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The Electronic News Net of the
SIAM Activity Group on Orthogonal Polynomials and Special Functions

http://math.nist.gov/opsf

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Topics:

Calendar of Events:

May 11-15, 2020—**Postponed due to COVID-19 outbreak.**
LMS-CMI Research School: *Methods for Random Matrix Theory and Applications*
University of Reading, Reading, UK
https://janivirtanen.wordpress.com/research-school-2020

May 18-22, 2020—**Postponed due to COVID-19 outbreak.**
Baylor Analysis Fest: *From Operator Theory to Orthogonal Polynomials, Combinatorics, and Number Theory*
Baylor University, Waco, Texas, USA
https://www.baylor.edu/math/conference

June 15–24, 2020—**Cancelled due to COVID-19 outbreak.**
Foundations of Computational Mathematics (FoCM2020)
Workshop on Approximation Theory, June 18–20
Organized by Albert Cohen, Peter Binev and Maria Charina
Workshop on Random Matrices, June 18–20
Organized by Ionna Dumitriu and Sheehan Olver
Workshop on Special Functions and Orthogonal Polynomials, June 22–24
Organized by Ana Loureiro, Francisco Marcellán and Andrei Martínez Finkelshtein
Simon Fraser University, Vancouver, Canada
http://focm-society.org/2020/index.html

July 6–10, 2020—**Cancelled with select virtual sessions due to COVID-19 outbreak.**
SIAM Annual Meeting, held jointly with CAIMS
(Canadian Applied and Industrial Mathematics Society)
The OPSF activity group has a track of sessions and one invited speaker (Andrei Martínez-Finkelshtein). There are minisymposia on orthogonal polynomials and asymptotic methods, random matrices, symbolic computation, integrable systems and combinatorics, basic hypergeometric series and q-orthogonal polynomials.
Sheraton Centre Toronto Hotel, Toronto, Ontario, Canada
https://www.siam.org/Conferences/CM/Main/an20

July 13–17, 2020—**Postponed due to COVID-19 outbreak.**
XXI Lluis Santaló School *Random and Deterministic Point Configurations*
Universidad Internacional Menéndez Pelayo, Santander, Spain
https://www.ub.edu/santalo20/

July 13–18, 2020—**Postponed due to COVID-19 outbreak.**
Combinatorics around the q-Onsager algebra, celebrating the 65$^{\text{th}}$ birthday of Paul Terwilliger
Satellite event of the 8$^{\text{th}}$ European Congress of Mathematics
which will be held the prior week in Portorož, Slovenia,
Kranjska Gora, Slovenia
https://conferences.famnit.upr.si/indico/event/15/overview

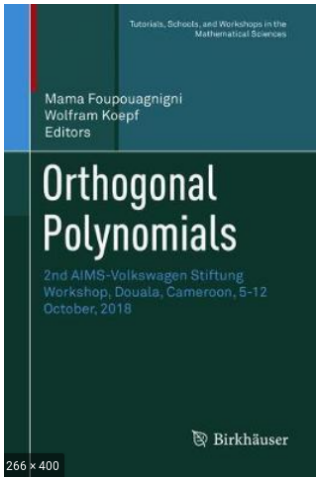
August 10–14, 2020—**Cancelled due to COVID-19 outbreak.**
OPSFA Summer School 2020
Radboud University, Nijmegen, The Netherlands
https://www.ru.nl/radboudsummerschool/courses/2020/opsfa-summer-school-2020

June 7–11, 2021—**New date due to COVID-19 outbreak.**
33$^{\text{rd}}$ International Colloquium on Group Theoretical Methods in Physics (Group33)
Cotonou, Benin
http://www.cipma.net/group33-cotonou-benin

June 20–26, 2021—**New date due to COVID-19 outbreak.**
8$^{\text{th}}$ European Congress of Mathematics (8ECM)
Mini-symposium on Orthogonal Polynomials and Special Functions
Organized by Paco Marcellán, Juan J. Moreno-Balcázar and Galina Filipuk,
Portorož, Slovenia
https://www.8ecm.si/minisymposia

July 6–9, 2021—**New date due to COVID-19 outbreak.**
Functional Analysis, Approximation Theory and Numerical Analysis (FAATNA)
Matera, Italy
http://web.unibas.it/faatna20/

From: Walter Van Assche (walter.vanassche@kuleuven.be)
Subject: Book Review by **Van Assche**: *Orthogonal Polynomials*, Douala, Cameroon



Orthogonal Polynomials, 2020, pp. x+683
2nd AIMS–Volkswagen Stiftung Workshop, 5–12 October 2018, Douala, Cameroon
Editors: Mama Foupouagnigni and Wolfram Koepf
Birkhäuser, Springer Nature, Switzerland
ISBN 978–3–030–36743–5 (eBook 978–3–030–36744–2)

The book under review contains the proceedings of a workshop, *Introduction to Orthogonal Polynomials and Applications*. This workshop was organized by AIMS–Cameroon and sponsored by Volkswagen Stiftung under guidance of Mama Foupouagnigni (AIMS–Cameroon & Université de Yaoundé I, Cameroon) and Wolfram Koepf (University of Kassel, Germany). This was the second workshop hosted by AIMS–Cameroon, with the first dealing with Computer Algebra and Applications.

AIMS (African Institute for Mathematical Sciences, Cameroon)

AIMS–Cameroon is one of six centres of the African Institute for Mathematical Sciences, a pan–African network of centres of excellence for postgraduate education, research and outreach in mathematical sciences. Mama Foupouagnigni was the founding Academic Director of AIMS–Cameroon and presently the Centre President of the AIMS–Cameroon campus in Limbe. Wolfram Koepf is a specialist in computer algebra who, over the past decades, has supervised many African students for their PhD in mathematics, often on topics involving orthogonal polynomials and computer algebra. The workshop took place in the Hotel Prince de Galles in Douala, the economic capital of Cameroon, from 5–12 October, 2018. There were 60 participants and 19 plenary speakers.

Since many of the potential African participants were not very familiar with the research field of Orthogonal Polynomials and Applications, it was decided to have a preliminary workshop (October 5–7) covering the basic aspects of the field. The main workshop (October 8–12) was aimed at introducing contemporary research topics. The present book reflects this concept of the workshop. It is divided into two parts: Part I gives an Introduction to Orthogonal Polynomials based on the talks given by Salifou Mboutngam, Maurice Kenfack Nangho, Daniel Duviol Tcheutia, Patrick Njionou Sadjang and Merlin Mouafo Wouodjié and the editors in the preliminary workshop, whereas Part II contains Recent Research Topics in Orthogonal Polynomials and Applications, which were presented by the plenary speakers, including Iván Area, Walter Van Assche, Gaspard Bangerezako, Hamza Chaggara, Mama Foupouagnigni, Jeff Geronimo, David Gómez–Ullate, Norbert Hounkonnou, Kerstin Jordaan, Wolfram Koepf, Ana Loureiro, Paco Marcellán, Sergei Suslov, and Luc Vinet.

Part I covers classical orthogonal polynomials (both continuous and discrete), generating functions, hypergeometric representations, and properties of the zeros of classical orthogonal poly–

nomials. There is also some emphasis on computer algebra for power series, summation and solutions of differential and difference equations. Part II covers contemporary research on orthogonal polynomials with topics such as the behaviour of zeros of orthogonal polynomials, properties of certain classes of semiclassical orthogonal polynomials and their relation with Painlevé equations, special functions and their applications in quantum mechanics, exceptional orthogonal polynomials and rational solutions of Painlevé equations, orthogonal polynomials of two variables and Fejér–Riesz factorization, hypergeometric multivariate orthogonal polynomials, Sobolev orthogonal polynomials, multiple orthogonal polynomials, applications in signal processing, spin chains, graphs and state revival, random matrices and quantum algebras.

The book is a valuable source for young researchers who are entering the research field of orthogonal polynomials. Of course it does not replace the classical books of Chihara (*An Introduction to Orthogonal Polynomials*), Szegő (*Orthogonal Polynomials*) or Ismail (*Classical and Quantum Orthogonal Polynomials in One Variable*), but it has the advantage that it has both an introductory part with the basic theory and an advanced part with some contemporary research on various aspects of orthogonal polynomials and their applications.

Participants and plenary speakers of the workshop in Douala, Cameroon.

From: Howard Cohl (Howard.Cohl@nist.gov)
Subject: Eight remembrances and communications to **Richard A. Askey**, Part II

Eight remembrances and communications to Richard Allen Askey (June 4, 1933—October 9, 2019)

by Cooper, Diaconis, Garvan, Gasper, Johnson, Krattenthaler, Olver and Ono

An obituary of Richard A. Askey appeared in OP-SF Net 26.6, published on November 15, 2019. Below are eight remembrances of Dick from some of his colleagues, students, and friends:

Shaun Cooper, Persi Diaconis, Cyndi & Frank Garvan, George Gasper, Warren Johnson, Christian Krattenthaler, Peter Olver and Ken Ono.

The following collection of eight individual contributions regarding Dick represent *part II* of a multi-part series selected from the Askey Liber Amicorum, a Friendship Book for Dick Askey. The Askey Liber Amicorum was described in OP-SF Net 27.1, published on January 15, 2020.

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Shaun Cooper, Massey University, Albany, New Zealand.

I met Dick Askey when I was a first-year graduate student at the University of Wisconsin. Having seen the advertisement for the weekly "Special Functions Seminar", I was intrigued and went along. When Dick noticed I was new, he asked my name and I was immediately made to feel welcome. The topic that semester was Epstein's zeta function, following Selberg and Chowla [1], and the next 50 minutes I can only describe as thrilling. I had never seen anything like it in terms of the insights and ideas that were revealed—I can remember thinking it was as if Dick had direct access to Riemann—and my brain had to work as fast as it could to try to keep up. At the end of my first year I passed my qualifying exams and Dick agreed to take me on as a student.

The Special Functions Seminar continued for every semester, and quite often Dick also taught a graduate course in Special Functions Math 805 that I also enrolled for. The other regular participants in the seminar were Ranjan Roy and fellow student Warren Johnson, both of whom gave clear and beautiful lectures. Many other mathematicians, especially visitors, joined the seminar from time to time. The seminar remains the best learning experience in all of my education at any level.

During the seminars, Dick would frequently mention unsolved problems. I solved one of them, something about a constant term in a Macdonald-type identity associated to the root system G_2, and expanded the topic to make it into a thesis.

A frequent subject of the seminar, which I just loved, was the mathematics of Srinivasa Ramanujan. One day, Dick gave me a copy of the monograph "Development of Elliptic Functions according to Ramanujan" that had just been written by K. Venkatachaliengar and suggested I edit it. The typographical errors were not too difficult to fix, but there were some other issues that were deeper, for which I lacked the skills and knowledge to correct. I kept coming back to the monograph over the years and I am delighted that the edited book was published by World Scientific in 2012 [2]. That makes it the most overdue homework assignment I have completed

by about 20 years!

One of the fun things about Dick's teaching was his informal definitions. His definition of "*to understand something*" for an undergraduate student, was "*to be able to explain it to a younger brother or sister in a way that they would get it.*". The definition he applied for himself, and that he encouraged graduate students to use, was "*to be able to give a lecture on it, in a year's time, without notes or preparation.*"

Dick expressed great personal interest in my progress. He made sure that I met mathematicians from other universities and countries and made it possible for me to attend several conferences. After I graduated and returned to work in New Zealand he was concerned that I did not become isolated and helped me make contacts that have been essential in my career development and for which I will always be grateful.

In conclusion, thank you Dick for everything. I learned so much.

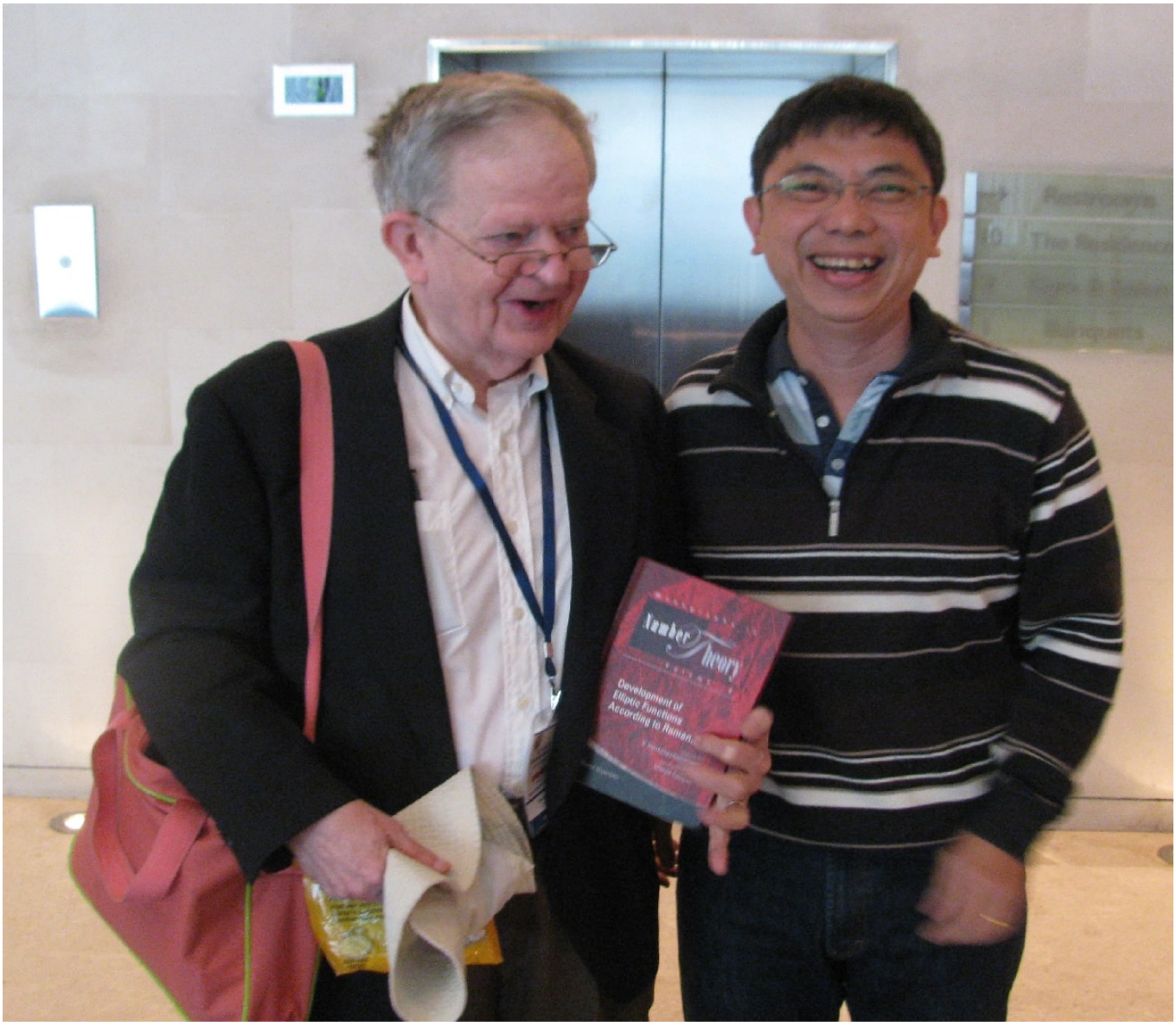
Shaun Cooper

References

[1] A. Selberg and S. Chowla. On Epstein's zeta-function. *Journal für die Reine und Angewandte Mathematik. [Crelle's Journal]*, 227:86–110, 1967.

[2] K. Venkatachaliengar. *Development of elliptic functions according to Ramanujan*, volume 6 of *Monographs in Number Theory*. World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2012. Edited, revised, and with a preface by Shaun Cooper.

Dick Askey and Shaun Cooper at the *Askey 80th Birthday Conference*, Madison, Wisconsin, USA in December 2013. Photo taken by Patsy Wang-Iverson.



Dick Askey and Heng Huat Chan at Ramanujan 125 at Delhi University, Delhi, India in December 2012. "*Dick was holding Cooper's book and he was obviously very proud of Cooper.*"

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Persi Diaconis, Stanford University, Stanford, California, USA.

Dear Friend Dick,

You are one of my heroes. Not just because of your wonderful work but because of your bravery under fire. As we both know, there was a long time when our math world just didn't know what to think about orthogonal polynomials: was it applied math, a corner of representation theory, or numerical analysis? Just what was it??

Anyway, it got "no respect". You kept soldiering on and beat the bastards at their own game. I recently taught an orthogonal polynomials course here at Stanford. I had 20+ grad students take it for credit—and quite a few of them really learned something about what my old colleagues Pólya, Szegő, and Dick Askey had to say!

Your steadfastness through 50 years of scrutiny, with humor and good taste, has really meant the world to those of us in your orbit.

All good things,
Persi Diaconis

Persi Diaconis and Dick Askey at the *Askey 80th Birthday Conference*, Madison, Wisconsin, USA in December 2013. Photo taken by Patsy Wang-Iverson.

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Cyndi & Frank Garvan, University of Florida, Gainesville, Florida, USA.

We were a young family. Gerard was a baby, Jeff was three, and Mike was six. Frank had finished his PhD and had interviewed for permanent positions at several universities with no luck in getting any job offers. At this time he was teaching at a branch campus of Penn State in York, Pennsylvania. He was resigned to stay there, he was the one we were dependent on to pay the bills. I will then never forget receiving a phone call from Frank. "*Cyndi, I just took a post doc job at University of Wisconsin.*" "*Frank,*" I said, "*In America it is custom to talk to your wife before making that kind of decision.*" "*Oh yeah,*" Frank said, "*but this is Dick Askey!*" He was so excited and happy and of course I would move to any place with him but I did think it worth mentioning that wives are generally involved.

So off we moved to Madison with three little boys and our worldly possessions packed in a U-Haul truck. We had no idea where to live but we did know how to camp. So we camped in Madison until we found a place to rent (our decision was probably a bit hasty but two weeks of camping with a baby and two small boys took its toll on our standards).

And then I got to meet Liz and Dick Askey. They were so kind to us. They invited us on excursions and to parties. The Askeys took us to see a play by Shakespeare at the outdoor American Players theatre. I remember the evening as magical. Being with such an intelligent and interesting couple who were taking time to show Frank and I the treasures of Madison.

Over the years and at many conference occasions, I had the great opportunity of spending time with Liz. We were in rooms pretty close to each other during the *Ramanujan's Centenary conference* at Urbana. Liz kept me sane while I was perpetually chasing my three young sons. She was always calm and upbeat with a fantastic sense of humor. I remember her vast interests in so many things. She took advantage of seeing whatever a locale had to offer (e.g., the special children's book collection at the University of Florida). In short, Liz was an inspiring role model for me. I have always looked up to her.

Dick Askey is one of my heroes for many reasons. He helped Frank's career at a very critical time. He was an amazing mentor to Frank. I can remember example after example of his thoughtfulness of others. I admired his concern about the inability of Freeman Dyson to attend a Florida conference and how he taught conference goers the tradition of sending a note to someone who could not attend. Whenever I spoke to Dick about my messy work in the world of medical research, he would wisely and respectfully remark that, "*Math is easy, People are hard.*"

I am grateful that I got to be a small part of the community that Dick and Liz, Bruce and Helen, and George and Joy built. A community that felt like family whenever we had the opportunity to convene and share the latest mathematical discoveries and just catch up with life's events. The world of mathematics is a world of beauty. The community built by the Askey's, Berndt's, and Andrews' is also a world of kindness.

This is where Cyndi asked me to add my part. Reading Cyndi's part has reminded me what we should remember. The q-world and the Ramanujan world is a very nice place to be and the leadership of Dick Askey, George Andrews, and Bruce Berndt has made it so. Repeating Cyndi I again thank Dick for bringing me to Wisconsin and introducing me to many

wonderful aspects of mathematics. Dick encouraged me to go over to Physics Dept and learn the computer algebra software REDUCE. This was the beginning for me in a life of q and experimental math. It was very clever for Dick to get me to referee a paper that was completely outside my experience. This pushed me into the problem of the Macdonald identities and surprisingly led me to tackle the F_4 case etc. Throughout my career sign-posts have magically appeared and I have trusted my instincts to follow them. In Wisconsin Dick showed me the signpost towards Dennis Stanton and Minnesota. Thanks again Dick for being a wonderful mentor and human being.

Dick, we are praying for your comfort and peace. Eleanor Roosevelt said that, "*Many people will walk in and out of your life. But only true friends will leave footprints on your heart.*" The Askeys have left footprints on the hearts of many of their friends in the world of q and Ramanujan's mathematics. We are all the richer for the presence they have had in our lives.

Cyndi and Frank Garvan, September 9, 2019

l to r: Dick Askey, Krishna Alladi, Frank Garvan, Michael Hirschhorn, at the *Combinatorics of q-Series and Partitions* conference in Honor of George Andrew's 75th Birthday at Tianjin, China in 2013.

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George Gasper, Northwestern University, Evanston, Illinois, USA.

Thinking of you and thanking you

My first correspondences with Dick started in the Spring of 1967 when he kindly mailed me copies of several of his pre 1968 papers, partially joint with R.P. Boas, I. Hirschman and S. Wainger, on ultraspherical expansions, transplantation theorems, mean summability and norm inequalities for some orthogonal series, etc. (see Askey and Hirschman [1], Askey and Wainger [2], and Gasper, Ismail, Koornwinder, Nevai, and Stanton [3], the Curriculum Vitae of Richard A. Askey is on pp. 19–29), and encouraged me to attend his Special Functions course, seminars, and talks during my PostDoc at the University of Wisconsin in Madison. During his talks and conversations, Dick was willing to point out many interesting open problems that he and others have been working on, encouraging others to also try to solve them. In particular, at one of his talks in February of 1968 he pointed to a complicated looking integral containing products and quotients of different sine functions that he and James Fitch needed to obtain a new shorter proof of a 1953 theorem of Turán on positivity of a certain trigonometric sum. I responded with just one word "*differentiate*" and after looking at the integral for a few seconds Dick said "*Yes, that will work!*". A few days later he handed me a preprint of the R. Askey, J. Fitch, and G. Gasper "On a positive trigonometric sum" one page paper, which was published later that year in [4]. Subsequent joint papers with Dick were a lot harder to do!

In Askey and Gasper [5], along with several other results, we used a sum of squares of Gegenbauer (ultraspherical) polynomials to prove that if $\alpha \geq -2$, then the sum of the Jacobi polynomials $P_k^{(\alpha,0)}(x), k = 0, 1, ..., n$, is non-negative for $-1 < x \leq 1$, $n \geq 0$, and is equal to zero only when $\alpha = -2$ and either $n = 1$ or $x = 1$, $n \geq 1$ (now called the Askey-Gasper inequality), which was then used to prove that the Cesáro $(C, \alpha + 2)$ means of the Jacobi series of a non-negative function are non-negative when $\alpha \geq -1/2$ and $\beta = 0$. Later, sums of squares of certain Jacobi polynomials were used in Gasper [6] to prove more general inequalities and, in particular, the non-negativity of a fractional derivative of order $1/2$ of the sum in Askey-Gasper inequality, and that the Cesáro $(C, \alpha + \beta + 2)$ means of the Jacobi series of a non-negative function are nonnegative when $\alpha, \beta \geq -1/2$. The latter is best possible in the sense that all of the Cesáro (C, λ) means are not necessarily non-negative when $\lambda < \alpha + \beta + 2$. Since, due to a delay in publication, the first proofs of Askey-Gasper inequality were presented in 1975 in Askey's Regional Conference book [7] and in my survey paper [8]. Dick and I were really surprised that the Askey-Gasper inequality, which was a fractional integral of order $1/2$ of a sharper inequality, sufficed for de Branges to complete his proof of the Bieberbach conjecture (and of the more general Robertson and Milin conjectures) in [9]. Also see the papers and comments in Baernstein II, Drasin, Duren, and Marden [10].

Concerning the Askey and Wilson groundbreaking Memoir *Some basic hypergeometric orthogonal polynomials that generalize Jacobi polynomials* [11] in which they derived the orthogonality of certain balanced ${}_4F_3$ hypergeometric series and ${}_4\phi_3$ basic hypergeometric series, I wish to thank Dick for telling me that it was after seeing the Saalschützian (which he changed to the simpler word "balanced") ${}_4F_3$ series representations for the Hahn polynomials in a preprint of my paper [12, (3.18)–(3.21)] that he decided to try to discover

what other orthogonal polynomials can be represented by balanced ${}_4F_3$ series and their q-analogues. It was while writing the first edition of the Gasper and Rahman *Basic Hypergeometric Series* book [13, 14], that Mizan Rahman and I decided to call the continuous ${}_4\phi_3$ orthogonal polynomials derived by Askey and Wilson in their Memoir the *Askey-Wilson polynomials*. Years later, Mizan and I derived some orthogonal multivariable generalizations of the Askey-Wilson polynomials in [15].

I also wish to thank Dick for introducing me to Special Functions, which he called *Useful Functions* in view of their applications, helping me transfer to a PostDoc position at the University of Toronto in order for my wife to satisfy a two year foreign residency requirement, recommending me for an Assistant Professor position, a Sloan Fellowship, an Associate Professor position and, later, a tenured Professor position at Northwestern University, and for many years of stimulating mathematical discussions. Most recently, last year he suggested to David C. Brown and Shaun William Davies, who in order to help make their work on Financing Efficiency of Securities-Based Crowdfunding mathematically rigorous needed a proof of a conjectured inequality for the quotient of products of certain hypergeometric series, that they contact me. For my subsequent proof and comments, see Addendum 1 in the Brown and Davies paper [16].

George Gasper
george@math.northwestern.edu

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[2] R. Askey and S. Wainger. Mean convergence of expansions in Laguerre and Hermite series. *American Journal of Mathematics*, 87:695–708, 1965.

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[5] R. Askey and G. Gasper. Positive Jacobi polynomial sums. II. *American Journal of Mathematics*, 98(3):709–737, 1976.

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Dick Askey and George Gasper.

l to r: Bruce Berndt, Atul Dixit, Walter Van Assche, Dick Askey, Alan Sokal, Christian Krattenthaler, George Andrews, Patsy Wang-Iverson at the *Askey 80th Birthday Conference*, Madison, Wisconsin, USA in December 2013.

Tom Koornwinder and Dennis Stanton listening to Alan Sokal at the *Askey 80th Birthday Conference*, Madison, Wisconsin, USA in December 2013.

Conference table with Dick Askey at the *Askey 80th Birthday Conference*, Madison, Wisconsin, in December 2013. Clockwise: Dick Askey, Howard Cohl, Roderick Wong, Mourad Ismail, Hans Volkmer, Martin Muldoon, and Ted Chihara.

Dick at the lectern, with Liz and family members at the family table during the banquet at the *Askey 80th Birthday Conference*, Madison, Wisconsin, USA in December 2013.

Photo at the Askey family table behind the lectern before Gasper spoke during the *Askey 80th Birthday Conference* banquet, Madison, Wisconsin, USA in December. l to r: David Foss (son-in-law), Suzanne Askey (daughter), Liz Askey (wife), Jim Zurlo (son), Kathy Zurlo (daughter-in-law).

Dick Askey, Alan Sokal, George Andrews, and Peter Duren at the *Askey 80th Birthday Conference*, Madison, Wisconsin, USA in December 2013.

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Warren Johnson, Connecticut College, New London, Connecticut, USA.

I was a directionless student in high school until falling in love with techniques of integration late in my senior year. While there were occasional highlights, such as Laplace transforms and residue calculus, and I did slowly learn to like some other parts of mathematics, my undergraduate years weren't much better. My first year of graduate school was so unpleasant that I was just about ready to abandon the idea of becoming a mathematician. When I took Dick's Special Functions course in my third semester, my life changed.

It was the first time I found someone who loved integrals as I did, and I saw immediately that Dick knew far more about them than anyone I'd ever met. On the first day he connected the beta integral to the gamma function using functional equations, a breathtaking argument that I have since used several times in my own classes. As a final project, he had me work through his paper "*More q-beta integrals*" (joint with Ranjan Roy) [1]. This taught me what it really meant to read a significant piece of mathematics, and the realization that I could actually do it was a tremendous thrill. In this context, I must also mention "*An Elementary Evaluation of a Beta Type Integral*" [2], one of my favorite papers. I think Dick was not entirely satisfied with the original evaluation of the Askey–Wilson integral, in spite of its importance. Here he evaluates a generalization by a *tour de force* of functional equations.

Dick also opened up the world of q to me, which became my second great love in mathematics. In his honor, I want to say a little about an elementary idea in integration by parts. Examples are scattered in the literature, but it seems that no one has ever written it up systematically. When I showed it to Dick some years ago, he told me he had not seen it before. That will surprise anyone who knows him.

I'll start with an integral that appears in three great old calculus books, with three different treatments. (Reading great old mathematics is another of Dick's gifts to me.) We can write

$$\begin{aligned}\int \frac{xe^x}{(x+1)^2}\,dx &= \int \frac{(x+1)e^x - e^x}{(x+1)^2}\,dx \\ &= \int \frac{e^x}{x+1}\,dx - \int \frac{e^x}{(x+1)^2}\,dx.\end{aligned} \tag{E}$$

If you know a lot about about integrals, these both look hopeless. If you only know a little, it is natural to integrate one of them by parts, say $u = e^x$ and $dv = dx/(x+1)^2$, so $du = e^x\,dx$ and $v = -1/(x+1)$. Then

$$\int \frac{xe^x}{(x+1)^2}\,dx = \int \frac{e^x}{x+1}\,dx - \left(\frac{-e^x}{x+1} + \int \frac{e^x}{x+1}\,dx\right) = \frac{e^x}{x+1} + C.$$

This is essentially Lacroix's solution [3, p. 94]. I like to describe it by saying that the integral *commits suicide*. Euler [4, p. 145, Section 233] more or less recognizes (E) as a quotient rule, and Bertrand [5, p. 10] integrates by parts at the beginning with $u = xe^x$ and $dv = dx/(x+1)^2$.

Many of the suicidal integrals I have seen are based on the fact that

$$\frac{d}{dx}\,\frac{\sin x}{1+\cos x} = \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2} = \frac{1}{1+\cos x},$$

and consequently

$$\int \frac{f'(x)\sin x + f(x)}{1+\cos x}\,dx = \frac{f(x)\sin x}{1+\cos x} + C.$$

The examples $f(x) = e^x$ and $f(x) = x$ are in several places in the literature. Edwards gives one or two others in his *Treatise on the Integral Calculus* [6, 7].

This is not the place to multiply examples. I'll conclude with one that I was constructing at the moment the power went out when Tropical Storm Irene hit the Connecticut shoreline in 2011. Rewrite

$$\int e^{x\sqrt{2}}\tan^3 x\,dx = \int e^{x\sqrt{2}}\tan x\sec^2 x\,dx - \int e^{x\sqrt{2}}\tan x\,dx,$$

and do *both* integrals by parts. In the first take $u = e^{x\sqrt{2}}$ and $dv = \tan x\sec^2 x\,dx$, so that $v = \frac{1}{2}\sec^2 x$ and $du = \sqrt{2}\,e^{x\sqrt{2}}\,dx$. In the second take $u = \tan x$ and $dv = e^{x\sqrt{2}}\,dx$, so that $du = \sec^2 x\,dx$ and $v = \frac{1}{\sqrt{2}}\,e^{x\sqrt{2}}$. Then we have

$$\begin{aligned}\int e^{x\sqrt{2}}\tan^3 x\,dx &= \frac{1}{2}\,e^{x\sqrt{2}}\sec^2 x - \frac{1}{\sqrt{2}}\int e^{x\sqrt{2}}\sec^2 x\,dx \\ &\qquad - \left(\frac{1}{\sqrt{2}}\,e^{x\sqrt{2}}\tan x - \frac{1}{\sqrt{2}}\int e^{x\sqrt{2}}\sec^2 x\,dx\right) \\ &= \frac{1}{2}\,e^{x\sqrt{2}}\sec^2 x - \frac{1}{\sqrt{2}}\,e^{x\sqrt{2}}\tan x + C.\end{aligned}$$

This is the simplest of a family of integrals. The next two members are

$$\int e^{x\sqrt{2}}\tan^2 x\sec^2 x\,dx \qquad \text{and} \qquad \int e^{x\sqrt{2}}\left(2\tan^5 x + \tan x\right)dx.$$

At least from my perspective, the late 1980s were not a great time for special functions at Wisconsin. There were strong postdocs (first Frank Garvan and then Peter Forrester), but interest among the other graduate students was minimal. Fortunately, I couldn't have cared less what they thought—if I hadn't learned all the mathematics I ought to have done by then, at least I had learned that. Things picked up in the early 1990s after Shaun Cooper came. But what made Dick unique is that he wasn't just trying to educate the students at Wisconsin, he was trying—and in a large measure succeeding—to educate the entire mathematical community. (Even this is not broad enough, since he often talked to people in other disciplines, especially physics.) He is irreplaceable.

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Obviously, I knew the name "Richard Askey" already as a student, through his work on orthogonal polynomials and q-series. I remember in particular studying the great Memoirs volume of his and Mourad Ismail [1] which made a big (and lasting) impression on me.

When I met Richard in person I do not remember for sure; it may have been at an AMS meeting in 1994 or at a Fields Institute Workshop in 1995. It goes without saying that I was struck by his personality. On the other hand, I can't report any specific encounters that made a direct impact on me or on my career (that may have been different behind the scenes, but that I do not know); although he was always very encouraging, our research interests were not close enough I guess.

Instead I want to quote Dominique Foata on Richard Askey. Dominique once told me: "*If, in a referee's report, you read somewhere "...and this is very important!", then this is a report by Richard Askey.*" (I don't know whether this is true; if so, I never got a report from Richard Askey ...) We all know Richard Askey as somebody who acts as a preacher and prophet, telling people which research problems they should look at, which methods may work, which research directions look promising, etc., frequently against the "main stream". Dominique added that Richard has **always** been like that, already as a young man! This is very impressive since this requires a lot of courage, but most importantly a most profound knowledge and taste. As an example, Dominique mentioned his bijective proof of the Mehler formula for Hermite polynomials. Dominique told me that he himself did not think greatly of it, he considered it a nice exercise, but nobody would be interested in it. Therefore, he did not even want to publish it. Richard insisted that he **had** to publish it. As we all know today, the consequence was a whole flood of research by so many people (continuing up to today) showing that there is much more combinatorics in analysis and leading to so much more insight than had existed before, and it made Dominique an invited speaker at the ICM in Warszawa in 1983.

No doubt, Richard Askey could also be very critical. As it fits to a personality of his caliber, this criticism did not stop in front of him. I have heard Richard tell the following at least twice in talks that he gave, but—unfortunately—it does not seem to be written down in one of his articles. Fortunately, Doron Zeilberger recorded it in his article that contains his proof of the refined alternating sign matrix conjecture [2, p. 63]. The story begins with Richard's firm belief that the determinantal formula for orthogonal polynomials in terms of the moments of the orthogonality measure is aesthetically pleasing but otherwise completely useless. This belief was destroyed by his student Jim Wilson who used exactly that formula to find the—what are now called—Wilson polynomials, and subsequently led to the discovery of the—what are now called—Askey-Wilson polynomials. After having told

this, Richard then gave the following advice to young people: "*If an authority in the field tells you that you should look at a certain thing, listen! If that authority tells you to* **not** *look at a certain thing,* **don't** *listen!*"

This is wisdom at its purest. Richard Askey stands and stood exactly for this.

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Given their mathematical proximity, I wish my father were still around to write this encomium to Dick. They were the best of friends—fellow toilers in the special function vineyards—despite their distinct styles and mathematical tastes, and despite not always seeing eye to eye. Dick is more the wild American, a devotee of the luxuriant jungle of the Bateman manuscript (although of course Bateman himself was originally British), while my father, being the true Englishman that he remained despite moving to the US in 1961, preferred the meticulously cultivated garden of Abramowitz and Stegun and, of course, the subsequent DLMF [1]. Nevertheless, Dick contributed to three of the chapters of the DLMF one of which he even coauthored with my father and Ranjan Roy and Roderick Wong—which was, in fact, their only collaborative effort. Also, I would not be surprised if Dick were the one who convinced my father to include the chapter on Meijer G–functions, which Dick co–wrote with Adri Olde Daalhuis, since my father was of the opinion that they were much too general to be of any use. (However, right after he died, I happened to ask Mathematica to evaluate a certain integral involving the Airy function, and was surprised when an answer came back in terms of a hypergeometric function. I subsequently discovered that the package that accomplished this was in fact based on Meijer G–functions. Sadly I was no longer able to get my father's opinion on this development.) Further, while certainly acknowledging their importance, my father was far less into orthogonal polynomials than Dick, preferring to concentrate his efforts on Airy, Bessel, (confluent) hypergeometric, Whittaker, and the like.

As a result of their connection, I knew Dick well before starting my own mathematical career, although my memory of when and how I first met him is cloudy. I do vividly remember the famous conference in Wisconsin in early spring, 1975, when I was still in graduate school, and where they were both in attendance and in top form, with many disagreements, and finally ending in an epic snowstorm that closed off the airport when we were due to depart. (Somehow this did not dissuade me from eventually going to Minnesota!) While I learned enough about special functions to use them on those rare occasions they showed up in my own work, I did not follow their lead (or that of my colleagues Willard Miller and Dennis Stanton, for that matter), preferring to toil in my own (somewhat unkempt, but more "symmetrical") mathematical garden. I, of course, ran into Dick on many occasions, particularly when returning to Wisconsin, and was always in awe of his brilliance, insight,

enthusiasm for mathematics, and strong opinions on many subjects, that he did not hesitate to state. And, while I am not close mathematically, I still regard him as one of my early inspirations as to what it would be like to be a true mathematician.

So both my wife Cheri (another mathematician, as is our son Sheehan) and I wish Dick the very best on this occasion. I wish I could be there to celebrate in person, but my continuing administrative duties and other travels are tying my hands at this time.

All the best, Dick!
Peter

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For Dick,

It was a pleasure to speak at the lovely retirement dinner in your honor in Madison many years ago. Although I no longer have my notes, I remember some of the thoughts I shared with our colleagues at the time. You have been a gift to mathematics and mathematicians (of all ages). Let me remind you of the significant roles that you have played in my life.

I first learned of Ramanujan thanks to the worldwide effort that you spearheaded in the early 1980s. My father was one of the many mathematicians who made a contribution for the Granlund bust, and he treasured the thank-you note that he received from Janaki Ammal. This note has served as a constant reminder of beauty in mathematics, and is now one of my most cherished possessions. In fact, the note is prominently displayed on the wall in my office, and it welcomes me each work day. Thank you.

I have many fond memories of your lectures on special functions, orthogonal polynomials, and their role in number theory. The flair with which you introduced Ramanujan's identities with your t-shirts was breathtaking. I wish we had captured one of these lectures on video so that young students entering the field today can share the wonder of these beautiful identities. Thank you.

Your encyclopedic knowledge of mathematical history was mind-blowing. The commentary you added at the end of lectures made us recognize the roles we played in our own research. Instead of concentrating on grants and numbers of papers, I learned from you that important research are like bricks. An important theorem has to fit well with the edifice of theorems laid before us by great mathematicians. This realization was important to me during my formative years. Thank you.

In addition to your role as a world class mathematician, I have to thank you for your investment in education. I remember the days you would read Dr. Seuss to your UW math education students. I thank you for taking a stand on critical issues which plague K-12 education. Although the challenges remain, I am happy to report that your voice persists

(certainly among the AMS leadership and the US National Committee for Mathematics). I am doing my best, as are many others. Thank you.

For so many reasons you have been a gift to mathematics and mathematicians.
Thank you.

With the deepest admiration.
Ken Ono

Topic #3 ——— OP - SF Net 27.3 ——— May 15, 2020

From: OP-SF Net Editors
Subject: Preprints in arXiv.org

The following preprints related to the fields of orthogonal polynomials and special functions were posted or cross-listed to one of the subcategories of arXiv.org during March and April 2020. This list has been separated into two categories.

OP-SF Net Subscriber E-Prints

http://arxiv.org/abs/2003.00398
On degenerate gamma functions
Taekyun Kim, Dae san Kim

http://arxiv.org/abs/2003.01646
Singular nonsymmetric Jack polynomials for some rectangular tableaux
Charles F. Dunkl

http://arxiv.org/abs/2003.01676
Hankel determinants of linear combinations of moments of orthogonal polynomials
Johann Cigler, Christian Krattenthaler

http://arxiv.org/abs/2003.01921
The Absent-Minded Passengers Problem: A Motivating Challenge Solved by Computer Algebra
Carsten Schneider

http://arxiv.org/abs/2003.04911
The Smallest Eigenvalue Distribution of the Jacobi Unitary Ensembles
Shulin Lyu, Yang Chen

http://arxiv.org/abs/2003.05335
On the Laguerre fractional integro-differentiation
Semyon Yakubovich

http://arxiv.org/abs/2003.06040
On modified kernel polynomials and classical type Sobolev orthogonal polynomials
Sergey M. Zagorodnyuk

http://arxiv.org/abs/2003.06668
Proof of the Chudnovsky's series for $1/\pi$
Jesús Guillera

http://arxiv.org/abs/2003.06726
A discrete and q-Asymptotic Iteration Method
Mourad E. H. Ismail, Nasser Saad

http://arxiv.org/abs/2003.07330
Hyponormal Toeplitz Operators on Weighted Bergman Spaces
Trieu Le, Brian Simanek

http://arxiv.org/abs/2003.07403
Evaluation of some non-elementary integrals involving the generalized hypergeometric function with some applications
Victor Nijimbere

http://arxiv.org/abs/2003.07517
The geometric distribution of Selmer groups of elliptic curves over function fields
Tony Feng, Aaron Landesman, Eric Rains

http://arxiv.org/abs/2003.07837
q-Orthogonal dualities for asymmetric particle systems
Gioia Carinci, Chiara Franceschini, Wolter Groenevelt

http://arxiv.org/abs/2003.08192
Some multivariate master polynomials for permutations, set partitions, and perfect matchings, and their continued fractions
Alan D. Sokal, Jiang Zeng

http://arxiv.org/abs/2003.08324
On Polynomial Solutions of Linear Differential Equations with Applications
Kyle R. Bryenton, Andrew R. Cameron, Keegan L. A. Kirk, Nasser Saad, Patrick Strongman, Nikita Volodin

http://arxiv.org/abs/2003.08327
Incomplete Symmetric Orthogonal Polynomials of Finite Type Generated by a Generalized Sturm-Liouville Theorem
Mohammad Masjed-Jamei, Zahra Moalemi, Nasser Saad

http://arxiv.org/abs/2003.08678
Solving the Dirichlet and Holmgren problems for a three-dimensional elliptic equation by the potential method
Tuhtasin Ergashev

http://arxiv.org/abs/2003.09558
The Heun-Racah and Heun-Bannai-Ito algebras
Geoffroy Bergeron, Nicolas Crampé, Satoshi Tsujimoto, Luc Vinet, Alexei Zhedanov

http://arxiv.org/abs/2003.09666
Double lowering operators on polynomials
Paul Terwilliger

http://arxiv.org/abs/2003.09668
Notes on the Leonard system classification
Paul Terwilliger

http://arxiv.org/abs/2003.10231
The positive Dressian equals the positive tropical Grassmannian
David Speyer, Lauren K. Williams

http://arxiv.org/abs/2003.11353
On Razamat's A_2 and A_3 kernel identities
Simon Ruijsenaars

http://arxiv.org/abs/2003.11616
Reflective prolate-spheroidal operators and the adelic Grassmannian
W. Riley Casper, F. Alberto Grünbaum, Milen Yakimov, Ignacio Zurrián

http://arxiv.org/abs/2003.11861
Multiplication operator and exceptional Jacobi polynomials
Á. P. Horváth

http://arxiv.org/abs/2003.12385
A Tutorial on the Basic Special Functions of Fractional Calculus
Francesco Mainardi

http://arxiv.org/abs/2003.12653
On a conjecture related to integer-valued polynomials
Victor J. W. Guo

http://arxiv.org/abs/2003.12894
A Sequence of Weighted Birman-Hardy-Rellich Inequalities with Logarithmic Refinements
Fritz Gesztesy, Lance L. Littlejohn, Isaac Michael, Michael M. H. Pang

http://arxiv.org/abs/2003.14238
Confined systems associated with the discrete Meixner polynomials
A. D. Alhaidari, T. J. Taiwo

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On the origins of Riemann-Hilbert problems in mathematics
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http://arxiv.org/abs/2004.00926
Refined asymptotics of the Riemann-Siegel theta function
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http://arxiv.org/abs/2004.01163
A Spectral Approach to the Shortest Path Problem
Stefan Steinerberger

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Uniform asymptotics of a Gauss hypergeometric function with two large parameters, V
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Jacobi matrices on trees generated by Angelesco systems: asymptotics of coefficients and essential spectrum
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A. Gil, J. Segura, N. M. Temme

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Árpád Baricz, Gergő Nemes

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Stefan Steinerberger

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Idempotent systems
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Malgorzata Stawiska

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Andrés Chirre, Oswaldo Velásquez Castañón

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D. J. Prabhakaran, K. Ranjith Kumar

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Parabolic Hilbert schemes via the Dunkl-Opdam subalgebra
Eugene Gorsky, José Simental, Monica Vazirani

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Elliptic curves with Galois-stable cyclic subgroups of order 4
Carl Pomerance, Edward F. Schaefer

Topic #4 ——— OP - SF Net 27.3 ——— May 15, 2020

From: OP-SF Net Editors
Subject: Submitting contributions to OP-SF NET and SIAM-OPSF (OP-SF Talk)

To contribute a news item to OP-SF NET, send e-mail to one of the OP-SF Editors howard.cohl@nist.gov, or spost@hawaii.edu.

Contributions to OP-SF NET 27.4 should be sent by July 1, 2020.

OP-SF NET is an electronic newsletter of the SIAM Activity Group on Special Functions and Orthogonal Polynomials. We disseminate your contributions on anything of interest to the special functions and orthogonal polynomials community. This includes announcements of conferences, forthcoming books, new software, electronic archives, research questions, and job openings as well as news about new appointments, promotions, research visitors, awards and prizes. OP-SF Net is transmitted periodically through a post to SIAM-OPSF (OP-SF Talk).

SIAM-OPSF (OP-SF Talk) is a listserv of the SIAM Activity Group on Special Functions and Orthogonal Polynomials, which facilitates communication among members, and friends of the Activity Group. See the previous Topic. To post an item to the listserv, send e-mail to siam-opsf@siam.org.

WWW home page of this Activity Group:
http://math.nist.gov/opsf
Information on joining SIAM and this activity group: service@siam.org

The elected Officers of the Activity Group (2020-2022) are:
Peter Alan Clarkson, Chair
Luc Vinet, Vice Chair
Andrei Martínez-Finkelshtein, Program Director
Teresa E. Pérez, Secretary and OP-SF Talk moderator

The appointed officers are:
Howard Cohl, OP-SF NET co-editor
Sarah Post, OP-SF NET co-editor
Diego Dominici, OP-SF Talk moderator
Bonita Saunders, Webmaster and OP-SF Talk moderator

Topic #5 ——— OP - SF Net 27.3 ——— May 15, 2020

From: OP-SF Net Editors
Subject: Thought of the Month by **John H. Conway**

"You know, people think mathematics is complicated. Mathematics is the simple bit. It's the stuff we can understand. It's cats that are complicated. I mean, what is it in those little molecules and stuff that make one cat behave differently than another, or that make a cat? And how do you define a cat? I have no idea."

John Horton Conway (26 December 1937 – 11 April 2020).