

## Funktionentheorie

$$\text{In}[1]:= \mathbf{u} = \frac{1 - \mathbf{x}^2 - \mathbf{y}^2}{1 + \mathbf{x}^2 + \mathbf{y}^2 - 2 \mathbf{x}}$$

$$\text{Out}[1]= \frac{-x^2 - y^2 + 1}{x^2 - 2x + y^2 + 1}$$

$$\text{In}[2]:= \mathbf{D}[\mathbf{u}, \mathbf{x}] \text{ // Together}$$

$$\text{Out}[2]= \frac{2(x^2 - 2x - y^2 + 1)}{(x^2 - 2x + y^2 + 1)^2}$$

$$\text{In}[3]:= \mathbf{D}[\mathbf{u}, \{\mathbf{x}, 2\}] \text{ // Together}$$

$$\text{Out}[3]= -\frac{4(x^3 - 3x^2 - 3xy^2 + 3x + 3y^2 - 1)}{(x^2 - 2x + y^2 + 1)^3}$$

$$\text{In}[4]:= \mathbf{D}[\mathbf{u}, \{\mathbf{x}, 2\}] + \mathbf{D}[\mathbf{u}, \{\mathbf{y}, 2\}] \text{ // Together}$$

$$\text{Out}[4]= 0$$

$$\text{In}[5]:= \mathbf{u} = \mathbf{Exp}[\mathbf{x}^2 - \mathbf{y}^2] * \mathbf{Cos}[2 \mathbf{x} \mathbf{y}]$$

$$\text{Out}[5]= e^{x^2-y^2} \cos(2xy)$$

$$\text{In}[6]:= \mathbf{D}[\mathbf{u}, \{\mathbf{x}, 2\}] \text{ // Together}$$

$$\text{Out}[6]= 2e^{x^2-y^2} (2x^2 \cos(2xy) - 2y^2 \cos(2xy) - 4xy \sin(2xy) + \cos(2xy))$$

$$\text{In}[7]:= \mathbf{D}[\mathbf{u}, \{\mathbf{x}, 2\}] + \mathbf{D}[\mathbf{u}, \{\mathbf{y}, 2\}] \text{ // Together}$$

$$\text{Out}[7]= 0$$

### ■ Umkehrfunktion der Joukowski-Funktion

```
In[8]:= Needs["Graphics`ComplexMap`"]
```

— *General::obspkg :*

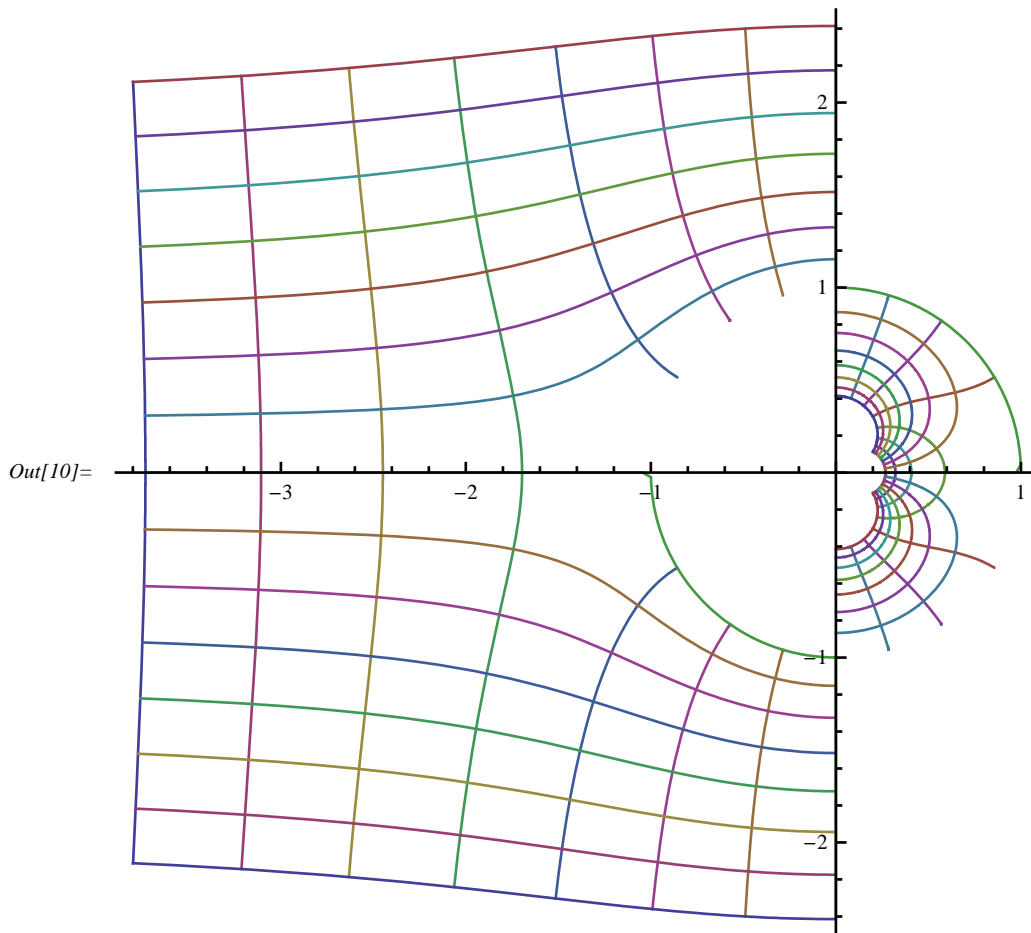
*Graphics`ComplexMap` is now obsolete. The legacy version being loaded may conflict with current Mathematica functionality.*

*See the Compatibility Guide for updating information. >>*

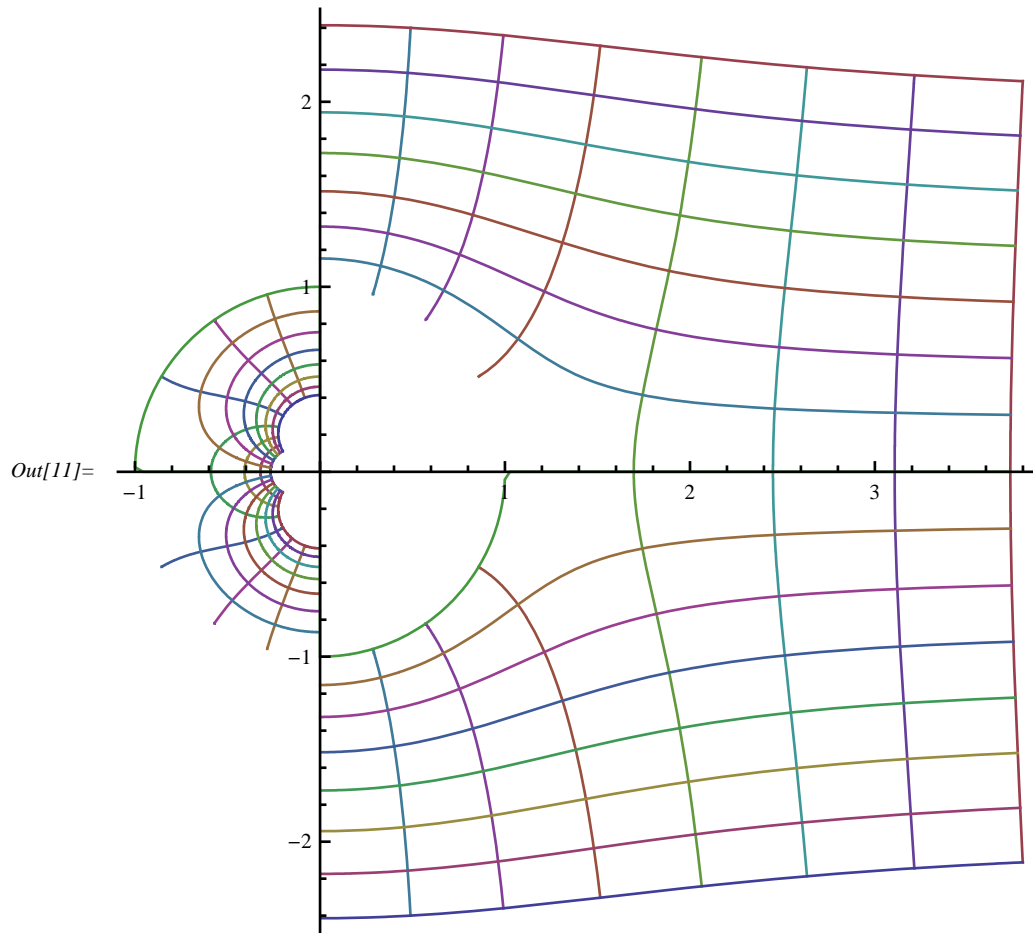
```
In[9]:= sol = Solve[w ==  $\frac{1}{2} \left( z + \frac{1}{z} \right)$ , z]
```

```
Out[9]= {{z ->  $w - \sqrt{w^2 - 1}$ }, {z ->  $\sqrt{w^2 - 1} + w$ }}
```

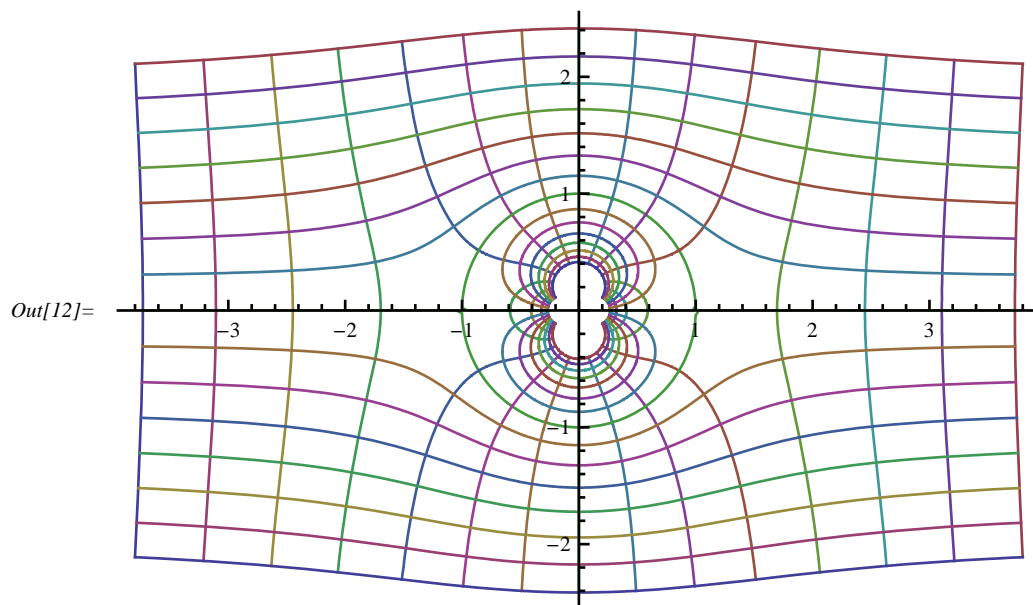
```
In[10]:= plot1 = CartesianMap[z /. sol[[1]] /. w -> # &, {-2, 2}, {-1, 1}]
```



```
In[11]:= plot2 = CartesianMap[z /. sol[[2]] /. w -> # &, {-2, 2}, {-1, 1}]
```



```
In[12]:= Show[plot1, plot2]
```



**■ Kurvenintegrale**

$$\int_K (z - z_0)^n dz$$

$$\text{In[13]:= int} = \int_0^{2\pi} ((z[t] - z_0)^n z'[t] /. \{z \rightarrow (z_0 + r e^{i\# \&})\}) dt$$

$$\text{Out[13]= } 0$$

$$\text{In[14]:= int} = \int_0^{2\pi} ((z[t] - z_0)^{-1} z'[t] /. \{z \rightarrow (z_0 + r e^{i\# \&})\}) dt$$

$$\text{Out[14]= } 2i\pi$$