

```
[> restart:
> read "ODE3solve.mpl":
    Package "Solving third-order holonomic differential equations", Maple 16
    Copyright 2017, Mouafo Wouodjie Merlin, University of Kassel
    Package "Hypergeometric Summation", Maple V - Maple 17
    Copyright 1998-2013, Wolfram Koepf, University of Kassel
```

(1)

[Here are the Maple implementations in chapter 5 related just to the 2F2 functions.

```
[> ##### THE EXPONENT DIFFERENCES #####
```

[In chapter 5, section 5.2 which is called "Exponent differences", we have the following Maple implementations:

```
> L22 := x^2*Dx^3+(-x^2+x*b2+x*x*b1)*Dx^2+(-x*a1-x*a2+b2*b1-x)*Dx-
a1*a2;
    L22 := x^2 Dx^3 + (x b1 + x b2 - x^2 + x) Dx^2 + (-x a1 - x a2 + b2 b1 - x) Dx - a1 a2
```

(2)

```
> gen_exp(L22,t,x=0);
    [[0, t=x], [-b1 + 1, t=x], [1 - b2, t=x]]
```

(3)

```
> gen_exp(L22,t,x=infinity);
    [[a1, t= 1/x], [a2, t= 1/x], [- 1/t - a1 - a2 + b1 + b2, t= 1/x]]
```

(4)

```
[> ##### EXAMPLE IN THE THESIS #####
```

[In chapter 5, section 5.4.4 which is called "Examples", those are the Maple implementations for the example that we have used in the 2F2 type solutions:

```
> F:=sumdiffeq(hyperterm([a1,a2],[b1,b2],x,i),i,J(x));
    F := (d^3 J(x) / dx^3) x^2 + (b1 + b2 - x + 1) (d^2 J(x) / dx^2) x - (x a1 + x a2 - b2 b1 + x) (d J(x) / dx) - a1 a2 J(x) = 0
```

(5)

```
> LA:=de2diffop(F,J(x));
    LA := x^2 Dx^3 + (b1 x + b2 x - x^2 + x) Dx^2 + (-a1 x - a2 x + b2 b1 - x) Dx - a1 a2
```

(6)

```
> L22:=subs({a1=1/7,a2=1/3,b1=1,b2=1/12},LA);
    L22 := x^2 Dx^3 + (25/12 x - x^2) Dx^2 + (-31 x / 21 + 1/12) Dx - 1/21
```

(7)

```
> f:= normal((2*(x-7)^4*(x-12))/(x-3)^2,expanded);
```

(8)

$$f := \frac{2x^5 - 80x^4 + 1260x^3 - 9800x^2 + 37730x - 57624}{x^2 - 6x + 9} \quad (8)$$

> L:=ChangeOfVariables(L22,f);

$$L := 84 (x-12)^2 (x-7)^3 (3x^2 - 32x - 3)^3 (x-3)^5 Dx^3 - 7 (x-12) (216x^9 - 13248x^8 + 344544x^7 - 4922601x^6 + 41680902x^5 - 208564471x^4 + 565736148x^3 - 609105879x^2 - 131038290x - 3472713) (x-3)^2 (x-7)^2 (3x^2 - 32x - 3)^2 Dx^2 - (x-3) (x-7) (3x^2 - 32x - 3) (11016x^{13} - 910656x^{12} + 33121152x^{11} - 692939925x^{10} + 9119049254x^9 - 76891672763x^8 + 394635286912x^7 - 944552894530x^6 - 1376352434380x^5 + 16184128819722x^4 - 41634936222768x^3 + 34489504598535x^2 + 7779400339662x + 210612181185) Dx - 8 (x-7)^4 (3x^2 - 32x - 3)^6 \quad (9)$$

> ext:=indets(L,{RootOf,name}) minus {x,Dx};

$$ext := \emptyset \quad (10)$$

> ext:= indets(map(s-> ReplirrRoot(s,{ }),ext),{RootOf,name});

$$ext := \emptyset \quad (11)$$

> extppp:={};

$$extppp := \emptyset \quad (12)$$

> E:= Singular(L,extppp);

$$E := \left[[x-7, 7], [x-3, 3], \left[x^2 - \frac{32}{3}x - 1, \text{RootOf}(3_Z^2 - 32_Z - 3) \right], [\infty, \infty], [x - 12, 12] \right] \quad (13)$$

> F:= NotAppSing(L,E,ext);

$$F := [[\infty, \infty], [x-12, 12], [x-7, 7], [x-3, 3]] \quad (14)$$

> Sirr:= irrsing2F2(L,t,F,ext);

$$Sirr := \left[[[\infty, \infty], [x-3, 3]], \left[\left[1, \frac{3}{7}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{51}{28} \right], \left[\frac{2}{3}, \frac{2}{7}, \frac{9216}{t^2} - \frac{5120}{t} + \frac{17}{14} \right] \right], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{23}{28}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{39}{28}, -\frac{4}{7} \right], \left[\frac{9216}{t^2} - \frac{5120}{t} + \frac{23}{42}, \frac{9216}{t^2} - \frac{5120}{t} + \frac{13}{14}, -\frac{8}{21} \right] \right], [3, 2], [1, 1], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{51}{28}, 1 \right], \left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{51}{28}, \frac{3}{7} \right], \left[\frac{3}{7}, 1 \right] \right], \left[\left[\frac{9216}{t^2} - \frac{5120}{t} + \frac{17}{14}, \frac{2}{3} \right], \left[\frac{9216}{t^2} - \frac{5120}{t} + \frac{17}{14}, \frac{2}{7} \right], \left[\frac{2}{7}, \frac{2}{3} \right] \right], [[-6t^3 + 136t^2 - 834t, -6t^3 + 136t^2 - 834t, 0], [9216t^2 - 5120t, 9216t^2 - 5120t, 0]], \left[\left[\frac{23}{28}, \frac{39}{28}, -\frac{4}{7} \right], \left[\frac{23}{42}, \frac{13}{14}, -\frac{8}{21} \right] \right], [[x-12, 12], [x-7, 7]], \left[\left[\left[0, 0, \frac{11}{12} \right], \left[0, \frac{11}{12}, \frac{11}{12} \right], [1, 1, 1], [0, 0], \left[\frac{11}{12}, 0 \right], \left[\frac{11}{12}, 0 \right] \right], 3 \right], \left[\left[0, 0, \frac{11}{3} \right], \left[0, \frac{11}{3}, \frac{11}{3} \right], [1, 1, 1], [0, 0], \left[\frac{11}{3}, 0 \right], \left[\frac{11}{3}, 0 \right] \right] \right] \quad (15)$$

$$0]], 3]]]]]$$

> Sreg:= regsingtrue2F2(L,t,Sirr[-1],ext);

$$Sreg := \left[\left[[x-12, 12], [x-7, 7] \right], \left[\left[0, 0, \frac{11}{12} \right], \left[0, 0, \frac{11}{3} \right] \right], \left[\left[0, \frac{11}{12}, \frac{11}{12} \right], \left[0, \frac{11}{3}, \frac{11}{3} \right] \right], \left[\left[0, 0 \right], \left[\frac{11}{12}, 0 \right], \left[\frac{11}{12}, 0 \right] \right], \left[\left[0, 0 \right], \left[\frac{11}{3}, 0 \right], \left[\frac{11}{3}, 0 \right] \right] \right] \quad (16)$$

> RSreg:= Sregseptrue2F2(L,Sreg,ext);

$$RSreg := \left[[], [], \left[[x-12, 12], [x-7, 7] \right], \left[\left[0, 0, \frac{11}{12} \right], \left[0, 0, \frac{11}{3} \right] \right], \left[\left[\left[\frac{11}{12}, \frac{11}{12} \right], [0] \right], \left[\left[\frac{11}{3}, \frac{11}{3} \right], [0] \right] \right] \right] \quad (17)$$

> R1:=IrrRegAppsing2F2(L,t,E,ext);

$$R1 := \left[\left[[\infty, \infty], [x-3, 3] \right], \left[\left[1, \frac{3}{7}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{51}{28} \right], \left[\frac{2}{3}, \frac{2}{7}, \frac{9216}{t^2} - \frac{5120}{t} + \frac{17}{14} \right] \right], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{23}{28}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{39}{28}, -\frac{4}{7} \right], \left[\frac{9216}{t^2} - \frac{5120}{t} + \frac{23}{42}, \frac{9216}{t^2} - \frac{5120}{t} + \frac{13}{14}, -\frac{8}{21} \right] \right], [3, 2], [1, 1], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{51}{28}, 1 \right], \left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{51}{28}, \frac{3}{7} \right], \left[\frac{3}{7}, 1 \right] \right], \left[\left[\frac{9216}{t^2} - \frac{5120}{t} + \frac{17}{14}, \frac{2}{3} \right], \left[\frac{9216}{t^2} - \frac{5120}{t} + \frac{17}{14}, \frac{2}{7} \right], \left[\frac{2}{7}, \frac{2}{3} \right] \right], [[-6t^3 + 136t^2 - 834t, -6t^3 + 136t^2 - 834t, 0], [9216t^2 - 5120t, 9216t^2 - 5120t, 0]], \left[\left[\frac{23}{28}, \frac{39}{28}, -\frac{4}{7} \right], \left[\frac{23}{42}, \frac{13}{14}, -\frac{8}{21} \right] \right], \left[[x-12, 12], [x-7, 7] \right], \left[\left[0, 0, \frac{11}{12} \right], \left[0, 0, \frac{11}{3} \right] \right], \left[\left[0, \frac{11}{12}, \frac{11}{12} \right], \left[0, \frac{11}{3}, \frac{11}{3} \right] \right], \left[\left[[0, 0], \left[\frac{11}{12}, 0 \right], \left[\frac{11}{12}, 0 \right] \right], \left[[0, 0], \left[\frac{11}{3}, 0 \right], \left[\frac{11}{3}, 0 \right] \right] \right], \left[[], [], \left[[x-12, 12], [x-7, 7] \right], \left[\left[0, 0, \frac{11}{12} \right], \left[0, 0, \frac{11}{3} \right] \right], \left[\left[\left[\frac{11}{12}, \frac{11}{12} \right], [0] \right], \left[\left[\frac{11}{3}, \frac{11}{3} \right], [0] \right] \right] \right], \left[\left[\left[x^2 - \frac{32}{3}x - 1, RootOf(3_Z^2 - 32_Z - 3) \right], [0, 2, 4], [2, 4, 2], [[2, 0], [4, 0], [4, 2]] \right], \left[[\infty, \infty], [x-12, 12], [x-7, 7], [x-3, 3] \right], \left[\left[1, \frac{3}{7}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{51}{28} \right], \left[0, 0, \frac{11}{12} \right], \left[0, 0, \frac{11}{3} \right], \left[\frac{2}{3}, \frac{2}{7}, \frac{9216}{t^2} - \frac{5120}{t} + \frac{17}{14} \right], \left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{23}{28}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{39}{28}, -\frac{4}{7} \right], \left[0, \frac{11}{12}, \frac{11}{12} \right], \left[0, \frac{11}{3}, \frac{11}{3} \right], \left[\frac{9216}{t^2} - \frac{5120}{t} + \frac{23}{42}, \frac{9216}{t^2} - \frac{5120}{t} + \frac{13}{14}, -\frac{8}{21} \right], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} \right] \right] \right] \quad (18)$$

$$+ \frac{51}{28}, 1], \left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{51}{28}, \frac{3}{7} \right], \left[\frac{3}{7}, 1 \right], \left[[0, 0], \left[\frac{11}{12}, 0 \right], \left[\frac{11}{12}, 0 \right] \right], \\ \left[[0, 0], \left[\frac{11}{3}, 0 \right], \left[\frac{11}{3}, 0 \right] \right], \left[\left[\frac{9216}{t^2} - \frac{5120}{t} + \frac{17}{14}, \frac{2}{3} \right], \left[\frac{9216}{t^2} - \frac{5120}{t} + \frac{17}{14}, \right. \right. \\ \left. \left. \frac{2}{7} \right], \left[\frac{2}{7}, \frac{2}{3} \right] \right], \left[[1, 1, 1], [1, 1, 1], [1, 1, 1], [1, 1, 1] \right] \Bigg] \Bigg]$$

> F1:= Hyp2F2Subst(L,x,t,R1[1],ext);

$$F1 := \left[-\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 1193x + 9984)}{(x-3)^2}, \right. \\ \left. -\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 6313x - 9984)}{(x-3)^2}, \right. \\ \left. \frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 6313x - 9984)}{(x-3)^2}, \right. \\ \left. \frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 1193x + 9984)}{(x-3)^2} \right] \quad (19)$$

> B:= Candichangvar2F2(F1,R1,ext);

$$B := \left\{ \left[-\frac{2(x-7)^4(x-12)}{(x-3)^2}, [1, 4], \left[[x-12, 12], [x-7, 7], \left[[0, 0, \frac{11}{12}], [0, 0, \right. \right. \right. \right. \\ \left. \left. \left. \frac{11}{3} \right], \left[[0, \frac{11}{12}, \frac{11}{12}], [0, \frac{11}{3}, \frac{11}{3}], \left[[0, 0], \left[\frac{11}{12}, 0 \right], \left[\frac{11}{12}, 0 \right], \left[[0, 0], \left[\frac{11}{3}, 0 \right], \right. \right. \right. \right. \\ \left. \left. \left. \left[\frac{11}{3}, 0 \right] \right] \right] \right], \left[\frac{2(x-7)^4(x-12)}{(x-3)^2}, [1, 4], \left[[x-12, 12], [x-7, 7], \left[[0, 0, \frac{11}{12}], \right. \right. \right. \right. \\ \left. \left. \left. [0, 0, \frac{11}{3}], \left[[0, \frac{11}{12}, \frac{11}{12}], [0, \frac{11}{3}, \frac{11}{3}], \left[[0, 0], \left[\frac{11}{12}, 0 \right], \left[\frac{11}{12}, 0 \right], \left[[0, 0], \right. \right. \right. \right. \\ \left. \left. \left. \left[\frac{11}{3}, 0 \right], \left[\frac{11}{3}, 0 \right] \right] \right] \right] \right\} \quad (20)$$

> find2F2ln(L,R1,F1,ext,x,t);

$$\left[\left[\left[\left[\left[-1, \frac{17}{21} \right], \left[1, \frac{17}{21} \right], \left[\frac{1}{3}, \frac{1}{7} \right], \left[\frac{2}{3}, \frac{10}{21} \right] \right], \left[1, \frac{1}{12} \right], \frac{2(x-7)^4(x-12)}{(x-3)^2} \right], \right. \right. \\ \left. \left[\left[\left[-1, \frac{17}{21} \right], \left[1, \frac{17}{21} \right], \left[\frac{1}{3}, \frac{1}{7} \right], \left[\frac{2}{3}, \frac{10}{21} \right] \right], \left[1, \frac{1}{12} \right], -\frac{2(x-7)^4(x-12)}{(x-3)^2} \right] \right] \quad (21)$$

> TIME :=time();
Hyp2F2Solutions(L);
time() - TIME;

TIME := 14.015

$$\left[\left[\left[\left[\left[\frac{1}{3}, \frac{1}{7} \right], \left[1, \frac{1}{12} \right], [0], [1] \right], \frac{2(x-7)^4(x-12)}{(x-3)^2} \right] \right] \right]$$

4.188

(22)

[Here are another examples related to the 2F2 type solutions. Those examples are not in my PhD thesis.

```
[> ##### THE INTEGER CASE #####

> F:=sumdiffEq(hyperterm([a1,a2],[b1,b2],x,k),k,J(x));
F := (d^3/dx^3 J(x)) x^2 + (b1 + b2 - x + 1) (d^2/dx^2 J(x)) x - (a1 x + a2 x - b2 b1 + x) (d/dx J(x)) - a1 a2 J(x) = 0 (23)

> LA:=de2diffEq(F,J(x));
LA := x^2 Dx^3 + (b1 x + b2 x - x^2 + x) Dx^2 + (-a1 x - a2 x + b2 b1 - x) Dx - a1 a2 (24)

> L22:=subs({a1=-1,a2=1/3,b1=1/4,b2=1/2},LA);
L22 := x^2 Dx^3 + (7/4 x - x^2) Dx^2 + (-x/3 + 1/8) Dx + 1/3 (25)

> f:= normal((2*(x-12)^4)/x,expanded);
f := (2 x^4 - 96 x^3 + 1728 x^2 - 13824 x + 41472) / x (26)

> L:=ChangeOfVariables(L22,f);
L := 8 x^4 (x+4)^3 (x-12)^3 Dx^3 - 6 x^2 (8 x^6 - 320 x^5 + 3968 x^4 - 6143 x^3 - 165880 x^2 + 442640 x + 2654208) (x+4)^2 (x-12)^2 Dx^2 + x (x+4) (x-12) (48 x^8 - 1536 x^7 + 12288 x^6 + 5 x^5 + 614480 x^4 - 8255136 x^3 - 42471168 x^2 + 339787008 x + 1274019840) Dx + 144 (x-12)^4 (x+4)^6 (27)

> ext:=indets(L,{RootOf,name}) minus {x,Dx};
ext := ∅ (28)

> ext:= indets(map(s-> ReplirrRoot(s,{ } ),ext),{RootOf,name});
ext := ∅ (29)

> extppp:={ };
extppp := ∅ (30)

> E:= Singular(L,extppp);
E := [[x, 0], [x + 4, -4], [∞, ∞], [x - 12, 12]] (31)

> F:= NotAppSing(L,E,ext);
F := [[x, 0], [∞, ∞]] (32)

> Sirr:= irrsing2F2(L,t,F,ext);
Sirr := [[ [x, 0], [∞, ∞] ], [ [-1, 1/3, -41472/t + 17/12], [-3, 1, -6/t^3 + 192/t^2 - 1728/t + 17/4] ], [ [-41472/t + 29/12, -41472/t + 13/12, 4/3], [-6/t^3 + 192/t^2 - 1728/t + 29/4, -6/t^3 + 192/t^2 - 1728/t + 13/4] ], [ [1, 3], [1, 1], [ [-41472/t + 17/12, -1], [-41472/t + 17/12, 1/3] ], [ [1/3, -1], [-6/t^3 + 192/t^2 - 1728/t + 17/4, -3], [-6/t^3 + 192/t^2 - 1728/t + 17/4, -3] ] ] (33)
```

$$+ \frac{17}{4}, 1], [1, -3]]], [[-41472 t, -41472 t, 0], [-6 t^3 + 192 t^2 - 1728 t, -6 t^3 + 192 t^2 - 1728 t, 0]], \left[\left[\frac{29}{12}, \frac{13}{12}, \frac{4}{3} \right], \left[\frac{29}{4}, \frac{13}{4}, 4 \right] \right], [[], []]$$

$$\begin{aligned} &> \text{Sreg} := \text{regsingtrue2F2}(\text{L}, \text{t}, \text{Sirr}[-1], \text{ext}); \\ &\quad \text{Sreg} := [] \end{aligned} \quad (34)$$

$$\begin{aligned} &> \text{RSreg} := \text{Sregseptrue2F2}(\text{L}, \text{Sreg}, \text{ext}); \\ &\quad \text{RSreg} := [[], [], []] \end{aligned} \quad (35)$$

$$> \text{R1} := \text{IrrRegAppsing2F2}(\text{L}, \text{t}, \text{E}, \text{ext}); \quad (36)$$

$$\begin{aligned} \text{R1} := & \left[\left[[x, 0], [\infty, \infty] \right], \left[\left[-1, \frac{1}{3}, -\frac{41472}{t} + \frac{17}{12} \right], \left[-3, 1, -\frac{6}{t^3} + \frac{192}{t^2} - \frac{1728}{t} + \frac{17}{4} \right] \right], \right. \\ & \left[\left[-\frac{41472}{t} + \frac{29}{12}, -\frac{41472}{t} + \frac{13}{12}, \frac{4}{3} \right], \left[-\frac{6}{t^3} + \frac{192}{t^2} - \frac{1728}{t} + \frac{29}{4}, -\frac{6}{t^3} + \frac{192}{t^2} - \frac{1728}{t} + \frac{13}{4} \right] \right], \\ & [1, 3], [1, 1], \left[\left[\left[-\frac{41472}{t} + \frac{17}{12}, -1 \right], \left[-\frac{41472}{t} + \frac{17}{12}, \frac{1}{3} \right] \right], \left[\frac{1}{3}, -1 \right] \right], \\ & \left[\left[-\frac{6}{t^3} + \frac{192}{t^2} - \frac{1728}{t} + \frac{17}{4}, -3 \right], \left[-\frac{6}{t^3} + \frac{192}{t^2} - \frac{1728}{t} + \frac{17}{4}, 1 \right] \right], \\ & [1, -3]]], [[-41472 t, -41472 t, 0], [-6 t^3 + 192 t^2 - 1728 t, -6 t^3 + 192 t^2 - 1728 t, 0]], \\ & \left[\left[\frac{29}{12}, \frac{13}{12}, \frac{4}{3} \right], \left[\frac{29}{4}, \frac{13}{4}, 4 \right] \right], [], [[], [], []], [[x - 12, 12], [x + 4, -4]], \\ & [[0, 2, 3], [0, 2, 4]], [[2, 3, 1], [2, 4, 2]], [[2, 0], [3, 0], [3, 2]], [[2, 0], [4, 0], [4, 2]]], \\ & \left[[x, 0], [\infty, \infty] \right], \left[\left[-1, \frac{1}{3}, -\frac{41472}{t} + \frac{17}{12} \right], \left[-3, 1, -\frac{6}{t^3} + \frac{192}{t^2} - \frac{1728}{t} + \frac{17}{4} \right] \right], \\ & \left[\left[-\frac{41472}{t} + \frac{29}{12}, -\frac{41472}{t} + \frac{13}{12}, \frac{4}{3} \right], \left[-\frac{6}{t^3} + \frac{192}{t^2} - \frac{1728}{t} + \frac{29}{4}, -\frac{6}{t^3} + \frac{192}{t^2} - \frac{1728}{t} + \frac{13}{4} \right] \right], \\ & \left[\left[\left[-\frac{41472}{t} + \frac{17}{12}, -1 \right], \left[-\frac{41472}{t} + \frac{17}{12}, \frac{1}{3} \right] \right], \left[\frac{1}{3}, -1 \right] \right], \\ & \left[\left[-\frac{6}{t^3} + \frac{192}{t^2} - \frac{1728}{t} + \frac{17}{4}, -3 \right], \left[-\frac{6}{t^3} + \frac{192}{t^2} - \frac{1728}{t} + \frac{17}{4}, 1 \right] \right], \\ & [1, -3]]], [[1, 1, 1], [1, 1, 1]] \end{aligned}$$

$$\begin{aligned} &> \text{F1} := \text{Hyp2F2Subst}(\text{L}, \text{x}, \text{t}, \text{R1}[1], \text{ext}); \\ \text{F1} := & \left[-\frac{2(x^4 - 48x^3 + 864x^2 + 20736)}{x}, \frac{2(x^4 - 48x^3 + 864x^2 - 20736)}{x}, \right. \\ & \left. -\frac{2(x^4 - 48x^3 + 864x^2 - 20736)}{x}, \frac{2(x^4 - 48x^3 + 864x^2 + 20736)}{x} \right] \end{aligned} \quad (37)$$

$$\begin{aligned} &> \text{find2F2Int}(\text{L}, \text{R1}, \text{F1}, \text{x}, \text{t}, \text{ext}); \\ & \left[\left[\left[\left[\frac{1}{2}, \frac{1}{6} \right], \left[\frac{1}{2}, \frac{5}{6} \right], \left[\frac{1}{6}, \frac{5}{6} \right] \right], \left[\frac{1}{2}, \frac{3}{4} \right] \right], \left[\left[\left[-1, \frac{1}{3} \right], \left[-1, \frac{2}{3} \right], \left[1, \frac{1}{3} \right], \left[1, \frac{2}{3} \right] \right], \right. \end{aligned} \quad (38)$$

$$\left[\left[\frac{1}{3}, \frac{2}{3} \right] \right], \left[\frac{1}{2}, \frac{1}{4} \right], \left[\left[\left[\frac{1}{12}, \frac{3}{4} \right], \left[\frac{1}{12}, \frac{5}{12} \right], \left[\frac{3}{4}, \frac{5}{12} \right] \right], \left[\frac{1}{4}, \frac{3}{4} \right] \right], -\frac{2(x-12)^4}{x} \right]$$

$$, \left[\left[\left[\left[\frac{1}{2}, \frac{1}{6} \right], \left[\frac{1}{2}, \frac{5}{6} \right], \left[\frac{1}{6}, \frac{5}{6} \right] \right], \left[\frac{1}{2}, \frac{3}{4} \right] \right], \left[\left[-1, \frac{1}{3} \right], \left[-1, \frac{2}{3} \right], \left[1, \frac{1}{3} \right], \left[1, \frac{2}{3} \right], \left[\frac{1}{3}, \frac{2}{3} \right] \right], \left[\frac{1}{2}, \frac{1}{4} \right], \left[\left[\left[\frac{1}{12}, \frac{3}{4} \right], \left[\frac{1}{12}, \frac{5}{12} \right], \left[\frac{3}{4}, \frac{5}{12} \right] \right], \left[\frac{1}{4}, \frac{3}{4} \right] \right], \frac{2(x-12)^4}{x} \right] \right]$$

```
> TIME :=time();
Hyp2F2Solutions(L);
time() - TIME;
```

TIME := 32.484

$$\left\{ \left[\left[\left[\left[\left[\frac{1}{2}, \frac{5}{6} \right], \left[\frac{1}{2}, \frac{3}{4} \right], \left[-\frac{3}{2x} \right], \left[\frac{x^3 (16x^4 - 768x^3 + 13824x^2 - 110595x + 331776) Dx^2}{16(x+4)^2} - \frac{1}{64(x+4)^3} ((384x^9 - 29184x^8 + 909312x^7 - 14450968x^6 + 111487648x^5 - 148774656x^4 - 3566840793x^3 + 18349590732x^2 + 18325315584x - 220150628352) x Dx) - \frac{1}{128(x+4)^3} (3(1024x^9 - 67584x^8 + 1769472x^7 - 22151408x^6 + 108532928x^5 + 311407104x^4 - 4983048183x^3 + 8155680852x^2 + 48913956864x - 97844723712)) \right] \right], \frac{2(x-12)^4}{x} \right] \right\}$$

4.578

(39)

```
> ##### THE LOGARITHMIC CASE #####
```

```
> F:=sumdiffeq(hyperterm([a1,a2],[b1,b2],x,k),k,J(x));
```

$$F := \left(\frac{d^3}{dx^3} J(x) \right) x^2 + (b1 + b2 - x + 1) \left(\frac{d^2}{dx^2} J(x) \right) x - (a1 x + a2 x - b2 b1 + x) \left(\frac{d}{dx} J(x) \right) - a1 a2 J(x) = 0 \quad (40)$$

```
> LA:=de2diffop(F,J(x));
```

$$LA := x^2 Dx^3 + (b1 x + b2 x - x^2 + x) Dx^2 + (-a1 x - a2 x + b2 b1 - x) Dx - a1 a2 \quad (41)$$

```
> L22:=subs({a1=1/7,a2=1/3,b1=1,b2=1},LA);
```

(42)

$$L22 := x^2 Dx^3 + (-x^2 + 3x) Dx^2 + \left(-\frac{31x}{21} + 1\right) Dx - \frac{1}{21} \quad (42)$$

> f:= normal((2*(x-7)^4*(x-12))/(x-3)^2,expanded);

$$f := \frac{2x^5 - 80x^4 + 1260x^3 - 9800x^2 + 37730x - 57624}{x^2 - 6x + 9} \quad (43)$$

> L:=ChangeOfVariables(L22,f);

$$L := 21(x-12)^2(x-7)^3(3x^2-32x-3)^3(x-3)^5Dx^3 - 21(x-12)(18x^9-1104x^8 + 28712x^7 - 410225x^6 + 3473634x^5 - 17382425x^4 + 47151620x^3 - 50766075x^2 - 10921392x - 289467)(x-3)^2(x-7)^2(3x^2-32x-3)^2Dx^2 - (x-3)(x-7)(3x^2-32x-3)(2754x^{13} - 227664x^{12} + 8280288x^{11} - 173235501x^{10} + 2279787608x^9 - 19223397689x^8 + 98663020692x^7 - 236150477066x^6 - 344149859746x^5 + 4046598267774x^4 - 10410230153988x^3 + 8623615206087x^2 + 1945121010840x + 52666278651)Dx - 2(x-7)^4(3x^2-32x-3)^6 \quad (44)$$

> ext:=indets(L,{RootOf,name}) minus {x,Dx};

$$ext := \emptyset \quad (45)$$

> ext:= indets(map(s-> ReplirrRoot(s,{ }),ext),{RootOf,name});

$$ext := \emptyset \quad (46)$$

> extppp:={};

$$extppp := \emptyset \quad (47)$$

> E:= Singular(L,extppp);

$$E := \left[[x-7, 7], [x-3, 3], \left[x^2 - \frac{32}{3}x - 1, \text{RootOf}(3_Z^2 - 32_Z - 3) \right], [\infty, \infty], [x - 12, 12] \right] \quad (48)$$

> F:= NotAppSing(L,E,ext);

$$F := [[\infty, \infty], [x-12, 12], [x-7, 7], [x-3, 3]] \quad (49)$$

> Sirr:= irrsing2F2(L,t,F,ext);

$$Sirr := \left[[[\infty, \infty], [x-3, 3]], \left[\left[1, \frac{3}{7}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{32}{7} \right], \left[\frac{2}{3}, \frac{2}{7}, \frac{9216}{t^2} - \frac{5120}{t} + \frac{64}{21} \right] \right], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{25}{7}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{29}{7}, -\frac{4}{7} \right], \left[\frac{9216}{t^2} - \frac{5120}{t} + \frac{50}{21}, \frac{9216}{t^2} - \frac{5120}{t} + \frac{58}{21}, -\frac{8}{21} \right] \right], [3, 2], [1, 1], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{32}{7}, 1 \right], \left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{32}{7}, \frac{3}{7} \right], \left[\frac{3}{7}, 1 \right] \right], \left[\left[\frac{9216}{t^2} - \frac{5120}{t} + \frac{64}{21}, \frac{2}{3} \right], \left[\frac{9216}{t^2} - \frac{5120}{t} + \frac{64}{21}, \frac{2}{7} \right], \left[\frac{2}{7}, \frac{2}{3} \right] \right], [[-6t^3 + 136t^2 - 834t, -6t^3 + 136t^2 - 834t, 0], [9216t^2 - 5120t, 9216t^2 - 5120t, 0], \left[\left[\frac{25}{7}, \frac{29}{7}, -\frac{4}{7} \right], \left[\frac{50}{21}, \frac{58}{21}, -\frac{8}{21} \right] \right], [[x-12, 12], [x-7, 7]], [[0, 0, 0], [0, 0, 0], [1, 1, 1], [[0, 0], [0, 0],$$

$$[0, 0], 4], [[0, 0, 0], [0, 0, 0], [1, 1, 1], [[0, 0], [0, 0], [0, 0]], 4]]]$$

$$\begin{aligned} & \textbf{> Sreg:= regsingtrue2F2(L,t,Sirr[-1],ext);} \\ \text{Sreg} &:= [[[x-12, 12], [x-7, 7]], [[0, 0, 0], [0, 0, 0]], [[0, 0, 0], [0, 0, 0]], [[[0, 0], [0, 0], [0, 0]], [[0, 0], [0, 0], [0, 0]]]] \end{aligned} \quad (51)$$

$$\begin{aligned} & \textbf{> RSreg:= Sregseptrue2F2(L,Sreg,ext);} \\ \text{RSreg} &:= [[], [], [[[x-12, 12], [x-7, 7]], [[0, 0, 0], [0, 0, 0]], [[[], [0, 0, 0]], [[], [0, 0, 0]]]]] \end{aligned} \quad (52)$$

$$\begin{aligned} & \textbf{> R1:=IrrRegAppsing2F2(L,t,E,ext);} \\ \text{R1} &:= \left[\left[[[\infty, \infty], [x-3, 3]], \left[\left[1, \frac{3}{7}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{32}{7} \right], \left[\frac{2}{3}, \frac{2}{7}, \frac{9216}{t^2} - \frac{5120}{t} + \frac{64}{21} \right] \right], \right. \\ & \quad \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{25}{7}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{29}{7}, -\frac{4}{7} \right], \right. \\ & \quad \left[\frac{9216}{t^2} - \frac{5120}{t} + \frac{50}{21}, \frac{9216}{t^2} - \frac{5120}{t} + \frac{58}{21}, -\frac{8}{21} \right], [3, 2], [1, 1], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{32}{7}, 1 \right], \right. \\ & \quad \left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{32}{7}, \frac{3}{7} \right], \left[\frac{3}{7}, 1 \right], \left[\frac{9216}{t^2} - \frac{5120}{t} + \frac{64}{21}, \frac{2}{7} \right], \left[\frac{9216}{t^2} - \frac{5120}{t} + \frac{64}{21}, \frac{2}{7} \right], \left[\frac{2}{7}, \frac{2}{3} \right] \Big], \\ & \quad [[-6t^3 + 136t^2 - 834t, -6t^3 + 136t^2 - 834t, 0], [9216t^2 - 5120t, 9216t^2 - 5120t, 0], \left[\left[\frac{25}{7}, \frac{29}{7}, -\frac{4}{7} \right], \left[\frac{50}{21}, \frac{58}{21}, -\frac{8}{21} \right] \right], \\ & \quad [[[x-12, 12], [x-7, 7]], [[0, 0, 0], [0, 0, 0]], [[0, 0, 0], [0, 0, 0]], [[[0, 0], [0, 0], [0, 0]], [[0, 0], [0, 0], [0, 0]]]], \\ & \quad [[[], [], [[[x-12, 12], [x-7, 7]], [[0, 0, 0], [0, 0, 0]], [[[], [0, 0, 0]], [[], [0, 0, 0]]]]], \left[\left[\left[x^2 - \frac{32}{3}x - 1, \text{RootOf}(3_Z^2 - 32_Z - 3) \right], \right. \right. \\ & \quad \left. \left[[0, 2, 4], [2, 4, 2], [[2, 0], [4, 0], [4, 2]] \right], \left[[[\infty, \infty], [x-12, 12], [x-7, 7], [x-3, 3]], \right. \right. \\ & \quad \left[\left[1, \frac{3}{7}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{32}{7} \right], [0, 0, 0], [0, 0, 0], \left[\frac{2}{3}, \frac{2}{7}, \frac{9216}{t^2} - \frac{5120}{t} + \frac{64}{21} \right], \right. \\ & \quad \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{25}{7}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{29}{7}, -\frac{4}{7} \right], [0, 0, 0], [0, 0, 0], \left[\frac{9216}{t^2} - \frac{5120}{t} + \frac{50}{21}, \frac{9216}{t^2} - \frac{5120}{t} + \frac{58}{21}, \right. \\ & \quad \left. -\frac{8}{21} \right], \left[\left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{32}{7}, 1 \right], \left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} + \frac{32}{7}, \frac{3}{7} \right], \left[\frac{3}{7}, 1 \right], \right. \\ & \quad \left. \left[\frac{9216}{t^2} - \frac{5120}{t} + \frac{64}{21}, \frac{2}{7} \right], \left[\frac{9216}{t^2} - \frac{5120}{t} + \frac{64}{21}, \frac{2}{7} \right], \left[\frac{2}{7}, \frac{2}{3} \right] \right], \\ & \quad [[1, 1, 1], [1, 1, 1], [1, 1, 1], [1, 1, 1]] \Big] \end{aligned} \quad (53)$$

```
> F1:= Hyp2F2Subst(L,x,t,R1[1],ext);
```

$$F1 := \left[-\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 1193x + 9984)}{(x-3)^2}, \right. \\ \left. -\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 6313x - 9984)}{(x-3)^2}, \right. \\ \left. \frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 6313x - 9984)}{(x-3)^2}, \right. \\ \left. \frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 1193x + 9984)}{(x-3)^2} \right] \quad (54)$$

```
> B:= Candichangvar2F2(F1,R1,ext);
```

$$B := \left\{ \left[-\frac{2(x-7)^4(x-12)}{(x-3)^2}, [1, 4], [[x-12, 12], [x-7, 7]], [[0, 0, 0], [0, 0, 0]], [[0, 0, 0], [0, 0, 0]], [[0, 0, 0], [0, 0, 0]], [[0, 0, 0], [0, 0, 0]] \right], \right. \\ \left. \left[\frac{2(x-7)^4(x-12)}{(x-3)^2}, [1, 4], [[x-12, 12], [x-7, 7]], [[0, 0, 0], [0, 0, 0]], [[0, 0, 0], [0, 0, 0]], [[0, 0, 0], [0, 0, 0]] \right] \right\} \quad (55)$$

```
> find2F2ln(L,R1,F1,ext,x,t);
```

$$\left[\left[\left[\left[-1, \frac{17}{21} \right], \left[1, \frac{17}{21} \right], \left[\frac{1}{3}, \frac{1}{7} \right], \left[\frac{2}{3}, \frac{10}{21} \right] \right], [1, 1], \frac{2(x-7)^4(x-12)}{(x-3)^2} \right], \left[\left[\left[-1, \frac{17}{21} \right], \left[1, \frac{17}{21} \right], \left[\frac{1}{3}, \frac{1}{7} \right], \left[\frac{2}{3}, \frac{10}{21} \right] \right], [1, 1], -\frac{2(x-7)^4(x-12)}{(x-3)^2} \right] \right] \quad (56)$$

```
> TIME :=time();
Hyp2F2Solutions(L);
time() - TIME;
```

$$TIME := 60.281$$

$$\left\{ \left[\left[\left[\left[\frac{1}{3}, \frac{1}{7} \right], [1, 1], [0, 1] \right], \frac{2(x-7)^4(x-12)}{(x-3)^2} \right] \right\} \right. \\ \left. 4.344 \right\} \quad (57)$$

```
> F:=sumdiffeq(hyperterm([a1,a2],[b1,b2],x,k),k,J(x));
```

$$F := \left(\frac{d^3}{dx^3} J(x) \right) x^2 + (b1 + b2 - x + 1) \left(\frac{d^2}{dx^2} J(x) \right) x - (a1 x + a2 x - b2 b1 + x) \left(\frac{d}{dx} J(x) \right) - a1 a2 J(x) = 0 \quad (58)$$

```
> LA:=de2diffop(F,J(x));
```

$$LA := x^2 Dx^3 + (b1 x + b2 x - x^2 + x) Dx^2 + (-a1 x - a2 x + b2 b1 - x) Dx - a1 a2 \quad (59)$$

$$\begin{aligned}
& > \text{L22} := \text{subs}(\{a1 = \text{RootOf}(x^2 + 7), a2 = 1/3, b1 = 1, b2 = 1/5\}, \text{LA}); \\
& \text{L22} := x^2 Dx^3 + \left(\frac{11}{5} x - x^2 \right) Dx^2 + \left(-\text{RootOf}(_Z^2 + 7) x - \frac{4x}{3} + \frac{1}{5} \right) Dx \\
& \quad - \frac{\text{RootOf}(_Z^2 + 7)}{3}
\end{aligned} \tag{60}$$

$$\begin{aligned}
& > f := \text{normal}((2*(x-7)^4*(x-12))/(x-3)^2, \text{expanded}); \\
& f := \frac{2x^5 - 80x^4 + 1260x^3 - 9800x^2 + 37730x - 57624}{x^2 - 6x + 9}
\end{aligned} \tag{61}$$

$$\begin{aligned}
& > \text{L} := \text{ChangeOfVariables}(\text{L22}, f); \\
& \text{L} := 15 (x-12)^2 (x-7)^3 (3x^2 - 32x - 3)^3 (x-3)^5 Dx^3 - 3 (x-12) (90x^9 - 5520x^8 \\
& \quad + 143560x^7 - 2051089x^6 + 17367186x^5 - 86903169x^4 + 235727812x^3 - 253798731x^2 \\
& \quad - 54600264x - 1447011) (x-7)^2 (x-3)^2 (3x^2 - 32x - 3)^2 Dx^2 - \frac{1}{67} ((2 \\
& \quad + 3 \text{RootOf}(_Z^2 + 7)) (3x^2 - 32x - 3) (73770769602 + 2717477809074x \\
& \quad - 113001595263 \text{RootOf}(_Z^2 + 7) - 21090673707732x^3 + 11373922732554x^2 \\
& \quad - 6700596947868x^4 + 46351148654x^9 - 409791725654x^8 + 2370971552688x^7 \\
& \quad - 8518937491772x^6 + 16220756582018x^5 + 54270x^{13} - 4486320x^{12} + 163607940x^{11} \\
& \quad - 3451507182x^{10} - 4174753151646 \text{RootOf}(_Z^2 + 7)x + 51300 \text{RootOf}(_Z^2 + 7)x^{11} \\
& \quad - 4426767 \text{RootOf}(_Z^2 + 7)x^{10} + 167228094 \text{RootOf}(_Z^2 + 7)x^9 \\
& \quad - 3632836539 \text{RootOf}(_Z^2 + 7)x^8 + 50038841988 \text{RootOf}(_Z^2 + 7)x^7 \\
& \quad - 453207655182 \text{RootOf}(_Z^2 + 7)x^6 + 2696724214608 \text{RootOf}(_Z^2 + 7)x^5 \\
& \quad - 10133483252598 \text{RootOf}(_Z^2 + 7)x^4 + 21575505129768 \text{RootOf}(_Z^2 + 7)x^3 \\
& \quad - 18587532086451 \text{RootOf}(_Z^2 + 7)x^2) (x-3) (x-7) Dx) - 10 \text{RootOf}(_Z^2 \\
& \quad + 7) (x-7)^4 (3x^2 - 32x - 3)^6
\end{aligned} \tag{62}$$

$$\begin{aligned}
& > \text{ext} := \text{indets}(\text{L}, \{\text{RootOf}, \text{name}\}) \text{ minus } \{x, Dx\}; \\
& \quad \text{ext} := \{\text{RootOf}(_Z^2 + 7)\}
\end{aligned} \tag{63}$$

$$\begin{aligned}
& > \text{ext} := \text{indets}(\text{map}(s \rightarrow \text{ReplirrRoot}(s, \{\}), \text{ext}), \{\text{RootOf}, \text{name}\}); \\
& \quad \text{ext} := \{\text{RootOf}(_Z^2 + 7)\}
\end{aligned} \tag{64}$$

$$\begin{aligned}
& > \text{extppp} := \{\}; \\
& \quad \text{extppp} := \emptyset
\end{aligned} \tag{65}$$

$$\begin{aligned}
& > \text{E} := \text{Singular}(\text{L}, \text{extppp}); \\
& \text{E} := \left[[x-7, 7], [x-3, 3], \left[x^2 - \frac{32}{3}x - 1, \text{RootOf}(3_Z^2 - 32_Z - 3) \right], [\infty, \infty], [x \right. \\
& \quad \left. - 12, 12] \right]
\end{aligned} \tag{66}$$

$$\begin{aligned}
& > \text{F} := \text{NotAppSing}(\text{L}, \text{E}, \text{ext}); \\
& \quad \text{F} := [[\infty, \infty], [x-12, 12], [x-7, 7], [x-3, 3]]
\end{aligned} \tag{67}$$

$$\begin{aligned}
& > \text{Sirr} := \text{irrsing2F2}(\text{L}, t, \text{F}, \text{ext}); \\
& \text{Sirr} := \left[[[\infty, \infty], [x-3, 3]], \left[\left[1, 3 \text{RootOf}(_Z^2 + 7), -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} \right. \right. \right.
\end{aligned} \tag{68}$$

$$\begin{aligned}
& -3 \operatorname{RootOf}(_Z^2 + 7) + \frac{13}{5} \Big], \Big[\frac{2}{3}, 2 \operatorname{RootOf}(_Z^2 + 7), \frac{9216}{t^2} - \frac{5120}{t} \\
& -2 \operatorname{RootOf}(_Z^2 + 7) + \frac{26}{15} \Big], \Big[\Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2 + 7) + \frac{8}{5}, \\
& -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 6 \operatorname{RootOf}(_Z^2 + 7) + \frac{13}{5}, 3 \operatorname{RootOf}(_Z^2 + 7) - 1 \Big], \Big[\frac{9216}{t^2} \\
& - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 + 7) + \frac{16}{15}, \frac{9216}{t^2} - \frac{5120}{t} - 4 \operatorname{RootOf}(_Z^2 + 7) + \frac{26}{15}, \\
& -\frac{2}{3} + 2 \operatorname{RootOf}(_Z^2 + 7) \Big], [3, 2], [1, 1], \Big[\Big[\Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2 \\
& + 7) + \frac{13}{5}, 1 \Big], \Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2 + 7) + \frac{13}{5}, 3 \operatorname{RootOf}(_Z^2 \\
& + 7) \Big], [3 \operatorname{RootOf}(_Z^2 + 7), 1] \Big], \Big[\Big[\frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 + 7) + \frac{26}{15}, \frac{2}{3} \Big], \\
& \Big[\frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 + 7) + \frac{26}{15}, 2 \operatorname{RootOf}(_Z^2 + 7) \Big], \Big[2 \operatorname{RootOf}(_Z^2 \\
& + 7), \frac{2}{3} \Big] \Big], [[-6t^3 + 136t^2 - 834t, -6t^3 + 136t^2 - 834t, 0], [9216t^2 - 5120t, \\
& 9216t^2 - 5120t, 0]], \Big[\Big[-3 \operatorname{RootOf}(_Z^2 + 7) + \frac{8}{5}, -6 \operatorname{RootOf}(_Z^2 + 7) + \frac{13}{5}, \\
& 3 \operatorname{RootOf}(_Z^2 + 7) - 1 \Big], \Big[-2 \operatorname{RootOf}(_Z^2 + 7) + \frac{16}{15}, -4 \operatorname{RootOf}(_Z^2 + 7) + \frac{26}{15}, -\frac{2}{3} \\
& + 2 \operatorname{RootOf}(_Z^2 + 7) \Big], \Big[[x - 12, 12], [x - 7, 7], \Big[\Big[0, 0, \frac{4}{5} \Big], \Big[0, \frac{4}{5}, \frac{4}{5} \Big], [1, 1, 1], \\
& \Big[0, 0], \Big[\frac{4}{5}, 0 \Big], \Big[\frac{4}{5}, 0 \Big], 3 \Big], \Big[\Big[0, 0, \frac{16}{5} \Big], \Big[0, \frac{16}{5}, \frac{16}{5} \Big], [1, 1, 1], \Big[0, 0], \Big[\frac{16}{5}, 0 \Big], \\
& \Big[\frac{16}{5}, 0 \Big], 3 \Big] \Big] \Big]
\end{aligned}$$

> Sreg:= regsingtrue2F2(L,t,Sirr[-1],ext);

$$Sreg := \Big[[x - 12, 12], [x - 7, 7], \Big[\Big[0, 0, \frac{4}{5} \Big], \Big[0, 0, \frac{16}{5} \Big], \Big[\Big[0, \frac{4}{5}, \frac{4}{5} \Big], \Big[0, \frac{16}{5}, \frac{16}{5} \Big], \Big[\Big[0, 0], \Big[\frac{4}{5}, 0 \Big], \Big[\frac{4}{5}, 0 \Big], \Big[0, 0], \Big[\frac{16}{5}, 0 \Big], \Big[\frac{16}{5}, 0 \Big] \Big] \Big] \Big] \quad (69)$$

> RSreg:= Sregseptrue2F2(L,Sreg,ext);

$$RSreg := \Big[[], [], \Big[[x - 12, 12], [x - 7, 7], \Big[\Big[0, 0, \frac{4}{5} \Big], \Big[0, 0, \frac{16}{5} \Big], \Big[\Big[\Big[\frac{4}{5}, \frac{4}{5} \Big], [0] \Big], \Big[\Big[\frac{16}{5}, \frac{16}{5} \Big], [0] \Big] \Big] \Big] \Big] \quad (70)$$

> Rl:=IrrRegAppsing2F2(L,t,E,ext);

$$Rl := \Big[\Big[[\infty, \infty], [x - 3, 3], \Big[\Big[1, 3 \operatorname{RootOf}(_Z^2 + 7), -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} \quad (71)$$

$$\begin{aligned}
& -3 \operatorname{RootOf}(_Z^2 + 7) + \frac{13}{5} \Big], \Big[\frac{2}{3}, 2 \operatorname{RootOf}(_Z^2 + 7), \frac{9216}{t^2} - \frac{5120}{t} \\
& -2 \operatorname{RootOf}(_Z^2 + 7) + \frac{26}{15} \Big], \Big[\Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2 + 7) + \frac{8}{5}, \\
& -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 6 \operatorname{RootOf}(_Z^2 + 7) + \frac{13}{5}, 3 \operatorname{RootOf}(_Z^2 + 7) - 1 \Big], \Big[\frac{9216}{t^2} \\
& - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 + 7) + \frac{16}{15}, \frac{9216}{t^2} - \frac{5120}{t} - 4 \operatorname{RootOf}(_Z^2 + 7) + \frac{26}{15}, \\
& -\frac{2}{3} + 2 \operatorname{RootOf}(_Z^2 + 7) \Big], [3, 2], [1, 1], \Big[\Big[\Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2 \\
& + 7) + \frac{13}{5}, 1 \Big], \Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2 + 7) + \frac{13}{5}, 3 \operatorname{RootOf}(_Z^2 \\
& + 7) \Big], [3 \operatorname{RootOf}(_Z^2 + 7), 1] \Big], \Big[\Big[\frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 + 7) + \frac{26}{15}, \frac{2}{3} \Big], \\
& \Big[\frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 + 7) + \frac{26}{15}, 2 \operatorname{RootOf}(_Z^2 + 7) \Big], \Big[2 \operatorname{RootOf}(_Z^2 \\
& + 7), \frac{2}{3} \Big] \Big], [[-6 t^3 + 136 t^2 - 834 t, -6 t^3 + 136 t^2 - 834 t, 0], [9216 t^2 - 5120 t, \\
& 9216 t^2 - 5120 t, 0]], \Big[\Big[-3 \operatorname{RootOf}(_Z^2 + 7) + \frac{8}{5}, -6 \operatorname{RootOf}(_Z^2 + 7) + \frac{13}{5}, \\
& 3 \operatorname{RootOf}(_Z^2 + 7) - 1 \Big], \Big[-2 \operatorname{RootOf}(_Z^2 + 7) + \frac{16}{15}, -4 \operatorname{RootOf}(_Z^2 + 7) + \frac{26}{15}, -\frac{2}{3} \\
& + 2 \operatorname{RootOf}(_Z^2 + 7) \Big] \Big], \Big[[[x - 12, 12], [x - 7, 7]], \Big[\Big[0, 0, \frac{4}{5} \Big], \Big[0, 0, \frac{16}{5} \Big] \Big], \Big[\Big[0, \frac{4}{5}, \\
& \frac{4}{5} \Big], \Big[0, \frac{16}{5}, \frac{16}{5} \Big] \Big], \Big[\Big[[0, 0], \Big[\frac{4}{5}, 0 \Big], \Big[\frac{4}{5}, 0 \Big] \Big], \Big[[0, 0], \Big[\frac{16}{5}, 0 \Big], \Big[\frac{16}{5}, 0 \Big] \Big] \Big], \Big[[], [], \\
& \Big[[[x - 12, 12], [x - 7, 7]], \Big[\Big[0, 0, \frac{4}{5} \Big], \Big[0, 0, \frac{16}{5} \Big] \Big], \Big[\Big[\Big[\frac{4}{5}, \frac{4}{5} \Big], [0] \Big], \Big[\Big[\frac{16}{5}, \frac{16}{5} \Big], \\
& [0] \Big] \Big] \Big], \Big[\Big[\Big[x^2 - \frac{32}{3} x - 1, \operatorname{RootOf}(3 _Z^2 - 32 _Z - 3) \Big], [[0, 2, 4], [2, 4, 2]], [[[2, \\
& 0], [4, 0], [4, 2]] \Big], \Big[[[\infty, \infty], [x - 12, 12], [x - 7, 7], [x - 3, 3]], \Big[\Big[1, 3 \operatorname{RootOf}(_Z^2 \\
& + 7), -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2 + 7) + \frac{13}{5} \Big], \Big[0, 0, \frac{4}{5} \Big], \Big[0, 0, \frac{16}{5} \Big], \Big[\frac{2}{3}, \\
& 2 \operatorname{RootOf}(_Z^2 + 7), \frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 + 7) + \frac{26}{15} \Big], \Big[\Big[-\frac{6}{t^3} + \frac{136}{t^2} \\
& - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2 + 7) + \frac{8}{5}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 6 \operatorname{RootOf}(_Z^2 + 7) + \frac{13}{5}, \\
& 3 \operatorname{RootOf}(_Z^2 + 7) - 1 \Big], \Big[0, \frac{4}{5}, \frac{4}{5} \Big], \Big[0, \frac{16}{5}, \frac{16}{5} \Big], \Big[\frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2
\end{aligned}$$

$$\begin{aligned}
& + 7) + \frac{16}{15}, \frac{9216}{t^2} - \frac{5120}{t} - 4 \operatorname{RootOf}(_Z^2 + 7) + \frac{26}{15}, -\frac{2}{3} + 2 \operatorname{RootOf}(_Z^2 + 7) \Big] \Big] \\
& , \left[\left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2 + 7) + \frac{13}{5}, 1 \right], \left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} \right. \right. \right. \\
& \left. \left. \left. - 3 \operatorname{RootOf}(_Z^2 + 7) + \frac{13}{5}, 3 \operatorname{RootOf}(_Z^2 + 7) \right], [3 \operatorname{RootOf}(_Z^2 + 7), 1] \right], \left[[0, 0], \right. \right. \\
& \left[\frac{4}{5}, 0 \right], \left[\frac{4}{5}, 0 \right] \Big], \left[[0, 0], \left[\frac{16}{5}, 0 \right], \left[\frac{16}{5}, 0 \right] \right], \left[\left[\frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 + 7) \right. \right. \\
& \left. \left. + \frac{26}{15}, \frac{2}{3} \right], \left[\frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 + 7) + \frac{26}{15}, 2 \operatorname{RootOf}(_Z^2 + 7) \right], \right. \\
& \left. \left[2 \operatorname{RootOf}(_Z^2 + 7), \frac{2}{3} \right] \right] \Big], [[1, 1, 1], [1, 1, 1], [1, 1, 1], [1, 1, 1]] \Big] \Big]
\end{aligned}$$

> F1:= Hyp2F2Subst(L,x,t,R1[1],ext);

$$\begin{aligned}
F1 := & \left[-\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 1193x + 9984)}{(x-3)^2}, \right. \\
& -\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 6313x - 9984)}{(x-3)^2}, \\
& \frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 6313x - 9984)}{(x-3)^2}, \\
& \left. \frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 1193x + 9984)}{(x-3)^2} \right]
\end{aligned} \tag{72}$$

> B:= Candichangvar2F2(F1,R1,ext);

$$\begin{aligned}
B := & \left\{ \left[-\frac{2(x-7)^4(x-12)}{(x-3)^2}, [1, 4], \left[[x-12, 12], [x-7, 7], \left[\left[0, 0, \frac{4}{5} \right], \left[0, 0, \frac{16}{5} \right] \right], \right. \right. \right. \\
& \left[\left[0, \frac{4}{5}, \frac{4}{5} \right], \left[0, \frac{16}{5}, \frac{16}{5} \right] \right], \left[\left[[0, 0], \left[\frac{4}{5}, 0 \right], \left[\frac{4}{5}, 0 \right] \right], \left[[0, 0], \left[\frac{16}{5}, 0 \right], \left[\frac{16}{5}, 0 \right] \right] \right] \right] \\
& , \left[\frac{2(x-7)^4(x-12)}{(x-3)^2}, [1, 4], \left[[x-12, 12], [x-7, 7], \left[\left[0, 0, \frac{4}{5} \right], \left[0, 0, \frac{16}{5} \right] \right], \right. \right. \\
& \left[\left[0, \frac{4}{5}, \frac{4}{5} \right], \left[0, \frac{16}{5}, \frac{16}{5} \right] \right], \left[\left[[0, 0], \left[\frac{4}{5}, 0 \right], \left[\frac{4}{5}, 0 \right] \right], \left[[0, 0], \left[\frac{16}{5}, 0 \right], \left[\frac{16}{5}, 0 \right] \right] \right] \right] \right\}
\end{aligned} \tag{73}$$

> find2F2ln(L,R1,F1,ext,x,t);

$$\begin{aligned}
& \left[\left[\left[\left[-1, \frac{2}{3} + \operatorname{RootOf}(_Z^2 + 7) \right], \left[1, \frac{2}{3} + \operatorname{RootOf}(_Z^2 + 7) \right], \left[\frac{1}{3}, \operatorname{RootOf}(_Z^2 + 7) \right], \right. \right. \right. \\
& \left[\frac{2}{3}, \operatorname{RootOf}(_Z^2 + 7) + \frac{1}{3} \right], \left[1, \frac{1}{5} \right], \frac{2(x-7)^4(x-12)}{(x-3)^2} \right], \left[\left[\left[-1, \frac{2}{3} \right. \right. \right. \\
& \left. \left. \left. + \operatorname{RootOf}(_Z^2 + 7) \right], \left[1, \frac{2}{3} + \operatorname{RootOf}(_Z^2 + 7) \right], \left[\frac{1}{3}, \operatorname{RootOf}(_Z^2 + 7) \right], \left[\frac{2}{3}, \right. \right. \right.
\end{aligned} \tag{74}$$

$$\begin{aligned}
L := & 5 (x-12)^2 (x-7)^3 (3x^2-32x-3)^3 (x-3)^5 Dx^3 - (x-12) (90x^9-5520x^8 \\
& + 143560x^7-2051089x^6+17367186x^5-86903169x^4+235727812x^3-253798731x^2 \\
& - 54600264x-1447011) (x-7)^2 (x-3)^2 (3x^2-32x-3)^2 Dx^2 \\
& + \frac{1}{68} ((3 \operatorname{RootOf}(_Z^2+2) + 3 \operatorname{RootOf}(_Z^2+1) + 1) (3x^2-32x-3) (\\
& -175251270978-6471836651196x+51300 \operatorname{RootOf}(_Z^2+2) \operatorname{RootOf}(_Z^2+1) x^{11} \\
& -4426767 \operatorname{RootOf}(_Z^2+2) \operatorname{RootOf}(_Z^2+1) x^{10}+167228094 \operatorname{RootOf}(_Z^2 \\
& +2) \operatorname{RootOf}(_Z^2+1) x^9+35830932919488x^3-28569620183706x^2 \\
& -11210832157908x^4-15458569476x^9+133804273846x^8-735567239592x^7 \\
& +2279254906908x^6-1900931214952x^5-18360x^{13}+1517760x^{12}-55281720x^{11} \\
& +1161786398x^{10}+6957921919410 \operatorname{RootOf}(_Z^2+2)x-5566337535528 \operatorname{RootOf}(_Z^2 \\
& +1)x-113001595263 \operatorname{RootOf}(_Z^2+2) \operatorname{RootOf}(_Z^2+1)
\end{aligned}
\tag{80}$$

$$\begin{aligned}
& -150668793684 \operatorname{RootOf}(_Z^2 + 1) - 3632836539 \operatorname{RootOf}(_Z^2 + 2) \operatorname{RootOf}(_Z^2 + 1) x^8 \\
& + 50038841988 \operatorname{RootOf}(_Z^2 + 2) \operatorname{RootOf}(_Z^2 + 1) x^7 - 453207655182 \operatorname{RootOf}(_Z^2 \\
& + 2) \operatorname{RootOf}(_Z^2 + 1) x^6 + 2696724214608 \operatorname{RootOf}(_Z^2 + 2) \operatorname{RootOf}(_Z^2 + 1) x^5 \\
& - 10133483252598 \operatorname{RootOf}(_Z^2 + 2) \operatorname{RootOf}(_Z^2 + 1) x^4 \\
& + 21575505129768 \operatorname{RootOf}(_Z^2 + 2) \operatorname{RootOf}(_Z^2 + 1) x^3 \\
& - 18587532086451 \operatorname{RootOf}(_Z^2 + 2) \operatorname{RootOf}(_Z^2 + 1) x^2 - 4174753151646 \operatorname{RootOf}(_Z^2 \\
& + 2) \operatorname{RootOf}(_Z^2 + 1) x + 188335992105 \operatorname{RootOf}(_Z^2 + 2) - 85500 \operatorname{RootOf}(_Z^2 \\
& + 2) x^{11} + 68400 \operatorname{RootOf}(_Z^2 + 1) x^{11} + 7377945 \operatorname{RootOf}(_Z^2 + 2) x^{10} \\
& - 5902356 \operatorname{RootOf}(_Z^2 + 1) x^{10} - 278713490 \operatorname{RootOf}(_Z^2 + 2) x^9 \\
& + 222970792 \operatorname{RootOf}(_Z^2 + 1) x^9 + 6054727565 \operatorname{RootOf}(_Z^2 + 2) x^8 \\
& - 4843782052 \operatorname{RootOf}(_Z^2 + 1) x^8 - 83398069980 \operatorname{RootOf}(_Z^2 + 2) x^7 \\
& + 66718455984 \operatorname{RootOf}(_Z^2 + 1) x^7 + 755346091970 \operatorname{RootOf}(_Z^2 + 2) x^6 \\
& - 604276873576 \operatorname{RootOf}(_Z^2 + 1) x^6 - 4494540357680 \operatorname{RootOf}(_Z^2 + 2) x^5 \\
& + 3595632286144 \operatorname{RootOf}(_Z^2 + 1) x^5 + 16889138754330 \operatorname{RootOf}(_Z^2 + 2) x^4 \\
& - 13511311003464 \operatorname{RootOf}(_Z^2 + 1) x^4 - 35959175216280 \operatorname{RootOf}(_Z^2 + 2) x^3 \\
& + 28767340173024 \operatorname{RootOf}(_Z^2 + 1) x^3 + 30979220144085 \operatorname{RootOf}(_Z^2 + 2) x^2 \\
& - 24783376115268 \operatorname{RootOf}(_Z^2 + 1) x^2 (x - 3) (x - 7) Dx - 10 \operatorname{RootOf}(_Z^2 \\
& + 2) \operatorname{RootOf}(_Z^2 + 1) (x - 7)^4 (3x^2 - 32x - 3)^6
\end{aligned}$$

$$\begin{aligned}
& \text{> ext:=indets(L,\{RootOf,name\}) minus \{x,Dx\};} \\
& \quad \text{ext := \{RootOf(_Z^2 + 1), RootOf(_Z^2 + 2)\}} \tag{81}
\end{aligned}$$

$$\begin{aligned}
& \text{> ext:= indets(map(s-> ReplirrRoot(s,\{\}),ext),\{RootOf,name\});} \\
& \quad \text{ext := \{RootOf(_Z^2 + 1), RootOf(_Z^2 + 2)\}} \tag{82}
\end{aligned}$$

$$\begin{aligned}
& \text{> extppp:=\{\};} \\
& \quad \text{extppp := } \emptyset \tag{83}
\end{aligned}$$

$$\begin{aligned}
& \text{> E:= Singular(L,extppp);} \\
& E := \left[[x - 7, 7], [x - 3, 3], \left[x^2 - \frac{32}{3}x - 1, \operatorname{RootOf}(3_Z^2 - 32_Z - 3) \right], [\infty, \infty], [x \right. \\
& \quad \left. - 12, 12] \right] \tag{84}
\end{aligned}$$

$$\begin{aligned}
& \text{> F:= NotAppSing(L,E,ext);} \\
& \quad F := [[\infty, \infty], [x - 12, 12], [x - 7, 7], [x - 3, 3]] \tag{85}
\end{aligned}$$

$$\begin{aligned}
& \text{> Sirr:= irrSing2F2(L,t,F,ext);} \\
& Sirr := \left[[[\infty, \infty], [x - 3, 3]], \left[\left[3 \operatorname{RootOf}(_Z^2 + 2), 3 \operatorname{RootOf}(_Z^2 + 1), -\frac{6}{t^3} + \frac{136}{t^2} \right. \right. \right. \\
& \quad \left. \left. - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2 + 2) - 3 \operatorname{RootOf}(_Z^2 + 1) + \frac{18}{5} \right], \left[2 \operatorname{RootOf}(_Z^2 + 2), \right. \right. \\
& \quad \left. \left. 2 \operatorname{RootOf}(_Z^2 + 1), \frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 + 2) - 2 \operatorname{RootOf}(_Z^2 + 1) + \frac{12}{5} \right] \right] \tag{86}
\end{aligned}$$

$$\begin{aligned}
& \left] , \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 6 \operatorname{RootOf}(_Z^2 + 2) - 3 \operatorname{RootOf}(_Z^2 + 1) + \frac{18}{5}, -\frac{6}{t^3} \right. \right. \\
& + \frac{136}{t^2} - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2 + 2) - 6 \operatorname{RootOf}(_Z^2 + 1) + \frac{18}{5}, 3 \operatorname{RootOf}(_Z^2 + 1) \\
& - 3 \operatorname{RootOf}(_Z^2 + 2) \left. \right] , \left[\frac{9216}{t^2} - \frac{5120}{t} - 4 \operatorname{RootOf}(_Z^2 + 2) - 2 \operatorname{RootOf}(_Z^2 + 1) \right. \\
& + \frac{12}{5}, \frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 + 2) - 4 \operatorname{RootOf}(_Z^2 + 1) + \frac{12}{5}, \\
& - 2 \operatorname{RootOf}(_Z^2 + 2) + 2 \operatorname{RootOf}(_Z^2 + 1) \left. \right] , [3, 2], [1, 1], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} \right. \right. \\
& - 3 \operatorname{RootOf}(_Z^2 + 2) - 3 \operatorname{RootOf}(_Z^2 + 1) + \frac{18}{5}, 3 \operatorname{RootOf}(_Z^2 + 2) \left. \right] , \left[-\frac{6}{t^3} + \frac{136}{t^2} \right. \\
& - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2 + 2) - 3 \operatorname{RootOf}(_Z^2 + 1) + \frac{18}{5}, 3 \operatorname{RootOf}(_Z^2 + 1) \left. \right] , \\
& [3 \operatorname{RootOf}(_Z^2 + 1), 3 \operatorname{RootOf}(_Z^2 + 2) \left. \right] , \left[\left[\frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 + 2) \right. \right. \\
& - 2 \operatorname{RootOf}(_Z^2 + 1) + \frac{12}{5}, 2 \operatorname{RootOf}(_Z^2 + 2) \left. \right] , \left[\frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 \right. \\
& + 2) - 2 \operatorname{RootOf}(_Z^2 + 1) + \frac{12}{5}, 2 \operatorname{RootOf}(_Z^2 + 1) \left. \right] , [2 \operatorname{RootOf}(_Z^2 + 1), \\
& 2 \operatorname{RootOf}(_Z^2 + 2) \left. \right] \left. \right] , [[-6t^3 + 136t^2 - 834t, -6t^3 + 136t^2 - 834t, 0], [9216t^2 \\
& - 5120t, 9216t^2 - 5120t, 0]], \left[\left[-6 \operatorname{RootOf}(_Z^2 + 2) - 3 \operatorname{RootOf}(_Z^2 + 1) + \frac{18}{5}, \right. \right. \\
& - 3 \operatorname{RootOf}(_Z^2 + 2) - 6 \operatorname{RootOf}(_Z^2 + 1) + \frac{18}{5}, 3 \operatorname{RootOf}(_Z^2 + 1) - 3 \operatorname{RootOf}(_Z^2 \\
& + 2) \left. \right] , \left[-4 \operatorname{RootOf}(_Z^2 + 2) - 2 \operatorname{RootOf}(_Z^2 + 1) + \frac{12}{5}, -2 \operatorname{RootOf}(_Z^2 + 2) \right. \\
& - 4 \operatorname{RootOf}(_Z^2 + 1) + \frac{12}{5}, -2 \operatorname{RootOf}(_Z^2 + 2) + 2 \operatorname{RootOf}(_Z^2 + 1) \left. \right] \left. \right] , \left[[x - 12, \right. \\
& 12], [x - 7, 7] \left. \right] , \left[\left[\left[0, 0, \frac{4}{5} \right], \left[0, \frac{4}{5}, \frac{4}{5} \right], [1, 1, 1], \left[[0, 0], \left[\frac{4}{5}, 0 \right], \left[\frac{4}{5}, 0 \right] \right], 3 \right], \left[\left[0, 0, \right. \right. \right. \\
& \left. \left. \frac{16}{5} \right], \left[0, \frac{16}{5}, \frac{16}{5} \right], [1, 1, 1], \left[[0, 0], \left[\frac{16}{5}, 0 \right], \left[\frac{16}{5}, 0 \right] \right], 3 \right] \left. \right] \left. \right]
\end{aligned}$$

> Sreg := regsingtrue2F2(L,t,Sirr[-1],ext);

$$\begin{aligned}
Sreg := & \left[[x - 12, 12], [x - 7, 7], \left[\left[0, 0, \frac{4}{5} \right], \left[0, 0, \frac{16}{5} \right] \right], \left[\left[0, \frac{4}{5}, \frac{4}{5} \right], \left[0, \frac{16}{5}, \frac{16}{5} \right] \right], \right. \\
& \left. \left[\left[[0, 0], \left[\frac{4}{5}, 0 \right], \left[\frac{4}{5}, 0 \right] \right], \left[[0, 0], \left[\frac{16}{5}, 0 \right], \left[\frac{16}{5}, 0 \right] \right] \right] \right]
\end{aligned} \tag{87}$$

> RSreg := Sregseptrue2F2(L,Sreg,ext);

$$\begin{aligned}
RSreg := & \left[[\], [\], \left[[x - 12, 12], [x - 7, 7] \right], \left[\left[0, 0, \frac{4}{5} \right], \left[0, 0, \frac{16}{5} \right] \right], \left[\left[\left[\frac{4}{5}, \frac{4}{5} \right], [0] \right], \right. \right. \\
& \left. \left. \left[\left[\frac{4}{5}, \frac{4}{5} \right], [0] \right] \right] \right]
\end{aligned} \tag{88}$$

$$\left[\left[\frac{16}{5}, \frac{16}{5} \right], [0] \right] \right]$$

> R1:=IrrRegAppsing2F2(L,t,E,ext);

$$\begin{aligned}
 RI := & \left[\left[[\infty, \infty], [x-3, 3] \right], \left[\left[3 \operatorname{RootOf}(_Z^2+2), 3 \operatorname{RootOf}(_Z^2+1), -\frac{6}{t^3} + \frac{136}{t^2} \right. \right. \right. \\
 & \left. \left. - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2+2) - 3 \operatorname{RootOf}(_Z^2+1) + \frac{18}{5} \right], \left[2 \operatorname{RootOf}(_Z^2+2), \right. \right. \\
 & \left. \left. 2 \operatorname{RootOf}(_Z^2+1), \frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2+2) - 2 \operatorname{RootOf}(_Z^2+1) + \frac{12}{5} \right] \right. \\
 & \left. \right], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 6 \operatorname{RootOf}(_Z^2+2) - 3 \operatorname{RootOf}(_Z^2+1) + \frac{18}{5}, -\frac{6}{t^3} \right. \right. \\
 & \left. \left. + \frac{136}{t^2} - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2+2) - 6 \operatorname{RootOf}(_Z^2+1) + \frac{18}{5}, 3 \operatorname{RootOf}(_Z^2+1) \right. \right. \\
 & \left. \left. - 3 \operatorname{RootOf}(_Z^2+2) \right], \left[\frac{9216}{t^2} - \frac{5120}{t} - 4 \operatorname{RootOf}(_Z^2+2) - 2 \operatorname{RootOf}(_Z^2+1) \right. \right. \\
 & \left. \left. + \frac{12}{5}, \frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2+2) - 4 \operatorname{RootOf}(_Z^2+1) + \frac{12}{5}, \right. \right. \\
 & \left. \left. -2 \operatorname{RootOf}(_Z^2+2) + 2 \operatorname{RootOf}(_Z^2+1) \right] \right], [3, 2], [1, 1], \left[\left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} \right. \right. \right. \\
 & \left. \left. - 3 \operatorname{RootOf}(_Z^2+2) - 3 \operatorname{RootOf}(_Z^2+1) + \frac{18}{5}, 3 \operatorname{RootOf}(_Z^2+2) \right], \left[-\frac{6}{t^3} + \frac{136}{t^2} \right. \right. \\
 & \left. \left. - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2+2) - 3 \operatorname{RootOf}(_Z^2+1) + \frac{18}{5}, 3 \operatorname{RootOf}(_Z^2+1) \right] \right], \\
 & \left[3 \operatorname{RootOf}(_Z^2+1), 3 \operatorname{RootOf}(_Z^2+2) \right], \left[\left[\frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2+2) \right. \right. \\
 & \left. \left. - 2 \operatorname{RootOf}(_Z^2+1) + \frac{12}{5}, 2 \operatorname{RootOf}(_Z^2+2) \right], \left[\frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 \right. \right. \\
 & \left. \left. + 2) - 2 \operatorname{RootOf}(_Z^2+1) + \frac{12}{5}, 2 \operatorname{RootOf}(_Z^2+1) \right], [2 \operatorname{RootOf}(_Z^2+1), \right. \\
 & \left. 2 \operatorname{RootOf}(_Z^2+2) \right] \right], [[-6t^3+136t^2-834t, -6t^3+136t^2-834t, 0], [9216t^2 \\
 & -5120t, 9216t^2-5120t, 0]], \left[\left[-6 \operatorname{RootOf}(_Z^2+2) - 3 \operatorname{RootOf}(_Z^2+1) + \frac{18}{5}, \right. \right. \\
 & \left. \left. -3 \operatorname{RootOf}(_Z^2+2) - 6 \operatorname{RootOf}(_Z^2+1) + \frac{18}{5}, 3 \operatorname{RootOf}(_Z^2+1) - 3 \operatorname{RootOf}(_Z^2 \right. \right. \\
 & \left. \left. + 2) \right], \left[-4 \operatorname{RootOf}(_Z^2+2) - 2 \operatorname{RootOf}(_Z^2+1) + \frac{12}{5}, -2 \operatorname{RootOf}(_Z^2+2) \right. \right. \\
 & \left. \left. - 4 \operatorname{RootOf}(_Z^2+1) + \frac{12}{5}, -2 \operatorname{RootOf}(_Z^2+2) + 2 \operatorname{RootOf}(_Z^2+1) \right] \right], \left[[x-12, \right.
 \end{aligned}
 \tag{89}$$

$$\begin{aligned}
& 12], [x-7, 7]], \left[\left[0, 0, \frac{4}{5} \right], \left[0, 0, \frac{16}{5} \right] \right], \left[\left[0, \frac{4}{5}, \frac{4}{5} \right], \left[0, \frac{16}{5}, \frac{16}{5} \right] \right], \left[\left[[0, 0], \left[\frac{4}{5}, 0 \right], \left[\frac{4}{5}, 0 \right] \right], \left[[0, 0], \left[\frac{16}{5}, 0 \right], \left[\frac{16}{5}, 0 \right] \right] \right], \left[[], [], \left[[x-12, 12], [x-7, 7] \right], \left[\left[0, 0, \frac{4}{5} \right], \left[0, 0, \frac{16}{5} \right] \right], \left[\left[\left[\frac{4}{5}, \frac{4}{5} \right], [0] \right], \left[\left[\frac{16}{5}, \frac{16}{5} \right], [0] \right] \right] \right], \left[\left[\left[x^2 - \frac{32}{3}x - 1, \right. \right. \right. \\
& \left. \left. \left. \text{RootOf}(3_Z^2 - 32_Z - 3) \right] \right], \left[[0, 2, 4], [2, 4, 2], [[2, 0], [4, 0], [4, 2]] \right], \left[[[\infty, \infty], [x-12, 12], [x-7, 7], [x-3, 3]], \left[\left[3 \text{RootOf}(_Z^2 + 2), 3 \text{RootOf}(_Z^2 + 1), -\frac{6}{t^3} \right. \right. \right. \\
& \left. \left. \left. + \frac{136}{t^2} - \frac{834}{t} - 3 \text{RootOf}(_Z^2 + 2) - 3 \text{RootOf}(_Z^2 + 1) + \frac{18}{5} \right], \left[0, 0, \frac{4}{5} \right], \left[0, 0, \frac{16}{5} \right], \left[2 \text{RootOf}(_Z^2 + 2), 2 \text{RootOf}(_Z^2 + 1), \frac{9216}{t^2} - \frac{5120}{t} - 2 \text{RootOf}(_Z^2 + 2) \right. \right. \\
& \left. \left. - 2 \text{RootOf}(_Z^2 + 1) + \frac{12}{5} \right] \right], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 6 \text{RootOf}(_Z^2 + 2) - 3 \text{RootOf}(_Z^2 + 1) + \frac{18}{5}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 \text{RootOf}(_Z^2 + 2) \right. \right. \\
& \left. \left. - 6 \text{RootOf}(_Z^2 + 1) + \frac{18}{5}, 3 \text{RootOf}(_Z^2 + 1) - 3 \text{RootOf}(_Z^2 + 2) \right], \left[0, \frac{4}{5}, \frac{4}{5} \right], \left[0, \frac{16}{5}, \frac{16}{5} \right], \left[\frac{9216}{t^2} - \frac{5120}{t} - 4 \text{RootOf}(_Z^2 + 2) - 2 \text{RootOf}(_Z^2 + 1) + \frac{12}{5}, \frac{9216}{t^2} - \frac{5120}{t} - 2 \text{RootOf}(_Z^2 + 2) - 4 \text{RootOf}(_Z^2 + 1) + \frac{12}{5}, -2 \text{RootOf}(_Z^2 + 2) \right. \right. \\
& \left. \left. + 2 \text{RootOf}(_Z^2 + 1) \right] \right], \left[\left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 \text{RootOf}(_Z^2 + 2) - 3 \text{RootOf}(_Z^2 + 1) + \frac{18}{5}, 3 \text{RootOf}(_Z^2 + 2) \right], \left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 \text{RootOf}(_Z^2 + 2) - 3 \text{RootOf}(_Z^2 + 1) + \frac{18}{5}, 3 \text{RootOf}(_Z^2 + 1) \right] \right], \left[3 \text{RootOf}(_Z^2 + 1), 3 \text{RootOf}(_Z^2 + 2) \right] \right], \left[[0, 0], \left[\frac{4}{5}, 0 \right], \left[\frac{4}{5}, 0 \right] \right], \left[[0, 0], \left[\frac{16}{5}, 0 \right], \left[\frac{16}{5}, 0 \right] \right], \left[\left[\frac{9216}{t^2} - \frac{5120}{t} - 2 \text{RootOf}(_Z^2 + 2) - 2 \text{RootOf}(_Z^2 + 1) + \frac{12}{5}, 2 \text{RootOf}(_Z^2 + 2) \right], \left[\frac{9216}{t^2} - \frac{5120}{t} - 2 \text{RootOf}(_Z^2 + 2) - 2 \text{RootOf}(_Z^2 + 1) + \frac{12}{5}, 2 \text{RootOf}(_Z^2 + 1) \right] \right], \left[2 \text{RootOf}(_Z^2 + 1), 2 \text{RootOf}(_Z^2 + 2) \right] \right], \left[[1, 1, 1], [1, 1, 1], [1, 1, 1], [1, 1, 1] \right] \right]
\end{aligned}$$

> F1:= Hyp2F2Subst(L,x,t,R1[1],ext);

$$F1 := \left[-\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 1193x + 9984)}{(x-3)^2}, \right. \\ \left. -\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 6313x - 9984)}{(x-3)^2}, \right. \\ \left. \frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 6313x - 9984)}{(x-3)^2}, \right. \\ \left. \frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 1193x + 9984)}{(x-3)^2} \right] \quad (90)$$

> B:= Candichangvar2F2(F1,R1,ext);

$$B := \left\{ \left[-\frac{2(x-7)^4(x-12)}{(x-3)^2}, [1, 4], \left[[x-12, 12], [x-7, 7], \left[\left[0, 0, \frac{4}{5} \right], \left[0, 0, \frac{16}{5} \right] \right], \right. \right. \right. \\ \left. \left[\left[0, \frac{4}{5}, \frac{4}{5} \right], \left[0, \frac{16}{5}, \frac{16}{5} \right], \left[\left[[0, 0], \left[\frac{4}{5}, 0 \right], \left[\frac{4}{5}, 0 \right] \right], \left[[0, 0], \left[\frac{16}{5}, 0 \right], \left[\frac{16}{5}, 0 \right] \right] \right] \right] \right. \\ \left. , \left[\frac{2(x-7)^4(x-12)}{(x-3)^2}, [1, 4], \left[[x-12, 12], [x-7, 7], \left[\left[0, 0, \frac{4}{5} \right], \left[0, 0, \frac{16}{5} \right] \right], \right. \right. \right. \\ \left. \left[\left[0, \frac{4}{5}, \frac{4}{5} \right], \left[0, \frac{16}{5}, \frac{16}{5} \right], \left[\left[[0, 0], \left[\frac{4}{5}, 0 \right], \left[\frac{4}{5}, 0 \right] \right], \left[[0, 0], \left[\frac{16}{5}, 0 \right], \left[\frac{16}{5}, 0 \right] \right] \right] \right] \right] \right\} \quad (91)$$

> find2F2ln(L,R1,F1,ext,x,t);

$$\left[\left[\left[\left[\left[\frac{1}{3} + \text{RootOf}(_Z^2 + 1), \frac{1}{3} + \text{RootOf}(_Z^2 + 2) \right], \left[\frac{2}{3} + \text{RootOf}(_Z^2 + 1), \frac{2}{3} + \text{RootOf}(_Z^2 + 2) \right], \left[\text{RootOf}(_Z^2 + 1), \text{RootOf}(_Z^2 + 2) \right] \right], \left[1, \frac{1}{5} \right] \right], \right. \right. \\ \left. \frac{2(x-7)^4(x-12)}{(x-3)^2} \right], \left[\left[\left[\left[\frac{1}{3} + \text{RootOf}(_Z^2 + 1), \frac{1}{3} + \text{RootOf}(_Z^2 + 2) \right], \left[\frac{2}{3} + \text{RootOf}(_Z^2 + 1), \frac{2}{3} + \text{RootOf}(_Z^2 + 2) \right], \left[\text{RootOf}(_Z^2 + 1), \text{RootOf}(_Z^2 + 2) \right] \right], \right. \right. \\ \left. \left[1, \frac{1}{5} \right] \right], -\frac{2(x-7)^4(x-12)}{(x-3)^2} \right] \quad (92)$$

> TIME :=time();
Hyp2F2Solutions(L);
time() - TIME;

TIME := 343.125

$$\left\{ \left[\left[\left[\left[\left[\text{RootOf}(_Z^2 + 1), \text{RootOf}(_Z^2 + 2) \right], \left[1, \frac{1}{5} \right], [0], [1] \right], \frac{2(x-7)^4(x-12)}{(x-3)^2} \right] \right] \right] \right\} \\ 28.343 \quad (93)$$

> F:=sumdiffeq(hyperterm([a1,a2],[b1,b2],x,k),k,J(x));

$$F := \left(\frac{d^3}{dx^3} J(x) \right) x^2 + (b1 + b2 - x + 1) \left(\frac{d^2}{dx^2} J(x) \right) x - (a1 x + a2 x - b2 b1 + x) \left(\frac{d}{dx} J(x) \right) - a1 a2 J(x) = 0 \quad (94)$$

> LA:=de2diffop(F,J(x));

$$LA := x^2 Dx^3 + (b1 x + b2 x - x^2 + x) Dx^2 + (-a1 x - a2 x + b2 b1 - x) Dx - a1 a2 \quad (95)$$

> L22:=subs({a1=1/7,b1=1,b2=1/5},LA);

$$L22 := x^2 Dx^3 + \left(\frac{11}{5} x - x^2 \right) Dx^2 + \left(-\frac{8}{7} x - a2 x + \frac{1}{5} \right) Dx - \frac{a2}{7} \quad (96)$$

> f:= normal((2*(x-7)^4*(x-12))/(x-3)^2,expanded);

$$f := \frac{2x^5 - 80x^4 + 1260x^3 - 9800x^2 + 37730x - 57624}{x^2 - 6x + 9} \quad (97)$$

> L:=ChangeOfVariables(L22,f);

$$L := 35 (x - 12)^2 (x - 7)^3 (3x^2 - 32x - 3)^3 (x - 3)^5 Dx^3 - 7 (x - 12) (90x^9 - 5520x^8 + 143560x^7 - 2051089x^6 + 17367186x^5 - 86903169x^4 + 235727812x^3 - 253798731x^2 - 54600264x - 1447011) (x - 7)^2 (x - 3)^2 (3x^2 - 32x - 3)^2 Dx^2 - (x - 7) (x - 3) (3x^2 - 32x - 3) (5670a2x^{13} - 468720a2x^{12} + 2700x^{13} + 17096940a2x^{11} - 223200x^{12} - 360913560a2x^{10} + 8101500x^{11} + 4854305050a2x^9 - 168420559x^{10} - 43067094280a2x^8 + 2181507538x^9 + 251198738280a2x^7 - 17682600603x^8 - 921604947760a2x^6 + 80699347476x^7 + 1882537448890a2x^5 - 86364656014x^6 - 1405877121600a2x^4 - 1201005762684x^5 - 700731721620a2x^3 + 7212132789354x^4 - 106333690680a2x^2 - 17114630206464x^3 - 6863234490a2x + 14406334309773x^2 - 163364040a2 + 3243762022158x + 87812337249) Dx - 10a2 (x - 7)^4 (3x^2 - 32x - 3)^6 \quad (98)$$

> ext:=indets(L,{RootOf,name}) minus {x,Dx};

$$ext := \{a2\} \quad (99)$$

> ext:= indets(map(s-> ReplirrRoot(s,{ }),ext),{RootOf,name});

$$ext := \{a2\} \quad (100)$$

> extppp:={};

$$extppp := \emptyset \quad (101)$$

> E:= Singular(L,extppp);

$$E := \left[[x - 7, 7], [x - 3, 3], \left[x^2 - \frac{32}{3}x - 1, \text{RootOf}(3_Z^2 - 32_Z - 3) \right], [\infty, \infty], [x - 12, 12] \right] \quad (102)$$

> F:= NotAppSing(L,E,ext);

$$F := [[\infty, \infty], [x - 12, 12], [x - 7, 7], [x - 3, 3]] \quad (103)$$

> Sirr:= irrSing2F2(L,t,F,ext);

$$Sirr := \left[[[\infty, \infty], [x - 3, 3]], \left[\left[\frac{3}{7}, 3a2, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a2 + \frac{111}{35} \right], \left[\frac{2}{7}, 2a2, \right. \right. \right] \quad (104)$$

$$\begin{aligned} & \left[\frac{9216}{t^2} - \frac{5120}{t} - 2a2 + \frac{74}{35} \right], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a2 + \frac{96}{35}, -\frac{6}{t^3} + \frac{136}{t^2} \right. \right. \\ & \left. \left. - \frac{834}{t} - 6a2 + \frac{111}{35}, -\frac{3}{7} + 3a2 \right], \left[\frac{9216}{t^2} - \frac{5120}{t} - 2a2 + \frac{64}{35}, \frac{9216}{t^2} - \frac{5120}{t} \right. \right. \\ & \left. \left. - 4a2 + \frac{74}{35}, 2a2 - \frac{2}{7} \right] \right], [3, 2], [1, 1], \left[\left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a2 + \frac{111}{35}, \frac{3}{7} \right], \right. \right. \\ & \left. \left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a2 + \frac{111}{35}, 3a2 \right], \left[3a2, \frac{3}{7} \right] \right], \left[\left[\frac{9216}{t^2} - \frac{5120}{t} - 2a2 \right. \right. \\ & \left. \left. + \frac{74}{35}, \frac{2}{7} \right], \left[\frac{9216}{t^2} - \frac{5120}{t} - 2a2 + \frac{74}{35}, 2a2 \right], \left[2a2, \frac{2}{7} \right] \right], \left[[-6t^3 + 136t^2 \right. \right. \\ & \left. \left. - 834t, -6t^3 + 136t^2 - 834t, 0] \right], [9216t^2 - 5120t, 9216t^2 - 5120t, 0] \right], \left[\left[-3a2 \right. \right. \\ & \left. \left. + \frac{96}{35}, -6a2 + \frac{111}{35}, -\frac{3}{7} + 3a2 \right], \left[-2a2 + \frac{64}{35}, -4a2 + \frac{74}{35}, 2a2 - \frac{2}{7} \right] \right], \left[[x \right. \right. \\ & \left. \left. - 12, 12], [x - 7, 7] \right], \left[\left[\left[0, 0, \frac{4}{5} \right], \left[0, \frac{4}{5}, \frac{4}{5} \right], [1, 1, 1], \left[[0, 0], \left[\frac{4}{5}, 0 \right], \left[\frac{4}{5}, 0 \right] \right], 3 \right], \right. \right. \\ & \left. \left[\left[0, 0, \frac{16}{5} \right], \left[0, \frac{16}{5}, \frac{16}{5} \right], [1, 1, 1], \left[[0, 0], \left[\frac{16}{5}, 0 \right], \left[\frac{16}{5}, 0 \right] \right], 3 \right] \right] \right] \end{aligned}$$

> Sreg:= regsingtrue2F2(L,t,Sirr[-1],ext);

$$Sreg := \left[[x - 12, 12], [x - 7, 7], \left[\left[0, 0, \frac{4}{5} \right], \left[0, 0, \frac{16}{5} \right] \right], \left[\left[0, \frac{4}{5}, \frac{4}{5} \right], \left[0, \frac{16}{5}, \frac{16}{5} \right] \right], \right. \quad (105)$$

$$\left. \left[\left[[0, 0], \left[\frac{4}{5}, 0 \right], \left[\frac{4}{5}, 0 \right] \right], \left[[0, 0], \left[\frac{16}{5}, 0 \right], \left[\frac{16}{5}, 0 \right] \right] \right] \right]$$

> RSreg:= Sregseptrue2F2(L,Sreg,ext);

$$RSreg := \left[[], [], \left[[x - 12, 12], [x - 7, 7], \left[\left[0, 0, \frac{4}{5} \right], \left[0, 0, \frac{16}{5} \right] \right], \left[\left[\left[\frac{4}{5}, \frac{4}{5} \right], [0] \right], \right. \quad (106)$$

$$\left. \left[\left[\frac{16}{5}, \frac{16}{5} \right], [0] \right] \right] \right]$$

> Rl:=IrrRegAppsing2F2(L,t,E,ext);

$$Rl := \left[\left[[\infty, \infty], [x - 3, 3] \right], \left[\left[\frac{3}{7}, 3a2, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a2 + \frac{111}{35} \right], \left[\frac{2}{7}, 2a2, \right. \quad (107)$$

$$\left. \frac{9216}{t^2} - \frac{5120}{t} - 2a2 + \frac{74}{35} \right], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a2 + \frac{96}{35}, -\frac{6}{t^3} + \frac{136}{t^2} \right. \right. \\ \left. \left. - \frac{834}{t} - 6a2 + \frac{111}{35}, -\frac{3}{7} + 3a2 \right], \left[\frac{9216}{t^2} - \frac{5120}{t} - 2a2 + \frac{64}{35}, \frac{9216}{t^2} - \frac{5120}{t} \right. \right. \\ \left. \left. - 4a2 + \frac{74}{35}, 2a2 - \frac{2}{7} \right] \right], [3, 2], [1, 1], \left[\left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a2 + \frac{111}{35}, \frac{3}{7} \right], \right. \right. \\ \left. \left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a2 + \frac{111}{35}, 3a2 \right], \left[3a2, \frac{3}{7} \right] \right], \left[\left[\frac{9216}{t^2} - \frac{5120}{t} - 2a2 \right. \right. \\ \left. \left. + \frac{74}{35}, \frac{2}{7} \right], \left[\frac{9216}{t^2} - \frac{5120}{t} - 2a2 + \frac{74}{35}, 2a2 \right], \left[2a2, \frac{2}{7} \right] \right], \left[[-6t^3 + 136t^2 \right. \right. \\ \left. \left. - 834t, -6t^3 + 136t^2 - 834t, 0] \right], [9216t^2 - 5120t, 9216t^2 - 5120t, 0] \right], \left[\left[-3a2 \right. \right. \\ \left. \left. + \frac{96}{35}, -6a2 + \frac{111}{35}, -\frac{3}{7} + 3a2 \right], \left[-2a2 + \frac{64}{35}, -4a2 + \frac{74}{35}, 2a2 - \frac{2}{7} \right] \right], \left[[x \right. \right. \\ \left. \left. - 12, 12], [x - 7, 7] \right], \left[\left[\left[0, 0, \frac{4}{5} \right], \left[0, \frac{4}{5}, \frac{4}{5} \right], [1, 1, 1], \left[[0, 0], \left[\frac{4}{5}, 0 \right], \left[\frac{4}{5}, 0 \right] \right], 3 \right], \right. \right. \\ \left. \left[\left[0, 0, \frac{16}{5} \right], \left[0, \frac{16}{5}, \frac{16}{5} \right], [1, 1, 1], \left[[0, 0], \left[\frac{16}{5}, 0 \right], \left[\frac{16}{5}, 0 \right] \right], 3 \right] \right] \right]$$

$$\begin{aligned}
& -834t, -6t^3 + 136t^2 - 834t, 0], [9216t^2 - 5120t, 9216t^2 - 5120t, 0]], \left[\left[-3a2 \right. \right. \\
& + \frac{96}{35}, -6a2 + \frac{111}{35}, -\frac{3}{7} + 3a2], \left[-2a2 + \frac{64}{35}, -4a2 + \frac{74}{35}, 2a2 - \frac{2}{7} \right] \right], \left[[x \right. \\
& - 12, 12], [x - 7, 7]], \left[\left[0, 0, \frac{4}{5} \right], \left[0, 0, \frac{16}{5} \right] \right], \left[\left[0, \frac{4}{5}, \frac{4}{5} \right], \left[0, \frac{16}{5}, \frac{16}{5} \right] \right], \left[\left[0, 0 \right], \right. \\
& \left. \left[\frac{4}{5}, 0 \right], \left[\frac{4}{5}, 0 \right] \right], \left[[0, 0], \left[\frac{16}{5}, 0 \right], \left[\frac{16}{5}, 0 \right] \right] \right], \left[[], [], \left[[x - 12, 12], [x - 7, 7] \right], \right. \\
& \left. \left[\left[0, 0, \frac{4}{5} \right], \left[0, 0, \frac{16}{5} \right] \right], \left[\left[\frac{4}{5}, \frac{4}{5} \right], [0] \right], \left[\left[\frac{16}{5}, \frac{16}{5} \right], [0] \right] \right] \right], \left[\left[\left[x^2 - \frac{32}{3}x - 1, \right. \right. \right. \\
& \left. \left. \left. \text{RootOf}(3Z^2 - 32Z - 3) \right] \right], [0, 2, 4], [2, 4, 2], [[2, 0], [4, 0], [4, 2]] \right], \left[[\infty, \right. \\
& \infty], [x - 12, 12], [x - 7, 7], [x - 3, 3]], \left[\left[\frac{3}{7}, 3a2, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a2 \right. \right. \\
& + \frac{111}{35} \right], \left[0, 0, \frac{4}{5} \right], \left[0, 0, \frac{16}{5} \right], \left[\frac{2}{7}, 2a2, \frac{9216}{t^2} - \frac{5120}{t} - 2a2 + \frac{74}{35} \right], \left[\left[-\frac{6}{t^3} \right. \right. \\
& + \frac{136}{t^2} - \frac{834}{t} - 3a2 + \frac{96}{35}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 6a2 + \frac{111}{35}, -\frac{3}{7} + 3a2 \right], \left[0, \right. \\
& \frac{4}{5}, \frac{4}{5} \right], \left[0, \frac{16}{5}, \frac{16}{5} \right], \left[\frac{9216}{t^2} - \frac{5120}{t} - 2a2 + \frac{64}{35}, \frac{9216}{t^2} - \frac{5120}{t} - 4a2 + \frac{74}{35}, \right. \\
& 2a2 - \frac{2}{7} \right], \left[\left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a2 + \frac{111}{35}, \frac{3}{7} \right], \left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a2 \right. \right. \right. \\
& + \frac{111}{35}, 3a2 \right], \left[3a2, \frac{3}{7} \right], \left[[0, 0], \left[\frac{4}{5}, 0 \right], \left[\frac{4}{5}, 0 \right] \right], \left[[0, 0], \left[\frac{16}{5}, 0 \right], \left[\frac{16}{5}, 0 \right] \right], \\
& \left[\left[\frac{9216}{t^2} - \frac{5120}{t} - 2a2 + \frac{74}{35}, \frac{2}{7} \right], \left[\frac{9216}{t^2} - \frac{5120}{t} - 2a2 + \frac{74}{35}, 2a2 \right], \left[2a2, \right. \right. \\
& \left. \left. \frac{2}{7} \right] \right], [[1, 1, 1], [1, 1, 1], [1, 1, 1], [1, 1, 1]] \right]
\end{aligned}$$

> F1:= Hyp2F2Subst(L,x,t,R1[1],ext);

$$\begin{aligned}
F1 := & \left[-\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 1193x + 9984)}{(x-3)^2}, \right. \\
& -\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 6313x - 9984)}{(x-3)^2}, \\
& \frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 6313x - 9984)}{(x-3)^2}, \\
& \left. \frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 1193x + 9984)}{(x-3)^2} \right]
\end{aligned} \tag{108}$$

> B:= Candichangvar2F2(F1,R1,ext);

(109)

$$B := \left\{ \left[-\frac{2(x-7)^4(x-12)}{(x-3)^2}, [1, 4], \left[[x-12, 12], [x-7, 7] \right], \left[\left[0, 0, \frac{4}{5} \right], \left[0, 0, \frac{16}{5} \right] \right], \right. \right. \\ \left. \left[\left[0, \frac{4}{5}, \frac{4}{5} \right], \left[0, \frac{16}{5}, \frac{16}{5} \right] \right], \left[\left[[0, 0], \left[\frac{4}{5}, 0 \right], \left[\frac{4}{5}, 0 \right] \right], \left[[0, 0], \left[\frac{16}{5}, 0 \right], \left[\frac{16}{5}, 0 \right] \right] \right] \right] \\ \left. , \left[\frac{2(x-7)^4(x-12)}{(x-3)^2}, [1, 4], \left[[x-12, 12], [x-7, 7] \right], \left[\left[0, 0, \frac{4}{5} \right], \left[0, 0, \frac{16}{5} \right] \right], \right. \right. \\ \left. \left[\left[0, \frac{4}{5}, \frac{4}{5} \right], \left[0, \frac{16}{5}, \frac{16}{5} \right] \right], \left[\left[[0, 0], \left[\frac{4}{5}, 0 \right], \left[\frac{4}{5}, 0 \right] \right], \left[[0, 0], \left[\frac{16}{5}, 0 \right], \left[\frac{16}{5}, 0 \right] \right] \right] \right] \right\} \quad (109)$$

```
> find2F2ln(L,R1,F1,ext,x,t);
```

$$\left[\left[\left[\left[\left[\frac{1}{7}, a2 \right], \left[\frac{10}{21}, \frac{1}{3} + a2 \right], \left[\frac{17}{21}, \frac{2}{3} + a2 \right] \right], \left[1, \frac{1}{5} \right], \frac{2(x-7)^4(x-12)}{(x-3)^2} \right], \left[\left[\left[\frac{1}{7}, \right. \right. \right. \right. \right. \\ \left. \left. \left. \left. a2 \right], \left[\frac{10}{21}, \frac{1}{3} + a2 \right], \left[\frac{17}{21}, \frac{2}{3} + a2 \right] \right], \left[1, \frac{1}{5} \right], -\frac{2(x-7)^4(x-12)}{(x-3)^2} \right] \right] \right] \quad (110)$$

```
> TIME :=time();
Hyp2F2Solutions(L);
time() - TIME;
```

$$TIME := 467.203$$

$$\left\{ \left[\left[\left[\left[\left[\frac{1}{7}, a2 \right], \left[1, \frac{1}{5} \right], [0], [1] \right], \frac{2(x-7)^4(x-12)}{(x-3)^2} \right] \right] \right] \right] \right\} \\ 23.437 \quad (111)$$

```
> F:=sumdiffeq(hyperterm([a1,a2],[b1,b2],x,k),k,J(x));
```

$$F := \left(\frac{d^3}{dx^3} J(x) \right) x^2 + (b1 + b2 - x + 1) \left(\frac{d^2}{dx^2} J(x) \right) x - (a1 x + a2 x - b2 b1 + x) \left(\frac{d}{dx} J(x) \right) - a1 a2 J(x) = 0 \quad (112)$$

```
> LA:=de2diffop(F,J(x));
```

$$LA := x^2 Dx^3 + (b1 x + b2 x - x^2 + x) Dx^2 + (-a1 x - a2 x + b2 b1 - x) Dx - a1 a2 \quad (113)$$

```
> L22:=subs({a1=RootOf(x^2+7),b1=1,b2=1/5},LA);
```

$$L22 := x^2 Dx^3 + \left(\frac{11}{5} x - x^2 \right) Dx^2 + \left(-RootOf(_Z^2 + 7) x - a2 x + \frac{1}{5} - x \right) Dx \\ - RootOf(_Z^2 + 7) a2 \quad (114)$$

```
> f:= normal((2*(x-7)^4*(x-12))/(x-3)^2,expanded);
```

$$f := \frac{2x^5 - 80x^4 + 1260x^3 - 9800x^2 + 37730x - 57624}{x^2 - 6x + 9} \quad (115)$$

```
> L:=ChangeOfVariables(L22,f);
```

$$L := 5(x-12)^2(x-7)^3(3x^2 - 32x - 3)^3(x-3)^5 Dx^3 - (x-12)(90x^9 - 5520x^8 \\ + 143560x^7 - 2051089x^6 + 17367186x^5 - 86903169x^4 + 235727812x^3 - 253798731x^2$$

$$\begin{aligned}
& -54600264 x - 1447011) (x-7)^2 (x-3)^2 (3x^2 - 32x - 3)^2 Dx^2 - (12547953567 \\
& + 463534640604 x - 23337720 \text{RootOf}(_Z^2 + 7) - 23337720 a2 - 2430646524972 x^3 \\
& - 980462070 a2 x + 2060217833859 x^2 + 1058996054022 x^4 + 810 a2 x^{13} - 66960 a2 x^{12} \\
& + 2442420 a2 x^{11} - 51559080 a2 x^{10} + 693472150 a2 x^9 - 6152442040 a2 x^8 \\
& + 35885534040 a2 x^7 - 131657849680 a2 x^6 + 268933921270 a2 x^5 \\
& - 200839588800 a2 x^4 - 100104531660 a2 x^3 - 15190527240 a2 x^2 + 212576484 x^9 \\
& - 1647165509 x^8 + 6401973348 x^7 + 6470456238 x^6 - 209991383422 x^5 + 270 x^{13} \\
& - 22320 x^{12} + 808440 x^{11} - 16694497 x^{10} - 980462070 \text{RootOf}(_Z^2 + 7) x \\
& + 810 \text{RootOf}(_Z^2 + 7) x^{13} - 66960 \text{RootOf}(_Z^2 + 7) x^{12} + 2442420 \text{RootOf}(_Z^2 \\
& + 7) x^{11} - 51559080 \text{RootOf}(_Z^2 + 7) x^{10} + 693472150 \text{RootOf}(_Z^2 + 7) x^9 \\
& - 6152442040 \text{RootOf}(_Z^2 + 7) x^8 + 35885534040 \text{RootOf}(_Z^2 + 7) x^7 \\
& - 131657849680 \text{RootOf}(_Z^2 + 7) x^6 + 268933921270 \text{RootOf}(_Z^2 + 7) x^5 \\
& - 200839588800 \text{RootOf}(_Z^2 + 7) x^4 - 100104531660 \text{RootOf}(_Z^2 + 7) x^3 \\
& - 15190527240 \text{RootOf}(_Z^2 + 7) x^2) (3x^2 - 32x - 3) (x-3) (x-7) Dx \\
& - 10 \text{RootOf}(_Z^2 + 7) a2 (x-7)^4 (3x^2 - 32x - 3)^6
\end{aligned}$$

> ext:=indets(L,{RootOf,name}) minus {x,Dx};

$$ext := \{a2, \text{RootOf}(_Z^2 + 7)\} \quad (117)$$

> ext:= indets(map(s-> ReplirrRoot(s,{ }),ext),{RootOf,name});

$$ext := \{a2, \text{RootOf}(_Z^2 + 7)\} \quad (118)$$

> extppp:={};

$$extppp := \emptyset \quad (119)$$

> E:= Singular(L,extppp);

$$E := \left[[x-7, 7], [x-3, 3], \left[x^2 - \frac{32}{3}x - 1, \text{RootOf}(3_Z^2 - 32_Z - 3) \right], [\infty, \infty], [x \right. \\ \left. - 12, 12] \right] \quad (120)$$

> F:= NotAppSing(L,E,ext);

$$F := [[\infty, \infty], [x-12, 12], [x-7, 7], [x-3, 3]] \quad (121)$$

> Sirr:= irrSing2F2(L,t,F,ext);

$$\begin{aligned}
Sirr := & \left[[[\infty, \infty], [x-3, 3]], \left[\left[3a2, 3\text{RootOf}(_Z^2 + 7), -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a2 \right. \right. \right. \\
& \left. \left. - 3\text{RootOf}(_Z^2 + 7) + \frac{18}{5} \right], \left[2a2, 2\text{RootOf}(_Z^2 + 7), \frac{9216}{t^2} - \frac{5120}{t} - 2a2 \right. \right. \\
& \left. \left. - 2\text{RootOf}(_Z^2 + 7) + \frac{12}{5} \right] \right], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 6a2 - 3\text{RootOf}(_Z^2 + 7) \right. \right. \\
& \left. \left. + \frac{18}{5}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a2 - 6\text{RootOf}(_Z^2 + 7) + \frac{18}{5}, -3a2 + 3\text{RootOf}(_Z^2 \right. \right. \\
& \left. \left. + 7) \right], \left[\frac{9216}{t^2} - \frac{5120}{t} - 4a2 - 2\text{RootOf}(_Z^2 + 7) + \frac{12}{5}, \frac{9216}{t^2} - \frac{5120}{t} - 2a2 \right. \right.
\end{aligned} \quad (122)$$

$$\begin{aligned}
& -4 \operatorname{RootOf}(_Z^2 + 7) + \frac{12}{5}, -2 a2 + 2 \operatorname{RootOf}(_Z^2 + 7) \Big], [3, 2], [1, 1], \Big[\Big[-\frac{6}{t^3} \\
& + \frac{136}{t^2} - \frac{834}{t} - 3 a2 - 3 \operatorname{RootOf}(_Z^2 + 7) + \frac{18}{5}, 3 a2 \Big], \Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} \\
& - 3 a2 - 3 \operatorname{RootOf}(_Z^2 + 7) + \frac{18}{5}, 3 \operatorname{RootOf}(_Z^2 + 7) \Big], [3 \operatorname{RootOf}(_Z^2 + 7), 3 a2] \Big], \\
& \Big[\Big[\frac{9216}{t^2} - \frac{5120}{t} - 2 a2 - 2 \operatorname{RootOf}(_Z^2 + 7) + \frac{12}{5}, 2 a2 \Big], \Big[\frac{9216}{t^2} - \frac{5120}{t} - 2 a2 \\
& - 2 \operatorname{RootOf}(_Z^2 + 7) + \frac{12}{5}, 2 \operatorname{RootOf}(_Z^2 + 7) \Big], [2 \operatorname{RootOf}(_Z^2 + 7), 2 a2] \Big] \Big], [[-6 t^3 \\
& + 136 t^2 - 834 t, -6 t^3 + 136 t^2 - 834 t, 0], [9216 t^2 - 5120 t, 9216 t^2 - 5120 t, 0]], \Big[\Big[\\
& -6 a2 - 3 \operatorname{RootOf}(_Z^2 + 7) + \frac{18}{5}, -3 a2 - 6 \operatorname{RootOf}(_Z^2 + 7) + \frac{18}{5}, -3 a2 \\
& + 3 \operatorname{RootOf}(_Z^2 + 7) \Big], \Big[-4 a2 - 2 \operatorname{RootOf}(_Z^2 + 7) + \frac{12}{5}, -2 a2 - 4 \operatorname{RootOf}(_Z^2 \\
& + 7) + \frac{12}{5}, -2 a2 + 2 \operatorname{RootOf}(_Z^2 + 7) \Big] \Big], \Big[[x - 12, 12], [x - 7, 7], \Big[\Big[0, 0, \frac{4}{5} \Big], \\
& \Big[0, \frac{4}{5}, \frac{4}{5} \Big], [1, 1, 1], \Big[[0, 0], \Big[\frac{4}{5}, 0 \Big], \Big[\frac{4}{5}, 0 \Big] \Big], 3 \Big], \Big[\Big[0, 0, \frac{16}{5} \Big], \Big[0, \frac{16}{5}, \frac{16}{5} \Big], [1, 1, \\
& 1], \Big[[0, 0], \Big[\frac{16}{5}, 0 \Big], \Big[\frac{16}{5}, 0 \Big] \Big], 3 \Big] \Big] \Big] \Big]
\end{aligned}$$

> Sreg:= regsingtrue2F2(L,t,Sirr[-1],ext);

$$Sreg := \Big[[x - 12, 12], [x - 7, 7], \Big[\Big[0, 0, \frac{4}{5} \Big], \Big[0, 0, \frac{16}{5} \Big] \Big], \Big[\Big[0, \frac{4}{5}, \frac{4}{5} \Big], \Big[0, \frac{16}{5}, \frac{16}{5} \Big] \Big], \quad (123) \\
\Big[\Big[[0, 0], \Big[\frac{4}{5}, 0 \Big], \Big[\frac{4}{5}, 0 \Big] \Big], \Big[[0, 0], \Big[\frac{16}{5}, 0 \Big], \Big[\frac{16}{5}, 0 \Big] \Big] \Big] \Big]$$

> RSreg:= Sregseptrue2F2(L,Sreg,ext);

$$RSreg := \Big[[], [], \Big[[x - 12, 12], [x - 7, 7], \Big[\Big[0, 0, \frac{4}{5} \Big], \Big[0, 0, \frac{16}{5} \Big] \Big], \Big[\Big[\Big[\frac{4}{5}, \frac{4}{5} \Big], [0] \Big], \quad (124) \\
\Big[\Big[\frac{16}{5}, \frac{16}{5} \Big], [0] \Big] \Big] \Big]$$

> R1:=IrrRegAppsing2F2(L,t,E,ext);

$$\begin{aligned}
R1 := & \Big[\Big[[\infty, \infty], [x - 3, 3], \Big[\Big[3 a2, 3 \operatorname{RootOf}(_Z^2 + 7), -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 a2 \\
& - 3 \operatorname{RootOf}(_Z^2 + 7) + \frac{18}{5} \Big], \Big[2 a2, 2 \operatorname{RootOf}(_Z^2 + 7), \frac{9216}{t^2} - \frac{5120}{t} - 2 a2 \\
& - 2 \operatorname{RootOf}(_Z^2 + 7) + \frac{12}{5} \Big] \Big], \Big[\Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 6 a2 - 3 \operatorname{RootOf}(_Z^2 + 7) \\
& + \frac{18}{5}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 a2 - 6 \operatorname{RootOf}(_Z^2 + 7) + \frac{18}{5}, -3 a2 + 3 \operatorname{RootOf}(_Z^2
\end{aligned} \quad (125)$$

$$\begin{aligned}
& + 7) \Big], \Big[\frac{9216}{t^2} - \frac{5120}{t} - 4a2 - 2 \operatorname{RootOf}(_Z^2 + 7) + \frac{12}{5}, \frac{9216}{t^2} - \frac{5120}{t} - 2a2 \\
& - 4 \operatorname{RootOf}(_Z^2 + 7) + \frac{12}{5}, -2a2 + 2 \operatorname{RootOf}(_Z^2 + 7) \Big], [3, 2], [1, 1], \Big[\Big[-\frac{6}{t^3} \\
& + \frac{136}{t^2} - \frac{834}{t} - 3a2 - 3 \operatorname{RootOf}(_Z^2 + 7) + \frac{18}{5}, 3a2 \Big], \Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} \\
& - 3a2 - 3 \operatorname{RootOf}(_Z^2 + 7) + \frac{18}{5}, 3 \operatorname{RootOf}(_Z^2 + 7) \Big], [3 \operatorname{RootOf}(_Z^2 + 7), 3a2] \Big], \\
& \Big[\Big[\frac{9216}{t^2} - \frac{5120}{t} - 2a2 - 2 \operatorname{RootOf}(_Z^2 + 7) + \frac{12}{5}, 2a2 \Big], \Big[\frac{9216}{t^2} - \frac{5120}{t} - 2a2 \\
& - 2 \operatorname{RootOf}(_Z^2 + 7) + \frac{12}{5}, 2 \operatorname{RootOf}(_Z^2 + 7) \Big], [2 \operatorname{RootOf}(_Z^2 + 7), 2a2] \Big] \Big], [[-6t^3 \\
& + 136t^2 - 834t, -6t^3 + 136t^2 - 834t, 0], [9216t^2 - 5120t, 9216t^2 - 5120t, 0]], \Big[\Big[\\
& -6a2 - 3 \operatorname{RootOf}(_Z^2 + 7) + \frac{18}{5}, -3a2 - 6 \operatorname{RootOf}(_Z^2 + 7) + \frac{18}{5}, -3a2 \\
& + 3 \operatorname{RootOf}(_Z^2 + 7) \Big], \Big[-4a2 - 2 \operatorname{RootOf}(_Z^2 + 7) + \frac{12}{5}, -2a2 - 4 \operatorname{RootOf}(_Z^2 \\
& + 7) + \frac{12}{5}, -2a2 + 2 \operatorname{RootOf}(_Z^2 + 7) \Big] \Big], \Big[[[x - 12, 12], [x - 7, 7]], \Big[\Big[0, 0, \frac{4}{5} \Big], \\
& \Big[0, 0, \frac{16}{5} \Big], \Big[\Big[0, \frac{4}{5}, \frac{4}{5} \Big], \Big[0, \frac{16}{5}, \frac{16}{5} \Big], \Big[\Big[0, 0], \Big[\frac{4}{5}, 0 \Big], \Big[\frac{4}{5}, 0 \Big], \Big[0, 0], \Big[\frac{16}{5}, 0 \Big], \\
& \Big[\frac{16}{5}, 0 \Big] \Big] \Big], \Big[[], [], \Big[[[x - 12, 12], [x - 7, 7]], \Big[\Big[0, 0, \frac{4}{5} \Big], \Big[0, 0, \frac{16}{5} \Big], \Big[\Big[\frac{4}{5}, \frac{4}{5} \Big], \\
& [0] \Big], \Big[\Big[\frac{16}{5}, \frac{16}{5} \Big], [0] \Big] \Big] \Big], \Big[\Big[\Big[x^2 - \frac{32}{3}x - 1, \operatorname{RootOf}(3_Z^2 - 32_Z - 3) \Big], [0, 2, \\
& 4], [[2, 4, 2]], [[2, 0], [4, 0], [4, 2]] \Big], \Big[[[\infty, \infty], [x - 12, 12], [x - 7, 7], [x - 3, \\
& 3]], \Big[\Big[3a2, 3 \operatorname{RootOf}(_Z^2 + 7), -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a2 - 3 \operatorname{RootOf}(_Z^2 + 7) \\
& + \frac{18}{5} \Big], \Big[0, 0, \frac{4}{5} \Big], \Big[0, 0, \frac{16}{5} \Big], \Big[2a2, 2 \operatorname{RootOf}(_Z^2 + 7), \frac{9216}{t^2} - \frac{5120}{t} - 2a2 \\
& - 2 \operatorname{RootOf}(_Z^2 + 7) + \frac{12}{5} \Big] \Big], \Big[\Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 6a2 - 3 \operatorname{RootOf}(_Z^2 + 7) \\
& + \frac{18}{5}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a2 - 6 \operatorname{RootOf}(_Z^2 + 7) + \frac{18}{5}, -3a2 + 3 \operatorname{RootOf}(_Z^2 \\
& + 7) \Big], \Big[0, \frac{4}{5}, \frac{4}{5} \Big], \Big[0, \frac{16}{5}, \frac{16}{5} \Big], \Big[\frac{9216}{t^2} - \frac{5120}{t} - 4a2 - 2 \operatorname{RootOf}(_Z^2 + 7) \\
& + \frac{12}{5}, \frac{9216}{t^2} - \frac{5120}{t} - 2a2 - 4 \operatorname{RootOf}(_Z^2 + 7) + \frac{12}{5}, -2a2 + 2 \operatorname{RootOf}(_Z^2
\end{aligned}$$

$$\left[-\frac{2(x-7)^4(x-12)}{(x-3)^2} \right]$$

```
> TIME :=time();
Hyp2F2Solutions(L);
time() - TIME;
```

TIME := 571.328

$$\left\{ \left[\left[\left[[a2, \text{RootOf}(_Z^2 + 7)] \right], \left[1, \frac{1}{5} \right], [0], [1] \right] \right], \frac{2(x-7)^4(x-12)}{(x-3)^2} \right\}$$

27.156

(129)

```
> F:=sumdiffop(hyperterm([a1,a2],[b1,b2],x,k),k,J(x));
```

$$F := \left(\frac{d^3}{dx^3} J(x) \right) x^2 + (b1 + b2 - x + 1) \left(\frac{d^2}{dx^2} J(x) \right) x - (a1 x + a2 x - b2 b1 + x) \left(\frac{d}{dx} J(x) \right) - a1 a2 J(x) = 0 \quad (130)$$

```
> LA:=de2diffop(F,J(x));
```

$$LA := x^2 Dx^3 + (b1 x + b2 x - x^2 + x) Dx^2 + (-a1 x - a2 x + b2 b1 - x) Dx - a1 a2 \quad (131)$$

```
> L22:=subs({a1=RootOf(x^2+2),b2=1},LA);
```

$$L22 := x^2 Dx^3 + (b1 x - x^2 + 2 x) Dx^2 + (-\text{RootOf}(_Z^2 + 2) x - a2 x + b1 - x) Dx - \text{RootOf}(_Z^2 + 2) a2 \quad (132)$$

```
> f:= normal((2*(x-7)^4*(x-12))/(x-3)^2,expanded);
```

$$f := \frac{2x^5 - 80x^4 + 1260x^3 - 9800x^2 + 37730x - 57624}{x^2 - 6x + 9} \quad (133)$$

```
> L:=ChangeOfVariables(L22,f);
```

$$L := (x-12)^2 (x-7)^3 (3x^2 - 32x - 3)^3 (x-3)^5 Dx^3 + (x-12) (x-7)^2 (x-3)^2 (3x^2 - 32x - 3)^2 (-18x^9 + 1104x^8 + 9b1x^6 - 28712x^7 - 246b1x^5 + 410216x^6 + 2239b1x^4 - 3473388x^5 - 7572b1x^3 + 17380186x^4 + 7911b1x^2 - 47144048x^3 + 1674b1x + 50758164x^2 + 81b1 + 10919718x + 289386) Dx^2 - (2509453224 + 92704113306x - 4667544a2 + 687447b1 - 486113761116x^3 + 14074074b1x - 196092414a2x + 412030693458x^2 + 211793329632x^4 + 162a2x^{13} - 13392a2x^{12} + 488484a2x^{11} - 10311816a2x^{10} + 138694430a2x^9 - 1230488408a2x^8 + 7177106808a2x^7 - 26331569936a2x^6 + 53786784254a2x^5 - 40167917760a2x^4 - 20020906332a2x^3 - 3038105448a2x^2 + 42515034x^9 - 329428120x^8 + 1280351044x^7 + 1294218556x^6 - 41997635114x^5 + 54x^{13} - 4464x^{12} + 161688x^{11} - 3338894x^{10} - 196092414\text{RootOf}(_Z^2 + 2)x - 27b1x^{10} + 1314b1x^9 - 24909b1x^8 + 218128b1x^7 - 636542b1x^6 - 3207852b1x^5 + 29405862b1x^4 - 77719392b1x^3 + 64366569b1x^2 - 4667544\text{RootOf}(_Z^2 + 2) + 162\text{RootOf}(_Z^2 + 2)x^{13} \quad (134)$$

$$\begin{aligned}
& -13392 \operatorname{RootOf}(_Z^2 + 2) x^{12} + 488484 \operatorname{RootOf}(_Z^2 + 2) x^{11} - 10311816 \operatorname{RootOf}(_Z^2 + 2) x^{10} \\
& + 138694430 \operatorname{RootOf}(_Z^2 + 2) x^9 - 1230488408 \operatorname{RootOf}(_Z^2 + 2) x^8 \\
& + 7177106808 \operatorname{RootOf}(_Z^2 + 2) x^7 - 26331569936 \operatorname{RootOf}(_Z^2 + 2) x^6 \\
& + 53786784254 \operatorname{RootOf}(_Z^2 + 2) x^5 - 40167917760 \operatorname{RootOf}(_Z^2 + 2) x^4 \\
& - 20020906332 \operatorname{RootOf}(_Z^2 + 2) x^3 - 3038105448 \operatorname{RootOf}(_Z^2 + 2) x^2 (3x^2 - 32x - 3) \\
& (x - 3) (x - 7) Dx - 2 \operatorname{RootOf}(_Z^2 + 2) a2 (x - 7)^4 (3x^2 - 32x - 3)^6
\end{aligned}$$

$$\begin{aligned}
& > \text{ext} := \text{indets}(\text{L}, \{\text{RootOf}, \text{name}\}) \text{ minus } \{\text{x}, \text{Dx}\}; \\
& \text{ext} := \{a2, b1, \operatorname{RootOf}(_Z^2 + 2)\} \tag{135}
\end{aligned}$$

$$\begin{aligned}
& > \text{ext} := \text{indets}(\text{map}(\text{s} \rightarrow \text{ReplirrRoot}(\text{s}, \{\}), \text{ext}), \{\text{RootOf}, \text{name}\}); \\
& \text{ext} := \{a2, b1, \operatorname{RootOf}(_Z^2 + 2)\} \tag{136}
\end{aligned}$$

$$\begin{aligned}
& > \text{extppp} := \{\}; \\
& \text{extppp} := \emptyset \tag{137}
\end{aligned}$$

$$\begin{aligned}
& > \text{E} := \text{Singular}(\text{L}, \text{extppp}); \\
& \text{E} := \left[[x - 7, 7], [x - 3, 3], \left[x^2 - \frac{32}{3}x - 1, \operatorname{RootOf}(3_Z^2 - 32_Z - 3) \right], [\infty, \infty], [x \right. \\
& \quad \left. - 12, 12] \right] \tag{138}
\end{aligned}$$

$$\begin{aligned}
& > \text{F} := \text{NotAppSing}(\text{L}, \text{E}, \text{ext}); \\
& \text{F} := [[\infty, \infty], [x - 12, 12], [x - 7, 7], [x - 3, 3]] \tag{139}
\end{aligned}$$

$$\begin{aligned}
& > \text{Sirr} := \text{irrsing2F2}(\text{L}, \text{t}, \text{F}, \text{ext}); \\
& \text{Sirr} := \left[[[\infty, \infty], [x - 3, 3]], \left[\left[3a2, 3 \operatorname{RootOf}(_Z^2 + 2), -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a2 \right. \right. \right. \\
& \quad \left. \left. - 3 \operatorname{RootOf}(_Z^2 + 2) + 3 + 3b1 \right], \left[2a2, 2 \operatorname{RootOf}(_Z^2 + 2), \frac{9216}{t^2} - \frac{5120}{t} - 2a2 \right. \right. \\
& \quad \left. \left. - 2 \operatorname{RootOf}(_Z^2 + 2) + 2 + 2b1 \right], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 6a2 - 3 \operatorname{RootOf}(_Z^2 + 2) \right. \right. \right. \\
& \quad \left. \left. + 2) + 3 + 3b1, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a2 - 6 \operatorname{RootOf}(_Z^2 + 2) + 3 + 3b1, -3a2 \right. \right. \\
& \quad \left. \left. + 3 \operatorname{RootOf}(_Z^2 + 2) \right], \left[\frac{9216}{t^2} - \frac{5120}{t} - 4a2 - 2 \operatorname{RootOf}(_Z^2 + 2) + 2 + 2b1, \right. \right. \\
& \quad \left. \left. \frac{9216}{t^2} - \frac{5120}{t} - 2a2 - 4 \operatorname{RootOf}(_Z^2 + 2) + 2 + 2b1, -2a2 + 2 \operatorname{RootOf}(_Z^2 + 2) \right] \right], \\
& [3, 2], [1, 1], \left[\left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a2 - 3 \operatorname{RootOf}(_Z^2 + 2) + 3 + 3b1, 3a2 \right], \left[\right. \right. \right. \\
& \quad \left. \left. -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a2 - 3 \operatorname{RootOf}(_Z^2 + 2) + 3 + 3b1, 3 \operatorname{RootOf}(_Z^2 + 2) \right], \right. \\
& \quad \left. \left[3 \operatorname{RootOf}(_Z^2 + 2), 3a2 \right] \right], \left[\left[\frac{9216}{t^2} - \frac{5120}{t} - 2a2 - 2 \operatorname{RootOf}(_Z^2 + 2) + 2 \right. \right. \\
& \quad \left. \left. + 2b1, 2a2 \right], \left[\frac{9216}{t^2} - \frac{5120}{t} - 2a2 - 2 \operatorname{RootOf}(_Z^2 + 2) + 2 + 2b1, 2 \operatorname{RootOf}(_Z^2 + 2) \right] \right] \tag{140}
\end{aligned}$$

$$+ 2) \Big], [2 \text{RootOf}(_Z^2 + 2), 2 a2] \Big] \Big], [[-6 t^3 + 136 t^2 - 834 t, -6 t^3 + 136 t^2 - 834 t, 0], [9216 t^2 - 5120 t, 9216 t^2 - 5120 t, 0]], [[-6 a2 - 3 \text{RootOf}(_Z^2 + 2) + 3 + 3 b1, -3 a2 - 6 \text{RootOf}(_Z^2 + 2) + 3 + 3 b1, -3 a2 + 3 \text{RootOf}(_Z^2 + 2)], [-4 a2 - 2 \text{RootOf}(_Z^2 + 2) + 2 + 2 b1, -2 a2 - 4 \text{RootOf}(_Z^2 + 2) + 2 + 2 b1, -2 a2 + 2 \text{RootOf}(_Z^2 + 2)]]], [[x - 12, 12], [x - 7, 7]], [[0, 0, -b1 + 1], [0, -b1 + 1, -b1 + 1], [1, 1, 1], [[0, 0], [-b1 + 1, 0], [-b1 + 1, 0]], 3], [[0, 0, -4 b1 + 4], [0, -4 b1 + 4, -4 b1 + 4], [1, 1, 1], [0, 0], [-4 b1 + 4, 0], [-4 b1 + 4, 0]], 3]]]]$$

$$\begin{aligned} & \textbf{> Sreg:= regsingtrue2F2(L,t,Sirr[-1],ext);} \\ \text{Sreg} &:= [[x - 12, 12], [x - 7, 7]], [[0, 0, -b1 + 1], [0, 0, -4 b1 + 4]], [[0, -b1 + 1, -b1 + 1], [0, -4 b1 + 4, -4 b1 + 4]], [[0, 0], [-b1 + 1, 0], [-b1 + 1, 0]], [[0, 0], [-4 b1 + 4, 0], [-4 b1 + 4, 0]]] \end{aligned} \quad (141)$$

$$\begin{aligned} & \textbf{> RSreg:= Sregseptrue2F2(L,Sreg,ext);} \\ \text{RSreg} &:= [[], []], [[x - 12, 12], [x - 7, 7]], [[0, 0, -b1 + 1], [0, 0, -4 b1 + 4]], [[[-b1 + 1, -b1 + 1], [0]], [-4 b1 + 4, -4 b1 + 4], [0]]] \end{aligned} \quad (142)$$

$$\begin{aligned} & \textbf{> R1:=IrrRegAppsing2F2(L,t,E,ext);} \\ \text{R1} &:= \left[\left[[\infty, \infty], [x - 3, 3] \right], \left[\left[3 a2, 3 \text{RootOf}(_Z^2 + 2), -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 a2 - 3 \text{RootOf}(_Z^2 + 2) + 3 + 3 b1 \right], \left[2 a2, 2 \text{RootOf}(_Z^2 + 2), \frac{9216}{t^2} - \frac{5120}{t} - 2 a2 - 2 \text{RootOf}(_Z^2 + 2) + 2 + 2 b1 \right] \right], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 6 a2 - 3 \text{RootOf}(_Z^2 + 2) + 3 + 3 b1, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 a2 - 6 \text{RootOf}(_Z^2 + 2) + 3 + 3 b1, -3 a2 + 3 \text{RootOf}(_Z^2 + 2) \right], \left[\frac{9216}{t^2} - \frac{5120}{t} - 4 a2 - 2 \text{RootOf}(_Z^2 + 2) + 2 + 2 b1, \frac{9216}{t^2} - \frac{5120}{t} - 2 a2 - 4 \text{RootOf}(_Z^2 + 2) + 2 + 2 b1, -2 a2 + 2 \text{RootOf}(_Z^2 + 2) \right] \right], \\ & [3, 2], [1, 1], \left[\left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 a2 - 3 \text{RootOf}(_Z^2 + 2) + 3 + 3 b1, 3 a2 \right], \left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 a2 - 3 \text{RootOf}(_Z^2 + 2) + 3 + 3 b1, 3 \text{RootOf}(_Z^2 + 2) \right], \right. \right. \\ & \left. \left[3 \text{RootOf}(_Z^2 + 2), 3 a2 \right] \right], \left[\left[\frac{9216}{t^2} - \frac{5120}{t} - 2 a2 - 2 \text{RootOf}(_Z^2 + 2) + 2 + 2 b1, 2 a2 \right], \left[\frac{9216}{t^2} - \frac{5120}{t} - 2 a2 - 2 \text{RootOf}(_Z^2 + 2) + 2 + 2 b1, 2 \text{RootOf}(_Z^2 + 2) \right] \right], \\ & [2 \text{RootOf}(_Z^2 + 2), 2 a2] \Big] \Big], [[-6 t^3 + 136 t^2 - 834 t, -6 t^3 + 136 t^2 - 834 t, 0], [9216 t^2 - 5120 t, 9216 t^2 - 5120 t, 0]], [[-6 a2 - 3 \text{RootOf}(_Z^2 + 2) + 3 + 3 b1, -3 a2 \end{aligned} \quad (143)$$

$$\begin{aligned}
& -6 \operatorname{RootOf}(_Z^2 + 2) + 3 + 3 b l, -3 a 2 + 3 \operatorname{RootOf}(_Z^2 + 2)], [-4 a 2 - 2 \operatorname{RootOf}(_Z^2 \\
& + 2) + 2 + 2 b l, -2 a 2 - 4 \operatorname{RootOf}(_Z^2 + 2) + 2 + 2 b l, -2 a 2 + 2 \operatorname{RootOf}(_Z^2 + 2)]] \\
&], [[[x - 12, 12], [x - 7, 7]], [[0, 0, -b l + 1], [0, 0, -4 b l + 4]], [[0, -b l + 1, -b l \\
& + 1], [0, -4 b l + 4, -4 b l + 4]], [[[0, 0], [-b l + 1, 0], [-b l + 1, 0]], [[0, 0], [-4 b l \\
& + 4, 0], [-4 b l + 4, 0]]]], [[], [], [[[x - 12, 12], [x - 7, 7]], [[0, 0, -b l + 1], [0, 0, \\
& -4 b l + 4]], [[[-b l + 1, -b l + 1], [0]], [[-4 b l + 4, -4 b l + 4], [0]]]]], \left[\left[\left[x^2 \right. \right. \right. \\
& \left. \left. \left. - \frac{32}{3} x - 1, \operatorname{RootOf}(3 _Z^2 - 32 _Z - 3) \right] \right], [[0, 2, 4]], [[2, 4, 2]], [[[2, 0], [4, 0], [4, \\
& 2]]] \right], \left[[[\infty, \infty], [x - 12, 12], [x - 7, 7], [x - 3, 3]], \left[\left[\left[3 a 2, 3 \operatorname{RootOf}(_Z^2 + 2), -\frac{6}{t^3} \right. \right. \right. \right. \\
& \left. \left. \left. + \frac{136}{t^2} - \frac{834}{t} - 3 a 2 - 3 \operatorname{RootOf}(_Z^2 + 2) + 3 + 3 b l \right], [0, 0, -b l + 1], [0, 0, -4 b l \\
& + 4], \left[2 a 2, 2 \operatorname{RootOf}(_Z^2 + 2), \frac{9216}{t^2} - \frac{5120}{t} - 2 a 2 - 2 \operatorname{RootOf}(_Z^2 + 2) + 2 \right. \right. \\
& \left. \left. + 2 b l \right] \right], \left[\left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 6 a 2 - 3 \operatorname{RootOf}(_Z^2 + 2) + 3 + 3 b l, -\frac{6}{t^3} + \frac{136}{t^2} \right. \right. \right. \\
& \left. \left. \left. - \frac{834}{t} - 3 a 2 - 6 \operatorname{RootOf}(_Z^2 + 2) + 3 + 3 b l, -3 a 2 + 3 \operatorname{RootOf}(_Z^2 + 2) \right], [0, -b l \\
& + 1, -b l + 1], [0, -4 b l + 4, -4 b l + 4], \left[\frac{9216}{t^2} - \frac{5120}{t} - 4 a 2 - 2 \operatorname{RootOf}(_Z^2 + 2) \right. \right. \\
& \left. \left. + 2 + 2 b l, \frac{9216}{t^2} - \frac{5120}{t} - 2 a 2 - 4 \operatorname{RootOf}(_Z^2 + 2) + 2 + 2 b l, -2 a 2 \right. \right. \\
& \left. \left. + 2 \operatorname{RootOf}(_Z^2 + 2) \right] \right], \left[\left[\left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 a 2 - 3 \operatorname{RootOf}(_Z^2 + 2) + 3 \right. \right. \right. \right. \\
& \left. \left. \left. + 3 b l, 3 a 2 \right], \left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 a 2 - 3 \operatorname{RootOf}(_Z^2 + 2) + 3 + 3 b l, \right. \right. \right. \\
& \left. \left. \left. 3 \operatorname{RootOf}(_Z^2 + 2) \right], [3 \operatorname{RootOf}(_Z^2 + 2), 3 a 2] \right], [[0, 0], [-b l + 1, 0], [-b l + 1, 0]], \\
& [[0, 0], [-4 b l + 4, 0], [-4 b l + 4, 0]], \left[\left[\frac{9216}{t^2} - \frac{5120}{t} - 2 a 2 - 2 \operatorname{RootOf}(_Z^2 + 2) \right. \right. \\
& \left. \left. + 2 + 2 b l, 2 a 2 \right], \left[\frac{9216}{t^2} - \frac{5120}{t} - 2 a 2 - 2 \operatorname{RootOf}(_Z^2 + 2) + 2 + 2 b l, \right. \right. \\
& \left. \left. 2 \operatorname{RootOf}(_Z^2 + 2) \right], [2 \operatorname{RootOf}(_Z^2 + 2), 2 a 2] \right] \right], [[1, 1, 1], [1, 1, 1], [1, 1, 1], [1, 1,
\end{aligned}$$

1]]]]

> F1:= Hyp2F2Subst(L,x,t,R1[1],ext);

$$F1 := \left[-\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 1193x + 9984)}{(x-3)^2}, \right. \\ \left. -\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 6313x - 9984)}{(x-3)^2}, \right. \\ \left. \frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 6313x - 9984)}{(x-3)^2}, \right. \\ \left. \frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 1193x + 9984)}{(x-3)^2} \right] \quad (144)$$

> B:= Candichangvar2F2(F1,R1,ext);

$$B := \left\{ \left[-\frac{2(x-7)^4(x-12)}{(x-3)^2}, [1, 4], [[x-12, 12], [x-7, 7]], [[0, 0, -b1+1], [0, 0, -4b1+4]], [[0, -b1+1, -b1+1], [0, -4b1+4, -4b1+4]], [[0, 0], [-b1+1, 0], [-b1+1, 0]], [[0, 0], [-4b1+4, 0], [-4b1+4, 0]] \right], \left[\frac{2(x-7)^4(x-12)}{(x-3)^2}, [1, 4], [[x-12, 12], [x-7, 7]], [[0, 0, -b1+1], [0, 0, -4b1+4]], [[0, -b1+1, -b1+1], [0, -4b1+4, -4b1+4]], [[0, 0], [-b1+1, 0], [-b1+1, 0]], [[0, 0], [-4b1+4, 0], [-4b1+4, 0]] \right] \right\} \quad (145)$$

> find2F2ln(L,R1,F1,ext,x,t);

$$\left[\left[\left[\left[a2, \text{RootOf}(_Z^2 + 2) \right], \left[\frac{1}{3} + a2, \frac{1}{3} + \text{RootOf}(_Z^2 + 2) \right], \left[\frac{2}{3} + a2, \frac{2}{3} + \text{RootOf}(_Z^2 + 2) \right] \right], [1, b1] \right], -\frac{2(x-7)^4(x-12)}{(x-3)^2}, \left[\left[\left[a2, \text{RootOf}(_Z^2 + 2) \right], \left[\frac{1}{3} + a2, \frac{1}{3} + \text{RootOf}(_Z^2 + 2) \right], \left[\frac{2}{3} + a2, \frac{2}{3} + \text{RootOf}(_Z^2 + 2) \right] \right], [1, b1] \right], \frac{2(x-7)^4(x-12)}{(x-3)^2} \right] \quad (146)$$

> TIME :=time();
Hyp2F2Solutions(L);
time() - TIME;

TIME := 709.171

$$\left[\left[\left[\left[a2, \text{RootOf}(_Z^2 + 2) \right], [1, b1], [0, 1] \right], \frac{2(x-7)^4(x-12)}{(x-3)^2} \right] \right]$$

26.954

(147)

$$\begin{aligned}
& > \mathbf{F} := \text{sumdiff}(\text{hyperterm}([a1, a2], [b1, b2], x, k), k, J(x)); \\
F &:= \left(\frac{d^3}{dx^3} J(x) \right) x^2 + (b1 + b2 - x + 1) \left(\frac{d^2}{dx^2} J(x) \right) x - (a1 x + a2 x - b2 b1 + x) \left(\frac{d}{dx} J(x) \right) - a1 a2 J(x) = 0 \quad (148)
\end{aligned}$$

$$\begin{aligned}
& > \mathbf{LA} := \text{de2diffop}(\mathbf{F}, J(x)); \\
LA &:= x^2 Dx^3 + (b1 x + b2 x - x^2 + x) Dx^2 + (-a1 x - a2 x + b2 b1 - x) Dx - a1 a2 \quad (149)
\end{aligned}$$

$$\begin{aligned}
& > \mathbf{L22} := \text{subs}(\{b2=1\}, LA); \\
L22 &:= x^2 Dx^3 + (b1 x - x^2 + 2 x) Dx^2 + (-a1 x - a2 x + b1 - x) Dx - a1 a2 \quad (150)
\end{aligned}$$

$$\begin{aligned}
& > \mathbf{f} := \text{normal}((2*(x-7)^4*(x-12))/(x-3)^2, \text{expanded}); \\
f &:= \frac{2x^5 - 80x^4 + 1260x^3 - 9800x^2 + 37730x - 57624}{x^2 - 6x + 9} \quad (151)
\end{aligned}$$

$$\begin{aligned}
& > \mathbf{L} := \text{ChangeOfVariables}(L22, f); \\
L &:= (x-12)^2 (x-7)^3 (3x^2 - 32x - 3)^3 (x-3)^5 Dx^3 + (x-12) (x-7)^2 (x-3)^2 (3x^2 - 32x - 3)^2 (-18x^9 + 1104x^8 + 9b1x^6 - 28712x^7 - 246b1x^5 + 410216x^6 \\
&+ 2239b1x^4 - 3473388x^5 - 7572b1x^3 + 17380186x^4 + 7911b1x^2 - 47144048x^3 \\
&+ 1674b1x + 50758164x^2 + 81b1 + 10919718x + 289386) Dx^2 - (x-7) (x-3) (3x^2 - 32x - 3) (162a1x^{13} + 162a2x^{13} - 13392a1x^{12} - 13392a2x^{12} + 54x^{13} \\
&+ 488484a1x^{11} + 488484a2x^{11} - 4464x^{12} - 10311816a1x^{10} - 10311816a2x^{10} \\
&- 27b1x^{10} + 161688x^{11} + 138694430a1x^9 + 138694430a2x^9 + 1314b1x^9 \\
&- 3338894x^{10} - 1230488408a1x^8 - 1230488408a2x^8 - 24909b1x^8 + 42515034x^9 \\
&+ 7177106808a1x^7 + 7177106808a2x^7 + 218128b1x^7 - 329428120x^8 \\
&- 26331569936a1x^6 - 26331569936a2x^6 - 636542b1x^6 + 1280351044x^7 \\
&+ 53786784254a1x^5 + 53786784254a2x^5 - 3207852b1x^5 + 1294218556x^6 \\
&- 40167917760a1x^4 - 40167917760a2x^4 + 29405862b1x^4 - 41997635114x^5 \\
&- 20020906332a1x^3 - 20020906332a2x^3 - 77719392b1x^3 + 211793329632x^4 \\
&- 3038105448a1x^2 - 3038105448a2x^2 + 64366569b1x^2 - 486113761116x^3 \\
&- 196092414a1x - 196092414a2x + 14074074b1x + 412030693458x^2 - 4667544a1 \\
&- 4667544a2 + 687447b1 + 92704113306x + 2509453224) Dx - 2a1a2 (x-7)^4 (3x^2 - 32x - 3)^6 \quad (152)
\end{aligned}$$

$$\begin{aligned}
& > \mathbf{ext} := \text{indets}(L, \{\text{RootOf}, \text{name}\}) \text{ minus } \{x, Dx\}; \\
& \quad \text{ext} := \{a1, a2, b1\} \quad (153)
\end{aligned}$$

$$\begin{aligned}
& > \mathbf{ext} := \text{indets}(\text{map}(s \rightarrow \text{ReplirrRoot}(s, \{\}), \text{ext}), \{\text{RootOf}, \text{name}\}); \\
& \quad \text{ext} := \{a1, a2, b1\} \quad (154)
\end{aligned}$$

$$\begin{aligned}
& > \mathbf{extppp} := \{\}; \\
& \quad \text{extppp} := \emptyset \quad (155)
\end{aligned}$$

$$\begin{aligned}
& > \mathbf{E} := \text{Singular}(L, \text{extppp}); \\
E &:= \left[[x-7, 7], [x-3, 3], \left[x^2 - \frac{32}{3}x - 1, \text{RootOf}(3_Z^2 - 32_Z - 3) \right], [\infty, \infty], [x \right. \quad (156)
\end{aligned}$$

$$-12, 12]]$$

> F:= NotAppSing(L,E,ext);

$$F := [[\infty, \infty], [x-12, 12], [x-7, 7], [x-3, 3]] \quad (157)$$

> Sirr:= irrSing2F2(L,t,F,ext);

$$\begin{aligned} \text{Sirr} := & \left[[[\infty, \infty], [x-3, 3]], \left[\left[3a1, 3a2, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a1 - 3a2 + 3 \right. \right. \right. \\ & + 3b1], \left[2a1, 2a2, \frac{9216}{t^2} - \frac{5120}{t} - 2a1 - 2a2 + 2 + 2b1 \right], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} \right. \right. \\ & - \frac{834}{t} - 6a1 - 3a2 + 3 + 3b1, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a1 - 6a2 + 3 + 3b1, -3a1 \\ & + 3a2], \left[\frac{9216}{t^2} - \frac{5120}{t} - 4a1 - 2a2 + 2 + 2b1, \frac{9216}{t^2} - \frac{5120}{t} - 2a1 - 4a2 + 2 \right. \\ & + 2b1, -2a1 + 2a2], [3, 2], [1, 1], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a1 - 3a2 + 3 \right. \right. \\ & + 3b1, 3a1], \left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a1 - 3a2 + 3 + 3b1, 3a2], [3a2, 3a1], \right. \\ & \left[\left[\frac{9216}{t^2} - \frac{5120}{t} - 2a1 - 2a2 + 2 + 2b1, 2a1], \left[\frac{9216}{t^2} - \frac{5120}{t} - 2a1 - 2a2 + 2 \right. \right. \\ & + 2b1, 2a2], [2a2, 2a1]]], [[-6t^3 + 136t^2 - 834t, -6t^3 + 136t^2 - 834t, 0], [9216t^2 \\ & - 5120t, 9216t^2 - 5120t, 0]], [[-6a1 - 3a2 + 3b1 + 3, -3a1 - 6a2 + 3b1 + 3, \\ & -3a1 + 3a2], [-4a1 - 2a2 + 2b1 + 2, -2a1 - 4a2 + 2b1 + 2, -2a1 + 2a2]], [[x \\ & - 12, 12], [x-7, 7]], [[0, 0, -b1 + 1], [0, -b1 + 1, -b1 + 1], [1, 1, 1], [[0, 0], [-b1 \\ & + 1, 0], [-b1 + 1, 0]], 3], [[0, 0, -4b1 + 4], [0, -4b1 + 4, -4b1 + 4], [1, 1, 1], [[0, \\ & 0], [-4b1 + 4, 0], [-4b1 + 4, 0]], 3]]]] \end{aligned} \quad (158)$$

> Sreg:= regSingtrue2F2(L,t,Sirr[-1],ext);

$$\text{Sreg} := [[x-12, 12], [x-7, 7], [[0, 0, -b1 + 1], [0, 0, -4b1 + 4]], [[0, -b1 + 1, -b1 + 1], [0, -4b1 + 4, -4b1 + 4]], [[0, 0], [-b1 + 1, 0], [-b1 + 1, 0]], [[0, 0], [-4b1 + 4, 0], [-4b1 + 4, 0]]] \quad (159)$$

> RSreg:= Sregseptrue2F2(L,Sreg,ext);

$$\text{RSreg} := [[], [], [[x-12, 12], [x-7, 7]], [[0, 0, -b1 + 1], [0, 0, -4b1 + 4]], [[[-b1 + 1, -b1 + 1], [0]], [[-4b1 + 4, -4b1 + 4], [0]]]] \quad (160)$$

> R1:=IrrRegAppsing2F2(L,t,E,ext);

$$\begin{aligned} \text{R1} := & \left[[[\infty, \infty], [x-3, 3]], \left[\left[3a1, 3a2, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a1 - 3a2 + 3 \right. \right. \right. \\ & + 3b1], \left[2a1, 2a2, \frac{9216}{t^2} - \frac{5120}{t} - 2a1 - 2a2 + 2 + 2b1 \right], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} \right. \right. \\ & - \frac{834}{t} - 6a1 - 3a2 + 3 + 3b1, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a1 - 6a2 + 3 + 3b1, -3a1 \end{aligned} \quad (161)$$

$$\begin{aligned}
& + 3 a2 \Big], \Big[\frac{9216}{t^2} - \frac{5120}{t} - 4 a1 - 2 a2 + 2 + 2 b1, \frac{9216}{t^2} - \frac{5120}{t} - 2 a1 - 4 a2 + 2 \\
& + 2 b1, -2 a1 + 2 a2 \Big], [3, 2], [1, 1], \Big[\Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 a1 - 3 a2 + 3 \\
& + 3 b1, 3 a1 \Big], \Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 a1 - 3 a2 + 3 + 3 b1, 3 a2 \Big], [3 a2, 3 a1] \Big], \\
& \Big[\Big[\frac{9216}{t^2} - \frac{5120}{t} - 2 a1 - 2 a2 + 2 + 2 b1, 2 a1 \Big], \Big[\frac{9216}{t^2} - \frac{5120}{t} - 2 a1 - 2 a2 + 2 \\
& + 2 b1, 2 a2 \Big], [2 a2, 2 a1] \Big], \Big[[-6 t^3 + 136 t^2 - 834 t, -6 t^3 + 136 t^2 - 834 t, 0], [9216 t^2 \\
& - 5120 t, 9216 t^2 - 5120 t, 0] \Big], \Big[[-6 a1 - 3 a2 + 3 b1 + 3, -3 a1 - 6 a2 + 3 b1 + 3, \\
& -3 a1 + 3 a2], [-4 a1 - 2 a2 + 2 b1 + 2, -2 a1 - 4 a2 + 2 b1 + 2, -2 a1 + 2 a2] \Big], \\
& \Big[[[x - 12, 12], [x - 7, 7]], [[0, 0, -b1 + 1], [0, 0, -4 b1 + 4]], [[0, -b1 + 1, -b1 \\
& + 1], [0, -4 b1 + 4, -4 b1 + 4]], [[0, 0], [-b1 + 1, 0], [-b1 + 1, 0]], [[0, 0], [-4 b1 \\
& + 4, 0], [-4 b1 + 4, 0]] \Big], \Big[[[x - 12, 12], [x - 7, 7]], [[0, 0, -b1 + 1], [0, 0, \\
& -4 b1 + 4]], [[[-b1 + 1, -b1 + 1], [0]], [[-4 b1 + 4, -4 b1 + 4], [0]]] \Big], \Big[\Big[x^2 \\
& - \frac{32}{3} x - 1, \text{RootOf}(3 _Z^2 - 32 _Z - 3) \Big], [[0, 2, 4]], [[2, 4, 2]], [[2, 0], [4, 0], [4, \\
& 2]] \Big], \Big[[[\infty, \infty], [x - 12, 12], [x - 7, 7], [x - 3, 3]], \Big[\Big[3 a1, 3 a2, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} \\
& - 3 a1 - 3 a2 + 3 + 3 b1 \Big], [0, 0, -b1 + 1], [0, 0, -4 b1 + 4], \Big[2 a1, 2 a2, \frac{9216}{t^2} \\
& - \frac{5120}{t} - 2 a1 - 2 a2 + 2 + 2 b1 \Big], \Big[\Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 6 a1 - 3 a2 + 3 + 3 b1, \\
& -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 a1 - 6 a2 + 3 + 3 b1, -3 a1 + 3 a2 \Big], [0, -b1 + 1, -b1 + 1], \\
& [0, -4 b1 + 4, -4 b1 + 4], \Big[\frac{9216}{t^2} - \frac{5120}{t} - 4 a1 - 2 a2 + 2 + 2 b1, \frac{9216}{t^2} - \frac{5120}{t} \\
& - 2 a1 - 4 a2 + 2 + 2 b1, -2 a1 + 2 a2 \Big], \Big[\Big[\Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 a1 - 3 a2 + 3 \\
& + 3 b1, 3 a1 \Big], \Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 a1 - 3 a2 + 3 + 3 b1, 3 a2 \Big], [3 a2, 3 a1] \Big], \\
& [[0, 0], [-b1 + 1, 0], [-b1 + 1, 0]], [[0, 0], [-4 b1 + 4, 0], [-4 b1 + 4, 0]], \Big[\Big[\frac{9216}{t^2}
\end{aligned}$$

$$\left[-\frac{5120}{t} - 2a1 - 2a2 + 2 + 2b1, 2a1 \right], \left[\frac{9216}{t^2} - \frac{5120}{t} - 2a1 - 2a2 + 2 + 2b1, \right. \\ \left. 2a2 \right], [2a2, 2a1]]], [[1, 1, 1], [1, 1, 1], [1, 1, 1], [1, 1, 1]]]]$$

> F1:= Hyp2F2Subst(L,x,t,R1[1],ext);

$$F1 := \left[-\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 1193x + 9984)}{(x-3)^2}, \right. \\ \left. -\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 6313x - 9984)}{(x-3)^2}, \right. \\ \left. \frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 6313x - 9984)}{(x-3)^2}, \right. \\ \left. \frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 1193x + 9984)}{(x-3)^2} \right] \quad (162)$$

> B:= Candichangvar2F2(F1,R1,ext);

$$B := \left\{ \left[-\frac{2(x-7)^4(x-12)}{(x-3)^2}, [1, 4], [[x-12, 12], [x-7, 7]], [[0, 0, -b1+1], [0, 0, \right. \right. \\ \left. \left. -4b1+4]], [[0, -b1+1, -b1+1], [0, -4b1+4, -4b1+4]], [[0, 0], [-b1+1, \right. \right. \\ \left. \left. 0], [-b1+1, 0]], [[0, 0], [-4b1+4, 0], [-4b1+4, 0]]] \right\}, \left[\frac{2(x-7)^4(x-12)}{(x-3)^2}, \right. \\ \left. [1, 4], [[x-12, 12], [x-7, 7]], [[0, 0, -b1+1], [0, 0, -4b1+4]], [[0, -b1+1, \right. \right. \\ \left. \left. -b1+1], [0, -4b1+4, -4b1+4]], [[0, 0], [-b1+1, 0], [-b1+1, 0]], [[0, 0], [\right. \right. \\ \left. \left. -4b1+4, 0], [-4b1+4, 0]]] \right\} \quad (163)$$

> find2F2ln(L,R1,F1,ext,x,t);

$$\left[\left[\left[\left[[a1, a2], \left[\frac{1}{3} + a1, \frac{1}{3} + a2 \right], \left[\frac{2}{3} + a1, \frac{2}{3} + a2 \right] \right], [1, b1] \right], -\frac{2(x-7)^4(x-12)}{(x-3)^2} \right], \right. \\ \left. \left[\left[\left[[a1, a2], \left[\frac{1}{3} + a1, \frac{1}{3} + a2 \right], \left[\frac{2}{3} + a1, \frac{2}{3} + a2 \right] \right], [1, b1] \right], \frac{2(x-7)^4(x-12)}{(x-3)^2} \right] \right] \quad (164)$$

> TIME :=time();
Hyp2F2Solutions(L);
time() - TIME;

TIME := 838.828

$$\left\{ \left[[[a1, a2], [1, b1], [0], [1]], \frac{2(x-7)^4(x-12)}{(x-3)^2} \right] \right\}$$

27.484

(165)

> ##### THE RATIONAL AND IRRATIONAL CASE #####

> F:=sumdiffeq(hyperterm([a1,a2],[b1,b2],x,k),k,J(x));

$$F := \left(\frac{d^3}{dx^3} J(x) \right) x^2 + (b1 + b2 - x + 1) \left(\frac{d^2}{dx^2} J(x) \right) x - (a1 x + a2 x - b2 b1 + x) \left(\frac{d}{dx} J(x) \right) - a1 a2 J(x) = 0 \quad (166)$$

> LA:=de2diffop(F,J(x));

$$LA := x^2 Dx^3 + (b1 x + b2 x - x^2 + x) Dx^2 + (-a1 x - a2 x + b2 b1 - x) Dx - a1 a2 \quad (167)$$

> L22:=subs({a1=1/7,a2=1/3,b1=1/12,b2=1/5},LA);

$$L22 := x^2 Dx^3 + \left(\frac{77}{60} x - x^2 \right) Dx^2 + \left(-\frac{31x}{21} + \frac{1}{60} \right) Dx - \frac{1}{21} \quad (168)$$

> f:= normal((2*(x-7)^4*(x-12))/(x-3)^2,expanded);

$$f := \frac{2x^5 - 80x^4 + 1260x^3 - 9800x^2 + 37730x - 57624}{x^2 - 6x + 9} \quad (169)$$

> L:=ChangeOfVariables(L22,f);

$$L := 420 (x-12)^2 (x-7)^3 (3x^2 - 32x - 3)^3 (x-3)^5 Dx^3 - 7 (x-12) (1080x^9 - 66240x^8 + 1722720x^7 - 24612573x^6 + 208392702x^5 - 1042714883x^4 + 2828317284x^3 - 3045149667x^2 - 655111098x - 17359677) (x-7)^2 (x-3)^2 (3x^2 - 32x - 3)^2 Dx^2 - (x-7) (x-3) (3x^2 - 32x - 3) (55080x^{13} - 4553280x^{12} + 165605760x^{11} - 3464715501x^{10} + 45596018902x^9 - 384473536987x^8 + 1973332229024x^7 - 4723699528786x^6 - 6878006703596x^5 + 80908852861866x^4 - 208149528128976x^3 + 172425749804847x^2 + 38892263379102x + 1052829699201) Dx - 40 (x-7)^4 (3x^2 - 32x - 3)^6 \quad (170)$$

> ext:=indets(L,{RootOf,name}) minus {x,Dx};

$$ext := \emptyset \quad (171)$$

> ext:= indets(map(s-> ReplirrRoot(s,{ }),ext),{RootOf,name});

$$ext := \emptyset \quad (172)$$

> extppp:={};

$$extppp := \emptyset \quad (173)$$

> E:= Singular(L,extppp);

$$E := \left[[x-7, 7], [x-3, 3], \left[x^2 - \frac{32}{3}x - 1, \text{RootOf}(3_Z^2 - 32_Z - 3) \right], [\infty, \infty], [x - 12, 12] \right] \quad (174)$$

> F:= NotAppSing(L,E,ext);

$$F := [[\infty, \infty], [x-12, 12], [x-7, 7], [x-3, 3]] \quad (175)$$

> Sirr:= irrsing2F2(L,t,F,ext);

$$Sirr := \left[[[\infty, \infty], [x-3, 3]], \left[\left[1, \frac{3}{7}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - \frac{81}{140} \right], \left[\frac{2}{3}, \frac{2}{7}, \frac{9216}{t^2} - \frac{5120}{t} - \frac{27}{70} \right] \right], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - \frac{221}{140}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - \frac{141}{140}, -\frac{4}{7} \right], \left[\frac{9216}{t^2} - \frac{5120}{t} - \frac{221}{210}, \frac{9216}{t^2} - \frac{5120}{t} - \frac{47}{70}, -\frac{8}{21} \right] \right], [3, 2], [1, 1], \left[\left[-\frac{6}{t^3} \right] \right] \quad (176)$$

$$\begin{aligned} & \left[\left[\left[\frac{16}{5}, \frac{11}{3} \right], \left[\left[\left[\frac{11}{12}, \frac{4}{5}, -\frac{7}{60} \right], [] \right], \left[\left[\frac{16}{5}, \frac{11}{3}, \frac{7}{15} \right], [] \right] \right], [], [] \right], \left[\left[\left[x^2 - \frac{32}{3} x \right. \right. \right. \\ & \left. \left. \left. - 1, \text{RootOf}(3 _Z^2 - 32 _Z - 3) \right] \right], [[0, 2, 4]], [[2, 4, 2]], [[[2, 0], [4, 0], [4, 2]]] \right], \\ & \left[[[\infty, \infty], [x - 12, 12], [x - 7, 7], [x - 3, 3]], \left[\left[1, \frac{3}{7}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - \frac{81}{140} \right], \right. \right. \\ & \left[0, \frac{11}{12}, \frac{4}{5} \right], \left[0, \frac{16}{5}, \frac{11}{3} \right], \left[\frac{2}{3}, \frac{2}{7}, \frac{9216}{t^2} - \frac{5120}{t} - \frac{27}{70} \right], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} \right. \right. \\ & \left. \left. - \frac{221}{140}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - \frac{141}{140}, -\frac{4}{7} \right], \left[\frac{11}{12}, \frac{4}{5}, -\frac{7}{60} \right], \left[\frac{16}{5}, \frac{11}{3}, \frac{7}{15} \right], \right. \\ & \left[\frac{9216}{t^2} - \frac{5120}{t} - \frac{221}{210}, \frac{9216}{t^2} - \frac{5120}{t} - \frac{47}{70}, -\frac{8}{21} \right], \left[\left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} \right. \right. \right. \\ & \left. \left. \left. - \frac{81}{140}, 1 \right], \left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - \frac{81}{140}, \frac{3}{7} \right], \left[\frac{3}{7}, 1 \right], \left[\left[\frac{11}{12}, 0 \right], \left[\frac{4}{5}, 0 \right], \left[\frac{4}{5}, \right. \right. \right. \\ & \left. \left. \left. \frac{11}{12} \right] \right], \left[\left[\frac{16}{5}, 0 \right], \left[\frac{11}{3}, 0 \right], \left[\frac{11}{3}, \frac{16}{5} \right] \right], \left[\left[\frac{9216}{t^2} - \frac{5120}{t} - \frac{27}{70}, \frac{2}{3} \right], \left[\frac{9216}{t^2} \right. \right. \\ & \left. \left. - \frac{5120}{t} - \frac{27}{70}, \frac{2}{7} \right], \left[\frac{2}{7}, \frac{2}{3} \right] \right], [[1, 1, 1], [1, 1, 1], [1, 1, 1], [1, 1, 1]] \right] \end{aligned}$$

> F1 := Hyp2F2Subst(L,x,t,R1[1],ext);

$$\begin{aligned} F1 := & \left[-\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 1193x + 9984)}{(x-3)^2}, \right. \\ & -\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 6313x - 9984)}{(x-3)^2}, \\ & \frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 6313x - 9984)}{(x-3)^2}, \\ & \left. \frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 1193x + 9984)}{(x-3)^2} \right] \end{aligned} \quad (180)$$

> find2F2RatIrr(L,R1,F1,x,t,ext);

$$\begin{aligned} & \left[\left[\left[\left[-1, \frac{17}{21} \right], \left[1, \frac{17}{21} \right], \left[\frac{1}{3}, \frac{1}{7} \right], \left[\frac{2}{3}, \frac{10}{21} \right] \right], \left[\frac{1}{5}, \frac{1}{12} \right], -\frac{2(x-7)^4(x-12)}{(x-3)^2} \right], \right. \\ & \left. \left[\left[\left[-1, \frac{17}{21} \right], \left[1, \frac{17}{21} \right], \left[\frac{1}{3}, \frac{1}{7} \right], \left[\frac{2}{3}, \frac{10}{21} \right] \right], \left[\frac{1}{5}, \frac{1}{12} \right], \frac{2(x-7)^4(x-12)}{(x-3)^2} \right] \right] \end{aligned} \quad (181)$$

> TIME :=time();
Hyp2F2Solutions(L);
time() - TIME;

TIME := 883.890

$$\left\{ \left[\left[\left[\left[\frac{1}{3}, \frac{1}{7} \right], \left[\frac{1}{5}, \frac{1}{12} \right], [0], [1] \right], \frac{2(x-7)^4(x-12)}{(x-3)^2} \right] \right\}$$

5.328

(182)

$$\begin{aligned}
 &> \mathbf{F} := \text{sumdiffeq}(\text{hyperterm}([\mathbf{a1}, \mathbf{a2}], [\mathbf{b1}, \mathbf{b2}], \mathbf{x}, \mathbf{k}), \mathbf{k}, \mathbf{J}(\mathbf{x})); \\
 \mathbf{F} &:= \left(\frac{d^3}{dx^3} J(x) \right) x^2 + (b1 + b2 - x + 1) \left(\frac{d^2}{dx^2} J(x) \right) x - (a1 x + a2 x - b2 b1 + x) \left(\frac{d}{dx} J(x) \right) - a1 a2 J(x) = 0 \quad (183)
 \end{aligned}$$

$$\begin{aligned}
 &> \mathbf{LA} := \text{de2diffop}(\mathbf{F}, \mathbf{J}(\mathbf{x})); \\
 \mathbf{LA} &:= x^2 Dx^3 + (b1 x + b2 x - x^2 + x) Dx^2 + (-a1 x - a2 x + b2 b1 - x) Dx - a1 a2 \quad (184)
 \end{aligned}$$

$$\begin{aligned}
 &> \mathbf{L22} := \text{subs}(\{\mathbf{a1}=1/7, \mathbf{a2}=1/3, \mathbf{b1}=1/12, \mathbf{b2}=\text{RootOf}(\mathbf{x}^2+1)\}, \mathbf{LA}); \\
 \mathbf{L22} &:= x^2 Dx^3 + \left(\frac{13x}{12} + \text{RootOf}(_Z^2 + 1) x - x^2 \right) Dx^2 + \left(-\frac{31x}{21} + \frac{\text{RootOf}(_Z^2 + 1)}{12} \right) Dx - \frac{1}{21} \quad (185)
 \end{aligned}$$

$$\begin{aligned}
 &> \mathbf{f} := \text{normal}((2*(\mathbf{x}-7)^4*(\mathbf{x}-12))/(\mathbf{x}-3)^2, \text{expanded}); \\
 \mathbf{f} &:= \frac{2x^5 - 80x^4 + 1260x^3 - 9800x^2 + 37730x - 57624}{x^2 - 6x + 9} \quad (186)
 \end{aligned}$$

$$\begin{aligned}
 &> \mathbf{L} := \text{ChangeOfVariables}(\mathbf{L22}, \mathbf{f}); \\
 \mathbf{L} &:= 84 (x-12)^2 (x-7)^3 (3x^2 - 32x - 3)^3 (x-3)^5 Dx^3 + 7 (-216x^9 + 13248x^8 + 108 \text{RootOf}(_Z^2 + 1) x^6 - 344544x^7 - 2952 \text{RootOf}(_Z^2 + 1) x^5 + 4922493x^6 + 26868 \text{RootOf}(_Z^2 + 1) x^4 - 41677950x^5 - 90864 \text{RootOf}(_Z^2 + 1) x^3 + 208537603x^4 + 94932 \text{RootOf}(_Z^2 + 1) x^2 - 565645284x^3 + 20088 \text{RootOf}(_Z^2 + 1) x + 609010947x^2 + 972 \text{RootOf}(_Z^2 + 1) + 131018202x + 3471741) (x - 12) (3x^2 - 32x - 3)^2 (x-3)^2 (x-7)^2 Dx^2 - (x-7) (x-3) (3x^2 - 32x - 3) (210554379504 + 7778215759860x - 41628647976552x^3 + 34484061301578x^2 + 16181181010536x^4 + 9119242412x^9 - 76895466056x^8 + 394674235528x^7 - 944786658564x^6 - 1375413567304x^5 + 11016x^{13} - 910656x^{12} + 33121152x^{11} - 692943894x^{10} + 1184579802 \text{RootOf}(_Z^2 + 1) x + 57801681 \text{RootOf}(_Z^2 + 1) + 3969 \text{RootOf}(_Z^2 + 1) x^{10} - 193158 \text{RootOf}(_Z^2 + 1) x^9 + 3793293 \text{RootOf}(_Z^2 + 1) x^8 - 38948616 \text{RootOf}(_Z^2 + 1) x^7 + 233764034 \text{RootOf}(_Z^2 + 1) x^6 - 938867076 \text{RootOf}(_Z^2 + 1) x^5 + 2947809186 \text{RootOf}(_Z^2 + 1) x^4 - 6288246216 \text{RootOf}(_Z^2 + 1) x^3 + 5443296957 \text{RootOf}(_Z^2 + 1) x^2) Dx - 8 (x - 7)^4 (3x^2 - 32x - 3)^6 \quad (187)
 \end{aligned}$$

$$\begin{aligned}
 &> \mathbf{ext} := \text{indets}(\mathbf{L}, \{\text{RootOf}, \text{name}\}) \text{ minus } \{\mathbf{x}, \mathbf{Dx}\}; \\
 \mathbf{ext} &:= \{\text{RootOf}(_Z^2 + 1)\} \quad (188)
 \end{aligned}$$

$$\begin{aligned}
 &> \mathbf{ext} := \text{indets}(\text{map}(\mathbf{s} \rightarrow \text{ReplirrRoot}(\mathbf{s}, \{\}), \mathbf{ext}), \{\text{RootOf}, \text{name}\}); \\
 \mathbf{ext} &:= \{\text{RootOf}(_Z^2 + 1)\} \quad (189)
 \end{aligned}$$

$$\begin{aligned}
 &> \mathbf{extppp} := \{\}; \\
 \mathbf{extppp} &:= \emptyset \quad (190)
 \end{aligned}$$

$$> \mathbf{E} := \text{Singular}(\mathbf{L}, \mathbf{extppp});$$

$$E := \left[[x-7, 7], [x-3, 3], \left[x^2 - \frac{32}{3}x - 1, \text{RootOf}(3_Z^2 - 32_Z - 3) \right], [\infty, \infty], [x - 12, 12] \right] \quad (191)$$

$$\begin{aligned} &> \mathbf{F := NotAppSing(L, E, ext);} \\ &\quad F := [[\infty, \infty], [x-12, 12], [x-7, 7], [x-3, 3]] \end{aligned} \quad (192)$$

$$\begin{aligned} &> \mathbf{Sirr := irrsing2F2(L, t, F, ext);} \\ \text{Sirr} := &\left[[[\infty, \infty], [x-3, 3]], \left[\left[1, \frac{3}{7}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - \frac{33}{28} + 3 \text{RootOf}(_Z^2 + 1) \right], \right. \\ &\left[\frac{2}{3}, \frac{2}{7}, \frac{9216}{t^2} - \frac{5120}{t} - \frac{11}{14} + 2 \text{RootOf}(_Z^2 + 1) \right], \left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - \frac{61}{28} \right. \\ &+ 3 \text{RootOf}(_Z^2 + 1), -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - \frac{45}{28} + 3 \text{RootOf}(_Z^2 + 1), -\frac{4}{7} \left. \right], \left[\frac{9216}{t^2} \right. \\ &- \frac{5120}{t} - \frac{61}{42} + 2 \text{RootOf}(_Z^2 + 1), \frac{9216}{t^2} - \frac{5120}{t} - \frac{15}{14} + 2 \text{RootOf}(_Z^2 + 1), \\ &- \frac{8}{21} \left. \right], [3, 2], [1, 1], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - \frac{33}{28} + 3 \text{RootOf}(_Z^2 + 1), 1 \right], \left[-\frac{6}{t^3} \right. \right. \\ &+ \frac{136}{t^2} - \frac{834}{t} - \frac{33}{28} + 3 \text{RootOf}(_Z^2 + 1), \frac{3}{7} \left. \right], \left[\frac{3}{7}, 1 \right], \left[\left[\frac{9216}{t^2} - \frac{5120}{t} - \frac{11}{14} \right. \right. \\ &+ 2 \text{RootOf}(_Z^2 + 1), \frac{2}{3} \left. \right], \left[\frac{9216}{t^2} - \frac{5120}{t} - \frac{11}{14} + 2 \text{RootOf}(_Z^2 + 1), \frac{2}{7} \right], \left[\frac{2}{7}, \right. \\ &\left. \frac{2}{3} \right] \left. \right], [[-6t^3 + 136t^2 - 834t, -6t^3 + 136t^2 - 834t, 0], [9216t^2 - 5120t, 9216t^2 \\ &- 5120t, 0]], \left[\left[-\frac{61}{28} + 3 \text{RootOf}(_Z^2 + 1), -\frac{45}{28} + 3 \text{RootOf}(_Z^2 + 1), -\frac{4}{7} \right], \left[-\frac{61}{42} \right. \right. \\ &+ 2 \text{RootOf}(_Z^2 + 1), -\frac{15}{14} + 2 \text{RootOf}(_Z^2 + 1), -\frac{8}{21} \left. \right], \left[[[x-12, 12], [x-7, 7]], \right. \\ &\left[\left[\left[0, \frac{11}{12}, 1 - \text{RootOf}(_Z^2 + 1) \right], \left[\frac{11}{12}, 1 - \text{RootOf}(_Z^2 + 1), \frac{1}{12} - \text{RootOf}(_Z^2 \right. \right. \\ &+ 1) \right], [1, 1, 1], \left[\left[\frac{11}{12}, 0 \right], [1 - \text{RootOf}(_Z^2 + 1), 0], \left[1 - \text{RootOf}(_Z^2 + 1), \frac{11}{12} \right] \right], 2 \left. \right] \\ &, \left[\left[0, \frac{11}{3}, 4 - 4 \text{RootOf}(_Z^2 + 1) \right], \left[\frac{11}{3}, 4 - 4 \text{RootOf}(_Z^2 + 1), \frac{1}{3} - 4 \text{RootOf}(_Z^2 \right. \right. \\ &+ 1) \right], [1, 1, 1], \left[\left[\frac{11}{3}, 0 \right], [4 - 4 \text{RootOf}(_Z^2 + 1), 0], \left[4 - 4 \text{RootOf}(_Z^2 + 1), \right. \right. \\ &\left. \left. \frac{11}{3} \right] \right], 2 \left. \right] \left. \right] \end{aligned} \quad (193)$$

$$\begin{aligned} &> \mathbf{Sreg := regsingtrue2F2(L, t, Sirr[-1], ext);} \\ \text{Sreg} := &\left[[[x-12, 12], [x-7, 7]], \left[\left[0, \frac{11}{12}, 1 - \text{RootOf}(_Z^2 + 1) \right], \left[0, \frac{11}{3}, 4 \right. \right. \right. \\ &- 4 \text{RootOf}(_Z^2 + 1) \left. \right], \left[\left[\frac{11}{12}, 1 - \text{RootOf}(_Z^2 + 1), \frac{1}{12} - \text{RootOf}(_Z^2 + 1) \right], \left[\frac{11}{3}, \right. \right. \end{aligned} \quad (194)$$

$$4 - 4 \operatorname{RootOf}(_Z^2 + 1), \frac{1}{3} - 4 \operatorname{RootOf}(_Z^2 + 1) \Big] \Big], \Big[\Big[\frac{11}{12}, 0 \Big], [1 - \operatorname{RootOf}(_Z^2 + 1), 0], \Big[1 - \operatorname{RootOf}(_Z^2 + 1), \frac{11}{12} \Big], \Big[\Big[\frac{11}{3}, 0 \Big], [4 - 4 \operatorname{RootOf}(_Z^2 + 1), 0], \Big[4 - 4 \operatorname{RootOf}(_Z^2 + 1), \frac{11}{3} \Big] \Big] \Big] \Big]$$

> RSreg:= Sregseptrue2F2(L,Sreg,ext);

$$RSreg := \Big[\Big[[x - 12, 12], [x - 7, 7] \Big], \Big[\Big[0, \frac{11}{12}, 1 - \operatorname{RootOf}(_Z^2 + 1) \Big], \Big[0, \frac{11}{3}, 4 - 4 \operatorname{RootOf}(_Z^2 + 1) \Big] \Big], \Big[\Big[\frac{11}{12}, 1 - \operatorname{RootOf}(_Z^2 + 1), \frac{1}{12} - \operatorname{RootOf}(_Z^2 + 1) \Big], [] \Big], \Big[\Big[\frac{11}{3}, 4 - 4 \operatorname{RootOf}(_Z^2 + 1), \frac{1}{3} - 4 \operatorname{RootOf}(_Z^2 + 1) \Big], [] \Big] \Big], [], [] \Big] \quad (195)$$

> R1:=IrrRegAppsing2F2(L,t,E,ext);

$$R1 := \Big[\Big[[\infty, \infty], [x - 3, 3] \Big], \Big[\Big[1, \frac{3}{7}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - \frac{33}{28} + 3 \operatorname{RootOf}(_Z^2 + 1) \Big], \Big[\frac{2}{3}, \frac{2}{7}, \frac{9216}{t^2} - \frac{5120}{t} - \frac{11}{14} + 2 \operatorname{RootOf}(_Z^2 + 1) \Big] \Big], \Big[\Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - \frac{61}{28} + 3 \operatorname{RootOf}(_Z^2 + 1), -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - \frac{45}{28} + 3 \operatorname{RootOf}(_Z^2 + 1), -\frac{4}{7} \Big], \Big[\frac{9216}{t^2} - \frac{5120}{t} - \frac{61}{42} + 2 \operatorname{RootOf}(_Z^2 + 1), \frac{9216}{t^2} - \frac{5120}{t} - \frac{15}{14} + 2 \operatorname{RootOf}(_Z^2 + 1), -\frac{8}{21} \Big] \Big], [3, 2], [1, 1], \Big[\Big[\Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - \frac{33}{28} + 3 \operatorname{RootOf}(_Z^2 + 1), 1 \Big], \Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - \frac{33}{28} + 3 \operatorname{RootOf}(_Z^2 + 1), \frac{3}{7} \Big], \Big[\frac{3}{7}, 1 \Big] \Big], \Big[\Big[\frac{9216}{t^2} - \frac{5120}{t} - \frac{11}{14} + 2 \operatorname{RootOf}(_Z^2 + 1), \frac{2}{3} \Big], \Big[\frac{9216}{t^2} - \frac{5120}{t} - \frac{11}{14} + 2 \operatorname{RootOf}(_Z^2 + 1), \frac{2}{7} \Big], \Big[\frac{2}{7}, \frac{2}{3} \Big] \Big], \Big[[-6t^3 + 136t^2 - 834t, -6t^3 + 136t^2 - 834t, 0], [9216t^2 - 5120t, 9216t^2 - 5120t, 0] \Big], \Big[\Big[-\frac{61}{28} + 3 \operatorname{RootOf}(_Z^2 + 1), -\frac{45}{28} + 3 \operatorname{RootOf}(_Z^2 + 1), -\frac{4}{7} \Big], \Big[-\frac{61}{42} + 2 \operatorname{RootOf}(_Z^2 + 1), -\frac{15}{14} + 2 \operatorname{RootOf}(_Z^2 + 1), -\frac{8}{21} \Big] \Big], \Big[[x - 12, 12], [x - 7, 7] \Big], \Big[\Big[0, \frac{11}{12}, 1 - \operatorname{RootOf}(_Z^2 + 1) \Big], \Big[0, \frac{11}{3}, 4 - 4 \operatorname{RootOf}(_Z^2 + 1) \Big] \Big], \Big[\Big[\frac{11}{12}, 1 - \operatorname{RootOf}(_Z^2 + 1), \frac{1}{12} - \operatorname{RootOf}(_Z^2 + 1) \Big], \Big[\frac{11}{3}, 4 - 4 \operatorname{RootOf}(_Z^2 + 1), \frac{1}{3} - 4 \operatorname{RootOf}(_Z^2 + 1) \Big] \Big], \Big[\Big[\frac{11}{12}, 0 \Big], [1 - \operatorname{RootOf}(_Z^2 + 1), 0], \Big[1 - \operatorname{RootOf}(_Z^2 + 1), \frac{11}{12} \Big] \Big] \Big] \quad (196)$$

$$\begin{aligned}
& \left[\frac{11}{12} \right] \Bigg], \left[\left[\frac{11}{3}, 0 \right], \left[4 - 4 \operatorname{RootOf}(_Z^2 + 1), 0 \right], \left[4 - 4 \operatorname{RootOf}(_Z^2 + 1), \frac{11}{3} \right] \right] \Bigg], \left[\left[\left[x \right. \right. \right. \\
& - 12, 12], [x - 7, 7], \left[\left[0, \frac{11}{12}, 1 - \operatorname{RootOf}(_Z^2 + 1) \right], \left[0, \frac{11}{3}, 4 - 4 \operatorname{RootOf}(_Z^2 \right. \right. \\
& + 1) \right] \Bigg], \left[\left[\left[\frac{11}{12}, 1 - \operatorname{RootOf}(_Z^2 + 1), \frac{1}{12} - \operatorname{RootOf}(_Z^2 + 1) \right], [\] \right], \left[\left[\frac{11}{3}, 4 \right. \right. \\
& - 4 \operatorname{RootOf}(_Z^2 + 1), \frac{1}{3} - 4 \operatorname{RootOf}(_Z^2 + 1) \right], [\] \Bigg], \left[[\] \right], \left[\left[\left[x^2 - \frac{32}{3} x - 1, \right. \right. \right. \\
& \operatorname{RootOf}(3 _Z^2 - 32 _Z - 3) \Bigg], [[0, 2, 4]], [[2, 4, 2]], [[[2, 0], [4, 0], [4, 2]]], \left[[[\infty, \right. \\
& \infty], [x - 12, 12], [x - 7, 7], [x - 3, 3]], \left[\left[1, \frac{3}{7}, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - \frac{33}{28} \right. \right. \\
& + 3 \operatorname{RootOf}(_Z^2 + 1) \Bigg], \left[0, \frac{11}{12}, 1 - \operatorname{RootOf}(_Z^2 + 1) \right], \left[0, \frac{11}{3}, 4 - 4 \operatorname{RootOf}(_Z^2 \right. \\
& + 1) \Bigg], \left[\frac{2}{3}, \frac{2}{7}, \frac{9216}{t^2} - \frac{5120}{t} - \frac{11}{14} + 2 \operatorname{RootOf}(_Z^2 + 1) \right] \Bigg], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} \right. \right. \\
& - \frac{61}{28} + 3 \operatorname{RootOf}(_Z^2 + 1), -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - \frac{45}{28} + 3 \operatorname{RootOf}(_Z^2 + 1), -\frac{4}{7} \Bigg], \\
& \left[\frac{11}{12}, 1 - \operatorname{RootOf}(_Z^2 + 1), \frac{1}{12} - \operatorname{RootOf}(_Z^2 + 1) \right], \left[\frac{11}{3}, 4 - 4 \operatorname{RootOf}(_Z^2 + 1), \right. \\
& \frac{1}{3} - 4 \operatorname{RootOf}(_Z^2 + 1) \Bigg], \left[\frac{9216}{t^2} - \frac{5120}{t} - \frac{61}{42} + 2 \operatorname{RootOf}(_Z^2 + 1), \frac{9216}{t^2} \right. \\
& - \frac{5120}{t} - \frac{15}{14} + 2 \operatorname{RootOf}(_Z^2 + 1), -\frac{8}{21} \Bigg], \left[\left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - \frac{33}{28} \right. \right. \right. \\
& + 3 \operatorname{RootOf}(_Z^2 + 1), 1 \Bigg], \left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - \frac{33}{28} + 3 \operatorname{RootOf}(_Z^2 + 1), \frac{3}{7} \right], \\
& \left[\frac{3}{7}, 1 \right] \Bigg], \left[\left[\frac{11}{12}, 0 \right], [1 - \operatorname{RootOf}(_Z^2 + 1), 0], \left[1 - \operatorname{RootOf}(_Z^2 + 1), \frac{11}{12} \right] \right] \Bigg], \left[\left[\frac{11}{3}, \right. \right. \\
& 0], [4 - 4 \operatorname{RootOf}(_Z^2 + 1), 0], \left[4 - 4 \operatorname{RootOf}(_Z^2 + 1), \frac{11}{3} \right] \Bigg], \left[\left[\frac{9216}{t^2} - \frac{5120}{t} \right. \right. \\
& - \frac{11}{14} + 2 \operatorname{RootOf}(_Z^2 + 1), \frac{2}{3} \Bigg], \left[\frac{9216}{t^2} - \frac{5120}{t} - \frac{11}{14} + 2 \operatorname{RootOf}(_Z^2 + 1), \frac{2}{7} \right], \\
& \left[\frac{2}{7}, \frac{2}{3} \right] \Bigg], [[1, 1, 1], [1, 1, 1], [1, 1, 1], [1, 1, 1]] \Bigg]
\end{aligned}$$

> F1:= Hyp2F2Subst(L,x,t,R1[1],ext);

$$\begin{aligned}
F1 := & \left[-\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 1193x + 9984)}{(x-3)^2}, \right. \\
& \left. -\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 6313x - 9984)}{(x-3)^2}, \right.
\end{aligned}$$

(197)

$$\left[\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 6313x - 9984)}{(x-3)^2}, \frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 1193x + 9984)}{(x-3)^2} \right]$$

> find2F2RatIrr(L,R1,F1,x,t,ext);

$$\left[\left[\left[\left[-1, \frac{17}{21} \right], \left[1, \frac{17}{21} \right], \left[\frac{1}{3}, \frac{1}{7} \right], \left[\frac{2}{3}, \frac{10}{21} \right] \right], \left[\frac{1}{12}, \text{RootOf}(_Z^2 + 1) \right] \right], \frac{2(x-7)^4(x-12)}{(x-3)^2} \right], \left[\left[\left[-1, \frac{17}{21} \right], \left[1, \frac{17}{21} \right], \left[\frac{1}{3}, \frac{1}{7} \right], \left[\frac{2}{3}, \frac{10}{21} \right] \right], \left[\frac{1}{12}, \text{RootOf}(_Z^2 + 1) \right] \right], -\frac{2(x-7)^4(x-12)}{(x-3)^2} \right] \quad (198)$$

**> TIME :=time();
Hyp2F2Solutions(L);
time() - TIME;**

TIME := 977.203

$$\left\{ \left[\left[\left[\left[\frac{1}{3}, \frac{1}{7} \right] \right], \left[\frac{1}{12}, \text{RootOf}(_Z^2 + 1) \right], [0], [1] \right], \frac{2(x-7)^4(x-12)}{(x-3)^2} \right] \right\}$$

21.218

(199)

> F:=sumdiffeq(hyperterm([a1,a2],[b1,b2],x,k),k,J(x));

$$F := \left(\frac{d^3}{dx^3} J(x) \right) x^2 + (b1 + b2 - x + 1) \left(\frac{d^2}{dx^2} J(x) \right) x - (a1 x + a2 x - b2 b1 + x) \left(\frac{d}{dx} J(x) \right) - a1 a2 J(x) = 0 \quad (200)$$

> LA:=de2diffop(F,J(x));

$$LA := x^2 Dx^3 + (b1 x + b2 x - x^2 + x) Dx^2 + (-a1 x - a2 x + b2 b1 - x) Dx - a1 a2 \quad (201)$$

> L22:=subs({a1=1/7,a2=RootOf(x^2+7),b1=RootOf(x^2+2)},LA);

$$L22 := x^2 Dx^3 + (\text{RootOf}(_Z^2 + 2) x + b2 x - x^2 + x) Dx^2 + \left(-\frac{8x}{7} - \text{RootOf}(_Z^2 + 7) x + b2 \text{RootOf}(_Z^2 + 2) \right) Dx - \frac{\text{RootOf}(_Z^2 + 7)}{7} \quad (202)$$

> f:= normal((2*(x-7)^4*(x-12))/(x-3)^2,expanded);

$$f := \frac{2x^5 - 80x^4 + 1260x^3 - 9800x^2 + 37730x - 57624}{x^2 - 6x + 9} \quad (203)$$

> L:=ChangeOfVariables(L22,f);

$$L := 7(x-12)^2(x-7)^3(3x^2-32x-3)^3(x-3)^5 Dx^3 + 7(-18x^9 + 1104x^8 + 9\text{RootOf}(_Z^2 + 2)x^6 + 9b2x^6 - 28712x^7 - 246\text{RootOf}(_Z^2 + 2)x^5 - 246b2x^5 + 410207x^6 + 2239\text{RootOf}(_Z^2 + 2)x^4 + 2239b2x^4 - 3473142x^5 - 7572\text{RootOf}(_Z^2 + 2)x^3 - 7572b2x^3 + 17377947x^4 + 7911\text{RootOf}(_Z^2 + 2)x^2 + 7911b2x^2$$

(204)

$$\begin{aligned}
& -47136476 x^3 + 1674 \operatorname{RootOf}(_Z^2 + 2) x + 1674 b_2 x + 50750253 x^2 + 81 \operatorname{RootOf}(_Z^2 \\
& + 2) + 81 b_2 + 10918044 x + 289305) (x - 12) (3 x^2 - 32 x - 3)^2 (x - 3)^2 (x \\
& - 7)^2 Dx^2 + \frac{1}{11} ((3 \operatorname{RootOf}(_Z^2 + 2) - 2) (17542236096 + 648337769352 x \\
& - 32672808 \operatorname{RootOf}(_Z^2 + 7) - 49009212 \operatorname{RootOf}(_Z^2 + 2) \operatorname{RootOf}(_Z^2 + 7) \\
& + 4832541 b_2 - 3420728430336 x^3 + 1701 x^{13} \operatorname{RootOf}(_Z^2 + 2) \operatorname{RootOf}(_Z^2 + 7) \\
& + 99375822 b_2 x + 2879361210222 x^2 + 7220745 b_2 \operatorname{RootOf}(_Z^2 + 2) \\
& + 1441388310336 x^4 + 436373252 x^9 - 3537928032 x^8 + 16154282484 x^7 \\
& - 17358665436 x^6 - 239863420776 x^5 + 540 x^{13} - 44640 x^{12} + 1620300 x^{11} \\
& - 33685586 x^{10} + 973049684670 \operatorname{RootOf}(_Z^2 + 2) x - 1372646898 \operatorname{RootOf}(_Z^2 + 7) x \\
& + 1134 \operatorname{RootOf}(_Z^2 + 7) x^{13} - 93744 \operatorname{RootOf}(_Z^2 + 7) x^{12} + 3419388 \operatorname{RootOf}(_Z^2 \\
& + 7) x^{11} - 72182712 \operatorname{RootOf}(_Z^2 + 7) x^{10} + 970861010 \operatorname{RootOf}(_Z^2 + 7) x^9 \\
& - 8613418856 \operatorname{RootOf}(_Z^2 + 7) x^8 + 50239747656 \operatorname{RootOf}(_Z^2 + 7) x^7 \\
& - 184320989552 \operatorname{RootOf}(_Z^2 + 7) x^6 + 376507489778 \operatorname{RootOf}(_Z^2 + 7) x^5 \\
& - 281175424320 \operatorname{RootOf}(_Z^2 + 7) x^4 - 140146344324 \operatorname{RootOf}(_Z^2 + 7) x^3 \\
& - 21266738136 \operatorname{RootOf}(_Z^2 + 7) x^2 + 26339848920 \operatorname{RootOf}(_Z^2 + 2) \\
& + 810 \operatorname{RootOf}(_Z^2 + 2) x^{13} - 66960 \operatorname{RootOf}(_Z^2 + 2) x^{12} + 2430450 \operatorname{RootOf}(_Z^2 \\
& + 2) x^{11} - 50526300 \operatorname{RootOf}(_Z^2 + 2) x^{10} + 654458700 \operatorname{RootOf}(_Z^2 + 2) x^9 \\
& - 5304908220 \operatorname{RootOf}(_Z^2 + 2) x^8 + 24211185970 \operatorname{RootOf}(_Z^2 + 2) x^7 \\
& - 25918888240 \operatorname{RootOf}(_Z^2 + 2) x^6 - 360253337220 \operatorname{RootOf}(_Z^2 + 2) x^5 \\
& + 2163453449580 \operatorname{RootOf}(_Z^2 + 2) x^4 - 5133964750740 \operatorname{RootOf}(_Z^2 + 2) x^3 \\
& + 4321538180820 \operatorname{RootOf}(_Z^2 + 2) x^2 - 108274068 \operatorname{RootOf}(_Z^2 + 2) \operatorname{RootOf}(_Z^2 \\
& + 7) x^{10} + 1456291515 \operatorname{RootOf}(_Z^2 + 2) \operatorname{RootOf}(_Z^2 + 7) x^9 \\
& - 12920128284 \operatorname{RootOf}(_Z^2 + 2) \operatorname{RootOf}(_Z^2 + 7) x^8 + 75359621484 \operatorname{RootOf}(_Z^2 \\
& + 2) \operatorname{RootOf}(_Z^2 + 7) x^7 - 276481484328 \operatorname{RootOf}(_Z^2 + 2) \operatorname{RootOf}(_Z^2 + 7) x^6 \\
& + 564761234667 \operatorname{RootOf}(_Z^2 + 2) \operatorname{RootOf}(_Z^2 + 7) x^5 - 421763136480 \operatorname{RootOf}(_Z^2 \\
& + 2) \operatorname{RootOf}(_Z^2 + 7) x^4 - 210219516486 \operatorname{RootOf}(_Z^2 + 2) \operatorname{RootOf}(_Z^2 + 7) x^3 \\
& - 31900107204 \operatorname{RootOf}(_Z^2 + 2) \operatorname{RootOf}(_Z^2 + 7) x^2 - 2058970347 \operatorname{RootOf}(_Z^2 \\
& + 2) \operatorname{RootOf}(_Z^2 + 7) x + 5985 \operatorname{RootOf}(_Z^2 + 2) b_2 x^8 - 312900 \operatorname{RootOf}(_Z^2 \\
& + 2) b_2 x^7 + 6372380 \operatorname{RootOf}(_Z^2 + 2) b_2 x^6 - 64110060 \operatorname{RootOf}(_Z^2 + 2) b_2 x^5 \\
& + 330475950 \operatorname{RootOf}(_Z^2 + 2) b_2 x^4 - 805136220 \operatorname{RootOf}(_Z^2 + 2) b_2 x^3 \\
& + 677508300 \operatorname{RootOf}(_Z^2 + 2) b_2 x^2 + 147884940 \operatorname{RootOf}(_Z^2 + 2) b_2 x + 2079 b_2 x^{10} \\
& - 101178 b_2 x^9 + 1965873 b_2 x^8 - 19299056 b_2 x^7 + 99992774 b_2 x^6 - 265875876 b_2 x^5 \\
& + 379556226 b_2 x^4 - 456696576 b_2 x^3 + 463840587 b_2 x^2 - 140616 x^{12} \operatorname{RootOf}(_Z^2 \\
& + 2) \operatorname{RootOf}(_Z^2 + 7) + 5129082 x^{11} \operatorname{RootOf}(_Z^2 + 2) \operatorname{RootOf}(_Z^2 + 7)) (3 x^2 - 32 x \\
& - 3) (x - 3) (x - 7) Dx) - 2 \operatorname{RootOf}(_Z^2 + 7) (x - 7)^4 (3 x^2 - 32 x - 3)^6
\end{aligned}$$

$$\begin{aligned} &> \text{ext} := \text{indets}(\mathbf{L}, \{\text{RootOf}, \text{name}\}) \text{ minus } \{\mathbf{x}, \mathbf{Dx}\}; \\ &\quad \text{ext} := \{b2, \text{RootOf}(_Z^2 + 2), \text{RootOf}(_Z^2 + 7)\} \end{aligned} \quad (205)$$

$$\begin{aligned} &> \text{ext} := \text{indets}(\text{map}(\mathbf{s} \rightarrow \text{ReplirrRoot}(\mathbf{s}, \{\}), \text{ext}), \{\text{RootOf}, \text{name}\}); \\ &\quad \text{ext} := \{b2, \text{RootOf}(_Z^2 + 2), \text{RootOf}(_Z^2 + 7)\} \end{aligned} \quad (206)$$

$$\begin{aligned} &> \text{extppp} := \{\}; \\ &\quad \text{extppp} := \emptyset \end{aligned} \quad (207)$$

$$\begin{aligned} &> \mathbf{E} := \text{Singular}(\mathbf{L}, \text{extppp}); \\ \mathbf{E} &:= \left[[x - 7, 7], [x - 3, 3], \left[x^2 - \frac{32}{3}x - 1, \text{RootOf}(3_Z^2 - 32_Z - 3) \right], [\infty, \infty], [x \right. \\ &\quad \left. - 12, 12] \right] \end{aligned} \quad (208)$$

$$\begin{aligned} &> \mathbf{F} := \text{NotAppSing}(\mathbf{L}, \mathbf{E}, \text{ext}); \\ &\quad \mathbf{F} := [[\infty, \infty], [x - 12, 12], [x - 7, 7], [x - 3, 3]] \end{aligned} \quad (209)$$

$$\begin{aligned} &> \text{Sirr} := \text{irrsing2F2}(\mathbf{L}, \mathbf{t}, \mathbf{F}, \text{ext}); \\ \text{Sirr} &:= \left[[[\infty, \infty], [x - 3, 3]], \left[\left[\frac{3}{7}, 3 \text{RootOf}(_Z^2 + 7), -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} \right. \right. \right. \end{aligned} \quad (210)$$

$$\left. - 3 \text{RootOf}(_Z^2 + 7) - \frac{3}{7} + 3b2 + 3 \text{RootOf}(_Z^2 + 2) \right], \left[\frac{2}{7}, 2 \text{RootOf}(_Z^2 + 7), \right.$$

$$\left. \frac{9216}{t^2} - \frac{5120}{t} - 2 \text{RootOf}(_Z^2 + 7) - \frac{2}{7} + 2b2 + 2 \text{RootOf}(_Z^2 + 2) \right], \left[\left[-\frac{6}{t^3} \right. \right.$$

$$\left. + \frac{136}{t^2} - \frac{834}{t} - 3 \text{RootOf}(_Z^2 + 7) - \frac{6}{7} + 3b2 + 3 \text{RootOf}(_Z^2 + 2), -\frac{6}{t^3} + \frac{136}{t^2} \right.$$

$$\left. - \frac{834}{t} - 6 \text{RootOf}(_Z^2 + 7) - \frac{3}{7} + 3b2 + 3 \text{RootOf}(_Z^2 + 2), 3 \text{RootOf}(_Z^2 + 7) \right.$$

$$\left. - \frac{3}{7} \right], \left[\frac{9216}{t^2} - \frac{5120}{t} - 2 \text{RootOf}(_Z^2 + 7) - \frac{4}{7} + 2b2 + 2 \text{RootOf}(_Z^2 + 2), \right.$$

$$\left. \frac{9216}{t^2} - \frac{5120}{t} - 4 \text{RootOf}(_Z^2 + 7) - \frac{2}{7} + 2b2 + 2 \text{RootOf}(_Z^2 + 2), -\frac{2}{7} \right.$$

$$\left. + 2 \text{RootOf}(_Z^2 + 7) \right], [3, 2], [1, 1], \left[\left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 \text{RootOf}(_Z^2 + 7) \right. \right. \right.$$

$$\left. - \frac{3}{7} + 3b2 + 3 \text{RootOf}(_Z^2 + 2), \frac{3}{7} \right], \left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 \text{RootOf}(_Z^2 + 7) \right.$$

$$\left. - \frac{3}{7} + 3b2 + 3 \text{RootOf}(_Z^2 + 2), 3 \text{RootOf}(_Z^2 + 7) \right], \left[3 \text{RootOf}(_Z^2 + 7), \frac{3}{7} \right],$$

$$\left[\left[\frac{9216}{t^2} - \frac{5120}{t} - 2 \text{RootOf}(_Z^2 + 7) - \frac{2}{7} + 2b2 + 2 \text{RootOf}(_Z^2 + 2), \frac{2}{7} \right], \right.$$

$$\left[\frac{9216}{t^2} - \frac{5120}{t} - 2 \text{RootOf}(_Z^2 + 7) - \frac{2}{7} + 2b2 + 2 \text{RootOf}(_Z^2 + 2), 2 \text{RootOf}(_Z^2 \right.$$

$$\left. + 7) \right], \left[2 \text{RootOf}(_Z^2 + 7), \frac{2}{7} \right], \left[[-6t^3 + 136t^2 - 834t, -6t^3 + 136t^2 - 834t, 0], \right.$$

$$\left. [9216t^2 - 5120t, 9216t^2 - 5120t, 0], \left[-3 \text{RootOf}(_Z^2 + 7) - \frac{6}{7} + 3b2 \right. \right.$$

$$\begin{aligned}
& + 3 \operatorname{RootOf}(_Z^2 + 2), -6 \operatorname{RootOf}(_Z^2 + 7) - \frac{3}{7} + 3 b2 + 3 \operatorname{RootOf}(_Z^2 + 2), \\
& 3 \operatorname{RootOf}(_Z^2 + 7) - \frac{3}{7} \Big], \Big[-2 \operatorname{RootOf}(_Z^2 + 7) - \frac{4}{7} + 2 b2 + 2 \operatorname{RootOf}(_Z^2 + 2), \\
& -4 \operatorname{RootOf}(_Z^2 + 7) - \frac{2}{7} + 2 b2 + 2 \operatorname{RootOf}(_Z^2 + 2), -\frac{2}{7} + 2 \operatorname{RootOf}(_Z^2 + 7) \Big], \\
& \Big[[x - 12, 12], [x - 7, 7], \Big[[0, 1 - b2, 1 - \operatorname{RootOf}(_Z^2 + 2)], [1 - b2, 1 \\
& - \operatorname{RootOf}(_Z^2 + 2), b2 - \operatorname{RootOf}(_Z^2 + 2)], [1, 1, 1], [1 - b2, 0], [1 - \operatorname{RootOf}(_Z^2 \\
& + 2), 0], [1 - \operatorname{RootOf}(_Z^2 + 2), 1 - b2] \Big], 2 \Big], \Big[[0, 4 - 4 b2, 4 - 4 \operatorname{RootOf}(_Z^2 + 2)], \\
& [4 - 4 b2, 4 - 4 \operatorname{RootOf}(_Z^2 + 2), 4 b2 - 4 \operatorname{RootOf}(_Z^2 + 2)], [1, 1, 1], [4 - 4 b2, 0], \\
& [4 - 4 \operatorname{RootOf}(_Z^2 + 2), 0], [4 - 4 \operatorname{RootOf}(_Z^2 + 2), 4 - 4 b2] \Big], 2 \Big] \Big]
\end{aligned}$$

> Sreg:= regsingtrue2F2(L,t,Sirr[-1],ext);

$$\begin{aligned}
Sreg := & \Big[[x - 12, 12], [x - 7, 7], \Big[[0, 1 - b2, 1 - \operatorname{RootOf}(_Z^2 + 2)], [0, 4 - 4 b2, 4 \\
& - 4 \operatorname{RootOf}(_Z^2 + 2)], \Big[[1 - b2, 1 - \operatorname{RootOf}(_Z^2 + 2), b2 - \operatorname{RootOf}(_Z^2 + 2)], [4 \\
& - 4 b2, 4 - 4 \operatorname{RootOf}(_Z^2 + 2), 4 b2 - 4 \operatorname{RootOf}(_Z^2 + 2)] \Big], \Big[[1 - b2, 0], [1 \\
& - \operatorname{RootOf}(_Z^2 + 2), 0], [1 - \operatorname{RootOf}(_Z^2 + 2), 1 - b2] \Big], [4 - 4 b2, 0], [4 \\
& - 4 \operatorname{RootOf}(_Z^2 + 2), 0], [4 - 4 \operatorname{RootOf}(_Z^2 + 2), 4 - 4 b2] \Big] \Big]
\end{aligned} \tag{211}$$

> RSreg:= Sregseptrue2F2(L,Sreg,ext);

$$\begin{aligned}
RSreg := & \Big[[x - 12, 12], [x - 7, 7], \Big[[0, 1 - b2, 1 - \operatorname{RootOf}(_Z^2 + 2)], [0, 4 - 4 b2, 4 \\
& - 4 \operatorname{RootOf}(_Z^2 + 2)], \Big[[1 - b2, 1 - \operatorname{RootOf}(_Z^2 + 2), b2 - \operatorname{RootOf}(_Z^2 + 2)], \\
& [] \Big], [4 - 4 b2, 4 - 4 \operatorname{RootOf}(_Z^2 + 2), 4 b2 - 4 \operatorname{RootOf}(_Z^2 + 2)], [] \Big], [], [] \Big]
\end{aligned} \tag{212}$$

> R1:=IrrRegAppsing2F2(L,t,E,ext);

$$\begin{aligned}
R1 := & \Bigg[\Big[[\infty, \infty], [x - 3, 3], \Bigg[\Big[\frac{3}{7}, 3 \operatorname{RootOf}(_Z^2 + 7), -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} \\
& - 3 \operatorname{RootOf}(_Z^2 + 7) - \frac{3}{7} + 3 b2 + 3 \operatorname{RootOf}(_Z^2 + 2) \Big], \Big[\frac{2}{7}, 2 \operatorname{RootOf}(_Z^2 + 7), \\
& \frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 + 7) - \frac{2}{7} + 2 b2 + 2 \operatorname{RootOf}(_Z^2 + 2) \Big] \Big], \Bigg[-\frac{6}{t^3} \\
& + \frac{136}{t^2} - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2 + 7) - \frac{6}{7} + 3 b2 + 3 \operatorname{RootOf}(_Z^2 + 2), -\frac{6}{t^3} + \frac{136}{t^2} \\
& - \frac{834}{t} - 6 \operatorname{RootOf}(_Z^2 + 7) - \frac{3}{7} + 3 b2 + 3 \operatorname{RootOf}(_Z^2 + 2), 3 \operatorname{RootOf}(_Z^2 + 7) \\
& - \frac{3}{7} \Big], \Bigg[\frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 + 7) - \frac{4}{7} + 2 b2 + 2 \operatorname{RootOf}(_Z^2 + 2), \\
& \frac{9216}{t^2} - \frac{5120}{t} - 4 \operatorname{RootOf}(_Z^2 + 7) - \frac{2}{7} + 2 b2 + 2 \operatorname{RootOf}(_Z^2 + 2), -\frac{2}{7} \\
& + 2 \operatorname{RootOf}(_Z^2 + 7) \Big] \Big], [3, 2], [1, 1], \Bigg[\Bigg[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2 + 7)
\end{aligned} \tag{213}$$

$$\begin{aligned}
& -\frac{3}{7} + 3b_2 + 3\text{RootOf}(_Z^2 + 2), \frac{3}{7} \Big], \Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3\text{RootOf}(_Z^2 + 7) \\
& -\frac{3}{7} + 3b_2 + 3\text{RootOf}(_Z^2 + 2), 3\text{RootOf}(_Z^2 + 7) \Big], \Big[3\text{RootOf}(_Z^2 + 7), \frac{3}{7} \Big], \\
& \Big[\Big[\frac{9216}{t^2} - \frac{5120}{t} - 2\text{RootOf}(_Z^2 + 7) - \frac{2}{7} + 2b_2 + 2\text{RootOf}(_Z^2 + 2), \frac{2}{7} \Big], \\
& \Big[\frac{9216}{t^2} - \frac{5120}{t} - 2\text{RootOf}(_Z^2 + 7) - \frac{2}{7} + 2b_2 + 2\text{RootOf}(_Z^2 + 2), 2\text{RootOf}(_Z^2 \\
& + 7) \Big], \Big[2\text{RootOf}(_Z^2 + 7), \frac{2}{7} \Big] \Big], \Big[[-6t^3 + 136t^2 - 834t, -6t^3 + 136t^2 - 834t, 0], \\
& [9216t^2 - 5120t, 9216t^2 - 5120t, 0] \Big], \Big[\Big[-3\text{RootOf}(_Z^2 + 7) - \frac{6}{7} + 3b_2 \\
& + 3\text{RootOf}(_Z^2 + 2), -6\text{RootOf}(_Z^2 + 7) - \frac{3}{7} + 3b_2 + 3\text{RootOf}(_Z^2 + 2), \\
& 3\text{RootOf}(_Z^2 + 7) - \frac{3}{7} \Big], \Big[-2\text{RootOf}(_Z^2 + 7) - \frac{4}{7} + 2b_2 + 2\text{RootOf}(_Z^2 + 2), \\
& -4\text{RootOf}(_Z^2 + 7) - \frac{2}{7} + 2b_2 + 2\text{RootOf}(_Z^2 + 2), -\frac{2}{7} + 2\text{RootOf}(_Z^2 + 7) \Big] \Big], \\
& \Big[[x - 12, 12], [x - 7, 7] \Big], \Big[[0, 1 - b_2, 1 - \text{RootOf}(_Z^2 + 2)], [0, 4 - 4b_2, 4 \\
& - 4\text{RootOf}(_Z^2 + 2)] \Big], \Big[[1 - b_2, 1 - \text{RootOf}(_Z^2 + 2), b_2 - \text{RootOf}(_Z^2 + 2)], [4 \\
& - 4b_2, 4 - 4\text{RootOf}(_Z^2 + 2), 4b_2 - 4\text{RootOf}(_Z^2 + 2)] \Big], \Big[[1 - b_2, 0], [1 \\
& - \text{RootOf}(_Z^2 + 2), 0], [1 - \text{RootOf}(_Z^2 + 2), 1 - b_2] \Big], \Big[[4 - 4b_2, 0], [4 \\
& - 4\text{RootOf}(_Z^2 + 2), 0], [4 - 4\text{RootOf}(_Z^2 + 2), 4 - 4b_2] \Big] \Big], \Big[[x - 12, 12], [x \\
& - 7, 7] \Big], \Big[[0, 1 - b_2, 1 - \text{RootOf}(_Z^2 + 2)], [0, 4 - 4b_2, 4 - 4\text{RootOf}(_Z^2 + 2)] \Big], \\
& \Big[[1 - b_2, 1 - \text{RootOf}(_Z^2 + 2), b_2 - \text{RootOf}(_Z^2 + 2)], [] \Big], \Big[[4 - 4b_2, 4 \\
& - 4\text{RootOf}(_Z^2 + 2), 4b_2 - 4\text{RootOf}(_Z^2 + 2)], [] \Big] \Big], \Big[[] \Big], \Big[\Big[\Big[x^2 - \frac{32}{3}x - 1, \\
& \text{RootOf}(3_Z^2 - 32_Z - 3) \Big] \Big], [0, 2, 4], [2, 4, 2], [[2, 0], [4, 0], [4, 2]] \Big], \Big[[[\infty, \\
& \infty], [x - 12, 12], [x - 7, 7], [x - 3, 3] \Big], \Big[\Big[\frac{3}{7}, 3\text{RootOf}(_Z^2 + 7), -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} \\
& - 3\text{RootOf}(_Z^2 + 7) - \frac{3}{7} + 3b_2 + 3\text{RootOf}(_Z^2 + 2) \Big], [0, 1 - b_2, 1 - \text{RootOf}(_Z^2
\end{aligned}$$

$$\begin{aligned}
& + 2)], [0, 4 - 4 b2, 4 - 4 \text{RootOf}(_Z^2 + 2)], \left[\frac{2}{7}, 2 \text{RootOf}(_Z^2 + 7), \frac{9216}{t^2} - \frac{5120}{t} \right. \\
& \left. - 2 \text{RootOf}(_Z^2 + 7) - \frac{2}{7} + 2 b2 + 2 \text{RootOf}(_Z^2 + 2) \right]], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} \right. \right. \\
& \left. \left. - 3 \text{RootOf}(_Z^2 + 7) - \frac{6}{7} + 3 b2 + 3 \text{RootOf}(_Z^2 + 2), -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} \right. \right. \\
& \left. \left. - 6 \text{RootOf}(_Z^2 + 7) - \frac{3}{7} + 3 b2 + 3 \text{RootOf}(_Z^2 + 2), 3 \text{RootOf}(_Z^2 + 7) - \frac{3}{7} \right]], [1 \right. \\
& \left. - b2, 1 - \text{RootOf}(_Z^2 + 2), b2 - \text{RootOf}(_Z^2 + 2)], [4 - 4 b2, 4 - 4 \text{RootOf}(_Z^2 \right. \\
& \left. + 2), 4 b2 - 4 \text{RootOf}(_Z^2 + 2)], \left[\frac{9216}{t^2} - \frac{5120}{t} - 2 \text{RootOf}(_Z^2 + 7) - \frac{4}{7} + 2 b2 \right. \right. \\
& \left. \left. + 2 \text{RootOf}(_Z^2 + 2), \frac{9216}{t^2} - \frac{5120}{t} - 4 \text{RootOf}(_Z^2 + 7) - \frac{2}{7} + 2 b2 \right. \right. \\
& \left. \left. + 2 \text{RootOf}(_Z^2 + 2), -\frac{2}{7} + 2 \text{RootOf}(_Z^2 + 7) \right]], \left[\left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} \right. \right. \right. \\
& \left. \left. \left. - 3 \text{RootOf}(_Z^2 + 7) - \frac{3}{7} + 3 b2 + 3 \text{RootOf}(_Z^2 + 2), \frac{3}{7} \right]], \left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} \right. \right. \right. \\
& \left. \left. \left. - 3 \text{RootOf}(_Z^2 + 7) - \frac{3}{7} + 3 b2 + 3 \text{RootOf}(_Z^2 + 2), 3 \text{RootOf}(_Z^2 + 7) \right]], \right. \\
& \left. \left[3 \text{RootOf}(_Z^2 + 7), \frac{3}{7} \right]], [[1 - b2, 0], [1 - \text{RootOf}(_Z^2 + 2), 0], [1 - \text{RootOf}(_Z^2 \right. \\
& \left. + 2), 1 - b2]], [[4 - 4 b2, 0], [4 - 4 \text{RootOf}(_Z^2 + 2), 0], [4 - 4 \text{RootOf}(_Z^2 + 2), 4 \right. \\
& \left. - 4 b2]], \left[\left[\frac{9216}{t^2} - \frac{5120}{t} - 2 \text{RootOf}(_Z^2 + 7) - \frac{2}{7} + 2 b2 + 2 \text{RootOf}(_Z^2 + 2), \right. \right. \\
& \left. \left. \frac{2}{7} \right]], \left[\frac{9216}{t^2} - \frac{5120}{t} - 2 \text{RootOf}(_Z^2 + 7) - \frac{2}{7} + 2 b2 + 2 \text{RootOf}(_Z^2 + 2), \right. \right. \\
& \left. \left. 2 \text{RootOf}(_Z^2 + 7) \right]], \left[2 \text{RootOf}(_Z^2 + 7), \frac{2}{7} \right]]], [[1, 1, 1], [1, 1, 1], [1, 1, 1], [1, 1, \right. \\
& \left. 1]]]
\end{aligned}$$

> F1:= Hyp2F2Subst(L,x,t,R1[1],ext);

$$\begin{aligned}
F1 := & \left[-\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 1193x + 9984)}{(x-3)^2}, \right. \\
& \left. -\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 6313x - 9984)}{(x-3)^2}, \right. \\
& \left. \frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 6313x - 9984)}{(x-3)^2}, \right.
\end{aligned} \tag{214}$$

$$\left[\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 1193x + 9984)}{(x-3)^2} \right]$$

> find2F2RatIrr(L,R1,F1,x,t,ext);

$$\left[\left[\left[\left[\left[\frac{1}{7}, \text{RootOf}(_Z^2 + 7) \right], \left[\frac{10}{21}, \text{RootOf}(_Z^2 + 7) + \frac{1}{3} \right], \left[\frac{17}{21}, \text{RootOf}(_Z^2 + 7) \right] \right] \right] \right] \right] \quad (215)$$

$$+ \frac{2}{3} \right], [b2, \text{RootOf}(_Z^2 + 2)], -\frac{2(x-7)^4(x-12)}{(x-3)^2}, \left[\left[\left[\frac{1}{7}, \text{RootOf}(_Z^2 + 7) \right], \left[\frac{10}{21}, \text{RootOf}(_Z^2 + 7) + \frac{1}{3} \right], \left[\frac{17}{21}, \text{RootOf}(_Z^2 + 7) + \frac{2}{3} \right] \right], [b2, \text{RootOf}(_Z^2 + 2)] \right], \frac{2(x-7)^4(x-12)}{(x-3)^2} \right]$$

**> TIME :=time();
Hyp2F2Solutions(L);
time() - TIME;**

$$TIME := 1117.734$$

$$\left\{ \left[\left[\left[\left[\frac{1}{7}, \text{RootOf}(_Z^2 + 7) \right], [b2, \text{RootOf}(_Z^2 + 2)], [0], [1] \right], \frac{2(x-7)^4(x-12)}{(x-3)^2} \right] \right\}$$

$$29.297$$

(216)

> F:=sumdiffeq(hyperterm([a1,a2],[b1,b2],x,k),k,J(x));

$$F := \left(\frac{d^3}{dx^3} J(x) \right) x^2 + (b1 + b2 - x + 1) \left(\frac{d^2}{dx^2} J(x) \right) x - (a1 x + a2 x - b2 b1 + x) \left(\frac{d}{dx} J(x) \right) - a1 a2 J(x) = 0 \quad (217)$$

> LA:=de2diffop(F,J(x));

$$LA := x^2 Dx^3 + (b1 x + b2 x - x^2 + x) Dx^2 + (-a1 x - a2 x + b2 b1 - x) Dx - a1 a2 \quad (218)$$

> L22:=subs({a1=1/7,a2=RootOf(x^2+1)},LA);

$$L22 := x^2 Dx^3 + (b1 x + b2 x - x^2 + x) Dx^2 + \left(-\frac{8x}{7} - \text{RootOf}(_Z^2 + 1) x + b2 b1 \right) Dx - \frac{\text{RootOf}(_Z^2 + 1)}{7} \quad (219)$$

> f:= normal((2*(x-7)^4*(x-12))/(x-3)^2,expanded);

$$f := \frac{2x^5 - 80x^4 + 1260x^3 - 9800x^2 + 37730x - 57624}{x^2 - 6x + 9} \quad (220)$$

> L:=ChangeOfVariables(L22,f);

$$L := 7(x-12)^2(x-7)^3(3x^2-32x-3)^3(x-3)^5 Dx^3 + 7(x-12)(x-7)^2(x-3)^2(3x^2-32x-3)^2(-18x^9 + 1104x^8 + 9b1x^6 + 9b2x^6 - 28712x^7 - 246b1x^5 - 246b2x^5 + 410207x^6 + 2239b1x^4 + 2239b2x^4 - 3473142x^5 - 7572b1x^3) \quad (221)$$

$$\begin{aligned}
& -7572 b2 x^3 + 17377947 x^4 + 7911 b1 x^2 + 7911 b2 x^2 - 47136476 x^3 + 1674 b1 x \\
& + 1674 b2 x + 50750253 x^2 + 81 b1 + 81 b2 + 10918044 x + 289305) Dx^2 \\
& - (17556687792 + 648633967884 x - 5103 b2 b1 + 4817232 b1 + 4817232 b2 \\
& - 3422295033192 x^3 + 98732844 b1 x + 98732844 b2 x + 2880722864124 x^2 \\
& + 1442136119832 x^4 + 436318064 x^9 - 3536845944 x^8 + 16143243708 x^7 \\
& - 17293696392 x^6 - 240113351352 x^5 + 540 x^{13} - 44640 x^{12} + 1620300 x^{11} \\
& - 33684452 x^{10} - 1372646898 \text{RootOf}(_Z^2 + 1) x + 378 b1 x^{10} - 18396 b1 x^9 \\
& + 360696 b1 x^8 - 3679592 b1 x^7 + 21656348 b1 x^6 - 83310192 b1 x^5 + 249269832 b1 x^4 \\
& - 522200952 b1 x^3 + 453884634 b1 x^2 - 567 b1 b2 x^{10} + 27594 b1 b2 x^9 \\
& - 535059 b1 b2 x^8 + 5206488 b1 b2 x^7 - 26112142 b1 b2 x^6 + 60855228 b1 b2 x^5 \\
& - 43428798 b1 b2 x^4 - 21834792 b1 b2 x^3 - 3318651 b1 b2 x^2 - 214326 b1 b2 x \\
& - 32672808 \text{RootOf}(_Z^2 + 1) + 1134 \text{RootOf}(_Z^2 + 1) x^{13} - 93744 \text{RootOf}(_Z^2 \\
& + 1) x^{12} + 3419388 \text{RootOf}(_Z^2 + 1) x^{11} - 72182712 \text{RootOf}(_Z^2 + 1) x^{10} \\
& + 970861010 \text{RootOf}(_Z^2 + 1) x^9 - 8613418856 \text{RootOf}(_Z^2 + 1) x^8 \\
& + 50239747656 \text{RootOf}(_Z^2 + 1) x^7 - 184320989552 \text{RootOf}(_Z^2 + 1) x^6 \\
& + 376507489778 \text{RootOf}(_Z^2 + 1) x^5 - 281175424320 \text{RootOf}(_Z^2 + 1) x^4 \\
& - 140146344324 \text{RootOf}(_Z^2 + 1) x^3 - 21266738136 \text{RootOf}(_Z^2 + 1) x^2 + 378 b2 x^{10} \\
& - 18396 b2 x^9 + 360696 b2 x^8 - 3679592 b2 x^7 + 21656348 b2 x^6 - 83310192 b2 x^5 \\
& + 249269832 b2 x^4 - 522200952 b2 x^3 + 453884634 b2 x^2) (3 x^2 - 32 x - 3) (x - 3) (x \\
& - 7) Dx - 2 \text{RootOf}(_Z^2 + 1) (x - 7)^4 (3 x^2 - 32 x - 3)^6
\end{aligned}$$

$$\begin{aligned}
& \text{> ext:=indets(L,{RootOf,name}) minus \{x,Dx\};} \\
& \text{ext := \{b1, b2, RootOf(_Z^2 + 1)\}} \tag{222}
\end{aligned}$$

$$\begin{aligned}
& \text{> ext:= indets(map(s-> ReplirrRoot(s,\{ \}),ext),\{RootOf,name\});} \\
& \text{ext := \{b1, b2, RootOf(_Z^2 + 1)\}} \tag{223}
\end{aligned}$$

$$\begin{aligned}
& \text{> extppp:=\{ \};} \\
& \text{extppp := } \emptyset \tag{224}
\end{aligned}$$

$$\begin{aligned}
& \text{> E:= Singular(L,extppp);} \\
& E := \left[[x-7, 7], [x-3, 3], \left[x^2 - \frac{32}{3} x - 1, \text{RootOf}(3 _Z^2 - 32 _Z - 3) \right], [\infty, \infty], [x \right. \\
& \left. - 12, 12] \right] \tag{225}
\end{aligned}$$

$$\begin{aligned}
& \text{> F:= NotAppSing(L,E,ext);} \\
& F := [[\infty, \infty], [x-12, 12], [x-7, 7], [x-3, 3]] \tag{226}
\end{aligned}$$

$$\begin{aligned}
& \text{> Sirr:= irrSing2F2(L,t,F,ext);} \\
& Sirr := \left[[[\infty, \infty], [x-3, 3]], \left[\left[\frac{3}{7}, 3 \text{RootOf}(_Z^2 + 1), -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} \right. \right. \right. \\
& \left. \left. - 3 \text{RootOf}(_Z^2 + 1) - \frac{3}{7} + 3 b1 + 3 b2 \right], \left[\frac{2}{7}, 2 \text{RootOf}(_Z^2 + 1), \frac{9216}{t^2} - \frac{5120}{t} \right] \right] \tag{227}
\end{aligned}$$

$$\begin{aligned}
& -2 \operatorname{RootOf}(_Z^2 + 1) - \frac{2}{7} + 2 b1 + 2 b2 \Big] \Big], \Big[\Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2 \\
& + 1) - \frac{6}{7} + 3 b1 + 3 b2, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 6 \operatorname{RootOf}(_Z^2 + 1) - \frac{3}{7} + 3 b1 \\
& + 3 b2, 3 \operatorname{RootOf}(_Z^2 + 1) - \frac{3}{7} \Big], \Big[\frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 + 1) - \frac{4}{7} + 2 b1 \\
& + 2 b2, \frac{9216}{t^2} - \frac{5120}{t} - 4 \operatorname{RootOf}(_Z^2 + 1) - \frac{2}{7} + 2 b1 + 2 b2, -\frac{2}{7} + 2 \operatorname{RootOf}(_Z^2 \\
& + 1) \Big] \Big], [3, 2], [1, 1], \Big[\Big[\Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2 + 1) - \frac{3}{7} + 3 b1 \\
& + 3 b2, \frac{3}{7} \Big], \Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2 + 1) - \frac{3}{7} + 3 b1 + 3 b2, \\
& 3 \operatorname{RootOf}(_Z^2 + 1) \Big], \Big[3 \operatorname{RootOf}(_Z^2 + 1), \frac{3}{7} \Big] \Big], \Big[\Big[\frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 \\
& + 1) - \frac{2}{7} + 2 b1 + 2 b2, \frac{2}{7} \Big], \Big[\frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 + 1) - \frac{2}{7} + 2 b1 \\
& + 2 b2, 2 \operatorname{RootOf}(_Z^2 + 1) \Big], \Big[2 \operatorname{RootOf}(_Z^2 + 1), \frac{2}{7} \Big] \Big] \Big], \Big[[-6 t^3 + 136 t^2 - 834 t, -6 t^3 \\
& + 136 t^2 - 834 t, 0], [9216 t^2 - 5120 t, 9216 t^2 - 5120 t, 0] \Big], \Big[\Big[-3 \operatorname{RootOf}(_Z^2 + 1) \\
& - \frac{6}{7} + 3 b1 + 3 b2, -6 \operatorname{RootOf}(_Z^2 + 1) - \frac{3}{7} + 3 b1 + 3 b2, 3 \operatorname{RootOf}(_Z^2 + 1) \\
& - \frac{3}{7} \Big], \Big[-2 \operatorname{RootOf}(_Z^2 + 1) - \frac{4}{7} + 2 b1 + 2 b2, -4 \operatorname{RootOf}(_Z^2 + 1) - \frac{2}{7} + 2 b1 \\
& + 2 b2, -\frac{2}{7} + 2 \operatorname{RootOf}(_Z^2 + 1) \Big] \Big], \Big[[x - 12, 12], [x - 7, 7], [[0, -b1 + 1, 1 \\
& - b2], [-b1 + 1, 1 - b2, b1 - b2], [1, 1, 1], [-b1 + 1, 0], [1 - b2, 0], [1 - b2, -b1 \\
& + 1]], 2], [[0, -4 b1 + 4, 4 - 4 b2], [-4 b1 + 4, 4 - 4 b2, 4 b1 - 4 b2], [1, 1, 1], [[\\
& -4 b1 + 4, 0], [4 - 4 b2, 0], [4 - 4 b2, -4 b1 + 4]], 2]]]
\end{aligned}$$

$$\begin{aligned}
& \text{> Sreg:= regsingtrue2F2(L,t,Sirr[-1],ext);} \\
& \text{Sreg := } [[[x - 12, 12], [x - 7, 7]], [[0, -b1 + 1, 1 - b2], [0, -4 b1 + 4, 4 - 4 b2]], [[-b1 \\
& + 1, 1 - b2, b1 - b2], [-4 b1 + 4, 4 - 4 b2, 4 b1 - 4 b2]], [[[-b1 + 1, 0], [1 - b2, 0], \\
& [1 - b2, -b1 + 1]], [-4 b1 + 4, 0], [4 - 4 b2, 0], [4 - 4 b2, -4 b1 + 4]]] \quad (228)
\end{aligned}$$

$$\begin{aligned}
& \text{> RSreg:= Sregseptrue2F2(L,Sreg,ext);} \\
& \text{RSreg := } [[[[x - 12, 12], [x - 7, 7]], [[0, -b1 + 1, 1 - b2], [0, -4 b1 + 4, 4 - 4 b2]], [[[\\
& -b1 + 1, 1 - b2, b1 - b2], [], [-4 b1 + 4, 4 - 4 b2, 4 b1 - 4 b2], []]], [], []] \quad (229)
\end{aligned}$$

$$\begin{aligned}
& \text{> R1:=IrrRegAppsing2F2(L,t,E,ext);} \\
& \text{R1 := } \Big[\Big[[\infty, \infty], [x - 3, 3], \Big[\Big[\frac{3}{7}, 3 \operatorname{RootOf}(_Z^2 + 1), -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} \\
& - 3 \operatorname{RootOf}(_Z^2 + 1) - \frac{3}{7} + 3 b1 + 3 b2 \Big], \Big[\frac{2}{7}, 2 \operatorname{RootOf}(_Z^2 + 1), \frac{9216}{t^2} - \frac{5120}{t} \end{aligned} \quad (230)$$

$$\begin{aligned}
& -2 \operatorname{RootOf}(_Z^2 + 1) - \frac{2}{7} + 2 b1 + 2 b2 \Big] \Big], \Big[\Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2 \\
& + 1) - \frac{6}{7} + 3 b1 + 3 b2, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 6 \operatorname{RootOf}(_Z^2 + 1) - \frac{3}{7} + 3 b1 \\
& + 3 b2, 3 \operatorname{RootOf}(_Z^2 + 1) - \frac{3}{7} \Big], \Big[\frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 + 1) - \frac{4}{7} + 2 b1 \\
& + 2 b2, \frac{9216}{t^2} - \frac{5120}{t} - 4 \operatorname{RootOf}(_Z^2 + 1) - \frac{2}{7} + 2 b1 + 2 b2, -\frac{2}{7} + 2 \operatorname{RootOf}(_Z^2 \\
& + 1) \Big] \Big], [3, 2], [1, 1], \Big[\Big[\Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2 + 1) - \frac{3}{7} + 3 b1 \\
& + 3 b2, \frac{3}{7} \Big], \Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2 + 1) - \frac{3}{7} + 3 b1 + 3 b2, \\
& 3 \operatorname{RootOf}(_Z^2 + 1) \Big], \Big[3 \operatorname{RootOf}(_Z^2 + 1), \frac{3}{7} \Big] \Big], \Big[\Big[\frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 \\
& + 1) - \frac{2}{7} + 2 b1 + 2 b2, \frac{2}{7} \Big], \Big[\frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 + 1) - \frac{2}{7} + 2 b1 \\
& + 2 b2, 2 \operatorname{RootOf}(_Z^2 + 1) \Big], \Big[2 \operatorname{RootOf}(_Z^2 + 1), \frac{2}{7} \Big] \Big] \Big], \Big[[-6 t^3 + 136 t^2 - 834 t, -6 t^3 \\
& + 136 t^2 - 834 t, 0], [9216 t^2 - 5120 t, 9216 t^2 - 5120 t, 0] \Big], \Big[\Big[-3 \operatorname{RootOf}(_Z^2 + 1) \\
& - \frac{6}{7} + 3 b1 + 3 b2, -6 \operatorname{RootOf}(_Z^2 + 1) - \frac{3}{7} + 3 b1 + 3 b2, 3 \operatorname{RootOf}(_Z^2 + 1) \\
& - \frac{3}{7} \Big], \Big[-2 \operatorname{RootOf}(_Z^2 + 1) - \frac{4}{7} + 2 b1 + 2 b2, -4 \operatorname{RootOf}(_Z^2 + 1) - \frac{2}{7} + 2 b1 \\
& + 2 b2, -\frac{2}{7} + 2 \operatorname{RootOf}(_Z^2 + 1) \Big] \Big] \Big], \Big[[x - 12, 12], [x - 7, 7], [[0, -b1 + 1, 1 \\
& - b2], [0, -4 b1 + 4, 4 - 4 b2]], [[-b1 + 1, 1 - b2, b1 - b2], [-4 b1 + 4, 4 - 4 b2, \\
& 4 b1 - 4 b2]], [[[-b1 + 1, 0], [1 - b2, 0], [1 - b2, -b1 + 1]], [[-4 b1 + 4, 0], [4 \\
& - 4 b2, 0], [4 - 4 b2, -4 b1 + 4]]], [[[x - 12, 12], [x - 7, 7], [[0, -b1 + 1, 1 \\
& - b2], [0, -4 b1 + 4, 4 - 4 b2]], [[[-b1 + 1, 1 - b2, b1 - b2], []], [[-4 b1 + 4, 4 \\
& - 4 b2, 4 b1 - 4 b2], []]]], [], []], \Big[\Big[\Big[x^2 - \frac{32}{3} x - 1, \operatorname{RootOf}(3 _Z^2 - 32 _Z - 3) \Big] \Big], \\
& [[0, 2, 4]], [[2, 4, 2]], [[2, 0], [4, 0], [4, 2]] \Big] \Big], \Big[[[\infty, \infty], [x - 12, 12], [x - 7, 7], [x \\
& - 3, 3]], \Big[\Big[\frac{3}{7}, 3 \operatorname{RootOf}(_Z^2 + 1), -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2 + 1) - \frac{3}{7}
\end{aligned}$$

$$\begin{aligned}
& + 3 b l + 3 b 2 \Big], [0, -b l + 1, 1 - b 2], [0, -4 b l + 4, 4 - 4 b 2], \Big[\frac{2}{7}, 2 \operatorname{RootOf}(_Z^2 \\
& + 1), \frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 + 1) - \frac{2}{7} + 2 b l + 2 b 2 \Big], \Big[\Big[-\frac{6}{t^3} + \frac{136}{t^2} \\
& - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2 + 1) - \frac{6}{7} + 3 b l + 3 b 2, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} \\
& - 6 \operatorname{RootOf}(_Z^2 + 1) - \frac{3}{7} + 3 b l + 3 b 2, 3 \operatorname{RootOf}(_Z^2 + 1) - \frac{3}{7} \Big], [-b l + 1, 1 \\
& - b 2, b l - b 2], [-4 b l + 4, 4 - 4 b 2, 4 b l - 4 b 2], \Big[\frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 \\
& + 1) - \frac{4}{7} + 2 b l + 2 b 2, \frac{9216}{t^2} - \frac{5120}{t} - 4 \operatorname{RootOf}(_Z^2 + 1) - \frac{2}{7} + 2 b l + 2 b 2, \\
& - \frac{2}{7} + 2 \operatorname{RootOf}(_Z^2 + 1) \Big], \Big[\Big[\Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2 + 1) - \frac{3}{7} \\
& + 3 b l + 3 b 2, \frac{3}{7} \Big], \Big[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 \operatorname{RootOf}(_Z^2 + 1) - \frac{3}{7} + 3 b l + 3 b 2, \\
& 3 \operatorname{RootOf}(_Z^2 + 1) \Big], \Big[3 \operatorname{RootOf}(_Z^2 + 1), \frac{3}{7} \Big] \Big], [[-b l + 1, 0], [1 - b 2, 0], [1 - b 2, \\
& -b l + 1]], [[-4 b l + 4, 0], [4 - 4 b 2, 0], [4 - 4 b 2, -4 b l + 4]], \Big[\Big[\frac{9216}{t^2} - \frac{5120}{t} \\
& - 2 \operatorname{RootOf}(_Z^2 + 1) - \frac{2}{7} + 2 b l + 2 b 2, \frac{2}{7} \Big], \Big[\frac{9216}{t^2} - \frac{5120}{t} - 2 \operatorname{RootOf}(_Z^2 + 1) \\
& - \frac{2}{7} + 2 b l + 2 b 2, 2 \operatorname{RootOf}(_Z^2 + 1) \Big], \Big[2 \operatorname{RootOf}(_Z^2 + 1), \frac{2}{7} \Big] \Big], [[1, 1, 1], [1, 1, \\
& 1], [1, 1, 1], [1, 1, 1]] \Big] \Big]
\end{aligned}$$

> F1:= Hyp2F2Subst(L,x,t,R1[1],ext);

$$\begin{aligned}
F1 := & \left[-\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 1193x + 9984)}{(x-3)^2}, \right. \\
& -\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 6313x - 9984)}{(x-3)^2}, \\
& \frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 6313x - 9984)}{(x-3)^2}, \\
& \left. \frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 1193x + 9984)}{(x-3)^2} \right]
\end{aligned} \tag{231}$$

> find2F2RatIrr(L,R1,F1,x,t,ext);

$$\left[\left[\left[\left[\frac{1}{7}, \operatorname{RootOf}(_Z^2 + 1) \right], \left[\frac{10}{21}, \operatorname{RootOf}(_Z^2 + 1) + \frac{1}{3} \right], \left[\frac{17}{21}, \operatorname{RootOf}(_Z^2 + 1) \right] \right] \right] \right] \tag{232}$$

$$\begin{aligned} & + \frac{2}{3} \Big] \Big\}, [b1, b2] \Big], - \frac{2 (x-7)^4 (x-12)}{(x-3)^2} \Big], \Big[\Big[\Big[\Big[\frac{1}{7}, RootOf(_Z^2+1) \Big], \Big[\frac{10}{21}, \\ & RootOf(_Z^2+1) + \frac{1}{3} \Big], \Big[\frac{17}{21}, RootOf(_Z^2+1) + \frac{2}{3} \Big] \Big\}, [b1, b2] \Big], \\ & \frac{2 (x-7)^4 (x-12)}{(x-3)^2} \Big] \Big] \end{aligned}$$

TIME := 1235.484

(233)

```
> F:=sumdiffee(hyperterm([a1,a2],[b1,b2],x,k),k,J(x));
```

$$J(x) \Big) - a1 \ a2 \ J(x) = 0$$

```
> LA:=de2diffop(F,J(x));
```

```
> f:= normal((2*(x-7)^4*(x-12))/(x-3)^2,expanded);
```

```
> L:=ChangeOfVariables(LA,f);
```

$$\begin{aligned} & -32x-3)^2(-18x^9+1104x^8+9b_1x^6+9b_2x^6-28712x^7-246b_1x^5-246b_2x^5 \\ & +410207x^6+2239b_1x^4+2239b_2x^4-3473142x^5-7572b_1x^3-7572b_2x^3 \\ & +17377947x^4+7911b_1x^2+7911b_2x^2-47136476x^3+1674b_1x+1674b_2x \\ & +50750253x^2+81b_1+81b_2+10918044x+289305)Dx^2-(x-7)(x-3)(3x^2 \\ & -32x-3)(162a_1x^{13}+162a_2x^{13}-13392a_1x^{12}-13392a_2x^{12}+54x^{13} \\ & +488484a_1x^{11}+488484a_2x^{11}-81b_1b_2x^{10}-4464x^{12}-10311816a_1x^{10} \\ & -10311816a_2x^{10}+3942b_1b_2x^9+54b_1x^{10}+54b_2x^{10}+161688x^{11} \\ & +138694430a_1x^9+138694430a_2x^9-76437b_1b_2x^8-2628b_1x^9-2628b_2x^9 \\ & -3338948x^{10}-1230488408a_1x^8-1230488408a_2x^8+743784b_1b_2x^7+51528b_1x^8 \\ & +51528b_2x^8+42517662x^9+7177106808a_1x^7+7177106808a_2x^7 \\ & -3730306b_1b_2x^6-525656b_1x^7-525656b_2x^7-329479648x^8-26331569936a_1x^6 \end{aligned}$$

$$\begin{aligned}
& -26331569936 a_2 x^6 + 8693604 b_1 b_2 x^5 + 3093764 b_1 x^6 + 3093764 b_2 x^6 \\
& + 1280876700 x^7 + 53786784254 a_1 x^5 + 53786784254 a_2 x^5 - 6204114 b_1 b_2 x^4 \\
& - 11901456 b_1 x^5 - 11901456 b_2 x^5 + 1291124792 x^6 - 40167917760 a_1 x^4 \\
& - 40167917760 a_2 x^4 - 3119256 b_1 b_2 x^3 + 35609976 b_1 x^4 + 35609976 b_2 x^4 \\
& - 41985733658 x^5 - 20020906332 a_1 x^3 - 20020906332 a_2 x^3 - 474093 b_1 b_2 x^2 \\
& - 74600136 b_1 x^3 - 74600136 b_2 x^3 + 211757719656 x^4 - 3038105448 a_1 x^2 \\
& - 3038105448 a_2 x^2 - 30618 b_1 b_2 x + 64840662 b_1 x^2 + 64840662 b_2 x^2 \\
& - 486039160980 x^3 - 196092414 a_1 x - 196092414 a_2 x - 729 b_2 b_1 + 14104692 b_1 x \\
& + 14104692 b_2 x + 411965852796 x^2 - 4667544 a_1 - 4667544 a_2 + 688176 b_1 \\
& + 688176 b_2 + 92690008614 x + 2508765048) Dx - 2 a_1 a_2 (x - 7)^4 (3 x^2 - 32 x - 3)^6
\end{aligned}$$

$$\begin{aligned}
& \text{> ext:=indets(L,{RootOf,name}) minus \{x,Dx\};} \\
& \text{ext := \{a1,a2,b1,b2\}} \tag{238}
\end{aligned}$$

$$\begin{aligned}
& \text{> ext:= indets(map(s-> ReplirrRoot(s,\{\}),ext),\{RootOf,name\});} \\
& \text{ext := \{a1,a2,b1,b2\}} \tag{239}
\end{aligned}$$

$$\begin{aligned}
& \text{> extppp:=\{\};} \\
& \text{extppp := } \emptyset \tag{240}
\end{aligned}$$

$$\begin{aligned}
& \text{> E:= Singular(L,extppp);} \\
& E := \left[[x-7, 7], [x-3, 3], \left[x^2 - \frac{32}{3} x - 1, \text{RootOf}(3 _Z^2 - 32 _Z - 3) \right], [\infty, \infty], [x \right. \\
& \left. - 12, 12] \right] \tag{241}
\end{aligned}$$

$$\begin{aligned}
& \text{> F:= NotAppSing(L,E,ext);} \\
& F := [[\infty, \infty], [x-12, 12], [x-7, 7], [x-3, 3]] \tag{242}
\end{aligned}$$

$$\begin{aligned}
& \text{> Sirr:= irrsing2F2(L,t,F,ext);} \\
& Sirr := \left[[[\infty, \infty], [x-3, 3]], \left[\left[3 a_1, 3 a_2, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 a_1 - 3 a_2 + 3 b_1 \right. \right. \right. \\
& \left. \left. + 3 b_2 \right], \left[2 a_1, 2 a_2, \frac{9216}{t^2} - \frac{5120}{t} - 2 a_1 - 2 a_2 + 2 b_1 + 2 b_2 \right], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} \right. \right. \right. \\
& \left. \left. - \frac{834}{t} - 6 a_1 - 3 a_2 + 3 b_1 + 3 b_2, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 a_1 - 6 a_2 + 3 b_1 \right. \right. \\
& \left. \left. + 3 b_2, -3 a_1 + 3 a_2 \right], \left[\frac{9216}{t^2} - \frac{5120}{t} - 4 a_1 - 2 a_2 + 2 b_1 + 2 b_2, \frac{9216}{t^2} - \frac{5120}{t} \right. \right. \\
& \left. \left. - 2 a_1 - 4 a_2 + 2 b_1 + 2 b_2, 2 a_2 - 2 a_1 \right], [3, 2], [1, 1], \left[\left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} \right. \right. \right. \right. \\
& \left. \left. - 3 a_1 - 3 a_2 + 3 b_1 + 3 b_2, 3 a_1 \right], \left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3 a_1 - 3 a_2 + 3 b_1 + 3 b_2, \right. \right. \\
& \left. \left. 3 a_2 \right], [3 a_2, 3 a_1] \right], \left[\left[\frac{9216}{t^2} - \frac{5120}{t} - 2 a_1 - 2 a_2 + 2 b_1 + 2 b_2, 2 a_1 \right], \left[\frac{9216}{t^2} \right. \right. \\
& \left. \left. - \frac{5120}{t} - 2 a_1 - 2 a_2 + 2 b_1 + 2 b_2, 2 a_2 \right], [2 a_2, 2 a_1] \right], [[-6 t^3 + 136 t^2 - 834 t,
\end{aligned}
\tag{243}$$

$-6t^3 + 136t^2 - 834t, 0], [9216t^2 - 5120t, 9216t^2 - 5120t, 0], [[-6al - 3a2 + 3bl + 3b2, -3al - 6a2 + 3bl + 3b2, -3al + 3a2], [-4al - 2a2 + 2bl + 2b2, -2al - 4a2 + 2bl + 2b2, 2a2 - 2al]], [[x - 12, 12], [x - 7, 7]], [[0, -bl + 1, 1 - b2], [-bl + 1, 1 - b2, bl - b2], [1, 1, 1], [[-bl + 1, 0], [1 - b2, 0], [1 - b2, -bl + 1]], 2], [[0, -4bl + 4, 4 - 4b2], [-4bl + 4, 4 - 4b2, 4bl - 4b2], [1, 1, 1], [[-4bl + 4, 0], [4 - 4b2, 0], [4 - 4b2, -4bl + 4]], 2]]]$

> Sreg:= regsingtrue2F2(L,t,Sirr[-1],ext);
 $Sreg := [[[x - 12, 12], [x - 7, 7]], [[0, -bl + 1, 1 - b2], [0, -4bl + 4, 4 - 4b2]], [[-bl + 1, 1 - b2, bl - b2], [-4bl + 4, 4 - 4b2, 4bl - 4b2]], [[[-bl + 1, 0], [1 - b2, 0], [1 - b2, -bl + 1]], [[-4bl + 4, 0], [4 - 4b2, 0], [4 - 4b2, -4bl + 4]]]]$ (244)

> RSreg:= Sregseptrue2F2(L,Sreg,ext);
 $RSreg := [[[x - 12, 12], [x - 7, 7]], [[0, -bl + 1, 1 - b2], [0, -4bl + 4, 4 - 4b2]], [[-bl + 1, 1 - b2, bl - b2], []], [[-4bl + 4, 4 - 4b2, 4bl - 4b2], []]], [], []]$ (245)

> R1:=IrrRegAppsing2F2(L,t,E,ext);
 $R1 := \left[\left[[\infty, \infty], [x - 3, 3] \right], \left[\left[3al, 3a2, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3al - 3a2 + 3bl + 3b2 \right], \left[2al, 2a2, \frac{9216}{t^2} - \frac{5120}{t} - 2al - 2a2 + 2bl + 2b2 \right], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 6al - 3a2 + 3bl + 3b2, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3al - 6a2 + 3bl + 3b2, -3al + 3a2 \right], \left[\frac{9216}{t^2} - \frac{5120}{t} - 4al - 2a2 + 2bl + 2b2, \frac{9216}{t^2} - \frac{5120}{t} - 2al - 4a2 + 2bl + 2b2, 2a2 - 2al \right] \right], [3, 2], [1, 1], \left[\left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3al - 3a2 + 3bl + 3b2, 3al \right], \left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3al - 3a2 + 3bl + 3b2, 3a2 \right], [3a2, 3al] \right], \left[\left[\frac{9216}{t^2} - \frac{5120}{t} - 2al - 2a2 + 2bl + 2b2, 2al \right], \left[\frac{9216}{t^2} - \frac{5120}{t} - 2al - 2a2 + 2bl + 2b2, 2a2 \right], [2a2, 2al] \right] \right], [[-6t^3 + 136t^2 - 834t, -6t^3 + 136t^2 - 834t, 0], [9216t^2 - 5120t, 9216t^2 - 5120t, 0], [[-6al - 3a2 + 3bl + 3b2, -3al - 6a2 + 3bl + 3b2, -3al + 3a2], [-4al - 2a2 + 2bl + 2b2, -2al - 4a2 + 2bl + 2b2, 2a2 - 2al]]], [[[x - 12, 12], [x - 7, 7]], [[0, -bl + 1, 1 - b2], [0, -4bl + 4, 4 - 4b2]], [[-bl + 1, 1 - b2, bl - b2], [-4bl + 4, 4 - 4b2, 4bl - 4b2]], [[[-bl + 1, 0], [1 - b2, 0], [1 - b2, -bl + 1]], [[-4bl + 4, 0], [4 - 4b2, 0], [4 - 4b2, -4bl + 4]]]]]$ (246)

$$\begin{aligned}
& -4b2, 0], [4 - 4b2, -4b1 + 4]]], [[[x - 12, 12], [x - 7, 7], [0, -b1 + 1, 1 \\
& -b2], [0, -4b1 + 4, 4 - 4b2]], [[[-b1 + 1, 1 - b2, b1 - b2], []], [[-4b1 + 4, 4 \\
& -4b2, 4b1 - 4b2], []]], [], []], \left[\left[\left[x^2 - \frac{32}{3}x - 1, \text{RootOf}(3_Z^2 - 32_Z - 3) \right] \right], \right. \\
& [[0, 2, 4]], [[2, 4, 2]], [[[2, 0], [4, 0], [4, 2]]] \left. \right], \left[[[\infty, \infty], [x - 12, 12], [x - 7, 7], [x \\
& -3, 3]], \left[\left[3a1, 3a2, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a1 - 3a2 + 3b1 + 3b2 \right], [0, -b1 + 1, 1 \\
& -b2], [0, -4b1 + 4, 4 - 4b2], \left[2a1, 2a2, \frac{9216}{t^2} - \frac{5120}{t} - 2a1 - 2a2 + 2b1 \right. \right. \\
& \left. \left. + 2b2 \right] \right], \left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 6a1 - 3a2 + 3b1 + 3b2, -\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} \right. \right. \\
& \left. \left. - 3a1 - 6a2 + 3b1 + 3b2, -3a1 + 3a2 \right] \right], [-b1 + 1, 1 - b2, b1 - b2], [-4b1 + 4, 4 \\
& -4b2, 4b1 - 4b2], \left[\frac{9216}{t^2} - \frac{5120}{t} - 4a1 - 2a2 + 2b1 + 2b2, \frac{9216}{t^2} - \frac{5120}{t} \right. \\
& \left. - 2a1 - 4a2 + 2b1 + 2b2, 2a2 - 2a1 \right], \left[\left[\left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a1 - 3a2 \right. \right. \right. \\
& \left. \left. + 3b1 + 3b2, 3a1 \right], \left[-\frac{6}{t^3} + \frac{136}{t^2} - \frac{834}{t} - 3a1 - 3a2 + 3b1 + 3b2, 3a2 \right], [3a2, \right. \\
& \left. 3a1] \right], [[-b1 + 1, 0], [1 - b2, 0], [1 - b2, -b1 + 1]], [[-4b1 + 4, 0], [4 - 4b2, 0], [4 \\
& -4b2, -4b1 + 4]], \left[\left[\frac{9216}{t^2} - \frac{5120}{t} - 2a1 - 2a2 + 2b1 + 2b2, 2a1 \right], \left[\frac{9216}{t^2} \right. \right. \\
& \left. \left. - \frac{5120}{t} - 2a1 - 2a2 + 2b1 + 2b2, 2a2 \right] \right], [2a2, 2a1] \left. \right], [[1, 1, 1], [1, 1, 1], [1, 1, \\
& 1], [1, 1, 1]] \left. \right] \left. \right]
\end{aligned}$$

> F1:= Hyp2F2Subst(L,x,t,R1[1],ext);

$$\begin{aligned}
F1 := & \left[-\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 1193x + 9984)}{(x-3)^2}, \right. \\
& -\frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 6313x - 9984)}{(x-3)^2}, \\
& \frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 6313x - 9984)}{(x-3)^2}, \\
& \left. \frac{2(x^5 - 40x^4 + 630x^3 - 2808x^2 + 1193x + 9984)}{(x-3)^2} \right]
\end{aligned} \tag{247}$$

```
> find2F2RatIrr(L,R1,F1,x,t,ext);
```

$$\left[\left[\left[\left[aI, a2 \right], \left[\frac{1}{3} + a2, aI + \frac{1}{3} \right], \left[\frac{2}{3} + a2, aI + \frac{2}{3} \right] \right], [bI, b2] \right], -\frac{2(x-7)^4(x-12)}{(x-3)^2} \right], \quad (248)$$

$$\left[\left[\left[[a1, a2], \left[\frac{1}{3} + a2, a1 + \frac{1}{3} \right], \left[\frac{2}{3} + a2, a1 + \frac{2}{3} \right] \right], [b1, b2] \right], \frac{2(x-7)^4(x-12)}{(x-3)^2} \right]$$

```
> TIME :=time();
Hyp2F2Solutions(L);
time() - TIME;
```

TIME := 1318.187

$$\left\{ \left[\left[\left[[a1, a2] \right], [b1, b2], [0], [1] \right], \frac{2(x-7)^4(x-12)}{(x-3)^2} \right] \right\}$$

27.375

(249)