Definition 2.13a

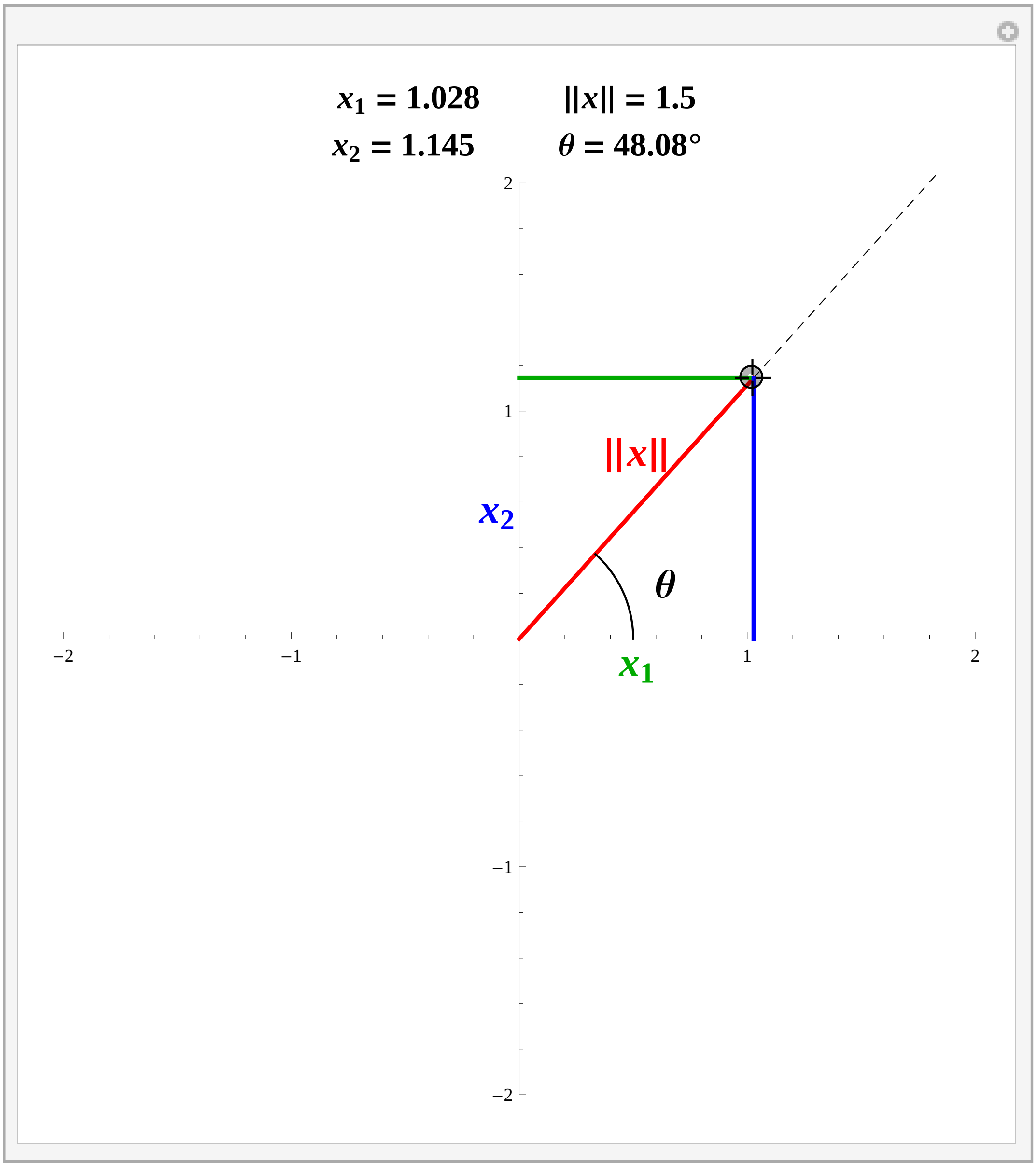
Die Abbildung

$$\langle\, .\,,\,.\,\rangle \;:\; E \times E \;\to\; \mathbb{R}$$

$$(x,\, y) \mapsto \langle x,\, y\rangle := x_1 \cdot y_1 + x_2 \cdot y_2 \;,$$

wobei $x := \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ und $y := \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

heißt *inneres Produkt* oder *Skalarprodukt* .


x_1 = 1.028
$||x||$ = 1.5
x_2 = 1.145
θ = 48.08°
2
1
−1
−2
−2
−1
1
2
$||x||$
x_2
θ
x_1

Dann gilt

$$\langle x, y \rangle = \|x\| \cdot \|y\| \cdot \cos(\alpha) ,$$

wobei α der von x und y eingeschlossene Winkel ist.

Beweis:
Mit

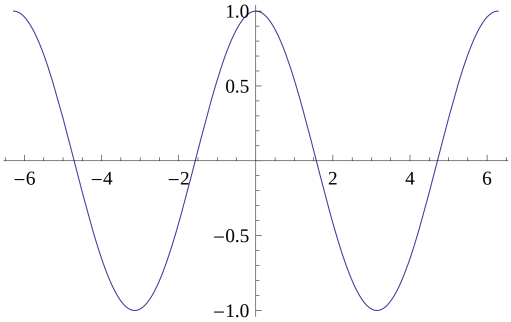
$$x_1 = \|x\| \cdot \cos(\beta) , x_2 = \|x\| \cdot \sin(\beta) , y_1 = \|y\| \cdot \cos(\gamma) , y_2 = \|y\| \cdot \sin(\gamma)$$

folgt

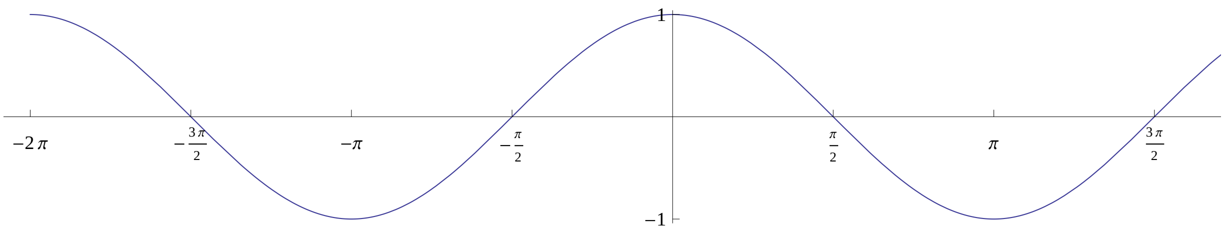
$$\begin{aligned}
\langle x, y \rangle = & \; x_1 \cdot y_1 + x_2 \cdot y_2 \\
= & \; \|x\| \cdot \cos(\beta) \cdot \|y\| \cdot \cos(\gamma) + \|x\| \cdot \sin(\beta) \cdot \|y\| \cdot \sin(\gamma) \\
= & \; \|x\| \cdot \|y\| \cdot \big(\cos(\beta) \cdot \cos(\gamma) + \sin(\beta) \cdot \sin(\gamma)\big) \\
= & \; \|x\| \cdot \|y\| \cdot \cos(\beta - \gamma) \\
= & \; \|x\| \cdot \|y\| \cdot \cos(\alpha) \quad .
\end{aligned}$$

Es gilt allgemein: $\cos(\alpha) = \cos(-\alpha)$.

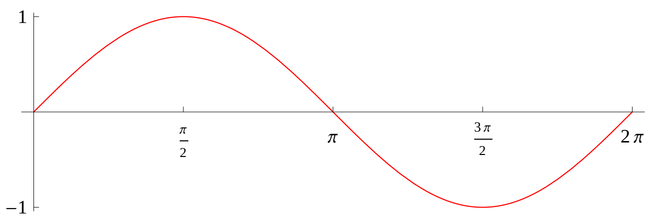
```
Plot[Cos[t], {t, -2 π, 2 π}]
```



```
g1 = Plot[Cos[t], {t, -2 π, 2 π},
  AspectRatio → Automatic,
  Ticks → {{-2 π, -3 π/2, -π, -π/2,
      {-0.08, "0", 0}, π/2, π, 3 π/2, 2 π}, {-1, 1}}]
```




```
g2 =
 Plot[Sin[t], {t, 0, 2 π}, AspectRatio → Automatic,
  Ticks → {{π/2, 3 π/2, π, 2 π}, {-1, 1}},
  PlotStyle → Red]
```




```
Show[g1, g2]
```