Differentialgleichungen

In[1]:=
```
DirectionField[DE_, y_[x_], {x_, a_, b_}, {y_, c_, d_}, options___] := Module[{g},
  g = DE[[2]] /. y[x] → y;
  VectorPlot[{1, g}, {x, a, b}, {y, c, d}, options]
 ]
```

■ Hausaufgabe 1: Bernoullische Differentialgleichung

In[2]:=
```
DE = y'[x] == f[x] y[x] + g[x] y[x]^a
```

Out[2]=
$$y'(x) = g(x)\, y(x)^a + f(x)\, y(x)$$

In[3]:=
```
f[x_] := 1/x; g[x_] := 1/x^3; a = 2;
```

In[4]:=
```
DSolve[DE, y[x], x]
```

Out[4]=
$$\left\{\left\{y(x) \to \frac{x^2}{c_1\, x + 1}\right\}\right\}$$

■ Sukzessive Lösung

In[5]:=
```
u[x] == 1/y[x]
```

Out[5]=
$$u(x) = \frac{1}{y(x)}$$

In[6]:=
```
DE2 = u'[x] == (1 - a) f[x] * u[x] + (1 - a) * g[x]
```

Out[6]=
$$u'(x) = -\frac{u(x)}{x} - \frac{1}{x^3}$$

In[17]:=
```
A[x_] := 1/x; B[x_] := -1/x^3;
```

In[18]:=
```
DSolve[DE2, u[x], x]
```

Out[18]=
$$\left\{\left\{u(x) \to \frac{c_1}{x} + \frac{1}{x^2}\right\}\right\}$$

In[20]:=
```
homogeneLösung = u[x] → C * Exp[∫ -A[x] 𝕕x]
```

Out[20]=
$$u(x) \to \frac{C}{x}$$

In[21]:=
```
DEhom = u'[x] == (1 - a) f[x] * u[x]
```

Out[21]=
$$u'(x) = -\frac{u(x)}{x}$$

In[22]:=
```
DSolve[DEhom, u[x], x]
```

Out[22]=
$$\left\{\left\{u(x) \to \frac{c_1}{x}\right\}\right\}$$

In[24]:=
```
spezielleLösung = u[x] → (Exp[∫ -A[x] 𝕕x] * ∫ B[x] Exp[∫ A[x] 𝕕x] 𝕕x)
```

Out[24]=
$$u(x) \to \frac{1}{x^2}$$

In[28]:=
```
Lösung = u[x] → (u[x] /. homogeneLösung) + (u[x] /. spezielleLösung)
```

Out[28]=
$$u(x) \to \frac{C}{x} + \frac{1}{x^2}$$

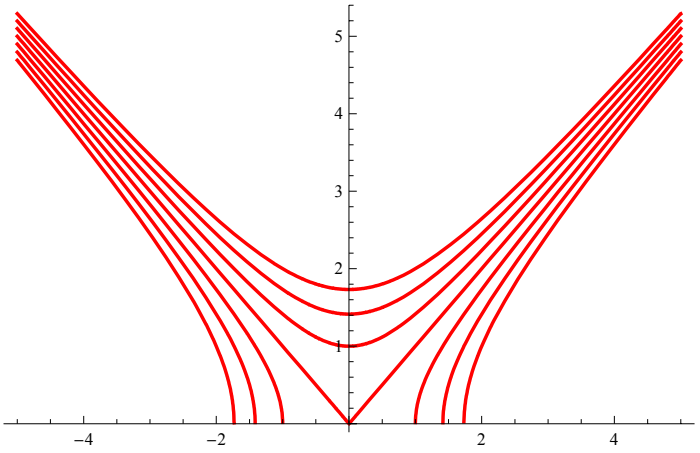
In[30]:=
```
y[x] == 1/u[x] /. Lösung // Together
```

Out[30]=
$$y(x) = \frac{x^2}{C\,x + 1}$$

■ Hausaufgabe 2: Orthogonaltrajektorien

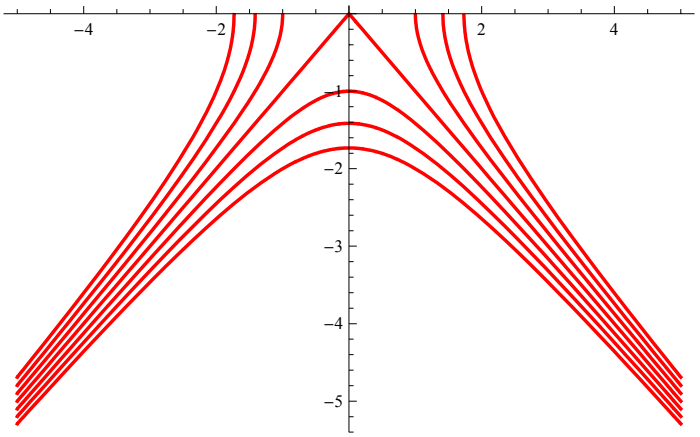
In[31]:=
```
plot1 = Plot[Evaluate[Table[Sqrt[x^2 + C], {C, -3, 3}]],
  {x, -5, 5}, PlotStyle → {{Thickness[0.005], RGBColor[1, 0, 0]}}]
```

Out[31]=



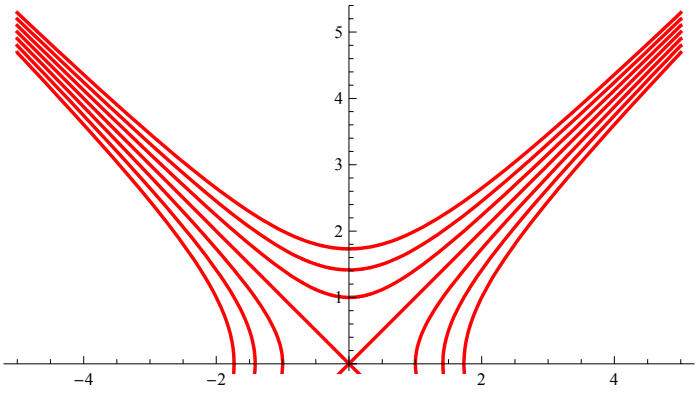
In[32]:=
```
plot2 = Plot[Evaluate[Table[-Sqrt[x^2 + C], {C, -3, 3}]],
  {x, -5, 5}, PlotStyle → {{Thickness[0.005], RGBColor[1, 0, 0]}}]
```

Out[32]=



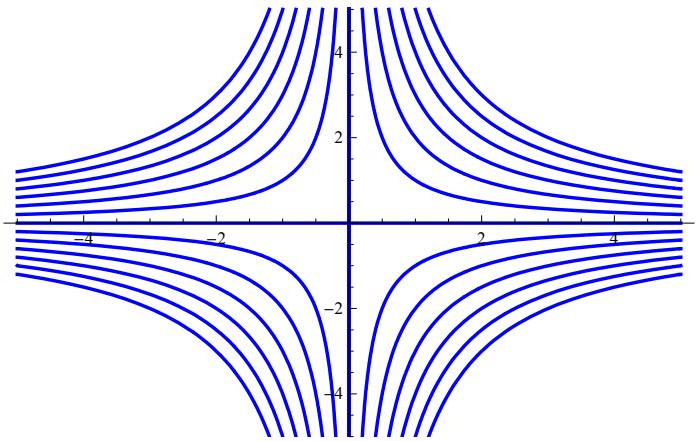
In[33]:=
```
Show[plot1, plot2, AspectRatio → Automatic]
```

Out[33]=



In[36]:=
```
plot5 = Plot[Evaluate[Table[K/x, {K, -6, 6}]], {x, -5, 5},
  PlotStyle → {{Thickness[0.005], RGBColor[0, 0, 1]}}, PlotRange → {-5, 5}]
```

Out[36]=



In[37]:=
```
Show[plot1, plot2, plot5, PlotRange → {-5, 5}, AspectRatio → Automatic]
```

Out[37]=