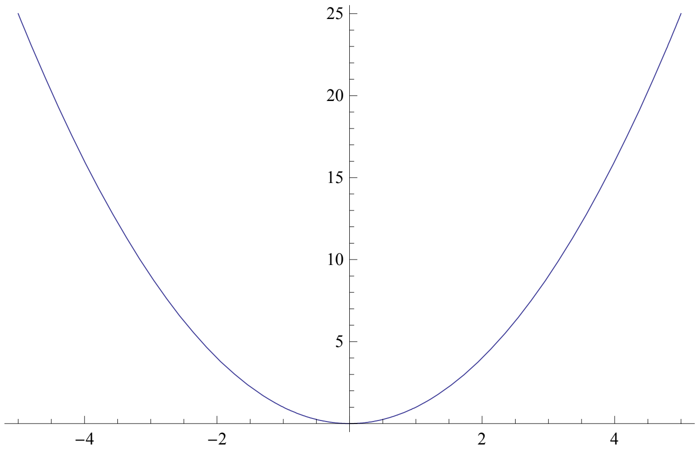
Funktionen

In[4]:= `Plot[x^2, {x, -5, 5}]`

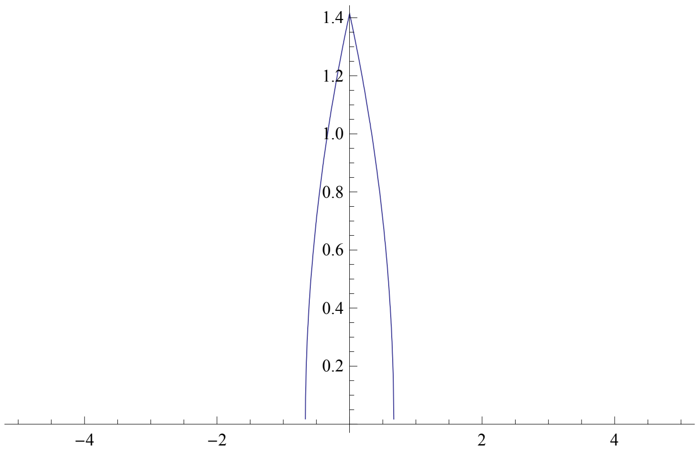
Out[4]=



■ Übung 3.1

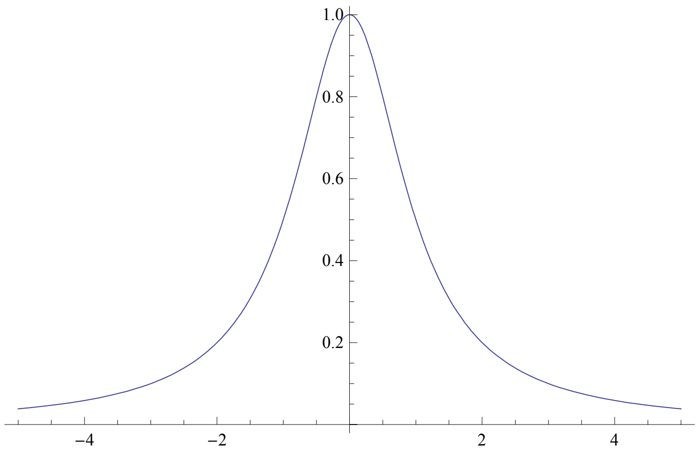
In[5]:= `Plot[Sqrt[2 - 3 Abs[x]], {x, -5, 5}]`

Out[5]=



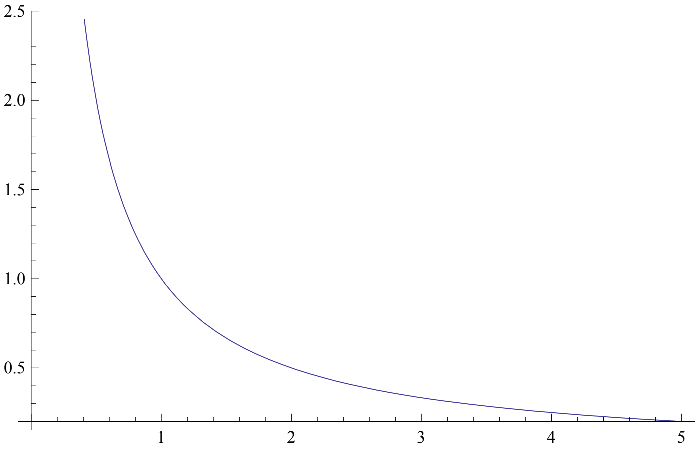
In[6]:= Plot$\left[\frac{1}{1+x^2}, \{x, -5, 5\}\right]$

Out[6]=



In[7]:= $f[x\_] := \frac{1}{1+x^2}$

In[8]:= `Plot[1/x, {x, 0, 5}]`

Out[8]=


In[9]:= `g[x_] := 1/x`

In[10]:= `g[f[x]]`

Out[10]= $x^2 + 1$

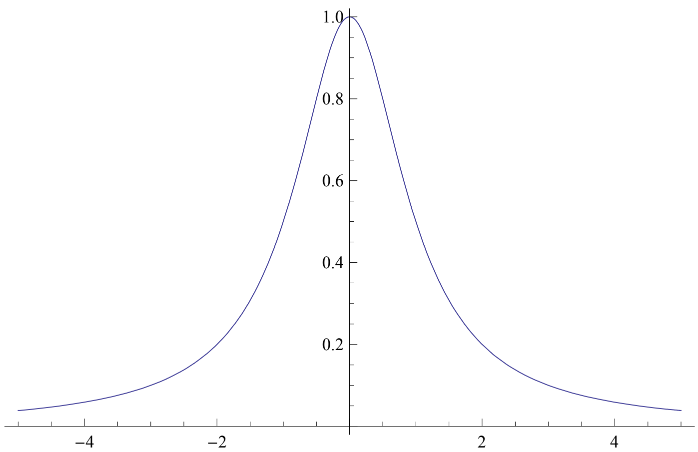
In[11]:= `f[g[x]] // Together`

Out[11]= $\frac{x^2}{x^2 + 1}$

- **Beispiele für Umkehrfunktionen, Stetigkeit und Unstetigkeit, Polstellen, Asymptoten**

- **Beispiel 3.8**

In[12]:= `plot1 = Plot[1/(x^2 + 1), {x, -5, 5}]`

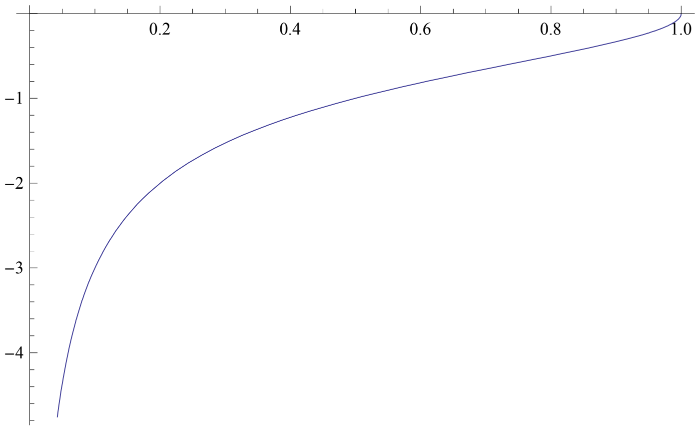
Out[12]=


In[13]:= `sol = Solve[y == 1/(x^2 + 1), x]`

Out[13]= $\left\{\left\{x \to -\frac{\sqrt{1-y}}{\sqrt{y}}\right\}, \left\{x \to \frac{\sqrt{1-y}}{\sqrt{y}}\right\}\right\}$

In[14]:=
```
plot2 = Plot[x /. sol[[1]], {y, 0, 1}]
```

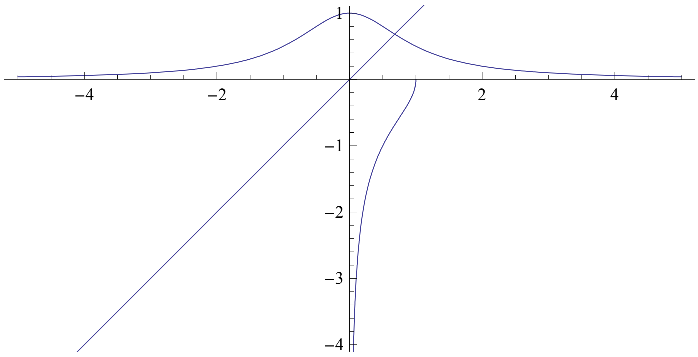
Out[14]=



In[15]:=
```
plot3 = Plot[x, {x, -5, 5}];
```

In[16]:=
```
Show[plot1, plot2, plot3, AspectRatio → Automatic, PlotRange → {-4, 1}]
```

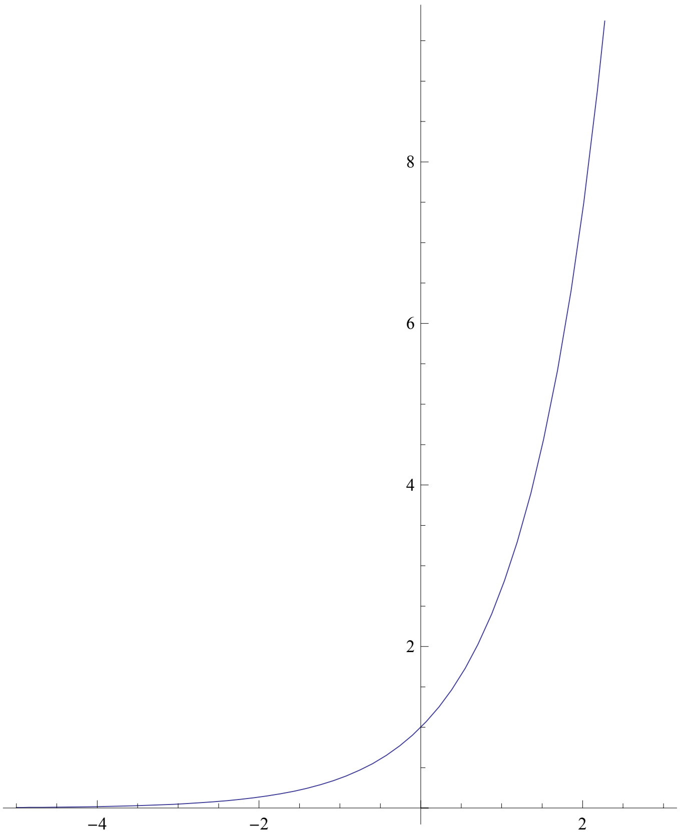
Out[16]=



■ Exponentialfunktion

In[17]:=
```
plot1 = Plot[Exp[x], {x, -5, 3}, AspectRatio → Automatic]
```

Out[17]=



■ Berechnung von e

In[18]:=
```
Exp[1] // N
```

Out[18]= 2.71828

In[19]:= $\sum_{k=0}^{7} \frac{1}{k!}$ // N

Out[19]= 2.71825

■ Logarithmusfunktion

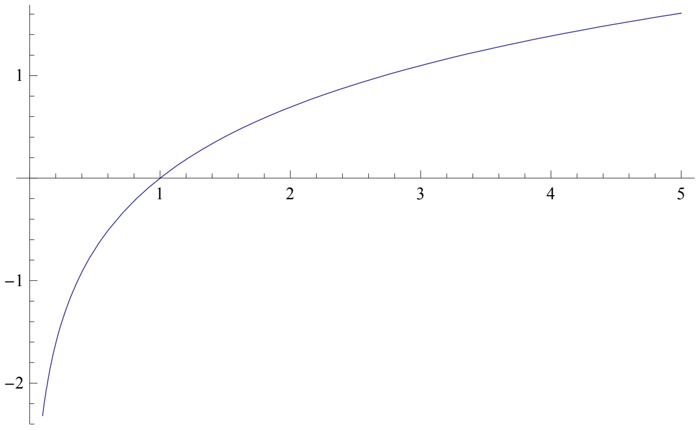
In[20]:=
```
Solve[Exp[x] == y, x]
```

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. ≫

Out[20]= $\{\{x \to \log(y)\}\}$

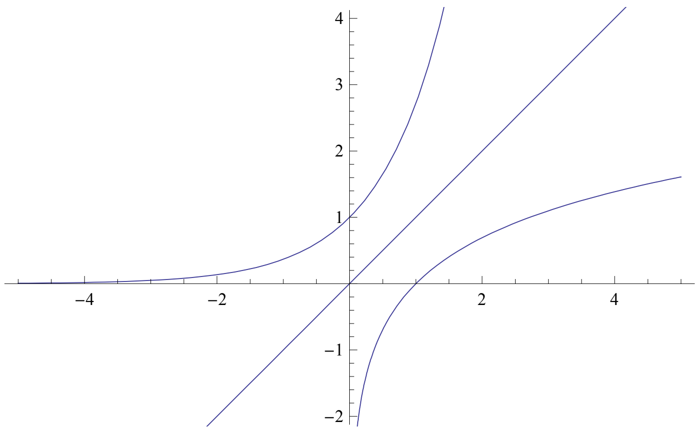
In[21]:= `plot2 = Plot[Log[x], {x, 0, 5}]`

Out[21]=



In[22]:= `Show[plot1, plot2, plot3, PlotRange → {-2, 4}]`

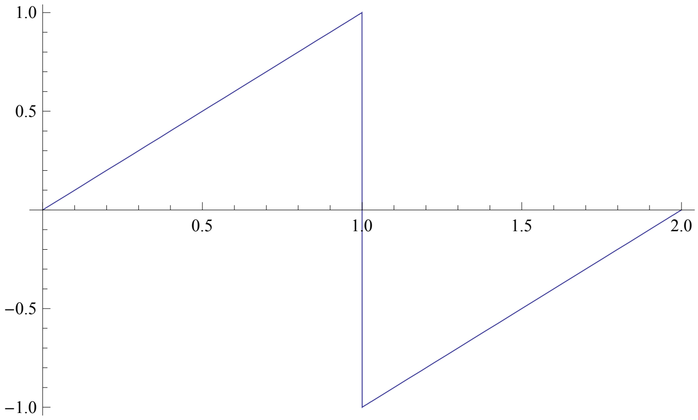
Out[22]=



- **Beispiel 3.15**

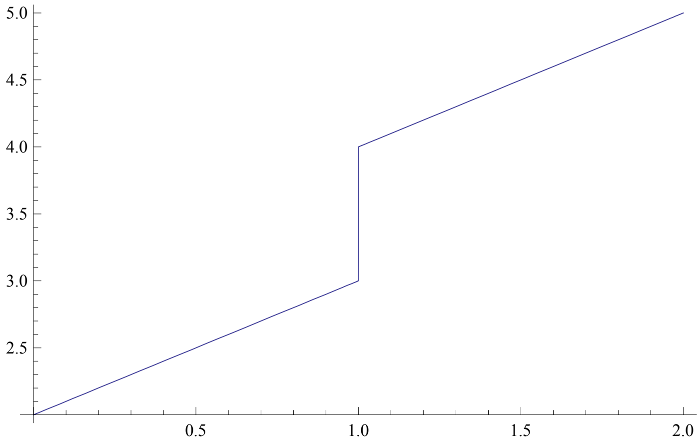
In[23]:= `Plot[If[x ≤ 1, x, x - 2], {x, 0, 2}]`

Out[23]=



In[24]:= `Plot[If[x ≤ 1, x + 2, x + 3], {x, 0, 2}]`

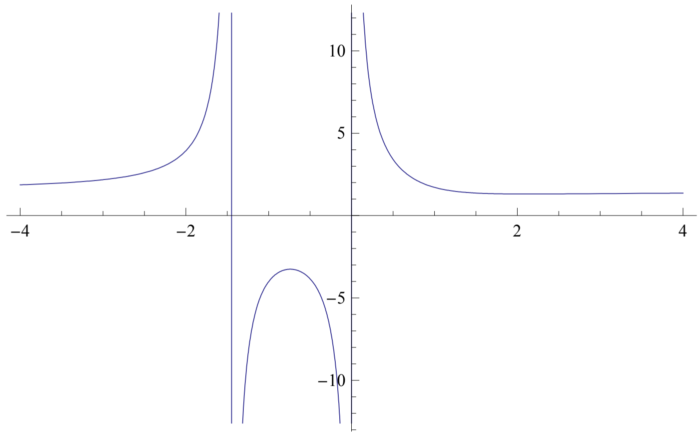
Out[24]=



■ **Beispiel 3.16**

In[25]:= $\texttt{Plot}\left[\frac{3x^4+2x^2+7}{2x^4+x^3+4x}, \{x, -4, 4\}\right]$

Out[25]=



■ **Pole**

In[26]:= $\texttt{sol = Solve}\left[2x^4+x^3+4x == 0, x\right]$

Out[26]= $\left\{\{x\to 0\}, \left\{x\to \frac{1}{6}\left(-1-\frac{1}{\sqrt[3]{217-12\sqrt{327}}}-\sqrt[3]{217-12\sqrt{327}}\right)\right\},\right.$

$\left\{x\to -\frac{1}{6}+\frac{1+i\sqrt{3}}{12\sqrt[3]{217-12\sqrt{327}}}+\frac{1}{12}\left(1-i\sqrt{3}\right)\sqrt[3]{217-12\sqrt{327}}\right\},$

$\left.\left\{x\to -\frac{1}{6}+\frac{1-i\sqrt{3}}{12\sqrt[3]{217-12\sqrt{327}}}+\frac{1}{12}\left(1+i\sqrt{3}\right)\sqrt[3]{217-12\sqrt{327}}\right\}\right\}$

In[27]:= `N[sol]`

Out[27]= $\{\{x\to 0.\}, \{x\to -1.45054\}, \{x\to 0.47527+1.07374\,i\}, \{x\to 0.47527-1.07374\,i\}\}$

■ **Nullstellen**

In[28]:= $\texttt{sol = Solve}\left[3x^4+2x^2+7 == 0, x\right]$

Out[28]= $\left\{\left\{x\to -\sqrt{-\frac{1}{3}-\frac{2i\sqrt{5}}{3}}\right\}, \left\{x\to \sqrt{-\frac{1}{3}-\frac{2i\sqrt{5}}{3}}\right\}, \left\{x\to -\sqrt{-\frac{1}{3}+\frac{2i\sqrt{5}}{3}}\right\}, \left\{x\to \sqrt{-\frac{1}{3}+\frac{2i\sqrt{5}}{3}}\right\}\right\}$

In[29]:= `N[sol]`

Out[29]= $\{\{x \to -0.77272 + 0.964588\, i\}, \{x \to 0.77272 - 0.964588\, i\}, \{x \to -0.77272 - 0.964588\, i\}, \{x \to 0.77272 + 0.964588\, i\}\}$

- **Asymptote**

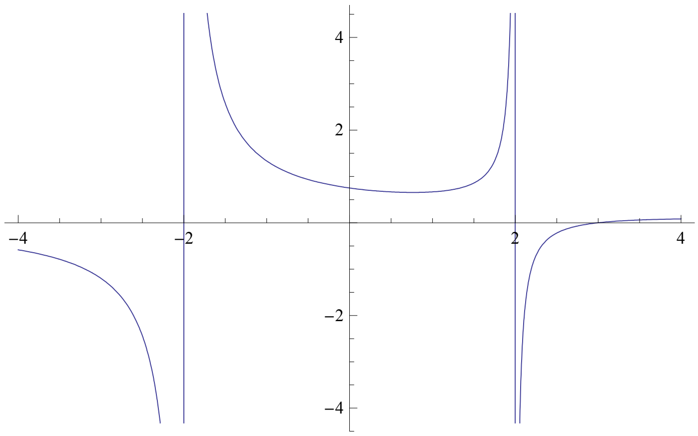
In[30]:= $\text{Limit}\left[\frac{3x^4 + 2x^2 + 7}{2x^4 + x^3 + 4x}, x \to \infty\right]$

Out[30]= $\frac{3}{2}$

- **Beispiel 3.17**

In[31]:= $\text{Plot}\left[\frac{1}{x+2} - \frac{1}{x^2-4}, \{x, -4, 4\}\right]$

Out[31]=



In[32]:= $\text{Limit}\left[\frac{1}{x+2} - \frac{1}{x^2-4}, x \to \infty\right]$

Out[32]= 0

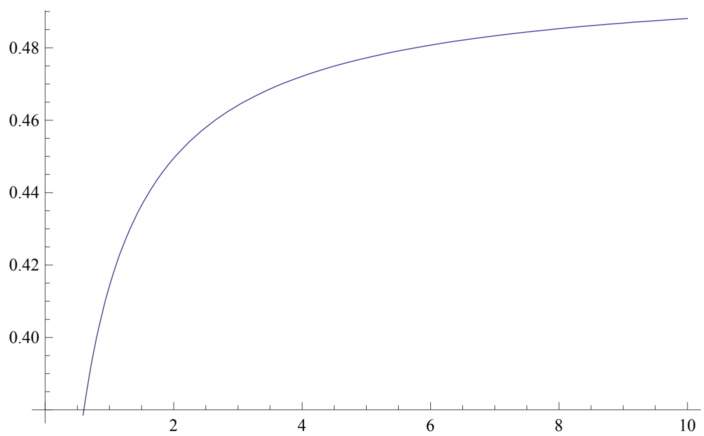
In[33]:= $\text{Limit}\left[\frac{1}{x+2} - \frac{1}{x^2-4}, x \to 2\right]$

Out[33]= $-\infty$

- **Beispiel 3.18**

In[34]:= $\text{Plot}\left[\sqrt{x^2 + x} - x, \{x, 0, 10\}\right]$

Out[34]=



- **natürlicher Definitionsbereich $\mathbb{R}_{\geq 0}$**

- **Asymptote**

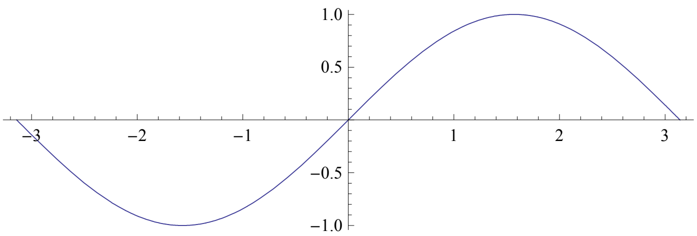
In[35]:= `Limit[Sqrt[x^2 + x] - x, x -> ∞]`

$$\text{Limit}\left[\sqrt{x^2 + x} - x,\ x \to \infty\right]$$

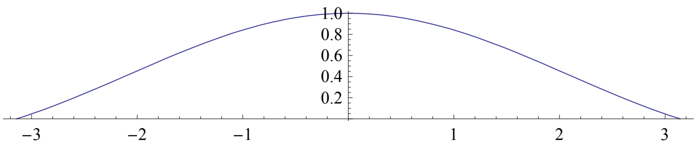
Out[35]= $\frac{1}{2}$

- **Beispiel 3.19**

In[36]:=
```
Plot[Sin[x], {x, -π, π}, AspectRatio → Automatic]
```

Out[36]=


In[37]:= $\text{Plot}\left[\frac{\text{Sin}[x]}{x},\ \{x, -\pi, \pi\},\ \text{AspectRatio} \to \text{Automatic}\right]$

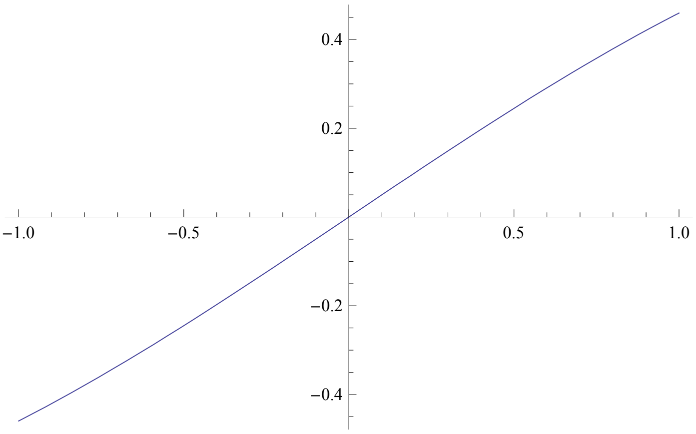
Out[37]=


In[38]:= $\text{Limit}\left[\frac{\text{Sin}[x]}{x},\ x \to 0\right]$

Out[38]= 1

- **Beispiel 3.20**

In[39]:= $\text{Plot}\left[\frac{1 - \text{Cos}[x]}{x},\ \{x, -1, 1\}\right]$

Out[39]=


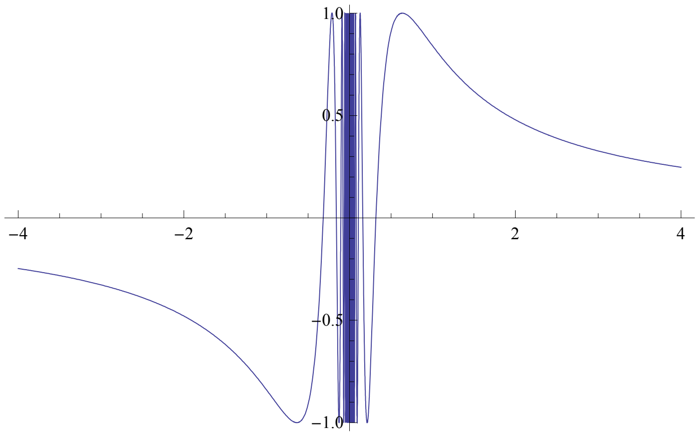
In[40]:= $\text{Limit}\left[\frac{1 - \text{Cos}[x]}{x},\ x \to 0\right]$

Out[40]= 0

- **Beispiel 3.21**

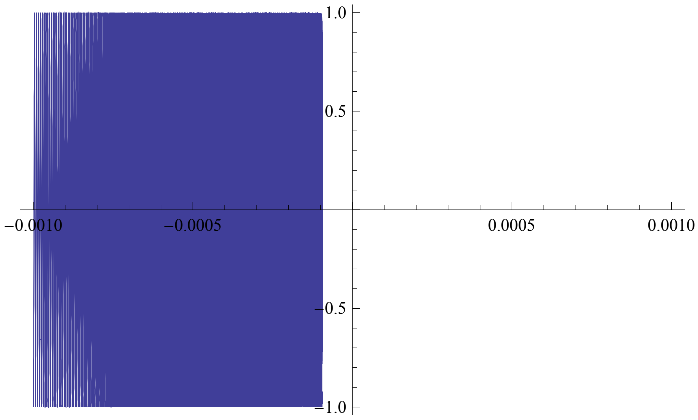
In[41]:=
```
Plot[Sin[1/x], {x, -4, 4}, PlotPoints → 1000]
```

Out[41]=



In[42]:=
```
Plot[Sin[1/x], {x, -0.001, 0.001}, PlotPoints → 1000]
```

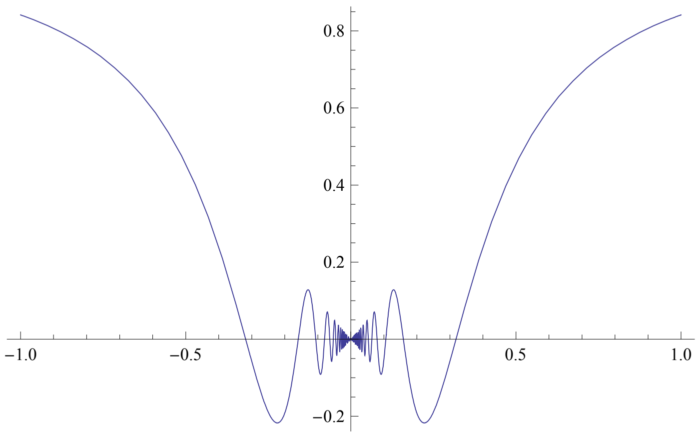
Out[42]=



- **$\sin\left(\frac{1}{x}\right)$ ist unstetig für $x = 0$**

In[43]:=
```
Plot[x Sin[1/x], {x, -1, 1}]
```

Out[43]=



In[44]:=
```
Limit[x Sin[1/x], x → 0]
```

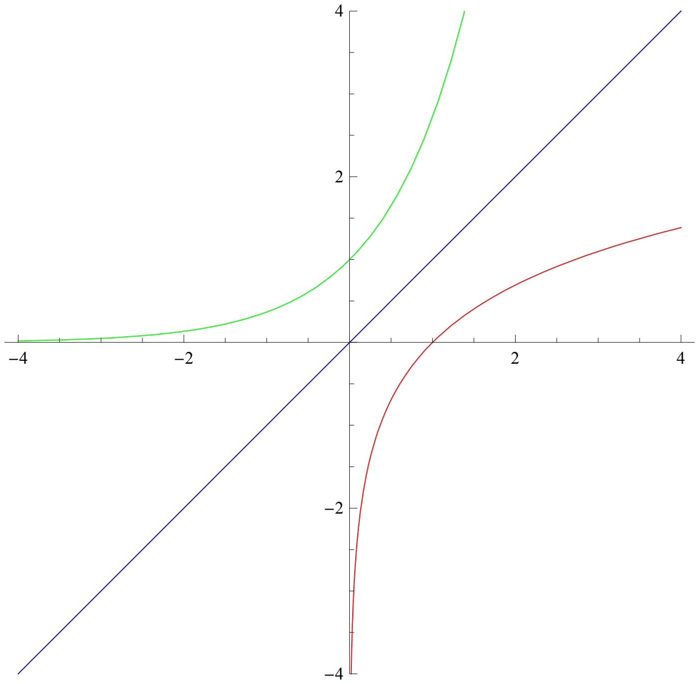
Out[44]= 0

- $x \sin\left(\frac{1}{x}\right)$ **ist stetig fortsetzbar nach** $\mathbb{R}$

- **Graphen von Logarithmus- und Exponentialfunktion**

In[45]:=
```
Plot[{Log[x], Exp[x], x}, {x, -4, 4}, AspectRatio → Automatic, PlotRange → {-4, 4},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 1, 0], RGBColor[0, 0, 1]}]
```

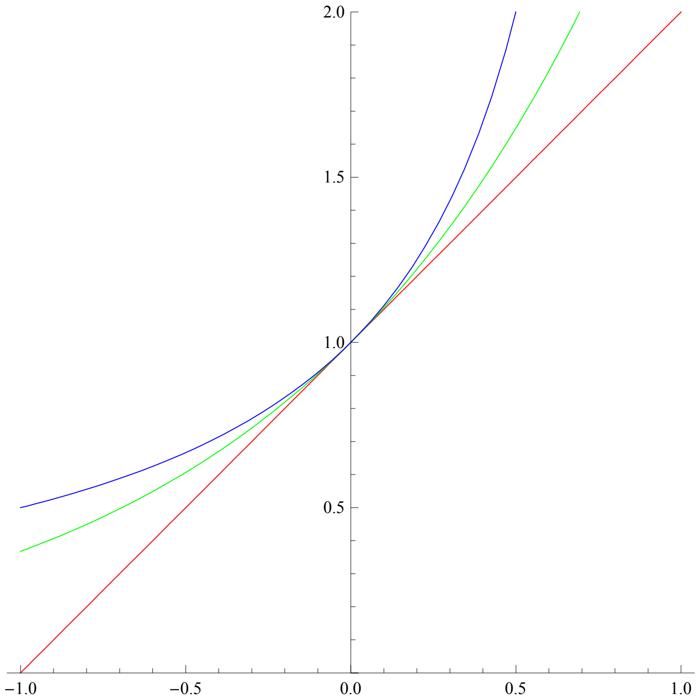
Out[45]=



- **Ungleichungen für die Exponentialfunktion**

In[46]:=
```
Plot[{1 + x, Exp[x], 1/(1 - x)}, {x, -1, 1},
 PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 1, 0], RGBColor[0, 0, 1]},
 AspectRatio → Automatic, PlotRange → {0, 2}]
```

Out[46]=



■ Ungleichungen für die Logarithmusfunktion

In[47]:=
```
Plot[{1 - 1/x, Log[x], x - 1}, {x, 0, 5},
  PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 1, 0], RGBColor[0, 0, 1]},
  AspectRatio → Automatic, PlotRange → {-2.5, 2.5}]
```

Out[47]=