Masjed-Jamei, Mohammad; **Koepf, Wolfram**
Two classes of special functions using Fourier transforms of generalized ultraspherical and generalized Hermite polynomials. (English) Zbl 1246.33005
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In this paper, one deals with the generalized ultraspherical (Gegenbauer) polynomials (GUP) which are orthogonal on $[-1, 1]$ with respect to the weight $|x|^{2a}(1 - x^2)^b$ and the generalized Hermite polynomials (GHP) which are orthogonal on $]-\infty, +\infty[$ with respect to $|x|^{2a}e^{-x^2}$. They are two basic examples of sieved orthogonal polynomials.

It is known that some orthogonal polynomial systems are mapped onto each other by the Fourier transform or other integral transforms such as the Mellin and Hankel transforms. For example, Koelink showed that the Jacobi and continuous Hahn polynomials can be mapped onto each other in such a way, and the orthogonality relations for the continuous Hahn polynomials then follow from the orthogonality relations of the Jacobi polynomials combined with the Parseval identity of Fourier analysis.

Motivated by Koelink's work, in this paper two new classes of orthogonal polynomials are introduced which are respectively Fourier transforms of the GUP and the GHP and their orthogonality relations are then obtained using Parseval's identity.

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