Ableitungsregeln

- **Linearität: Summen- und Konstantenregel**

In[38]:=
```
f = Sum[x^k, {k, 0, 10}]
```

Out[38]= $x^{10} + x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$

In[39]:=
```
D[f, x]
```

Out[39]= $10\,x^9 + 9\,x^8 + 8\,x^7 + 7\,x^6 + 6\,x^5 + 5\,x^4 + 4\,x^3 + 3\,x^2 + 2\,x + 1$

- **Produktregel**

In[40]:=
```
D[u[x] * v[x], x]
```

Out[40]= $v(x)\,u'(x) + u(x)\,v'(x)$

- **Erweiterung auf mehrere Faktoren**

In[41]:=
```
D[u[x] * v[x] * w[x], x]
```

Out[41]= $v(x)\,w(x)\,u'(x) + u(x)\,w(x)\,v'(x) + u(x)\,v(x)\,w'(x)$

- **Quotientenregel**

In[42]:=
```
D[u[x] / v[x], x]
```

Out[42]= $\frac{u'(x)}{v(x)} - \frac{u(x)\,v'(x)}{v(x)^2}$

In[43]:=
```
Together[D[u[x] / v[x], x]]
```

Out[43]= $\frac{v(x)\,u'(x) - u(x)\,v'(x)}{v(x)^2}$

- **Kettenregel**

In[44]:=
```
Clear[f]
```

In[45]:=
```
D[g[f[x]], x]
```

Out[45]= $f'(x)\,g'(f(x))$

- **Erweiterung auf tiefere Verkettung**

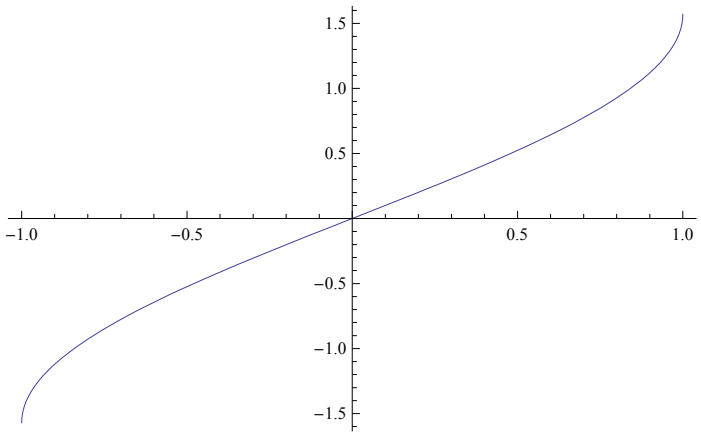
In[46]:=
```
D[h[g[f[x]]], x]
```

Out[46]= $f'(x)\,g'(f(x))\,h'(g(f(x)))$

■ Beispiel 4.12

In[47]:=
```
Plot[ArcSin[x], {x, -1, 1}]
```

Out[47]=



In[48]:=
```
D[ArcSin[x], x]
```

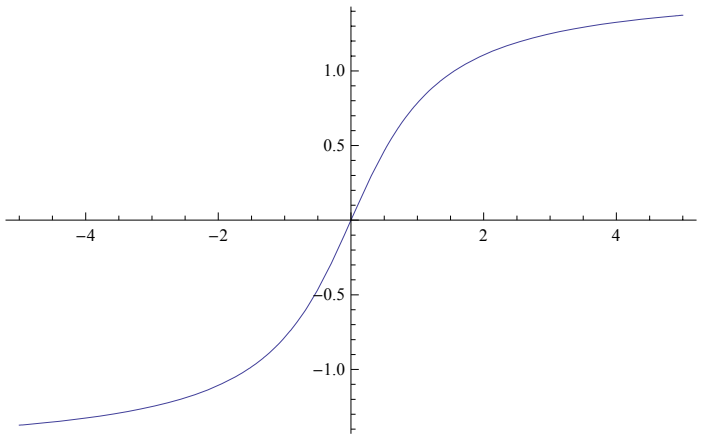
Out[48]=

$$\frac{1}{\sqrt{1-x^2}}$$

■ Beispiel 4.13

In[49]:=
```
Plot[ArcTan[x], {x, -5, 5}]
```

Out[49]=



In[50]:=
```
D[ArcTan[x], x]
```

Out[50]=

$$\frac{1}{x^2+1}$$

■ Kurvendiskussion

In[51]:=
```
f[x_] := x Exp[-x]
```

In[52]:=
```
D[f[x], {x, n}]
```

Out[52]=

$$\frac{\partial^n (e^{-x} x)}{\partial x^n}$$

In[53]:=
```
ableitungen = (-1)^n (x - n) Exp[-x]
```

Out[53]= $(-1)^n e^{-x} (x-n)$

In[54]:=
```
ableitungen /. {n → 0}
```

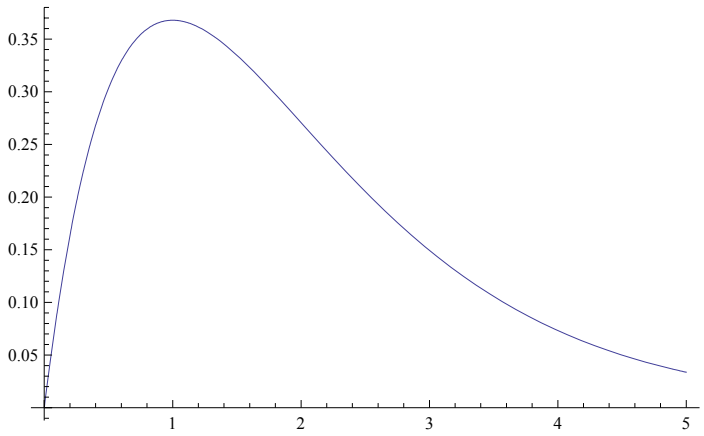
Out[54]= $e^{-x} x$

In[55]:=
```
Factor[D[ableitungen, x]]
```

Out[55]= $(-1)^n e^{-x} (n-x+1)$

In[56]:=
```
Plot[f[x], {x, 0, 5}, PlotRange → All]
```

Out[56]=



In[57]:=
```
ableitung = Factor[D[f[x], x]]
```

Out[57]= $-e^{-x}(x-1)$

In[58]:=
```
f[1]
```

Out[58]= $\frac{1}{e}$

In[59]:=
```
zweiteableitung = Factor[D[f[x], {x, 2}]]
```

Out[59]= $e^{-x}(x-2)$

In[60]:=
```
f[2]
```

Out[60]= $\frac{2}{e^2}$

In[61]:=
```
Limit[f[x], x → ∞]
```

Out[61]= 0

■ Satz von Rolle

In[62]:=
```
f = x^3 - 4 x^2 + 3 x + 2
```

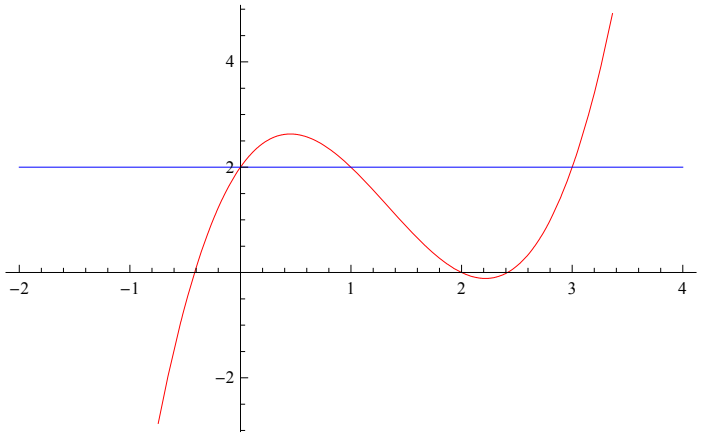
Out[62]= $x^3 - 4x^2 + 3x + 2$

In[63]:=
```
Solve[f == 2, x]
```

Out[63]= $\{\{x \to 0\}, \{x \to 1\}, \{x \to 3\}\}$

In[64]:=
```
Plot[{f, 2}, {x, -2, 4}, PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 0, 1]}]
```

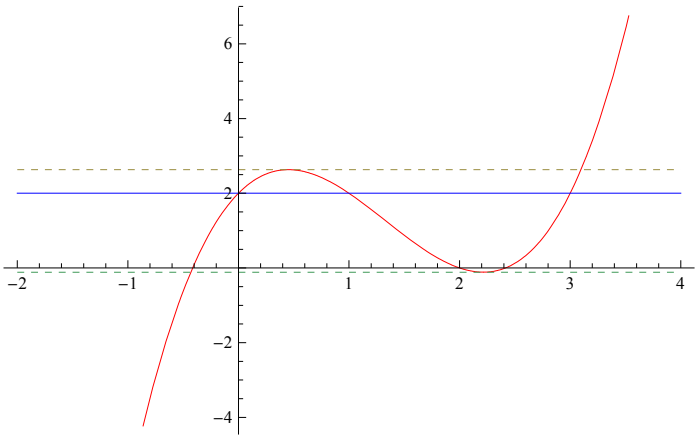
Out[64]=



■ Wo sind die vom Satz von Rolle prognostizierten Stellen?

In[65]:=
```
Plot[Evaluate[{f, 2, Apply[Sequence, f /. Solve[D[f, x] == 0, x]]}], {x, -2, 4}, PlotStyle →
  {RGBColor[1, 0, 0], RGBColor[0, 0, 1], Dashing[{0.01, 0.01}], Dashing[{0.01, 0.01}]}]
```

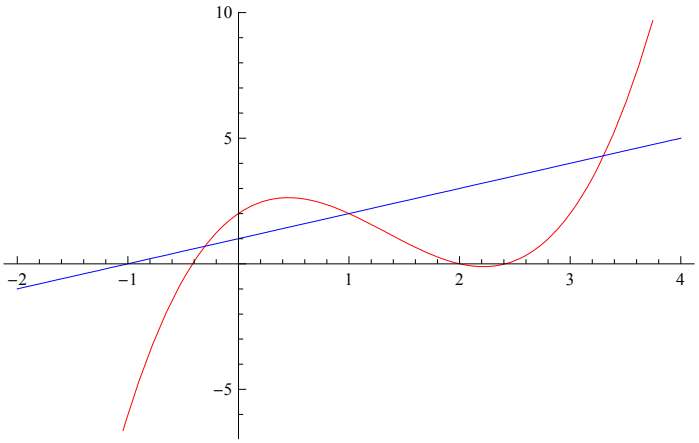
Out[65]=



■ Mittelwertsatz

In[66]:=
```
Plot[{f, 1 + x}, {x, -2, 4}, PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 0, 1]}]
```

Out[66]=



■ Wo sind die vom Mittelwertsatz prognostizierten Stellen?

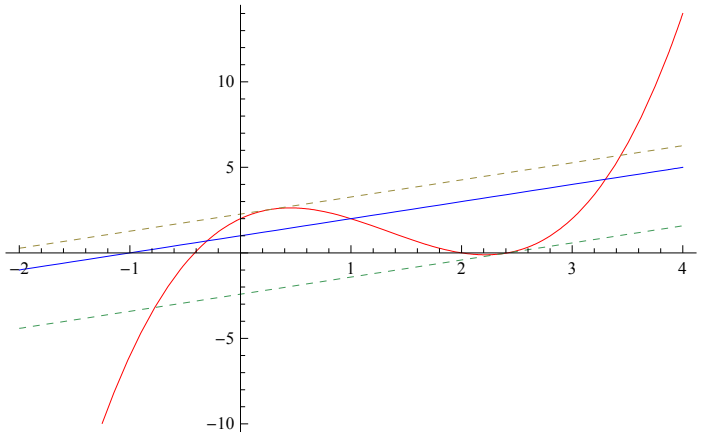
In[67]:=
```
sol = Solve[D[f, x] == 1, x]
```

Out[67]= $\left\{\left\{x \to \frac{1}{3}\left(4-\sqrt{10}\right)\right\}, \left\{x \to \frac{1}{3}\left(4+\sqrt{10}\right)\right\}\right\}$

In[68]:=
```
Tangente[f_, x_, x0_] := (f /. x → x0) + (D[f, x] /. x → x0) * (x - x0)
```

In[69]:=
```
Plot[Evaluate[{f, 1 + x, Tangente[f, x, x /. sol[[1]]], Tangente[f, x, x /. sol[[2]]]}],
 {x, -2, 4}, PlotStyle →
  {RGBColor[1, 0, 0], RGBColor[0, 0, 1], Dashing[{0.01, 0.01}], Dashing[{0.01, 0.01}]}]
```

Out[69]=



- **Beispiel 4.15**

In[70]:=
```
Limit[(x^2 + x - 2)/(x^2 - 1), x → 1]
```

Out[70]= $\frac{3}{2}$

- **Beispiel 4.16**

In[71]:=
```
Limit[(1 - Cos[3 x])/(1 - Cos[x]), x → 0]
```

Out[71]= 9

- **Grenzwerte Übung 4.10**

In[72]:=
```
Limit[x/(1 - Exp[-x]), x → 0]
```

Out[72]= 1

In[73]:=
```
Limit[(Sin[x] - x)/(x^2 Sin[x]), x → 0]
```

Out[73]= $-\frac{1}{6}$

In[74]:=
```
Limit[Sin[5 x]/Tan[3 x], x → 0]
```

Out[74]= $\frac{5}{3}$