Folgen

- **Wir erklären die Folge b_n wie in der Vorlesung**

In[2]:=
```
b[1] = 0; b[2] = 1; b[n_] := b[n] = 1/2 (b[n - 1] + b[n - 2])
```

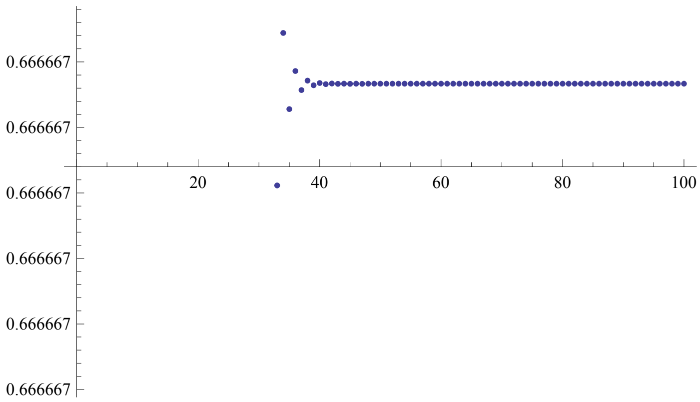
In[3]:=
```
Table[b[n], {n, 1, 10}]
```

Out[3]= $\left\{0, 1, \frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{11}{16}, \frac{21}{32}, \frac{43}{64}, \frac{85}{128}, \frac{171}{256}\right\}$

- **und stellen b_n graphisch dar:**

In[4]:=
```
plotb = ListPlot[Table[b[n], {n, 1, 100}]]
```

Out[4]=



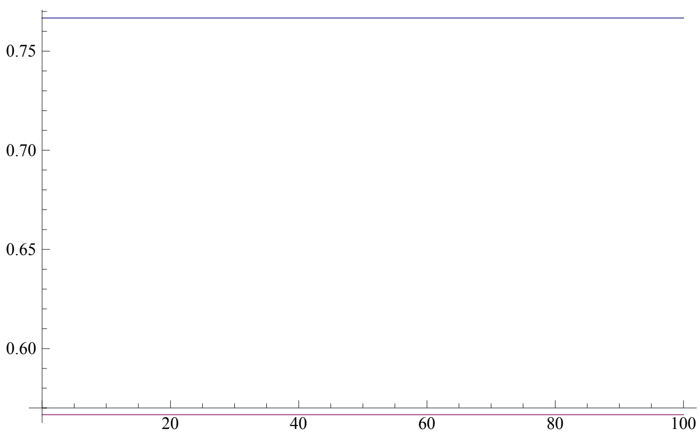
- **Welchen Grenzwert hat b_n?**

In[5]:=
```
ϵ = 0.1
```

Out[5]= 0.1

In[6]:=
```
ploteps = Plot[{2 / 3 + ϵ, 2 / 3 - ϵ}, {x, 0, 100}]
```

Out[6]=

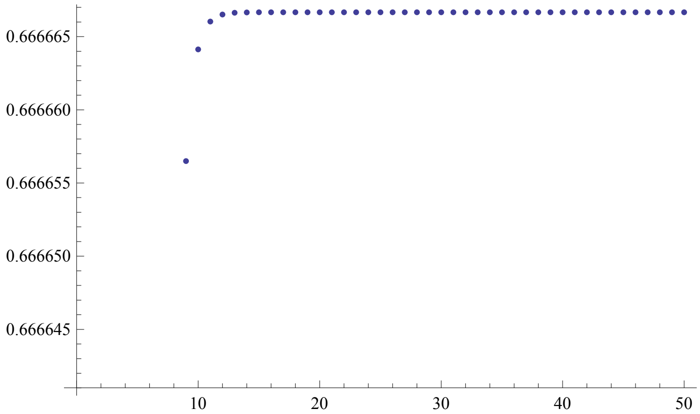


In[7]:= `Show[plotb, ploteps, PlotRange → All]`

Out[7]=

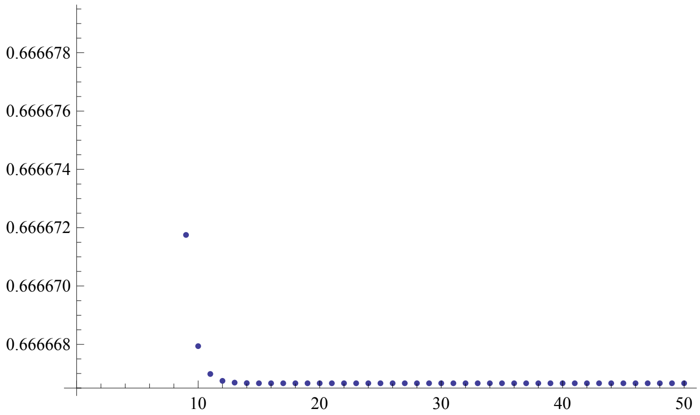

- **Die Teilfolge b_{2n-1} der ungeraden b_n ist monoton wachsend,**

In[8]:= `ListPlot[Table[b[n], {n, 1, 100, 2}]]`

Out[8]=


- **und die Teilfolge b_{2n} der geraden b_n ist monoton fallend.**

In[9]:= `ListPlot[Table[b[n], {n, 2, 100, 2}]]`

Out[9]=


- **Es gilt die Gleichung $\frac{1}{3} b_n + \frac{2}{3} b_{n+1} = \frac{2}{3}$:**

In[10]:= `Table[1/3 b[n] + 2/3 b[n + 1], {n, 1, 100}]`

$$\text{In[10]:=}\ \texttt{Table}\left[\frac{1}{3}\,\texttt{b[n]} + \frac{2}{3}\,\texttt{b[n+1]},\ \{\texttt{n}, 1, 100\}\right]$$

Out[10]=

$$\left\{\frac{2}{3}, \frac{2}{3},\right.$$
$$\frac{2}{3}, \frac{2}{3},$$
$$\left.\frac{2}{3}, \frac{2}{3}\right\}$$

- **Konvergenzgeschwindigkeit der Folge b_n:**

$$\text{In[11]:=}\ \texttt{Table}\left[\texttt{Abs}\left[\texttt{b[n]} - \frac{2}{3}\right] < \frac{1}{2^{n-1}},\ \{\texttt{n}, 2, 100\}\right]$$

Out[11]= {True, True, True, True, True, True, True, True, True, True, True, True, True, True, True, True, True, True, True, True,
True, True, True, True, True, True, True, True, True, True, True, True, True, True, True, True, True, True, True, True,
True, True, True, True, True, True, True, True, True, True, True, True, True, True, True, True, True, True, True, True,
True, True, True, True, True, True, True, True, True, True, True, True, True, True, True, True, True, True, True, True,
True, True, True, True, True, True, True, True, True, True, True, True, True, True, True, True, True, True, True}

- **Einige Grenzwerte:**

$$\text{In[12]:=}\ \texttt{Limit}\left[\frac{1}{n},\ n \to \infty\right]$$

Out[12]= 0

- **folgt aus dem Archimedischen Prinzip, und**

$$\text{In[13]:=}\ \texttt{Limit}\left[\frac{1}{2^n},\ n \to \infty\right]$$

Out[13]= 0

- **ist eine Teilfolge mit demselben Grenzwert.**
- **Eine weitere wichtige Folge**

$$\text{In[14]:=}\ \texttt{a[n\_]} := \left(1 + \frac{1}{n}\right)^n$$

In[15]:= `Table[a[n], {n, 1, 10}]`

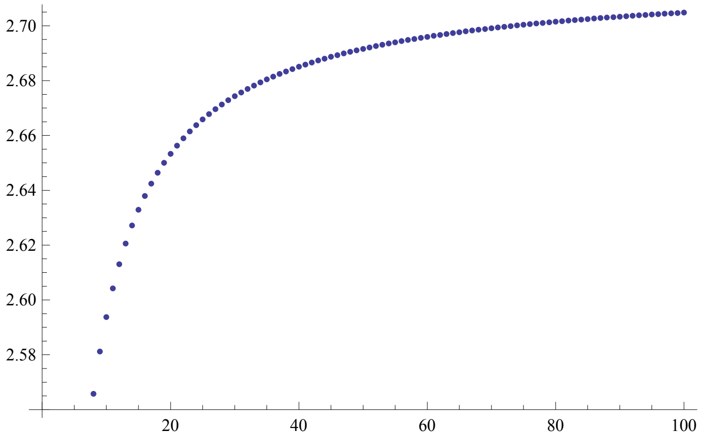
$$\text{Out[15]=}\ \left\{2, \frac{9}{4}, \frac{64}{27}, \frac{625}{256}, \frac{7776}{3125}, \frac{117\,649}{46\,656}, \frac{2\,097\,152}{823\,543}, \frac{43\,046\,721}{16\,777\,216}, \frac{1\,000\,000\,000}{387\,420\,489}, \frac{25\,937\,424\,601}{10\,000\,000\,000}\right\}$$

In[16]:= `Table[N[a[n]], {n, 1, 10}]`

Out[16]= {2., 2.25, 2.37037, 2.44141, 2.48832, 2.52163, 2.5465, 2.56578, 2.58117, 2.59374}

In[17]:= `plota = ListPlot[Table[a[n], {n, 1, 100}]]`

Out[17]=



In[18]:= $\text{Limit}\left[\left(1+\frac{1}{n}\right)^{n},\ n\to\infty\right]$

Out[18]= e

In[19]:= `N[%]`

Out[19]= 2.71828

In[20]:= $\text{Limit}\left[\left(1+\frac{x}{n}\right)^{n},\ n\to\infty\right]$

Out[20]= e^{x}

- **Mehrfache Zinsauszahlung pro Jahr**

In[21]:= $\text{Table}\left[\left(1+\frac{x}{n}\right)^{n}\ /.\ \{x\to 0.09\},\ \{n,1,12\}\right]$

Out[21]= {1.09, 1.09202, 1.09273, 1.09308, 1.0933, 1.09344, 1.09355, 1.09362, 1.09369, 1.09373, 1.09377, 1.09381}

- **Tägliche Zinsauszahlung**

In[22]:= $\left(1+\frac{x}{n}\right)^{n}\ /.\ \{x\to 0.09\}\ /.\ \{n\to 365\}$

Out[22]= 1.09416

- **Die Grenzwerte**

In[23]:= $\text{Limit}\left[\sqrt[n]{c},\ n\to\infty\right]$

Out[23]= 1

- **und**

In[24]:= $\text{Limit}\left[\sqrt[n]{n},\ n\to\infty\right]$

Out[24]= 1

- **sind bei der Betrachtung von Reihen wichtig, genauso wie der Grenzwert:**

In[25]:= $\text{Limit}\left[\frac{x^{n}}{n!},\ n\to\infty\right]$

Out[25]= 0

In[26]:= $\text{Table}\left[\frac{1}{n!}\ //\ N,\ \{n,1,10\}\right]$

Out[26]= $\{1., 0.5, 0.166667, 0.0416667, 0.00833333, 0.00138889, 0.000198413, 0.0000248016, 2.75573\times10^{-6}, 2.75573\times10^{-7}\}$

- **Reihen:**

In[27]:= $\sum_{k=0}^{\infty} q^k$

Out[27]= $\frac{1}{1-q}$

In[28]:= $\sum_{k=1}^{\infty} \frac{1}{k}$

Sum::div : Sum does not converge. ≫

Out[28]= $\sum_{k=1}^{\infty} \frac{1}{k}$

In[29]:= $\sum_{k=1}^{\infty} \frac{1}{k\,(k+1)}$

Out[29]= 1

In[30]:= $\text{Apart}\left[\frac{1}{k\,(k+1)}\right]$

Out[30]= $\frac{1}{k} - \frac{1}{k+1}$

In[31]:= $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}$

Out[31]= $\log(2)$

In[32]:= $\sum_{k=0}^{\infty} \frac{x^k}{k!}$

Out[32]= e^x

In[33]:= $\sum_{k=0}^{\infty} \frac{1}{k!}$

Out[33]= e

In[34]:= `N[%]`

Out[34]= 2.71828

In[35]:= $s = \sum_{k=1}^{\infty} \frac{1}{k^3}$

Out[35]= $\zeta(3)$

In[36]:= `N[%]`

Out[36]= 1.20206