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Two classes of special functions using Fourier transforms of generalized ultraspherical and generalized Hermite polynomials. (English summary)

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Let $U_n^{(a,b)}(x)$ be the generalized ultraspherical polynomials orthogonal with respect to the weight function $|x|^{2a}(1-x^2)^b$, and let $H_n^{(a)}$ be the generalized Hermite polynomials, first introduced by N. Ya. Sonin in 1880, orthogonal with respect to the weight $|x|^{2a}e^{-x^2}$. By using the Fourier transform on $U_n^{(a,b)}(x)$ and $H_n^{(a)}$ the authors introduce two new classes of orthogonal functions and then obtain their orthogonality relations by the aid of Parseval's identity.

Reviewed by *Boris Petrovich Osilenker*

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