```
> restart;
```

Auf was kommt es bei einem Computeralgebrasystem an?

```
> 40!;
```

815915283247897734345611269596115894272000000000

```
> binomial(123,45);
```

8966473191018617158916954970192684

```
> 40!/binomial(123,45);
```

$$\frac{2595835018726623874037043324518400000000}{285268404472916876134028573}$$

```
> evalf(Pi,100);
```

3.141592653589793238462643383279502884197169399375105820974944592307816406286208998628034825342117068

```
> p:=(x+y)^10-(x-y)^10;
```

$$p := (x+y)^{10} - (x-y)^{10}$$

```
> expand(p);
```

$$20\,x^9\,y + 240\,x^7\,y^3 + 504\,x^5\,y^5 + 240\,x^3\,y^7 + 20\,x\,y^9$$

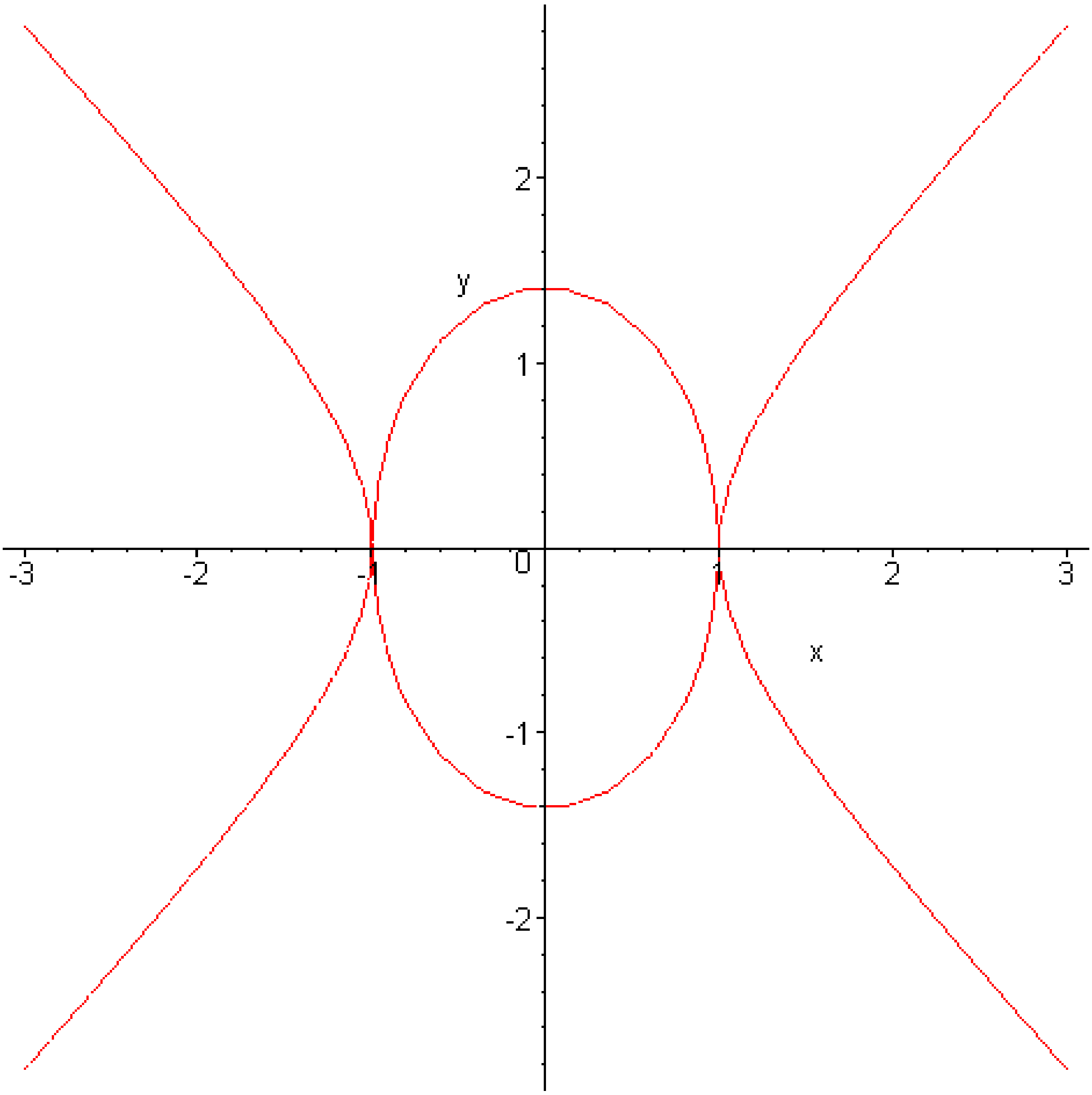
```
> factor(p);
```

$$4\,x\,y\,(5\,y^4 + 10\,x^2\,y^2 + x^4)\,(y^4 + 10\,x^2\,y^2 + 5\,x^4)$$

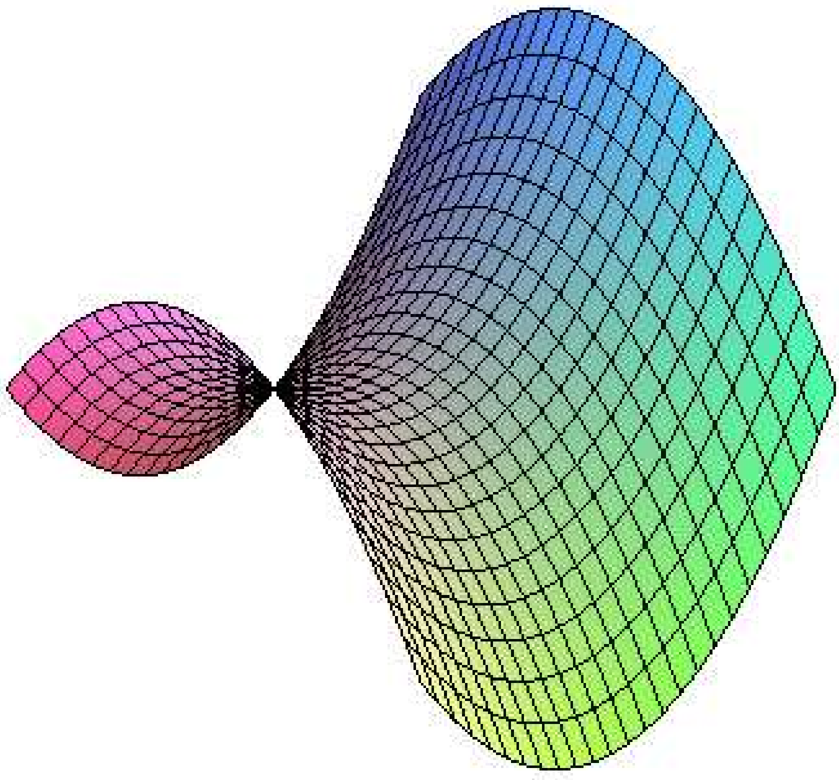
```
> solve({x^2+y^2/2=1,-x^2+y^2+1=0},{x,y});
```

$$\{y=0, x=1\}, \{y=0, x=-1\}$$

```
> plots[implicitplot]({x^2+y^2/2=1,-x^2+y^2+1=0},x=-3..3,y=-3..3);
```




```
> plot3d(x^2-y^2,x=-1..1,y=-1..1);
```



>

Berechnung der Rekursionskoeffizienten

Wir betrachten die drei höchsten Koeffozienten des orthogonalen Polynoms:

```
> p:=k[n]*x^n+kstrich[n]*x^(n-1)+kstrichstrich[n]*x^(n-2);
```

$$p := k_n\, x^n + kstrich_n\, x^{(n-1)} + kstrichstrich_n\, x^{(n-2)}$$

Wir erklären die Polynome σ und τ mit beliebigen Koeffizienten a,b,c,d,e:

```
> sigma:=a*x^2+b*x+c;
tau:=d*x+e;
```

$$\sigma := a\, x^2 + b\, x + c$$

$$\tau := d\, x + e$$

Das Polynom erfüllt die Differentialgleichung DE=0 mit:

```
> DE:=sigma*diff(p,x$2)+tau*diff(p,x)+lambda[n]*p;
```

$$DE := (a\,x^2+b\,x+c)\left(\frac{k_n\,x^n\,n^2}{x^2}-\frac{k_n\,x^n\,n}{x^2}+\frac{kstrich_n\,x^{(n-1)}\,(n-1)^2}{x^2}-\frac{kstrich_n\,x^{(n-1)}\,(n-1)}{x^2}+\frac{kstrichstrich_n\,x^{(n-2)}\,(n-2)^2}{x^2}-\frac{kstrichstrich_n\,x^{(n-2)}\,(n-2)}{x^2}\right)+(d\,x+e)\left(\frac{k_n\,x^n\,n}{x}+\frac{kstrich_n\,x^{(n-1)}\,(n-1)}{x}+\frac{kstrichstrich_n\,x^{(n-2)}\,(n-2)}{x}\right)+\lambda_n\,(k_n\,x^n+kstrich_n\,x^{(n-1)}+kstrichstrich_n\,x^{(n-2)})$$

Wir sortieren nach Potenzen von x:

```
> de:=collect(simplify(DE/x^(n-4)),x);
```

$$de := (a\,k_n\,n^2+d\,k_n\,n+\lambda_n\,k_n-a\,k_n\,n)\,x^4+(a\,kstrich_n\,n^2-d\,kstrich_n-b\,k_n\,n+\lambda_n\,kstrich_n+b\,k_n\,n^2-3\,a\,kstrich_n\,n+e\,k_n\,n+d\,kstrich_n\,n+2\,a\,kstrich_n)\,x^3+(\lambda_n\,kstrichstrich_n+2\,b\,kstrich_n+d\,kstrichstrich_n\,n+a\,kstrichstrich_n\,n^2+6\,a\,kstrichstrich_n-5\,a\,kstrichstrich_n\,n+c\,k_n\,n^2-c\,k_n\,n+b\,kstrich_n\,n^2-3\,b\,kstrich_n\,n+e\,kstrich_n\,n-2\,d\,kstrichstrich_n-e\,kstrich_n)\,x^2+(2\,c\,kstrich_n+c\,kstrich_n\,n^2-3\,c\,kstrich_n\,n+e\,kstrichstrich_n\,n+6\,b\,kstrichstrich_n+b\,kstrichstrich_n\,n^2-5\,b\,kstrichstrich_n\,n-2\,e\,kstrichstrich_n)\,x+6\,c\,kstrichstrich_n-5\,c\,kstrichstrich_n\,n+c\,kstrichstrich_n\,n^2$$

Koeffizientenvergleich beim höchsten Koeffizienten liefert die bereits erwähnte Gleichung für λ:

```
> rule1:=lambda[n]=solve(coeff(de,x,4),lambda[n]);
```

$$rule1 := \lambda_n=-n\,(a\,n+d-a)$$

Wir setzen dies ein:

```
> de:=expand(subs(rule1,de));
```

$$de := x^3\,e\,k_n\,n+x^2\,b\,kstrich_n\,n^2-4\,x^2\,a\,kstrichstrich_n\,n+x^2\,c\,k_n\,n^2+2\,x^3\,a\,kstrich_n+6\,x^2\,a\,kstrichstrich_n+2\,x^2\,b\,kstrich_n+6\,x\,b\,kstrichstrich_n+2\,x\,c\,kstrich_n+6\,c\,kstrichstrich_n-x^3\,d\,kstrich_n-2\,x^2\,d\,kstrichstrich_n-x^2\,e\,kstrich_n-2\,x\,e\,kstrichstrich_n-2\,x^3\,a\,kstrich_n\,n+x^3\,b\,k_n\,n^2-x^3\,b\,k_n\,n-3\,x^2\,b\,kstrich_n\,n+x\,b\,kstrichstrich_n\,n^2-5\,x\,b\,kstrichstrich_n\,n-x^2\,c\,k_n\,n+x\,c\,kstrich_n\,n^2-3\,x\,c\,kstrich_n\,n+c\,kstrichstrich_n\,n^2-5\,c\,kstrichstrich_n\,n+x^2\,e\,kstrich_n\,n$$

$+ x\, e\, kstrichstrich_n\, n$

und machen Koeffizientenvergleich beim zweithöchsten Koeffizienten. Dies liefert k'[n] als rationales Vielfaches von k[n]:

```
> rule2:=kstrich[n]=solve(coeff(de,x,3),kstrich[n]);
```

$$rule2 := kstrich_n = \frac{k_n\, n\, (e - b + b\, n)}{2\, a\, n + d - 2\, a}$$

Koeffizientenvergleich beim zweithöchsten Koeffizienten gibt k"[n] ebenfalls als rationales Vielfaches von k[n]:

```
> rule3:=kstrichstrich[n]=solve(coeff(subs(rule2,de),x,2),kstrichstrich[n]);
```

$$rule3 := kstrichstrich_n = \frac{1}{2} k_n\, n\, (-4\, c\, a\, n - c\, d + 2\, c\, a + 2\, c\, n^2\, a + c\, n\, d + 3\, b\, e - 2\, b^2 + 5\, b^2\, n - e^2 - 5\, e\, b\, n - 4\, b^2\, n^2 + 2\, b\, n^2\, e + b^2\, n^3 + e^2\, n)/((2\, a\, n + d - 2\, a)\, (-3\, a + d + 2\, a\, n))$$

Wir betrachten den monischen Fall

```
> k[n]:=1;
```

$$k_n := 1$$

so dass gilt:

```
> rule2;
```

$$kstrich_n = \frac{n\, (e - b + b\, n)}{2\, a\, n + d - 2\, a}$$

```
> rule3;
```

$$kstrichstrich_n = n\, (-4\, c\, a\, n - c\, d + 2\, c\, a + 2\, c\, n^2\, a + c\, n\, d + 3\, b\, e - 2\, b^2 + 5\, b^2\, n - e^2 - 5\, e\, b\, n - 4\, b^2\, n^2 + 2\, b\, n^2\, e + b^2\, n^3 + e^2\, n)/(2\, (2\, a\, n + d - 2\, a)\, (-3\, a + d + 2\, a\, n))$$

Wir wollen nun die Koeffizienten $\beta(n)$ und $\gamma(n)$ in der Rekursionsgleichung RE=0 bestimmen:

```
> RE:=P(n+1)-(x-beta[n])*P(n)+gamma[n]*P(n-1);
```

$$RE := \mathrm{P}(n+1) - (x - \beta_n)\, \mathrm{P}(n) + \gamma_n\, \mathrm{P}(n-1)$$

```
> RE:=subs({P(n)=p,P(n+1)=subs(n=n+1,p),P(n-1)=subs(n=n-1,p)},RE);
```

$$RE := x^{(n+1)} + kstrich_{n+1}\, x^n + kstrichstrich_{n+1}\, x^{(n-1)} - (x - \beta_n)\, (x^n + kstrich_n\, x^{(n-1)} + kstrichstrich_n\, x^{(n-2)}) + \gamma_n\, (x^{(n-1)} + kstrich_{n-1}\, x^{(n-2)} + kstrichstrich_{n-1}\, x^{(n-3)})$$

Wie substitutieren die berechneten Formeln:

```
> RE:=subs({rule2,subs(n=n+1,rule2),subs(n=n-1,rule2),rule3,subs(n=n+1,rule3),subs(n=n-1,rule3)},RE);
```

$$RE := x^{(n+1)} + \frac{(n+1)(e-b+b(n+1))\,x^{n}}{2a(n+1)+d-2a} + (n+1)(-4ca(n+1)-cd+2ca$$
$$+2c(n+1)^2a+c(n+1)d+3be-2b^2+5b^2(n+1)-e^2-5eb(n+1)$$
$$-4b^2(n+1)^2+2b(n+1)^2e+b^2(n+1)^3+e^2(n+1))\,x^{(n-1)}/(2$$
$$(2a(n+1)+d-2a)(-3a+d+2a(n+1))) - (x-\beta_n)\Bigg(x^{n}$$
$$+\frac{n(e-b+bn)\,x^{(n-1)}}{2an+d-2a} + n(-4can-cd+2ca+2cn^2a+cnd+3be-2b^2$$
$$+5b^2n-e^2-5ebn-4b^2n^2+2bn^2e+b^2n^3+e^2n)\,x^{(n-2)}/(2(2an+d-2a)$$
$$(-3a+d+2an))\Bigg) + \gamma_n\Bigg(x^{(n-1)} + \frac{(n-1)(e-b+b(n-1))\,x^{(n-2)}}{2a(n-1)+d-2a} + (n-1)($$
$$-4ca(n-1)-cd+2ca+2c(n-1)^2a+c(n-1)d+3be-2b^2$$
$$+5b^2(n-1)-e^2-5eb(n-1)-4b^2(n-1)^2+2b(n-1)^2e+b^2(n-1)^3$$
$$+e^2(n-1))\,x^{(n-3)}/(2(2a(n-1)+d-2a)(-3a+d+2a(n-1)))\Bigg)$$

```
> re:=simplify(numer(normal(RE))/x^(n-3));
```

$$re := -336x^3ea^4dn^3+2192x^3\beta_na^6n^2-96x^2n^2ebd^3a+80x^2n^6b^2a^3d$$
$$-504x^2n^5b^2a^3d-2084\gamma_nb^2n^3a^4+2644\gamma_nb^2n^4a^4+770x\gamma_nbd^3a^2n^2$$
$$-336x^2n^2b^2d^2a^2-560x^2n^4eba^3d+392x\beta_nn^3ca^5+236x\beta_nn^3e^2da^3$$
$$-396x^2n^3eba^2d^2+1162x^2n^4b^2a^3d+22\gamma_ne^2d^2a^2-480\gamma_ncn^4a^3d^2$$
$$+548x^2n^2bea^4+740x^2n^3b^2a^4+2396x\gamma_nba^3n^3d^2+24x^2\gamma_nd^5an$$
$$-464x\beta_nn^5cda^4-240x^2\beta_nn^2ba^5+972x\beta_nn^4cda^4-416x^2\beta_nn^5ea^5$$
$$+920\gamma_nebn^5a^4-40x^2nb^2da^3+584x\beta_nn^5ca^5+2248x^2\beta_nn^4ba^4d$$
$$+x\beta_nn^4b^2d^4+320x^2\gamma_na^3n^3d^3+80x\gamma_nen^3d^3a^2+2192x^2\gamma_na^5nd$$
$$+106\gamma_ne^2da^3n-188x^3ea^2d^3n+128x^3\beta_na^6n^6-1168x^3bn^4da^4$$
$$+856x^3ea^5n+656x^2n^4ca^5-1064x^2n^4b^2a^4-2400x^3\beta_na^5n^4d$$
$$+2x\beta_nn^3bed^4-416x\gamma_nbn^3d^3a^2+96x^2n^5ca^4d-496\gamma_nbn^5eda^3$$
$$+4x^2n^3b^2d^4+160x\gamma_nen^4d^2a^3+23\gamma_nb^2n^2d^4+548x^3ea^4d+1096x^3bn^2a^5$$
$$-704x\beta_nn^4ca^5+89x\beta_nn^2e^2d^2a^2+120x^3\beta_nd^4a^2n^2+10x\beta_nn^3cd^4a$$
$$+70\gamma_nca^2d^3-614x\gamma_ned^2a^3n+1360x^3bn^4a^5-30x^3ed^4a$$
$$-396x^3bn^2d^3a^2-265\gamma_ncd^3a^2n+124\gamma_ne^2a^4n^2+486x^2n^2b^2a^3d$$

$+ 96\, x^{2}\, n^{4}\, e\, b\, a^{2}\, d^{2} + 2\, x^{3}\, \beta_{n}\, d^{6} + 2\, x^{3}\, e\, d^{5} - 240\, x^{3}\, e\, a^{5} + 2\, x^{2}\, \gamma_{n}\, d^{6}$

$- 82\, x\, \gamma_{n}\, b\, n^{2}\, d^{4}\, a - 176\, \gamma_{n}\, b^{2}\, n^{7}\, a^{4} + 20\, x^{2}\, \beta_{n}\, n^{2}\, e\, d^{4}\, a + 2\, x^{2}\, \beta_{n}\, n\, e\, d^{5}$

$+ 24\, x\, \beta_{n}\, n^{4}\, e^{2}\, d^{2}\, a^{2} + x\, \beta_{n}\, n^{2}\, e^{2}\, d^{4} + 64\, x\, \gamma_{n}\, e\, n^{6}\, a^{5} + 160\, x\, \gamma_{n}\, b\, n^{6}\, a^{4}\, d$

$+ 389\, x\, \beta_{n}\, n^{4}\, b^{2}\, d^{2}\, a^{2} + 944\, x^{2}\, \beta_{n}\, n^{4}\, e\, a^{5} - 450\, x^{3}\, e\, a^{3}\, d^{2} - 554\, \gamma_{n}\, b\, e\, d\, a^{3}\, n$

$+ 569\, \gamma_{n}\, b\, e\, d^{2}\, a^{2}\, n - 186\, x^{2}\, n^{2}\, c\, d^{3}\, a^{2} + 12\, \gamma_{n}\, b^{2}\, d^{4} + 680\, x^{2}\, n^{4}\, b\, e\, a^{4}$

$+ 1124\, x^{2}\, n^{3}\, e\, b\, a^{3}\, d - 560\, \gamma_{n}\, c\, n^{5}\, a^{4}\, d - 2700\, x^{2}\, \gamma_{n}\, a^{4}\, d^{2}\, n - 960\, x^{3}\, \beta_{n}\, a^{6}\, n^{5}$

$- 240\, x^{2}\, n^{5}\, c\, a^{5} - 936\, x\, \beta_{n}\, n^{5}\, b^{2}\, a^{4} + 8\, x\, \beta_{n}\, n^{5}\, b^{2}\, d^{3}\, a - 1148\, x\, \beta_{n}\, n^{4}\, b^{2}\, d\, a^{3}$

$- 332\, x\, \beta_{n}\, n^{2}\, b^{2}\, d\, a^{3} + 4\, x\, \gamma_{n}\, b\, d^{5} + 224\, x^{3}\, b\, n^{4}\, d^{2}\, a^{3} - 2700\, x^{3}\, \beta_{n}\, d^{2}\, a^{4}\, n$

$+ 32\, x\, \beta_{n}\, n^{7}\, b\, e\, a^{4} + 1540\, \gamma_{n}\, c\, a^{4}\, n^{4}\, d + 160\, x\, \gamma_{n}\, e\, n^{5}\, a^{4}\, d + 48\, x^{3}\, e\, n^{2}\, d^{3}\, a^{2}$

$+ 19\, \gamma_{n}\, e\, b\, n\, d^{4} + 384\, x^{2}\, \gamma_{n}\, a^{5}\, n^{5}\, d + 1008\, x\, \gamma_{n}\, e\, a^{5}\, n^{4} + 3492\, x\, \gamma_{n}\, b\, a^{4}\, n^{2}\, d$

$- 325\, x\, \beta_{n}\, n^{2}\, b\, e\, d^{2}\, a^{2} + 40\, x^{2}\, n^{3}\, e\, b\, a\, d^{3} - 1184\, x^{2}\, n^{3}\, b^{2}\, a^{3}\, d + 16\, x\, \beta_{n}\, n^{4}\, b\, e\, d^{3}\, a$

$+ 548\, x^{2}\, \gamma_{n}\, a^{4}\, d^{2} + 384\, x^{3}\, \beta_{n}\, a^{5}\, n^{5}\, d - 362\, \gamma_{n}\, b^{2}\, n^{2}\, d^{3}\, a - 8\, \gamma_{n}\, b^{2}\, n^{3}\, d^{4}$

$- 64\, x^{3}\, e\, a^{5}\, n^{4} + 64\, x^{3}\, b\, n^{6}\, a^{5} - 158\, x^{2}\, n\, b\, e\, d^{2}\, a^{2} - 1800\, x^{2}\, \beta_{n}\, n^{4}\, b\, a^{5}$

$- 288\, x^{2}\, \beta_{n}\, n^{3}\, b\, d^{3}\, a^{2} + 240\, x^{2}\, \beta_{n}\, n^{2}\, e\, a^{5} - 922\, x^{2}\, \beta_{n}\, n^{2}\, b\, a^{3}\, d^{2} - 224\, x\, \beta_{n}\, n^{6}\, c\, a^{5}$

$- 2272\, x^{2}\, \beta_{n}\, n^{3}\, b\, a^{4}\, d - 102\, x^{2}\, n^{2}\, e^{2}\, d^{2}\, a^{2} - 24\, \gamma_{n}\, e^{2}\, a^{4}\, n + 24\, \gamma_{n}\, b^{2}\, n^{6}\, d^{2}\, a^{2}$

$+ 80\, \gamma_{n}\, c\, n^{6}\, a^{4}\, d + 696\, \gamma_{n}\, b^{2}\, a^{3}\, n\, d + 355\, \gamma_{n}\, c\, d^{3}\, a^{2}\, n^{2} + 20\, x\, \beta_{n}\, n\, e^{2}\, d\, a^{3}$

$- 1360\, \gamma_{n}\, c\, a^{5}\, n^{4} + 2896\, x\, \gamma_{n}\, b\, a^{5}\, n^{3} - 3600\, x^{2}\, \gamma_{n}\, a^{6}\, n^{3} - 208\, x^{2}\, \beta_{n}\, n^{2}\, e\, d^{3}\, a^{2}$

$+ 32\, x\, \beta_{n}\, n^{7}\, c\, a^{5} - 740\, \gamma_{n}\, b^{2}\, d^{2}\, a^{2}\, n + 58\, x^{2}\, n\, b^{2}\, d^{2}\, a^{2} - 44\, x\, \gamma_{n}\, b\, d^{4}\, a$

$+ 80\, \gamma_{n}\, c\, n^{5}\, a^{3}\, d^{2} + 96\, x^{2}\, n^{5}\, e\, b\, a^{3}\, d - 152\, x\, \beta_{n}\, n^{4}\, e^{2}\, d\, a^{3} + 2\, \gamma_{n}\, e^{2}\, d^{4}$

$+ 1096\, x^{2}\, \beta_{n}\, n^{3}\, b\, a^{5} - 960\, x^{2}\, \gamma_{n}\, a^{6}\, n^{5} - 168\, \gamma_{n}\, e^{2}\, n^{4}\, a^{3}\, d + 1020\, x\, \gamma_{n}\, e\, d^{2}\, a^{3}\, n^{2}$

$+ 24\, \gamma_{n}\, e^{2}\, n^{4}\, a^{2}\, d^{2} - 272\, \gamma_{n}\, b\, n^{6}\, e\, a^{4} + 1448\, \gamma_{n}\, e\, b\, n^{3}\, a^{4} + 80\, x\, \gamma_{n}\, b\, n^{4}\, d^{3}\, a^{2}$

$+ 64\, \gamma_{n}\, b\, n^{6}\, e\, a^{3}\, d + 128\, x^{3}\, b\, n^{3}\, d^{3}\, a^{2} + 1096\, x^{3}\, e\, a^{4}\, d\, n^{2} - 1372\, x^{3}\, e\, a^{4}\, n\, d$

$+ 36\, x^{3}\, b\, n^{2}\, d^{4}\, a - 2228\, x^{3}\, b\, a^{4}\, n^{2}\, d + 784\, x^{3}\, e\, a^{3}\, d^{2}\, n + 24\, x^{3}\, \beta_{n}\, d^{5}\, a\, n$

$- 408\, x^{3}\, e\, a^{3}\, d^{2}\, n^{2} - 480\, x^{3}\, \beta_{n}\, a^{6}\, n - 3600\, x^{3}\, \beta_{n}\, a^{6}\, n^{3} + 16\, x^{3}\, e\, d^{4}\, a\, n$

$- 56\, x^{3}\, b\, a\, n\, d^{4} + 1492\, x^{3}\, b\, a^{3}\, n^{2}\, d^{2} + 668\, x^{3}\, b\, a^{4}\, n\, d - 664\, x^{3}\, b\, a^{3}\, n\, d^{2}$

$+ 288\, x^{3}\, b\, a^{2}\, n\, d^{3} + 64\, x^{3}\, e\, n^{3}\, d^{2}\, a^{3} + 32\, x^{3}\, e\, n^{4}\, d\, a^{4} + 192\, x^{3}\, b\, n^{5}\, d\, a^{4}$

$+ 2512\, x^{3}\, b\, n^{3}\, d\, a^{4} - 1032\, x^{3}\, b\, n^{3}\, d^{2}\, a^{3} + 4\, x^{3}\, b\, n\, d^{5} - 240\, x^{3}\, b\, a^{5}\, n$

$- 1800\, x^{3}\, b\, n^{3}\, a^{5} - 944\, x^{3}\, e\, a^{5}\, n^{2} + 170\, x^{3}\, e\, a^{2}\, d^{3} - 480\, x^{3}\, b\, n^{5}\, a^{5} + 416\, x^{3}\, e\, a^{5}\, n^{3}$

$- 300\, x^{3}\, \beta_{n}\, d^{4}\, a^{2}\, n + 2192\, x^{3}\, \beta_{n}\, a^{5}\, n\, d + 170\, x^{3}\, \beta_{n}\, d^{4}\, a^{2} - 30\, x^{3}\, a\, \beta_{n}\, d^{5}$

$$+ 548\, x^3\, \beta_n\, d^2\, a^4 - 240\, x^3\, \beta_n\, a^5\, d - 450\, x^3\, \beta_n\, d^3\, a^3 + 2720\, x^3\, \beta_n\, a^6\, n^4$$
$$+ 5\, x\, \beta_n\, n^2\, b^2\, d^4 + 96\, \gamma_n\, c\, a^5\, n - 1200\, x^3\, \beta_n\, d^3\, a^3\, n^2 + 1360\, x^3\, \beta_n\, d^3\, a^3\, n$$
$$+ 5440\, x^3\, \beta_n\, a^5\, n^3\, d - 2400\, x^3\, \beta_n\, a^4\, n^3\, d^2 - 5400\, x^3\, \beta_n\, a^5\, n^2\, d + 4080\, x^3\, \beta_n\, a^4\, n^2\, d^2$$
$$+ 480\, x^3\, \beta_n\, a^4\, n^4\, d^2 + 320\, x^3\, \beta_n\, a^3\, n^3\, d^3 - 480\, x^2\, \beta_n\, n^6\, b\, a^5 + \gamma_n\, b^2\, n^4\, d^4$$
$$+ 548\, x^2\, n^3\, b^2\, d^2\, a^2 - 668\, \gamma_n\, e\, b\, n^2\, a^4 - 372\, x\, \beta_n\, n^2\, c\, d^2\, a^3 - 120\, x\, \beta_n\, n^2\, b\, e\, a^4$$
$$- 80\, x\, \beta_n\, n^2\, c\, a^5 - 992\, x^2\, \beta_n\, n^5\, b\, d\, a^4 + 40\, x\, \beta_n\, n^4\, c\, d^3\, a^2 + 60\, \gamma_n\, b\, e\, d^3\, a$$
$$- 886\, x^2\, n^2\, b\, e\, a^3\, d - 30\, x^2\, \gamma_n\, d^5\, a + 170\, x^2\, \gamma_n\, d^4\, a^2 + 80\, x\, \beta_n\, n^5\, c\, d^2\, a^3$$
$$- 228\, \gamma_n\, b^2\, n^5\, d^2\, a^2 - 22\, x\, a\, \beta_n\, n^2\, c\, d^4 - 3\, \gamma_n\, c\, n\, d^5 + 120\, x^2\, n\, c\, d^3\, a^2$$
$$- 328\, \gamma_n\, b^2\, n^6\, a^3\, d - 120\, x^2\, n^3\, e^2\, a^3\, d - 20\, x^2\, n\, b^2\, d^3\, a + 2\, x^2\, n\, b^2\, d^4$$
$$+ 10\, x\, a\, \beta_n\, n\, e^2\, d^3 - 3160\, x\, \gamma_n\, b\, a^5\, n^4 - 100\, \gamma_n\, c\, a^3\, d^2 - 46\, x^2\, a\, \beta_n\, n^2\, b\, d^4$$
$$+ 20\, x\, a\, \beta_n\, n\, b^2\, d^3 + 32\, \gamma_n\, b\, n^7\, e\, a^4 + 1356\, \gamma_n\, b^2\, n^5\, a^3\, d - 42\, x\, a\, \beta_n\, n^4\, b^2\, d^3$$
$$+ 2\, \gamma_n\, b\, n^3\, e\, d^4 - 110\, \gamma_n\, b\, e\, d^2\, a^2 + 40\, \gamma_n\, c\, n^4\, a^2\, d^3 - 200\, \gamma_n\, c\, n^3\, d^3\, a^2$$
$$- 1156\, x\, \gamma_n\, b\, a^4\, n\, d + 80\, x\, a\, \beta_n\, n^3\, b^2\, d^3 - 282\, \gamma_n\, e^2\, n^2\, d\, a^3 - 864\, x\, \gamma_n\, e\, a^4\, n^4\, d$$
$$+ 1350\, x\, \gamma_n\, b\, a^3\, n\, d^2 - 1588\, \gamma_n\, b^2\, n^3\, a^2\, d^2 + 50\, \gamma_n\, c\, d^4\, a\, n + 3460\, \gamma_n\, b^2\, n^3\, a^3\, d$$
$$+ 16\, \gamma_n\, b^2\, n^8\, a^4 - 848\, x^2\, n^2\, c\, a^4\, d + 44\, x^2\, n\, b\, e\, d^3\, a + 214\, x^2\, n\, b\, e\, a^3\, d$$
$$+ 80\, x^2\, n^2\, b^2\, d^3\, a - 26\, x^2\, n\, c\, d^4\, a - 4\, x^2\, n\, b\, e\, d^4 - 120\, x^2\, n\, b\, e\, a^4 + 160\, x^2\, n\, c\, d\, a^4$$
$$- 232\, x^2\, n\, c\, d^2\, a^3 - 240\, x^2\, n^5\, e\, b\, a^4 + 32\, x^2\, n^6\, e\, b\, a^4 + 112\, x^2\, n^4\, c\, d^2\, a^3$$
$$+ 6\, x^2\, n^2\, e\, b\, d^4 - 342\, x^2\, n^4\, b^2\, d^2\, a^2 + 644\, x^2\, n^2\, c\, d^2\, a^3 - 492\, x^2\, n^3\, c\, d^2\, a^3$$
$$+ 24\, x^2\, n^3\, e^2\, d^2\, a^2 + 72\, x^2\, n^5\, b^2\, d^2\, a^2 + 100\, x^2\, n\, e^2\, d^2\, a^2 + 266\, x^2\, n^2\, e^2\, d\, a^3$$
$$+ 12\, x^2\, n^2\, e^2\, d^3\, a + 18\, x^2\, n^2\, c\, d^4\, a - 24\, x^2\, n\, e^2\, d^3\, a + 64\, x^2\, n^3\, c\, d^3\, a^2$$
$$- 900\, x^2\, n^3\, b\, e\, a^4 - 88\, x^2\, n^3\, b^2\, d^3\, a - 768\, x^2\, n^3\, c\, a^5 + 320\, x^2\, n^2\, c\, a^5 - 6\, x^2\, n^2\, b^2\, d^4$$
$$- 200\, x^2\, n^2\, b^2\, a^4 + 32\, x^2\, n^6\, c\, a^5 - 174\, x^2\, n\, e^2\, d\, a^3 + 120\, x^2\, n\, e^2\, a^4 + 2\, x^2\, n\, c\, d^5$$
$$- 228\, x^2\, n^2\, e^2\, a^4 - 248\, x^2\, n^6\, b^2\, a^4 + 32\, x^2\, n^7\, b^2\, a^4 + 740\, x^2\, n^5\, b^2\, a^4 - 24\, x^2\, n^4\, e^2\, a^4$$
$$+ 2\, x^2\, n\, e^2\, d^4 + 132\, x^2\, n^3\, e^2\, a^4 + 1416\, x^2\, \beta_n\, n^3\, e\, a^4\, d - 6\, x\, \gamma_n\, b\, n\, d^5$$
$$+ 78\, x\, \beta_n\, n\, c\, d^2\, a^3 - 49\, x\, \beta_n\, n\, c\, d^3\, a^2 + 582\, x\, \beta_n\, n^3\, c\, d^2\, a^3 + 145\, x\, \beta_n\, n^2\, c\, d^3\, a^2$$
$$- 136\, x\, \beta_n\, n^2\, e^2\, d\, a^3 - 29\, x\, \beta_n\, n\, e^2\, d^2\, a^2 - 856\, x^2\, \beta_n\, n^2\, e\, a^4\, d + 352\, x\, \beta_n\, n^2\, c\, a^4\, d$$
$$- 2\, x\, \gamma_n\, e\, d^5 + 480\, x^2\, n^2\, b\, e\, d^2\, a^2 + 16\, x^2\, n^4\, e^2\, a^3\, d + 1136\, x^2\, n^3\, c\, a^4\, d$$
$$+ 28\, x^2\, n^4\, b^2\, d^3\, a + 12\, x\, \beta_n\, n\, c\, d^4\, a - 900\, x\, \beta_n\, n^3\, c\, d\, a^4 - 368\, x\, \beta_n\, n^4\, c\, d^2\, a^3$$
$$- 136\, x\, \beta_n\, n^3\, c\, d^3\, a^2 - 156\, x\, \beta_n\, n^5\, b^2\, d^2\, a^2 - 248\, x\, \beta_n\, n^6\, b^2\, d\, a^3$$
$$- 118\, x^2\, \beta_n\, n\, b\, a^2\, d^3 - 120\, x^2\, \beta_n\, n\, b\, a^4\, d + 214\, x^2\, \beta_n\, n\, b\, a^3\, d^2 + 756\, x\, \beta_n\, n^5\, b^2\, d\, a^3$$

$$
\begin{aligned}
&+976\,x^2\,\beta_n\,n^2\,b\,a^4\,d+265\,x\,\beta_n\,n^2\,b^2\,d^2\,a^2+900\,x\,\beta_n\,n^3\,b^2\,d\,a^3\\
&+708\,x^2\,\beta_n\,n^2\,e\,d^2\,a^3-214\,x^2\,\beta_n\,n\,e\,d^2\,a^3+118\,x^2\,\beta_n\,n\,e\,d^3\,a^2-60\,x\,\beta_n\,n\,b\,e\,d\,a^3\\
&-58\,x\,\beta_n\,n\,b^2\,d^2\,a^2+120\,x^2\,\beta_n\,n\,e\,a^4\,d-240\,x\,\beta_n\,n^6\,b\,e\,a^4-40\,x\,\beta_n\,n\,c\,a^4\,d\\
&+548\,x\,\beta_n\,n^3\,b\,e\,a^4+2720\,x^2\,\gamma_n\,a^6\,n^4-784\,x^2\,\beta_n\,n^4\,b\,d^2\,a^3-900\,x\,\beta_n\,n^4\,b\,e\,a^4\\
&+680\,x\,\beta_n\,n^5\,e\,b\,a^4-832\,x^2\,\beta_n\,n^4\,e\,d\,a^4+196\,x\,\beta_n\,n^4\,e^2\,a^4+516\,x\,\beta_n\,n^6\,b^2\,a^4\\
&-144\,x\,\beta_n\,n^7\,b^2\,a^4-96\,x\,\beta_n\,n^5\,e^2\,a^4+40\,x\,\beta_n\,n^2\,e^2\,a^4-856\,x^2\,\beta_n\,n^3\,e\,a^5\\
&+80\,x\,\beta_n\,n^2\,b^2\,a^4+900\,x\,\beta_n\,n^4\,b^2\,a^4-156\,x\,\beta_n\,n^3\,e^2\,a^4+x\,\beta_n\,n^2\,c\,d^5\\
&+1360\,x^2\,\beta_n\,n^5\,b\,a^5-x\,\beta_n\,n\,e^2\,d^4+32\,x\,\beta_n\,n^7\,b^2\,d\,a^3-84\,x\,\beta_n\,n^3\,e^2\,d^2\,a^2\\
&+24\,x\,\beta_n\,n^6\,b^2\,d^2\,a^2-18\,x\,\beta_n\,n^2\,e^2\,d^3\,a+80\,x\,\beta_n\,n^6\,c\,d\,a^4-432\,x\,\beta_n\,n^3\,b^2\,a^4\\
&-66\,x\,\beta_n\,n^2\,b^2\,d^3\,a-2\,x\,\beta_n\,n\,b^2\,d^4-4\,x\,\beta_n\,n^3\,b^2\,d^4+1332\,x^2\,\beta_n\,n^3\,b\,a^3\,d^2\\
&+326\,x^2\,\beta_n\,n^2\,b\,a^2\,d^3+26\,x^2\,\beta_n\,n\,b\,a\,d^4-464\,x\,\beta_n\,n^3\,b^2\,d^2\,a^2\\
&-624\,x^2\,\beta_n\,n^3\,e\,d^2\,a^3-26\,x^2\,\beta_n\,n\,e\,d^4\,a+160\,x^2\,\beta_n\,n^5\,e\,d\,a^4+160\,x^2\,\beta_n\,n^4\,e\,d^2\,a^3\\
&+64\,x^2\,\beta_n\,n^6\,e\,a^5+80\,x^2\,\beta_n\,n^3\,e\,d^3\,a^2-2\,x^2\,\beta_n\,n\,b\,d^5+160\,x^2\,\beta_n\,n^5\,b\,d^2\,a^3\\
&+80\,x^2\,\beta_n\,n^4\,b\,d^3\,a^2+20\,x^2\,\beta_n\,n^3\,b\,d^4\,a+160\,x^2\,\beta_n\,n^6\,b\,d\,a^4+128\,x^2\,\gamma_n\,a^6\,n^6\\
&-60\,x\,a\,\beta_n\,n^3\,b\,e\,d^3-11\,\gamma_n\,b\,n^2\,e\,d^4+16\,x\,\beta_n\,n^8\,b^2\,a^4+16\,x\,\beta_n\,n^6\,e^2\,a^4\\
&-30\,x\,a\,\beta_n\,n\,b\,e\,d^3+74\,x\,a\,\beta_n\,n^2\,e\,b\,d^3-240\,x^2\,\gamma_n\,a^5\,d+87\,x\,\beta_n\,n\,b\,e\,d^2\,a^2\\
&+928\,x\,\beta_n\,n^4\,e\,b\,a^3\,d-400\,x\,\beta_n\,n^5\,b\,e\,a^3\,d+448\,x\,\beta_n\,n^2\,b\,e\,a^3\,d\\
&-980\,x\,\beta_n\,n^3\,b\,e\,a^3\,d+430\,x\,\beta_n\,n^3\,e\,b\,d^2\,a^2-240\,x\,\beta_n\,n^4\,b\,e\,d^2\,a^2\\
&+40\,x\,\beta_n\,n\,b^2\,d\,a^3+864\,\gamma_n\,b^2\,n^2\,a^4-1916\,\gamma_n\,b^2\,n^5\,a^4+608\,x\,\gamma_n\,e\,a^5\,n^2\\
&-1144\,x\,\gamma_n\,e\,a^5\,n^3-3\,\gamma_n\,e^2\,n\,d^4-544\,\gamma_n\,c\,a^5\,n^2+1208\,\gamma_n\,c\,a^5\,n^3\\
&-1200\,x^2\,\gamma_n\,d^3\,a^3\,n^2+4080\,x^2\,\gamma_n\,a^4\,d^2\,n^2-2100\,\gamma_n\,c\,a^4\,n^3\,d-105\,\gamma_n\,e^2\,d^2\,a^2\,n\\
&+167\,\gamma_n\,e^2\,d^2\,a^2\,n^2-2910\,\gamma_n\,b^2\,n^4\,a^3\,d+1464\,\gamma_n\,c\,a^4\,n^2\,d-300\,x^2\,\gamma_n\,d^4\,a^2\,n\\
&+32\,\gamma_n\,e^2\,n^5\,a^3\,d+851\,\gamma_n\,b^2\,n^4\,d^2\,a^2+1360\,x^2\,\gamma_n\,d^3\,a^3\,n-70\,\gamma_n\,b^2\,n^4\,d^3\,a\\
&-1160\,\gamma_n\,c\,a^3\,n^2\,d^2-598\,x\,\gamma_n\,b\,d^3\,a^2\,n+120\,x^2\,\gamma_n\,d^4\,a^2\,n^2-2400\,x^2\,\gamma_n\,d^2\,a^4\,n^3\\
&+\gamma_n\,e^2\,n^2\,d^4+3416\,x\,\gamma_n\,b\,a^4\,n^4\,d+264\,\gamma_n\,b^2\,d^3\,a\,n+34\,\gamma_n\,e^2\,d^3\,a\,n\\
&-42\,x\,\gamma_n\,e\,d^4\,a\,n+106\,x\,\gamma_n\,b\,d^4\,a\,n-472\,\gamma_n\,c\,a^4\,n\,d-1184\,x\,\gamma_n\,b\,n^5\,a^4\,d\\
&-4848\,x\,\gamma_n\,b\,a^4\,n^3\,d+2\,\gamma_n\,c\,d^5-450\,x^2\,\gamma_n\,d^3\,a^3+2192\,x^2\,\gamma_n\,a^6\,n^2-480\,x^2\,\gamma_n\,a^6\,n\\
&-5\,x\,\beta_n\,n^2\,e\,b\,d^4+32\,x\,\beta_n\,n^5\,e^2\,a^3\,d+64\,x\,\beta_n\,n^6\,b\,e\,a^3\,d+8\,x\,\beta_n\,n^3\,e^2\,d^3\,a\\
&+3\,x\,\beta_n\,n\,b\,e\,d^4+48\,x\,\beta_n\,n^5\,b\,e\,d^2\,a^2+2\,x^2\,\beta_n\,n^2\,b\,d^5+64\,x^2\,\beta_n\,n^7\,b\,a^5\\
&-x\,\beta_n\,n\,c\,d^5-100\,\gamma_n\,b\,n^3\,e\,d^3\,a+160\,x\,\gamma_n\,b\,n^5\,d^2\,a^3+48\,\gamma_n\,b\,n^5\,e\,d^2\,a^2
\end{aligned}
$$

$-2234\,\gamma_n\,b^2\,n^2\,a^3\,d-1472\,x\,\gamma_n\,e\,a^4\,n^2\,d+548\,x\,\gamma_n\,e\,a^4\,n\,d+1688\,x\,\gamma_n\,e\,a^4\,n^3\,d$
$-194\,\gamma_n\,b\,e\,d^3\,a\,n+232\,\gamma_n\,b^2\,n^3\,d^3\,a-72\,\gamma_n\,b^2\,d\,a^3+132\,\gamma_n\,b^2\,d^2\,a^2$
$-12\,\gamma_n\,e^2\,d\,a^3-60\,x\,\gamma_n\,e\,a^4\,d-244\,x\,\gamma_n\,b\,a^3\,d^2+164\,x\,\gamma_n\,b\,a^2\,d^3+120\,x\,\gamma_n\,b\,a^4\,d$
$-82\,x\,\gamma_n\,e\,a^2\,d^3-10\,\gamma_n\,b\,e\,d^4-72\,\gamma_n\,b^2\,d^3\,a-2400\,x^2\,\gamma_n\,a^5\,n^4\,d-12\,\gamma_n\,e^2\,d^3\,a$
$-20\,\gamma_n\,c\,d^4\,a-5400\,x^2\,\gamma_n\,a^5\,n^2\,d+22\,x\,\gamma_n\,e\,d^4\,a+5440\,x^2\,\gamma_n\,a^5\,n^3\,d$
$+64\,x\,\gamma_n\,b\,n^7\,a^5+824\,\gamma_n\,c\,a^5\,n^5+16\,\gamma_n\,e^2\,n^6\,a^4+48\,\gamma_n\,c\,a^4\,d-28\,\gamma_n\,b^2\,n\,d^4$
$+122\,x\,\gamma_n\,e\,a^3\,d^2+218\,\gamma_n\,b\,n^2\,e\,d^3\,a+796\,\gamma_n\,b^2\,n^6\,a^4-1045\,\gamma_n\,b\,n^2\,e\,d^2\,a^2$
$-2654\,x\,\gamma_n\,b\,a^3\,n^2\,d^2-688\,x\,\gamma_n\,e\,d^2\,a^3\,n^3+8\,\gamma_n\,b^2\,n^5\,d^3\,a+20\,x\,\gamma_n\,e\,n^2\,d^4\,a$
$-2184\,\gamma_n\,b\,n^3\,e\,d\,a^3+874\,\gamma_n\,b\,n^3\,e\,d^2\,a^2+1488\,\gamma_n\,b\,n^4\,e\,d\,a^3+8\,\gamma_n\,e^2\,n^3\,d^3\,a$
$+20\,x\,\gamma_n\,b\,n^3\,d^4\,a-40\,\gamma_n\,c\,n^2\,d^4\,a+10\,\gamma_n\,c\,n^3\,d^4\,a+60\,\gamma_n\,b\,e\,d\,a^3$
$+1549\,\gamma_n\,b^2\,d^2\,a^2\,n^2+324\,\gamma_n\,e^2\,n^3\,d\,a^3-108\,\gamma_n\,e^2\,n^3\,d^2\,a^2-256\,x\,\gamma_n\,e\,d^3\,a^2\,n^2$
$+1090\,\gamma_n\,c\,d^2\,a^3\,n^3+570\,\gamma_n\,c\,d^2\,a^3\,n+258\,x\,\gamma_n\,e\,d^3\,a^2\,n-30\,\gamma_n\,e^2\,n^2\,d^3\,a$
$+1622\,\gamma_n\,b\,e\,d\,a^3\,n^2-96\,\gamma_n\,e^2\,n^5\,a^4-568\,x^2\,n^4\,c\,a^4\,d+2\,x\,\gamma_n\,e\,n\,d^5$
$-1336\,x\,\gamma_n\,b\,a^5\,n^2+1840\,x\,\gamma_n\,b\,a^5\,n^5-544\,x\,\gamma_n\,b\,n^6\,a^5-416\,x\,\gamma_n\,e\,a^5\,n^5$
$-120\,x\,\gamma_n\,e\,a^5\,n+480\,x^2\,\gamma_n\,a^4\,n^4\,d^2+2\,x\,\gamma_n\,b\,n^2\,d^5-256\,\gamma_n\,c\,n^6\,a^5+32\,\gamma_n\,c\,n^7\,a^5$
$+\gamma_n\,c\,n^2\,d^5-144\,\gamma_n\,b^2\,a^4\,n-240\,\gamma_n\,e^2\,a^4\,n^3+220\,\gamma_n\,e^2\,a^4\,n^4+240\,x\,\gamma_n\,b\,a^5\,n$
$+32\,\gamma_n\,b^2\,n^7\,a^3\,d-1580\,\gamma_n\,e\,b\,n^4\,a^4+120\,\gamma_n\,b\,e\,a^4\,n-336\,\gamma_n\,b\,n^4\,e\,d^2\,a^2$
$-1008\,x\,\gamma_n\,b\,n^4\,d^2\,a^3+16\,\gamma_n\,b\,n^4\,e\,d^3\,a$

Koeffizientenvergleich beim höchsten Koeffizienten liefert:

```
> rule4:=beta[n]=factor(solve(coeff(re,x,3),beta[n]));
```

$$rule4 := \beta_n = -\frac{2\,a\,b\,n^2-2\,a\,b\,n-2\,a\,e+2\,b\,n\,d+e\,d}{(2\,a\,n+d-2\,a)\,(d+2\,a\,n)}$$

und beim zweithöchsten Koeffizienten erhalten wir:

```
>
rule5:=gamma[n]=factor(subs(rule4,solve(coeff(re,x,2),gamma[n])))
;
```

$$rule5 := \gamma_n = -(a\,n-2\,a+d)\,(4\,a^2\,n^2\,c-8\,a^2\,n\,c+4\,a^2\,c-a\,b^2\,n^2+2\,a\,b^2\,n+4\,a\,c\,n\,d+a\,e^2-a\,b^2-4\,a\,c\,d-n\,b^2\,d-b\,e\,d+c\,d^2+b^2\,d)\,n\Big/\big((-3\,a+d+2\,a\,n)\,(d-a+2\,a\,n)\,(2\,a\,n+d-2\,a)^2\big)$$

\>

Orthogonale Polynom-Lösungen von Rekursionsgleichungen

```
> read "hsum6.mpl";
```

Package "Hypergeometric Summation", Maple V - Maple 8

```
> read "retode.mpl";
```

Package "REtoDE", Maple V - Maple 8

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Erstes Beispiel

```
> RE:=P(n+2)-(x-n-1)*P(n+1)+alpha*(n+1)^2*P(n)=0;
```

$$RE := \mathrm{P}(n+2) - (x-n-1)\,\mathrm{P}(n+1) + \alpha\,(n+1)^2\,\mathrm{P}(n) = 0$$

```
> REtoDE(RE,P(n),x);
```

Warning: parameters have the values , $\{e=0, a=0, b=2c, \alpha=\frac{1}{4}, c=c, d=-4c\}$

$$\left[\frac{1}{2}(2x+1)\left(\frac{\partial^2}{\partial x^2}\mathrm{P}(n,x)\right) - 2x\left(\frac{\partial}{\partial x}\mathrm{P}(n,x)\right) + 2n\,\mathrm{P}(n,x) = 0, \left[I=\left[\frac{-1}{2}, \infty\right], \rho(x) = 2\,\mathrm{e}^{(-2x)}\right], \frac{k_{n+1}}{k_n} = 1\right]$$

```
> REtodiscreteDE(RE,P(n),x);
```

Warning: parameters have the values , $\{a=0, b=-\frac{1}{2}f\,d - \frac{1}{2}d, e=-g\,d, c=\frac{1}{2}g\,d\,f + \frac{1}{2}g\,d - \frac{1}{4}f^2\,d + \frac{1}{4}d, f=f, d=d, \alpha=\frac{-1+f^2}{4f^2}, g=g\}$

$$\left[\frac{1}{2}\,\frac{(f+2fx-1)\,\Delta(\mathrm{Nabla}(\mathrm{P}(n,fx+g),x),x)}{f} - \frac{2x\,\Delta(\mathrm{P}(n,fx+g),x)}{f+1} + \frac{2n\,\mathrm{P}(n,fx+g)}{(f+1)f} = 0, \left[\sigma(x) = \frac{f}{2} + x - g - \frac{1}{2}, \sigma(x)+\tau(x) = -\frac{(f-1)(-1+2g-f-2x)}{2(f+1)}\right], \rho(x) = \left(\frac{f-1}{f+1}\right)^x, \frac{k_{n+1}}{k_n} = \frac{1}{f}\right]$$

Zweites Beispiel

```
> RE:=P(n+2)-x*P(n+1)+alpha*q^n*(q^(n+1)-1)*P(n)=0;
```

$$RE := \mathrm{P}(n+2) - \mathrm{P}(n+1)\,x + \alpha\,q^n\,(q^{(n+1)}-1)\,\mathrm{P}(n) = 0$$

```
> REtoqDE(RE,P(n),q,x);
```

Warning: parameters have the values ,

$\{a=-d\,q+d, c=-\alpha\,q\,d+\alpha\,d, b=0, e=0, d=d\}$

$$\left[\left(x^2+\alpha\right)\mathrm{Dq}\left(\mathrm{Dq}\left(\mathrm{P}(n,x),\frac{1}{q},x\right),q,x\right)-\frac{x\,\mathrm{Dq}(\mathrm{P}(n,x),q,x)}{q-1}+\frac{q\left(-1+q^n\right)\mathrm{P}(n,x)}{(q-1)^2\,q^n}=0,\right.$$

$$\left.\frac{\rho(q\,x)}{\rho(x)}=\frac{\alpha}{q^2\,x^2+\alpha},\frac{k_{n+1}}{k_n}=1\right]$$

>