findBesSqRootIrr(L,R1,info1,x,t,T,ext);
```

$$\Bigg\{\Bigg[\Big[RootOf(\_Z^2 - 2) + \frac{1}{2}\Big], \frac{x^2 - 14}{(x^2 - 12)\,(x^2 - 3)}\Bigg]\Bigg\} \tag{121}$$

```
> TIME :=time();
  BesSqRootSolutions(L);
  time() - TIME;
```

$$TIME := 17.984$$

$$\Bigg\{\Bigg[RootOf(\_Z^2 - 2) + \frac{1}{2}, [0], [1], \frac{x^2 - 14}{(x^2 - 12)\,(x^2 - 3)}\Bigg]\Bigg\}$$

$$0.453 \tag{122}$$

```
>          ####### THE RATIONAL CASE #######
```

```
> eq:=HolonomicDE(BesselI(nu, sqrt(x))^2,Y(x));
```

$$eq := -Y(x) + \left(-2\,v^2 - 2\,x + 2\right)\left(\frac{d}{dx}\,Y(x)\right) + 2\left(\frac{d^3}{dx^3}\,Y(x)\right)x^2 + 6\left(\frac{d^2}{dx^2}\,Y(x)\right)x \tag{123}$$

```
> LBB:=de2diffop(eq, Y(x));
```

$$LBB := 2\,x^2\,Dx^3 + 6\,x\,Dx^2 + \left(-2\,v^2 - 2\,x + 2\right)Dx - 1 \tag{124}$$

```
> LBB:=subs(nu=1/7,LBB);
```

$$LBB := 2\,x^2\,Dx^3 + 6\,x\,Dx^2 + \left(\frac{96}{49} - 2\,x\right)Dx - 1 \tag{125}$$

```
> f:=(x-1)*(x-7)/((x-12)*(x-14));
```

$$f := \frac{(x - 1)\,(x - 7)}{(x - 12)\,(x - 14)} \tag{126}$$

```
> L:=ChangeOfVariables(LBB,f);
```

$$L := 49\,Dx^3\,(x - 1)^2\,(x - 7)^2\,(x - 12)^4\,(x - 14)^4\,(9\,x^2 - 161\,x + 581)^2 + 147\,(18\,x^5 \tag{127}$$

$$- 789\,x^4 + 13272\,x^3 - 107191\,x^2 + 423878\,x - 697270)\,Dx^2\,(x - 1)\,(x - 7)\,(x$$

$$- 12)^3\,(x - 14)^3\,(9\,x^2 - 161\,x + 581) + (23814\,x^{12} - 2706858\,x^{11} + 137566491\,x^{10}$$

$$- 4143372774\,x^9 + 82541763612\,x^8 - 1147831469442\,x^7 + 11442049022705\,x^6$$

$$-82484931576558\,x^5 + 427350098776818\,x^4 - 1555362004770784\,x^3$$
$$+\, 3794891808698508\,x^2 - 5644752368493168\,x + 3979949284819172\big)\,Dx\,(x-12)\,(x$$
$$-\,14) + 196\,\big(9\,x^2 - 161\,x + 581\big)^5$$

```
> ext:=indets(L,{RootOf,name}) minus {x,Dx};
```
$$ext := \varnothing \qquad\qquad (128)$$

```
> ext:= indets(map(s-> ReplirrRoot(s,{}),ext),{RootOf,name});
```
$$ext := \varnothing \qquad\qquad (129)$$

```
> extppp:={};
```
$$extppp := \varnothing \qquad\qquad (130)$$

```
> E:= Singular(L,extppp);
```
$$E := \Big[\,[x-1,1],\,\Big[x^2 - \frac{161}{9}\,x + \frac{581}{9},\,RootOf\big(9\,\_Z^2 - 161\,\_Z + 581\big)\Big],\,[x-7,7],\,[x-14, \qquad (131)$$
$$14],\,[x-12,12]\Big]$$

```
> F:=NotAppSing(L,E,ext);
```
$$F := [\,[x-1,1],\,[x-12,12],\,[x-14,14],\,[x-7,7]\,] \qquad\qquad (132)$$

```
> Sirr:= irrsingBesSqRoot(L,t,F,ext);
```
$$Sirr := \Bigg[\,[[x-12,12],\,[x-14,14]],\,\Bigg[\Bigg[\frac{1}{2},\,-\frac{I\sqrt{110}}{2\sqrt{t}}+\frac{1}{2},\,\frac{I\sqrt{110}}{2\sqrt{t}}+\frac{1}{2}\Bigg],\,\Bigg[\frac{1}{2},\,\frac{\sqrt{182}}{2\sqrt{t}} \qquad (133)$$
$$+\,\frac{1}{2},\,-\frac{\sqrt{182}}{2\sqrt{t}}+\frac{1}{2}\Bigg]\Bigg],\,\Bigg[\Bigg[\frac{-\dfrac{I}{2}\sqrt{110}}{\sqrt{t}},\,\frac{\dfrac{I}{2}\sqrt{110}}{\sqrt{t}},\,\frac{I\sqrt{110}}{\sqrt{t}}\Bigg],\,\Bigg[\frac{\sqrt{182}}{2\sqrt{t}},\,-\frac{\sqrt{182}}{2\sqrt{t}},$$
$$-\,\frac{\sqrt{182}}{\sqrt{t}}\Bigg]\Bigg],\,\Big[\frac{1}{2},\,\frac{1}{2}\Big],\,[2,2],\,\Bigg[\Bigg[-\frac{I\sqrt{110}}{2\sqrt{t}}+\frac{1}{2},\,\frac{1}{2}\Bigg],\,\Bigg[\frac{I\sqrt{110}}{2\sqrt{t}}+\frac{1}{2},\,\frac{1}{2}\Bigg],\,\Bigg[\frac{I\sqrt{110}}{2\sqrt{t}}$$
$$+\,\frac{1}{2},\,-\frac{I\sqrt{110}}{2\sqrt{t}}+\frac{1}{2}\Bigg]\Bigg],\,\Bigg[\Bigg[\frac{\sqrt{182}}{2\sqrt{t}}+\frac{1}{2},\,\frac{1}{2}\Bigg],\,\Bigg[-\frac{\sqrt{182}}{2\sqrt{t}}+\frac{1}{2},\,\frac{1}{2}\Bigg],\,\Bigg[-\frac{\sqrt{182}}{2\sqrt{t}}+\frac{1}{2},$$
$$\frac{\sqrt{182}}{2\sqrt{t}}+\frac{1}{2}\Bigg]\Bigg],\,\Bigg[\Bigg[-\frac{I}{2}\sqrt{110}\,t,\,\frac{I}{2}\sqrt{110}\,t,\,I\sqrt{110}\,t\Bigg],\,\Bigg[\frac{\sqrt{182}\,t}{2},\,-\frac{\sqrt{182}\,t}{2},$$
$$-\sqrt{182}\,t\Bigg]\Bigg],\,[[0,0,0],\,[0,0,0]]\Bigg],\,\Bigg[[[x-1,1],\,[x-7,7]],\,\Bigg[\Big[0,\,\frac{1}{7},\,-\frac{1}{7}\Big],\,\Big[\frac{1}{7},\,-\frac{1}{7},$$
$$-\,\frac{2}{7}\Big],\,[1,1,1],\,\Bigg[\Big[\frac{1}{7},0\Big],\,\Big[-\frac{1}{7},0\Big],\,\Big[-\frac{1}{7},\,\frac{1}{7}\Big]\Bigg],\,2\Bigg],\,\Bigg[\Big[0,\,\frac{1}{7},\,-\frac{1}{7}\Big],\,\Big[\frac{1}{7},\,-\frac{1}{7},\,-\frac{2}{7}\Big],\,[1,$$
$$1,1],\,\Bigg[\Big[\frac{1}{7},0\Big],\,\Big[-\frac{1}{7},0\Big],\,\Big[-\frac{1}{7},\,\frac{1}{7}\Big]\Bigg],\,2\Bigg]\Bigg]\Bigg]$$

```
> Sreg:=regsingtrueBesSqRoot(L,t,Sirr[-1],ext);
```
$$Sreg := \Bigg[\,[[x-1,1],\,[x-7,7]],\,\Bigg[\Big[0,\,\frac{1}{7},\,-\frac{1}{7}\Big],\,\Big[0,\,\frac{1}{7},\,-\frac{1}{7}\Big]\Bigg],\,\Bigg[\Big[\frac{1}{7},\,-\frac{1}{7},\,-\frac{2}{7}\Big],\,\Big[\frac{1}{7}, \qquad (134)$$
$$-\,\frac{1}{7},\,-\frac{2}{7}\Big]\Bigg],\,\Bigg[\Big[\frac{1}{7},0\Big],\,\Big[-\frac{1}{7},0\Big],\,\Big[-\frac{1}{7},\,\frac{1}{7}\Big]\Bigg],\,\Bigg[\Big[\frac{1}{7},0\Big],\,\Big[-\frac{1}{7},0\Big],\,\Big[-\frac{1}{7},\,\frac{1}{7}\Big]\Bigg]\Bigg]\Bigg]$$

```
> NRemSreg:=SregseptrueBesSqRoot(L,Sreg,ext)[1];
```

$$NRemSreg := \left[[[x-1,1],[x-7,7]], \left[\left[0, \frac{1}{7}, -\frac{1}{7} \right], \left[0, \frac{1}{7}, -\frac{1}{7} \right] \right], \left[\left[\left[\frac{1}{7}, -\frac{1}{7}, -\frac{2}{7} \right], \right. \right. \right.$$ **(135)**

$$\left. \left. [\,] \right], \left[\left[\frac{1}{7}, -\frac{1}{7}, -\frac{2}{7} \right], [\,] \right] \right] \right]$$

```
> LogSreg:=SregseptrueBesSqRoot(L,Sreg,ext)[3];
```

$$LogSreg := [\,]$$ **(136)**

```
> RemSreg:=SregseptrueBesSqRoot(L,Sreg,ext)[2];
```

$$RemSreg := [\,]$$ **(137)**

```
> R1:=IrrRegAppsingBesSqRoot(L,t,E,ext);
```

$$R1 := \left[\left[[[x-12,12],[x-14,14]], \left[\left[\frac{1}{2}, -\frac{I\sqrt{110}}{2\sqrt{t}} + \frac{1}{2}, \frac{I\sqrt{110}}{2\sqrt{t}} + \frac{1}{2} \right], \left[\frac{1}{2}, \frac{\sqrt{182}}{2\sqrt{t}} \right. \right. \right. \right.$$ **(138)**

$$\left. \left. + \frac{1}{2}, -\frac{\sqrt{182}}{2\sqrt{t}} + \frac{1}{2} \right] \right], \left[\left[\frac{-\frac{I}{2}\sqrt{110}}{\sqrt{t}}, \frac{\frac{I}{2}\sqrt{110}}{\sqrt{t}}, \frac{I\sqrt{110}}{\sqrt{t}} \right], \left[\frac{\sqrt{182}}{2\sqrt{t}}, -\frac{\sqrt{182}}{2\sqrt{t}}, \right. \right.$$

$$\left. \left. -\frac{\sqrt{182}}{\sqrt{t}} \right] \right], \left[\frac{1}{2}, \frac{1}{2} \right], [2,2], \left[\left[\left[-\frac{I\sqrt{110}}{2\sqrt{t}} + \frac{1}{2}, \frac{1}{2} \right], \left[\frac{I\sqrt{110}}{2\sqrt{t}} + \frac{1}{2}, \frac{1}{2} \right], \left[\frac{I\sqrt{110}}{2\sqrt{t}} \right. \right. \right.$$

$$\left. \left. + \frac{1}{2}, -\frac{I\sqrt{110}}{2\sqrt{t}} + \frac{1}{2} \right] \right], \left[\left[\frac{\sqrt{182}}{2\sqrt{t}} + \frac{1}{2}, \frac{1}{2} \right], \left[-\frac{\sqrt{182}}{2\sqrt{t}} + \frac{1}{2}, \frac{1}{2} \right], \left[-\frac{\sqrt{182}}{2\sqrt{t}} + \frac{1}{2}, \right. \right.$$

$$\left. \left. \frac{\sqrt{182}}{2\sqrt{t}} + \frac{1}{2} \right] \right] \right], \left[\left[-\frac{I}{2}\sqrt{110}\, t, \frac{I}{2}\sqrt{110}\, t, I\sqrt{110}\, t \right], \left[\frac{\sqrt{182}\, t}{2}, -\frac{\sqrt{182}\, t}{2}, \right. \right.$$

$$\left. \left. -\sqrt{182}\, t \right] \right], [[0,0,0],[0,0,0]] \right], \left[[[x-1,1],[x-7,7]], \left[\left[0, \frac{1}{7}, -\frac{1}{7} \right], \left[0, \frac{1}{7}, \right. \right. \right.$$

$$\left. \left. -\frac{1}{7} \right] \right], \left[\left[\frac{1}{7}, -\frac{1}{7}, -\frac{2}{7} \right], \left[\frac{1}{7}, -\frac{1}{7}, -\frac{2}{7} \right] \right], \left[\left[\left[\frac{1}{7}, 0 \right], \left[-\frac{1}{7}, 0 \right], \left[-\frac{1}{7}, \frac{1}{7} \right] \right], \left[\left[\frac{1}{7}, 0 \right], \right. \right.$$

$$\left. \left. \left[-\frac{1}{7}, 0 \right], \left[-\frac{1}{7}, \frac{1}{7} \right] \right] \right] \right], \left[[[x-1,1],[x-7,7]], \left[\left[0, \frac{1}{7}, -\frac{1}{7} \right], \left[0, \frac{1}{7}, -\frac{1}{7} \right] \right], \right.$$

$$\left. \left[\left[\left[\frac{1}{7}, -\frac{1}{7}, -\frac{2}{7} \right], [\,] \right], \left[\left[\frac{1}{7}, -\frac{1}{7}, -\frac{2}{7} \right], [\,] \right] \right], [\,], [\,], \left[\left[\left[x^2 - \frac{161}{9}x + \frac{581}{9}, \right. \right. \right.$$

$$\left. \left. \left. RootOf\left(9\_Z^2 - 161\_Z + 581\right) \right] \right], [[0,2,4]], [[2,4,2]], [[[2,0],[4,0],[4,2]]] \right], \left[[[x \right.$$

$$-1, 1], [x - 12, 12], [x - 14, 14], [x - 7, 7]], \left[\left[0, \frac{1}{7}, -\frac{1}{7}\right], \left[\frac{1}{2}, -\frac{I\sqrt{110}}{2\sqrt{t}} + \frac{1}{2},\right.\right.$$

$$\left.\frac{I\sqrt{110}}{2\sqrt{t}} + \frac{1}{2}\right], \left[\frac{1}{2}, \frac{\sqrt{182}}{2\sqrt{t}} + \frac{1}{2}, -\frac{\sqrt{182}}{2\sqrt{t}} + \frac{1}{2}\right], \left[0, \frac{1}{7}, -\frac{1}{7}\right]\right], \left[\left[\frac{1}{7}, -\frac{1}{7}, -\frac{2}{7}\right],\right.$$

$$\left[\frac{-\frac{I}{2}\sqrt{110}}{\sqrt{t}}, \frac{\frac{I}{2}\sqrt{110}}{\sqrt{t}}, \frac{I\sqrt{110}}{\sqrt{t}}\right], \left[\frac{\sqrt{182}}{2\sqrt{t}}, -\frac{\sqrt{182}}{2\sqrt{t}}, -\frac{\sqrt{182}}{\sqrt{t}}\right], \left[\frac{1}{7}, -\frac{1}{7}, -\frac{2}{7}\right]\right]$$

$$, \left[\left[\frac{1}{7}, 0\right], \left[-\frac{1}{7}, 0\right], \left[-\frac{1}{7}, \frac{1}{7}\right]\right], \left[\left[-\frac{I\sqrt{110}}{2\sqrt{t}} + \frac{1}{2}, \frac{1}{2}\right], \left[\frac{I\sqrt{110}}{2\sqrt{t}} + \frac{1}{2}, \frac{1}{2}\right],\right.$$

$$\left[\frac{I\sqrt{110}}{2\sqrt{t}} + \frac{1}{2}, -\frac{I\sqrt{110}}{2\sqrt{t}} + \frac{1}{2}\right]\right], \left[\left[\frac{\sqrt{182}}{2\sqrt{t}} + \frac{1}{2}, \frac{1}{2}\right], \left[-\frac{\sqrt{182}}{2\sqrt{t}} + \frac{1}{2}, \frac{1}{2}\right], \left[\right.$$

$$\left.-\frac{\sqrt{182}}{2\sqrt{t}} + \frac{1}{2}, \frac{\sqrt{182}}{2\sqrt{t}} + \frac{1}{2}\right]\right], \left[\left[\frac{1}{7}, 0\right], \left[-\frac{1}{7}, 0\right], \left[-\frac{1}{7}, \frac{1}{7}\right]\right]\right], [[1, 1, 1], [1, 2, 2],$$

$$[1, 2, 2], [1, 1, 1]]\Big]\Big]$$

```
> info1:=SirrBesSqRootinfo1(L,R1[1],R1[2],x,t,ext);
```

$$info1 := \left[\left[\left[12, x - 12, \left[-\frac{55}{2(x-12)}\right], 1, \varnothing, \varnothing\right], \left[14, x - 14, \left[\frac{91}{2(x-14)}\right], 1, \varnothing, \varnothing\right]\right], 2,\right. \qquad \textbf{(139)}$$

$$4, (x - 12)(x - 14), 1\Big]$$

```
> findBesSqRootRat(L,R1,info1,x,t,T,ext);
```

$$\left\{\left[\left[\frac{1}{7}\right], \frac{(x-1)(x-7)}{(x-12)(x-14)}\right]\right\} \qquad \textbf{(140)}$$

```
> TIME :=time();
  BesSqRootSolutions(L);
  time() - TIME;
```

$$TIME := 19.859$$

$$\left\{\left[\frac{1}{7}, [0], [1], \frac{(x-1)(x-7)}{(x-12)(x-14)}\right]\right\}$$

$$0.250 \qquad \textbf{(141)}$$