MR2500371 (2010j:33011) 33C10 (34A05 34A25 68W30)
Debeerst, Ruben (D-UKSL); **van Hoeij, Mark** (1-FLS); **Koepf, Wolfram** (D-UKSL)
Solving differential equations in terms of Bessel functions. (English summary)
ISSAC 2008, 39–46, *ACM*, *New York*, 2008.

For a second-order linear differential equation $L(y)=0$, with coefficients in a differential field K, the question arises whether $L(y)$ can be transformed to the Bessel operator $L_B(y)$, by means of transformations: (i) $x\to f(x)$; (ii) $y\to\exp(\int r)\cdot y$; (iii) $y\to r_0y+r_1y'$, where $'$ is d/dx and $f,r,r_0,r_1\in K$. The authors present an algorithm that determines whether and how this can be done. Hence it determines solutions of $L(y)=0$, if they exist, of the form

$$\exp\left(\int r\,dx\right)(r_0B_\nu(f(x))+r_1B'_\nu(f(x))),$$

where B_ν denotes a Bessel function. This represents an important extension of earlier work where only two of the transformations (i),(ii),(iii) are considered. The algorithm leaves undecided the question of existence of solutions of this form when f is replaced by $f^{1/2}$. The paper includes a link to a Maple implementation of the algorithm and to the first author's Master's thesis ["Solving differential equations in terms of Bessel functions", Univ. Kassel, Kassel, 2007, http://www.mathematik.uni-kassel.de/~debeerst/master/master.pdf], where many more details are provided.

{For the entire collection see MR2516402 (2010c:68003)}

Reviewed by *M. E. Muldoon*