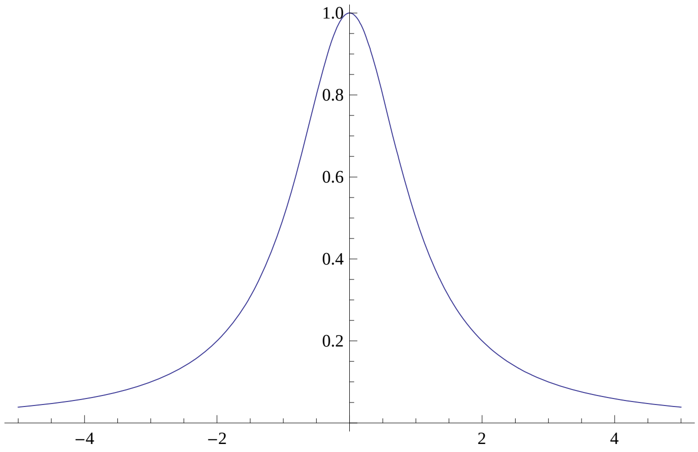
# Funktionentheorie

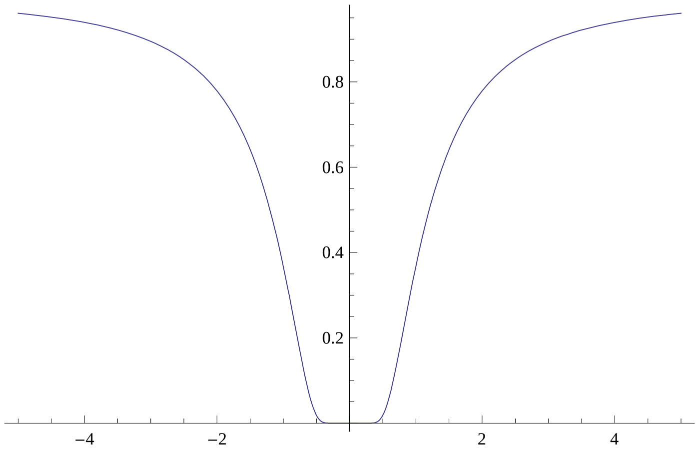
In[1]:= Plot$\left[\frac{1}{1+x^2}, \{x, -5, 5\}\right]$

Out[1]=



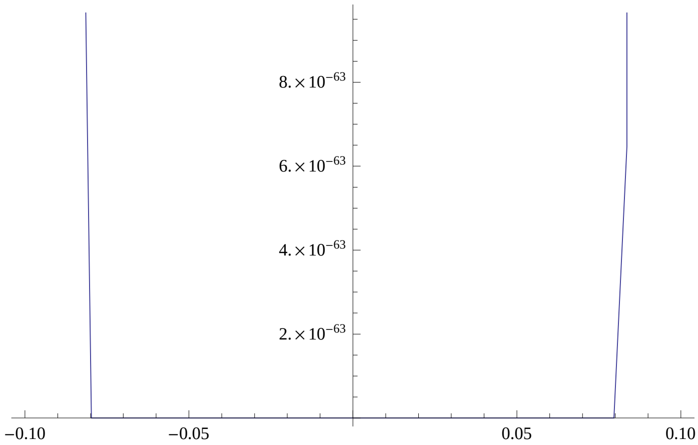

In[2]:= Plot$\left[\text{Exp}\left[-\frac{1}{x^2}\right], \{x, -5, 5\}\right]$

Out[2]=




In[3]:= Plot$\left[\text{Exp}\left[-\frac{1}{x^2}\right], \{x, -0.1, 0.1\}\right]$

Out[3]=



## Reelle Darstellung komplexer Abbildungen

In[4]:=
```
f = z^2
```
Out[4]= $z^2$

In[5]:=
```
ComplexExpand[f, z]
```
Out[5]= $2\,i\,\mathrm{Im}(z)\,\mathrm{Re}(z) - \mathrm{Im}(z)^2 + \mathrm{Re}(z)^2$

In[6]:=
```
ComplexExpand[Re[f /. z → x + ⅈ y]]
```
Out[6]= $x^2 - y^2$

In[7]:=
```
Realteil[f_] := ComplexExpand[Re[f /. z → x + ⅈ y]]

Imaginärteil[f_] := ComplexExpand[Im[f /. z → x + ⅈ y]]
```

In[9]:=
```
Realteil[z^2]
```
Out[9]= $x^2 - y^2$

In[10]:=
```
Imaginärteil[z^2]
```
Out[10]= $2\,x\,y$

In[11]:=
```
Realteil[z^3]
```
Out[11]= $x^3 - 3\,x\,y^2$

In[12]:=
```
Imaginärteil[z^3]
```
Out[12]= $3\,x^2\,y - y^3$

In[13]:=
```
Realteil[ⅇ^z]
```
Out[13]= $e^x \cos(y)$

In[14]:=
```
Imaginärteil[ⅇ^z]
```
Out[14]= $e^x \sin(y)$

In[15]:=
```
Realteil[(1 + z)/(1 - z)]
```
Out[15]= $-\frac{x^2}{(1-x)^2 + y^2} - \frac{y^2}{(1-x)^2 + y^2} + \frac{1}{(1-x)^2 + y^2}$

In[16]:=
```
Imaginärteil[(1 + z)/(1 - z)]
```
Out[16]= $\frac{2\,y}{(1-x)^2 + y^2}$

In[17]:=
```
Clear[Realteil, Imaginärteil]

Realteil[f_] := Simplify[ComplexExpand[Re[f /. z → x + ⅈ y], TargetFunctions → Conjugate]]

Imaginärteil[f_] :=
 Simplify[ComplexExpand[Im[f /. z → x + ⅈ y], TargetFunctions → Conjugate]]
```

In[20]:=
```
Realteil[(1 + z)/(1 - z)]
```
Out[20]= $-\frac{x^2 + y^2 - 1}{x^2 - 2\,x + y^2 + 1}$

In[21]:= 
```
Imaginärteil[(1 + z)/(1 - z)]
```

Out[21]= $\frac{2y}{x^2 - 2x + y^2 + 1}$

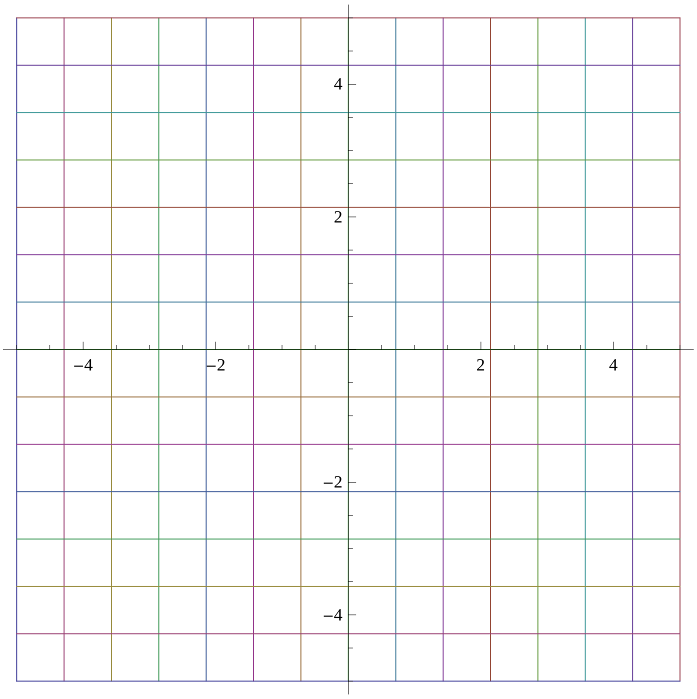
## ■ Graphische Darstellung komplexer Abbildungen

In[22]:= 
```
Needs["Graphics`ComplexMap`"]
```

General::obspkg :
Graphics`ComplexMap` is now obsolete. The legacy version being loaded may conflict with current Mathematica functionality. See the Compatibility Guide for updating information. ≫

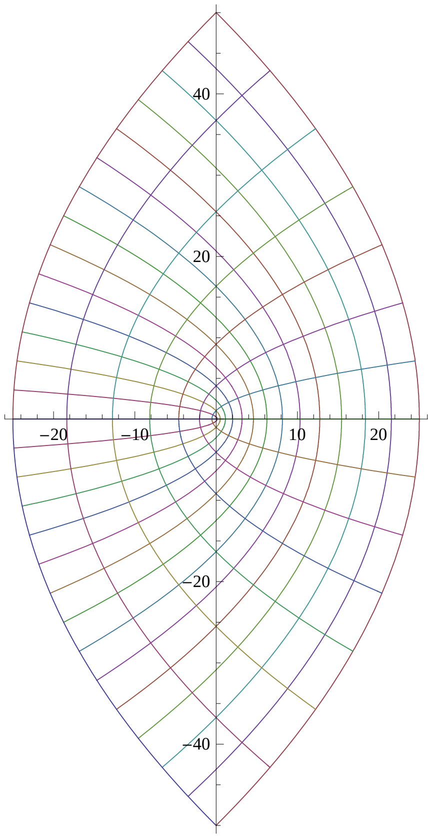
In[23]:= 
```
CartesianMap[Identity, {-5, 5}, {-5, 5}]
```

Out[23]=



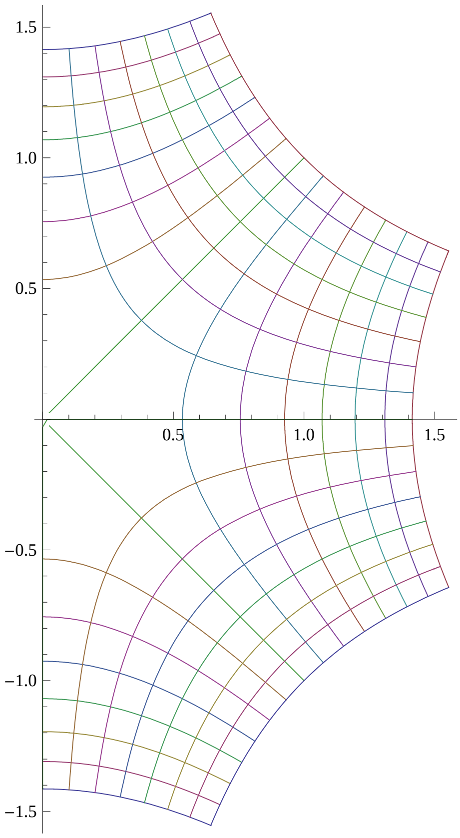
In[24]:= **CartesianMap[#$^2$ &, {0, 5}, {-5, 5}]**

Out[24]=



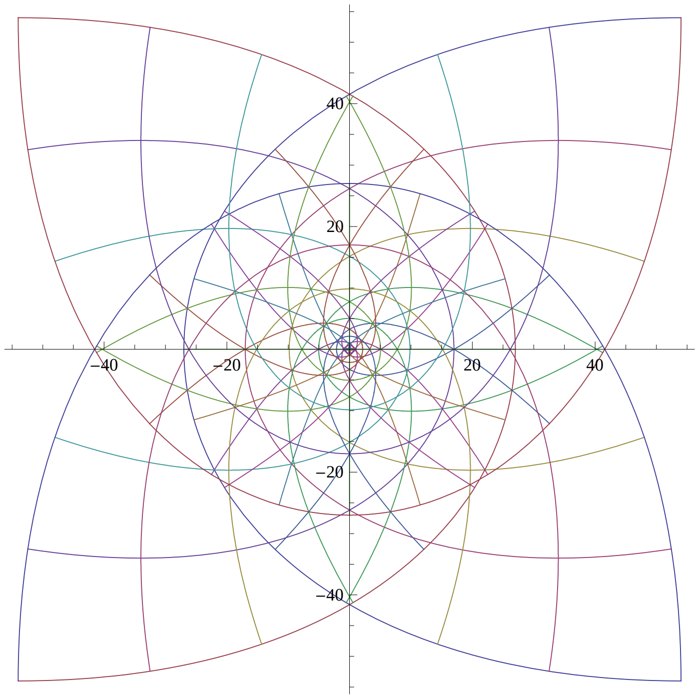

In[25]:= **CartesianMap[$\sqrt{\#}$ &, {-2, 2}, {-2, 2}]**

Out[25]=



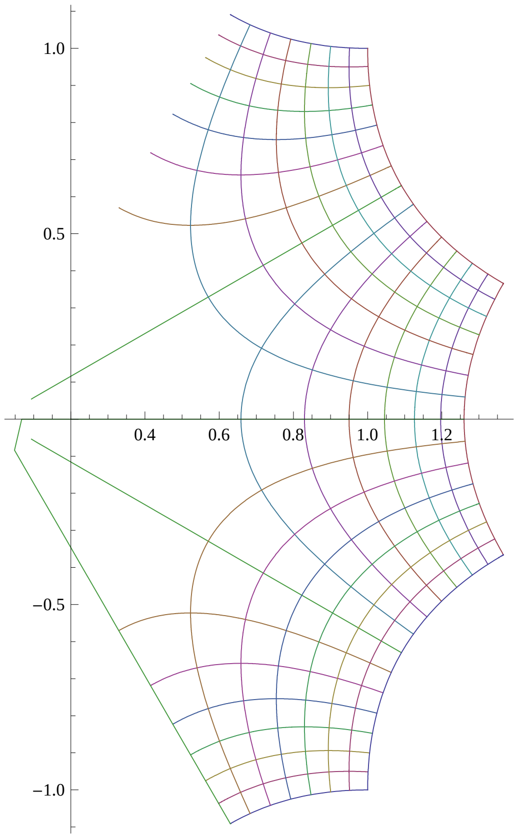
In[26]:= `CartesianMap[#^3 &, {-3, 3}, {-3, 3}]`

Out[26]=



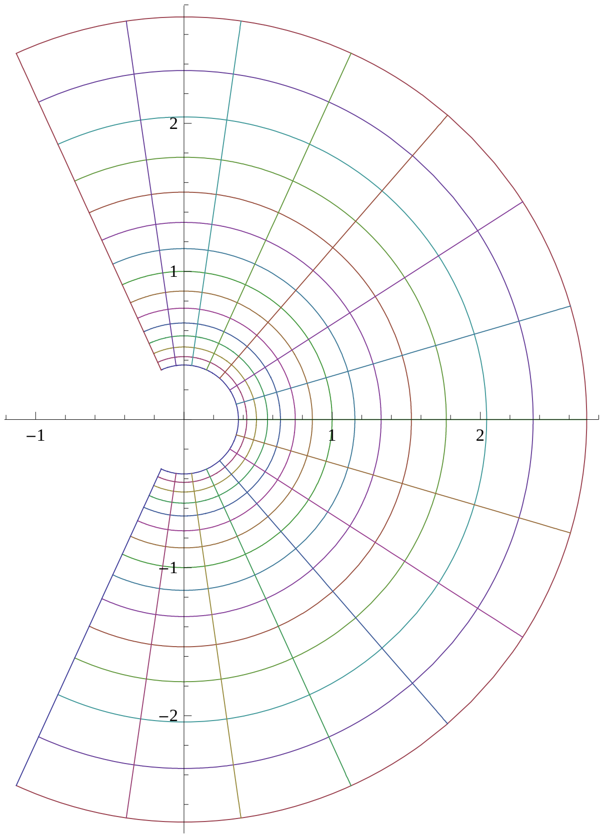

In[27]:= `CartesianMap[∛# &, {-2, 2}, {-2, 2}]`

Out[27]=



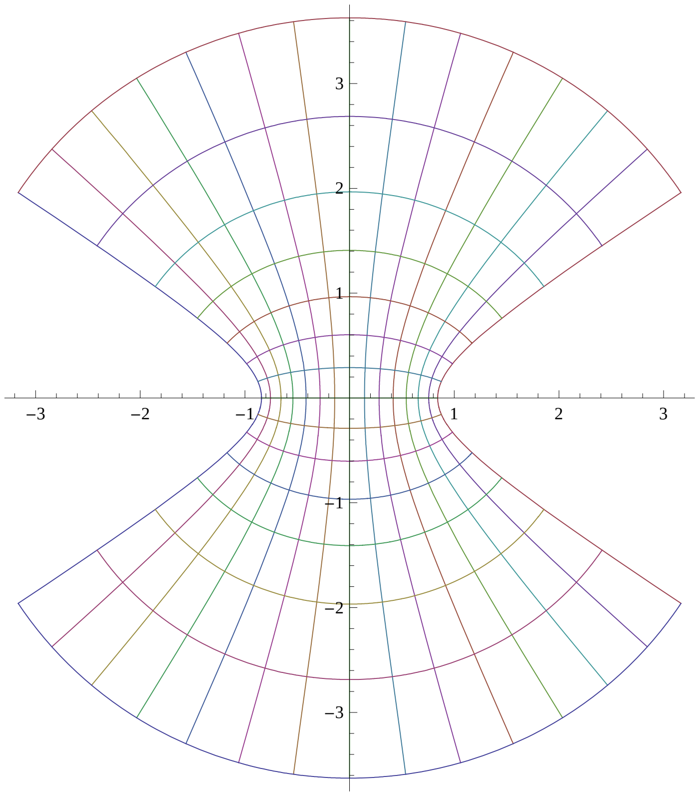
In[28]:= **CartesianMap[Exp, {-1, 1}, {-2, 2}]**

Out[28]=



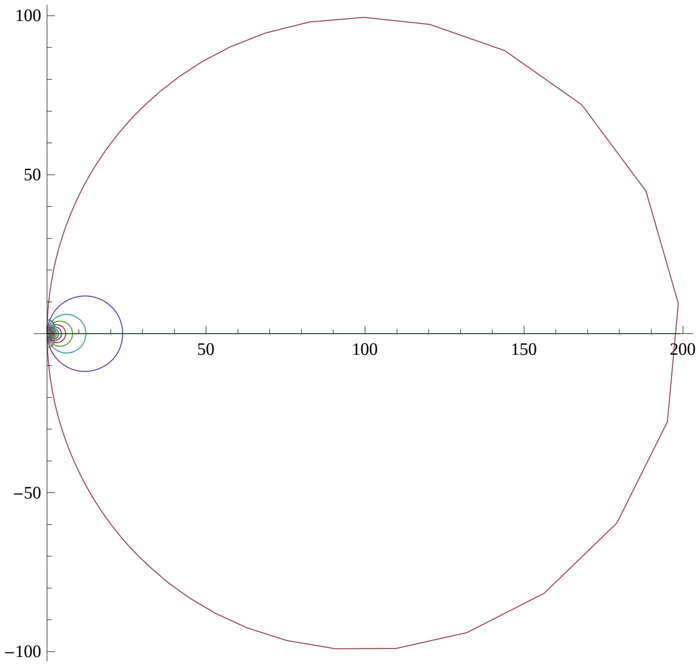

In[29]:= **CartesianMap[Sin, {-1, 1}, {-2, 2}]**

Out[29]=



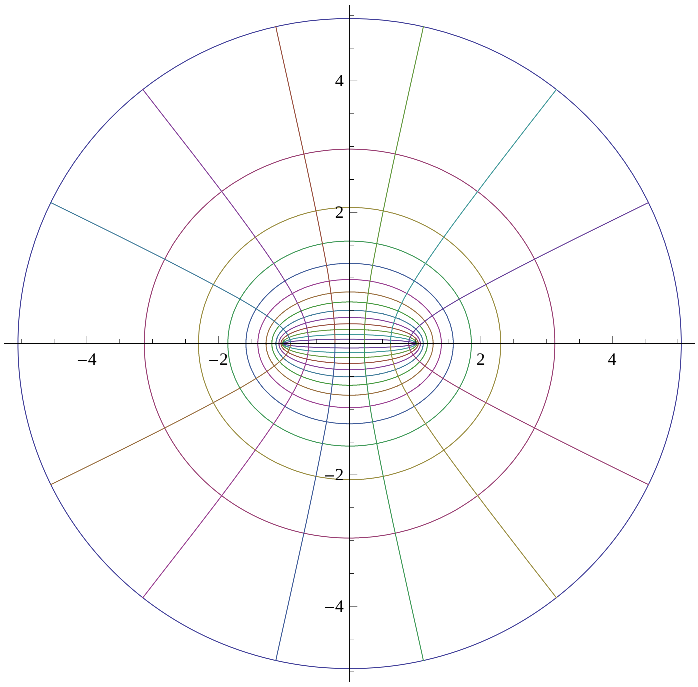
In[30]:= PolarMap$\left[\frac{1+\#}{1-\#}\right.$ &, {0, 0.99}, {−π, π}]

Out[30]=



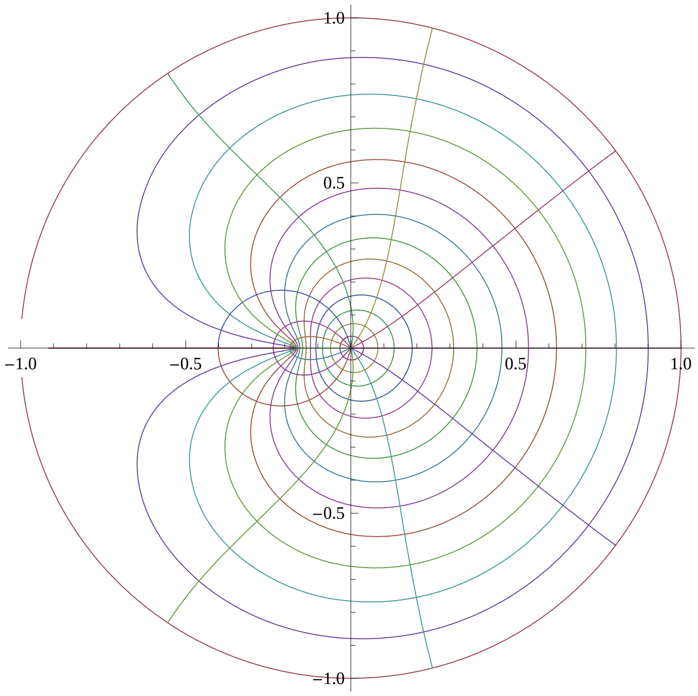

In[31]:= PolarMap$\left[\frac{1}{2}\left(\# + \frac{1}{\#}\right)\right.$ &, {0.1, 1}, {0, 2 π}]

Out[31]=



In[32]:= PolarMap$\left[\frac{2\,\#}{\left(1-\#+\sqrt{1+\#^2}\right)^2}\right.$ &, {0, 1}, {0, 2 π}, PlotRange → All$\left.\right]$

Out[32]=

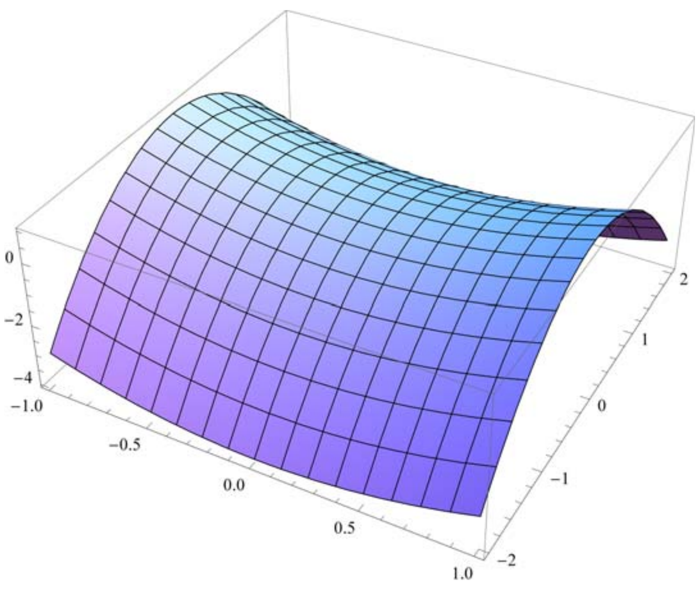



- **Realteil-, Imaginärteil- und Betragsgraphiken**

- **Die Quadratfunktion**

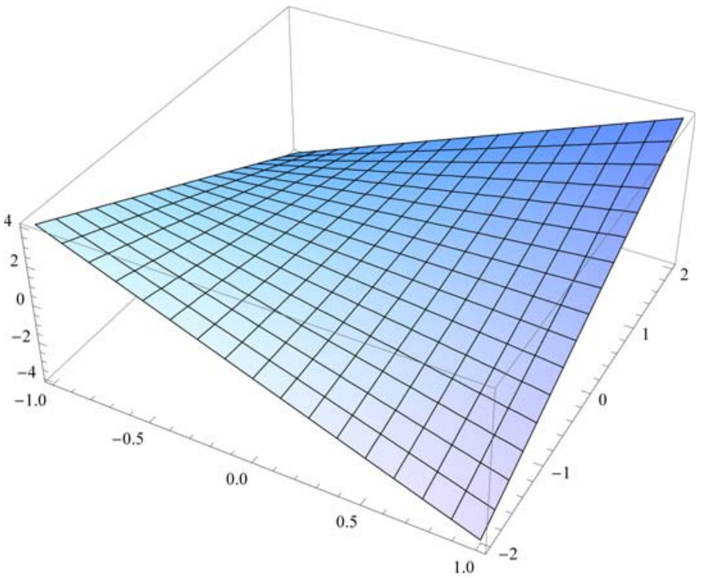
In[33]:= Plot3D[Evaluate[Re[$z^2$ /. z → x + ⅈ y]], {x, -1, 1}, {y, -2, 2}]

Out[33]=



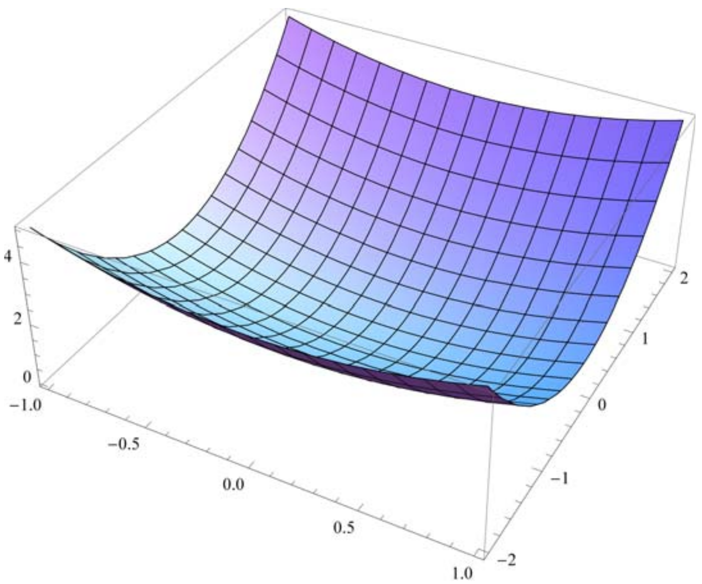
In[34]:= `Plot3D[Evaluate[Im[z^2 /. z → x + ⅈ y]], {x, -1, 1}, {y, -2, 2}]`

Out[34]=



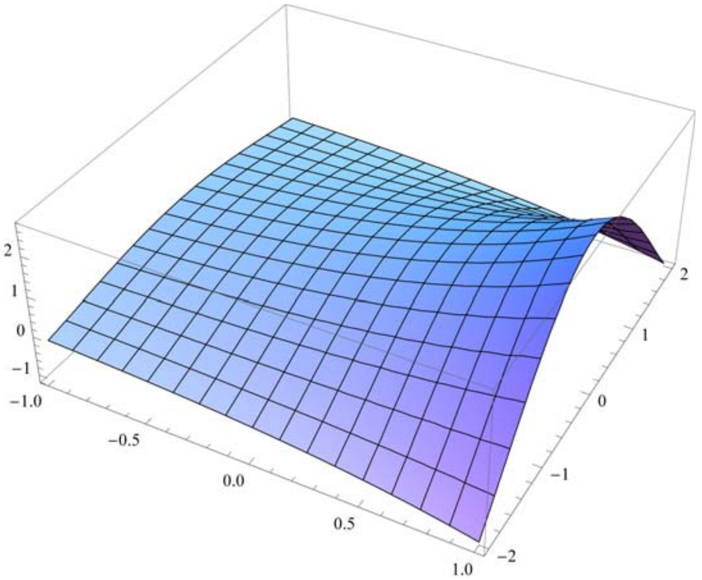

In[35]:= `Plot3D[Evaluate[Abs[z^2 /. z → x + ⅈ y]], {x, -1, 1}, {y, -2, 2}]`

Out[35]=

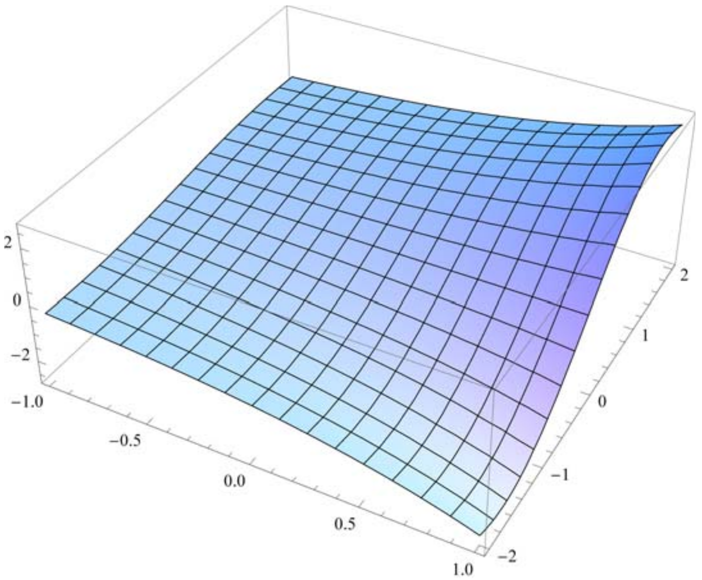


## ■ Die Exponentialfunktion

In[36]:= `Plot3D[Evaluate[Re[e^z /. z → x + i y]], {x, -1, 1}, {y, -2, 2}]`

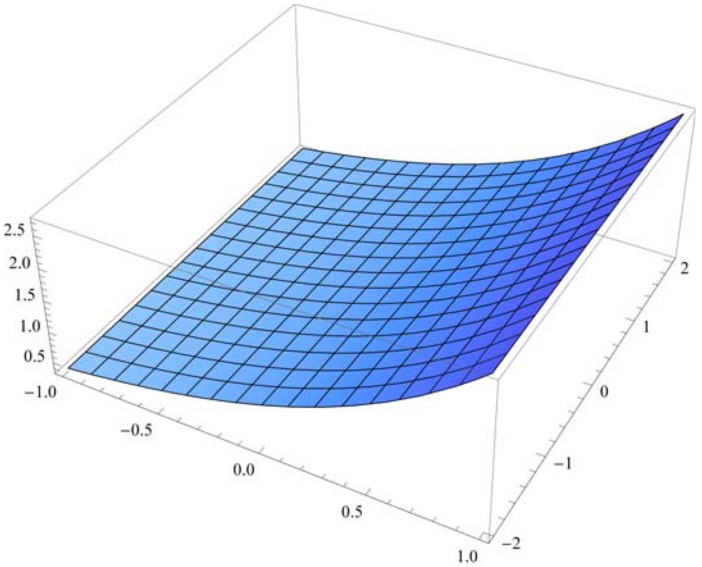
Out[36]=



In[37]:= `Plot3D[Evaluate[Im[e^z /. z → x + i y]], {x, -1, 1}, {y, -2, 2}]`

Out[37]=


In[38]:= `Plot3D[Evaluate[Abs[ⅇ^z /. z → x + ⅈ y]], {x, -1, 1}, {y, -2, 2}]`

Out[38]=




## ▪ Differenzierbarkeit komplexer Funktionen

In[39]:= `D[z^n, z]`

Out[39]= $n\, z^{n-1}$

In[40]:= `D[ⅇ^z, z]`

Out[40]= $e^{z}$

In[41]:= `D[Conjugate[z], z]`

Out[41]= Conjugate′($z$)

In[42]:= `D[Abs[z]^2, z]`

Out[42]= $2\,|z|$ Abs′($z$)

In[43]:= `D[z * Conjugate[z], z]`

Out[43]= $z$ Conjugate′($z$) + $z$

In[44]:= `D[Arg[z], z]`

Out[44]= arg′($z$)