Differentialgleichungen

```
DirectionField[DE_, y_[x_], {x_, a_, b_},
  {y_, c_, d_}, options___] := Module[{g},
  g = DE[[2 ]] /. y[x] → y;
  VectorPlot[{1, g}, {x, a, b}, {y, c, d}, options]
 ]
```

Hausaufgabe

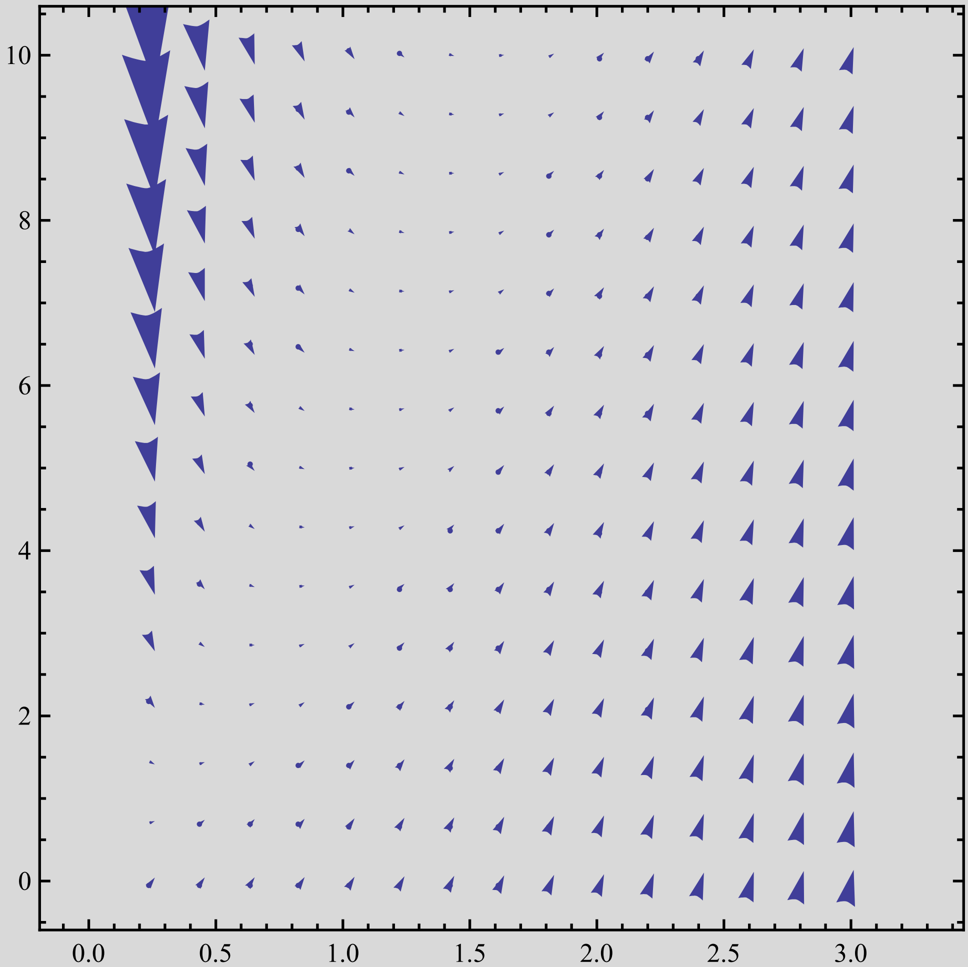
```
DE = y'[x] == -y[x]/x + x^2 + 4
```

$$y'(x) == x^2 - \frac{y(x)}{x} + 4$$

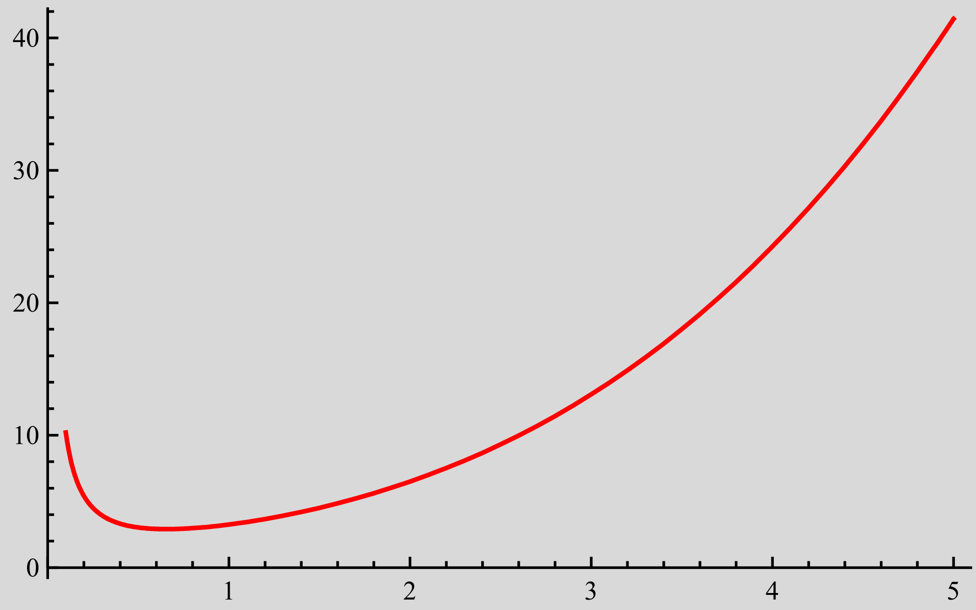
```
DSolve[{DE, y[1] == 13/4}, y[x], x]
```

$$\left\{\left\{y(x) \to \frac{x^4 + 8x^2 + 4}{4x}\right\}\right\}$$

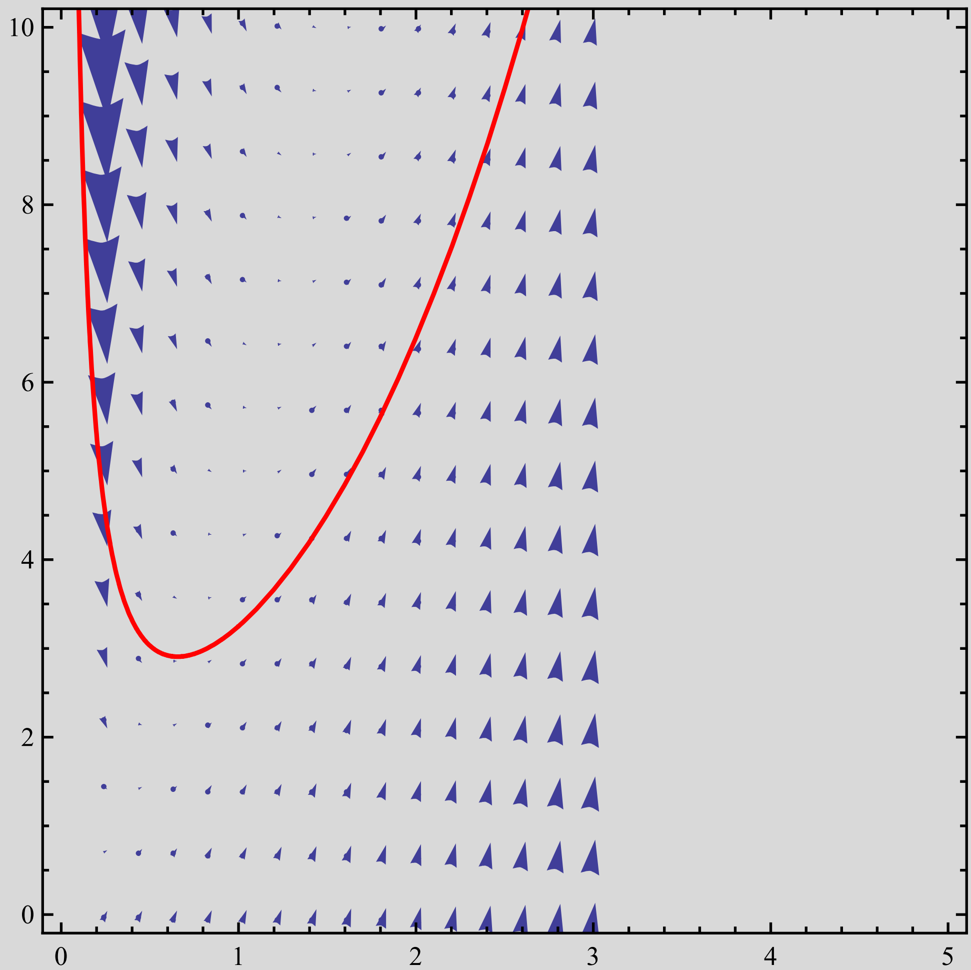
```
plot1 =
 DirectionField[DE, y[x], {x, 0.25, 3}, {y, 0, 10}, Frame → True]
```



```
plot2 = Plot[Evaluate[y[x] /. DSolve[{DE, y[1] == 13/4}, y[x], x][[1]]],
   {x, 0.1, 5}, PlotStyle → {Thickness[0.005], RGBColor[1, 0, 0]}]
```



```
Show[plot1, plot2, PlotRange → {0, 10}]
```



Linear transformierte Differentialgleichungen

Beispiel 2.4

```
DE = y'[x] == (2 x + y[x])^2
```

$$y'(x) = (y(x) + 2x)^2$$

```
DSolve[DE, y[x], x]
```

$$\left\{\left\{y(x) \to \frac{1}{c_1 e^{2 i \sqrt{2} x} - \frac{i}{2\sqrt{2}}} - 2x - i\sqrt{2}\right\}\right\}$$

nach Transformation

```
DE2 = u'[x] == 2 + (u[x])^2
```

$u'(x) = u(x)^2 + 2$

```
DSolve[DE2, u[x], x]
```

$\{\{u(x) \to \sqrt{2}\ \tan(\sqrt{2}\ c_1 + \sqrt{2}\ x)\}\}$

Einsetzen der selbst bestimmten Lösung:

```
lösung = y[x] → √2 Tan[√2 (x + C)] - 2 x
```

$y(x) \to \sqrt{2}\ \tan(\sqrt{2}\ (C + x)) - 2\,x$

```
DE /. {lösung, D[lösung, x]}
```

$2\sec^2(\sqrt{2}\ (C + x)) - 2 = 2\tan^2(\sqrt{2}\ (C + x))$

```
DE /. {lösung, D[lösung, x]} // Simplify
```

True

■ Bernoullische Differentialgleichung

```
Clear[f, g, a]
```

```
DE = y'[x] == f[x] y[x] + g[x] y[x]^a
```

$y'(x) = g(x)\ y(x)^a + f(x)\ y(x)$

```
DSolve[DE, y[x], x]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. ≫

$$\left\{\left\{y(x)\to\left(c_1\,e^{(1-a)\int_1^x f(K[1])\,dK[1]} - a\,e^{(1-a)\int_1^x f(K[1])\,dK[1]}\int_1^x g(K[2])\,e^{-(1-a)\int_1^{K[2]} f(K[1])\,dK[1]}\,dK[2] + e^{(1-a)\int_1^x f(K[1])\,dK[1]}\int_1^x g(K[2])\,e^{-(1-a)\int_1^{K[2]} f(K[1])\,dK[1]}\,dK[2]\right)^{\frac{1}{1-a}}\right\}\right\}$$

```
f[x_] := 4/x; g[x_] := 1; a = 1/2;
```

```
DSolve[DE, y[x], x]
```

$$\left\{\left\{y(x)\to\frac{1}{4}\left(4\,c_1^2\,x^4 - 4\,c_1\,x^3 + x^2\right)\right\}\right\}$$

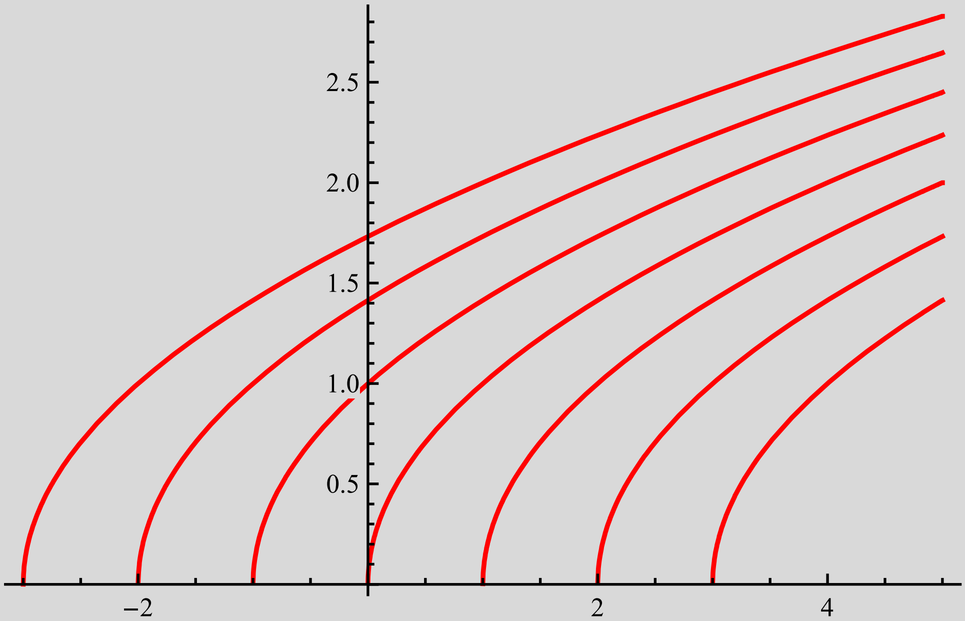
```
f[x_] := 1/x; g[x_] := 1/x^3; a = 2;
```

```
DSolve[DE, y[x], x]
```

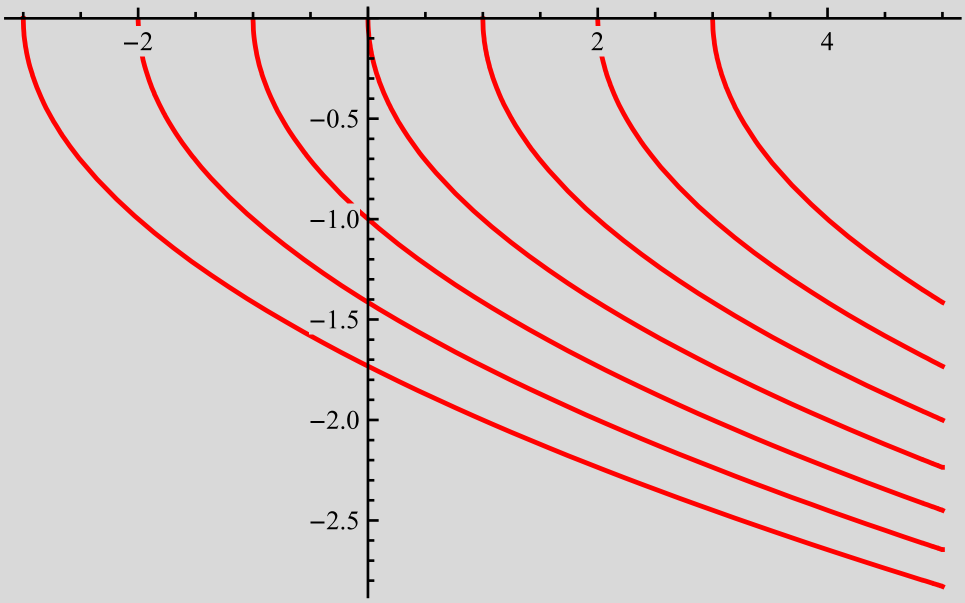
$$\left\{\left\{y(x)\to\frac{x^2}{c_1\,x+1}\right\}\right\}$$

■ Orthogonaltrajektorien

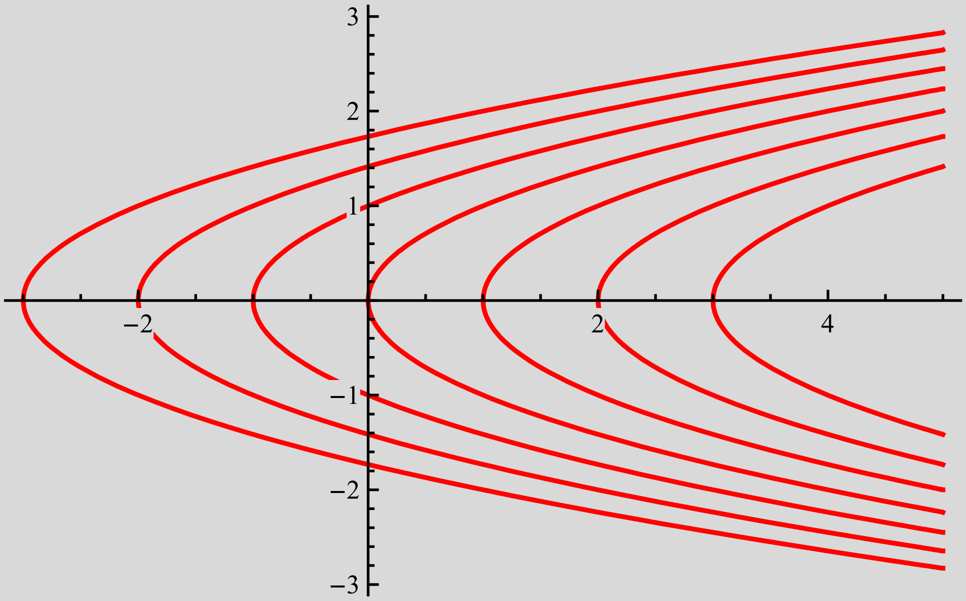
```
plot1 = Plot[Evaluate[Table[√(x + C), {C, -3, 3}]], {x, -3, 5},
  PlotStyle → {{Thickness[0.005], RGBColor[1, 0, 0]}}]
```



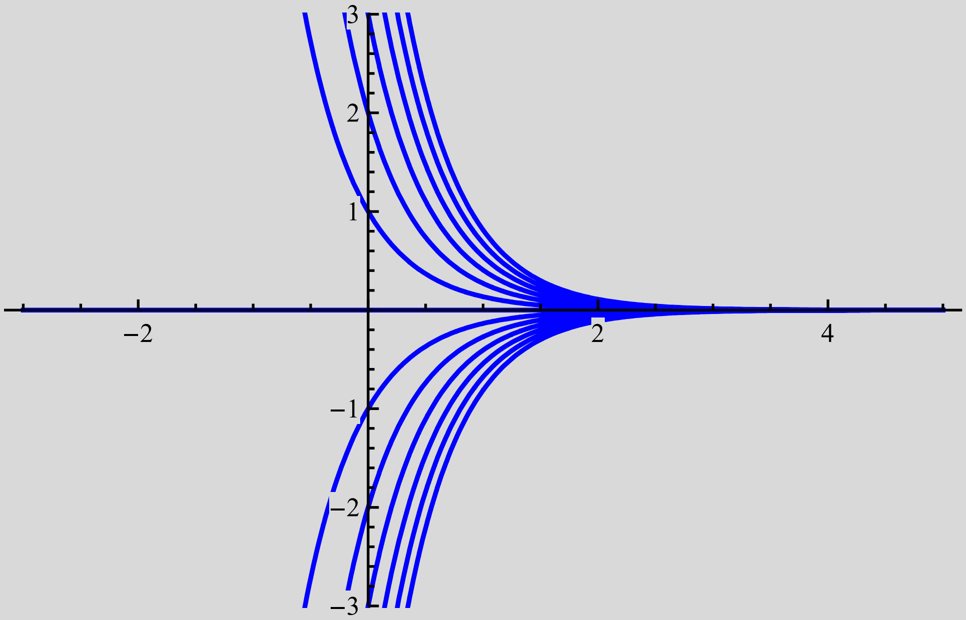

```
plot2 = Plot[Evaluate[Table[-√(x + C), {C, -3, 3}]], {x, -3, 5},
  PlotStyle → {{Thickness[0.005], RGBColor[1, 0, 0]}}]
```



```
Show[plot1, plot2, PlotRange → {-3, 3}]
```




```
plot3 = Plot[Evaluate[Table[C Exp[-2 x], {C, -6, 6}]], {x, -3, 5},
  PlotStyle → {{Thickness[0.005], RGBColor[0, 0, 1]}},
  PlotRange → {-3, 3}]
```



```
Show[plot1, plot2, plot3, PlotRange → {-3, 3}]
```