Funktionentheorie: Kurvenintegrale

- $\int_Q z^2 \, dz$
- Γ_1

```
∫_0^1 (z[t]^2 z'[t] /. {z → (# &)}) ⅆt
```

$\frac{1}{3}$

- Γ_2

```
∫_0^1 (z[t]^2 z'[t] /. {z → (1 + # ⅈ &)}) ⅆt
```

$-1 + \frac{2 i}{3}$

- **Summe**

```
int1 = ∫_0^1 (z[t]^2 z'[t] /. {z → (# &)}) ⅆt +
  ∫_0^1 (z[t]^2 z'[t] /. {z → (1 + # ⅈ &)}) ⅆt
```

$-\frac{2}{3} + \frac{2 i}{3}$

- Γ_3

```
∫_0^1 (z[t]^2 z'[t] /. {z → (# ⅈ &)}) ⅆt
```

$-\frac{i}{3}$

- Γ_4

```
∫_0^1 (z[t]^2 z'[t] /. {z → (# + ⅈ &)}) ⅆt
```

$-\frac{2}{3} + i$

- **Summe**

```
int2 = ∫_0^1 (z[t]^2 z'[t] /. {z → (# ⅈ &)}) ⅆt +
  ∫_0^1 (z[t]^2 z'[t] /. {z → (# + ⅈ &)}) ⅆt
```

$-\frac{2}{3}+\frac{2i}{3}$

```
int1 - int2
```

0

- $\int_Q$ **Conjugate[z]^2 dz**

```
int1 = ∫_0^1 (Conjugate[z[t]]^2 z'[t] /. {z → (# &)}) ⅆt +
  ∫_0^1 (Conjugate[z[t]]^2 z'[t] /. {z → (1 + # ⅈ &)}) ⅆt
```

$\frac{4}{3}+\frac{2i}{3}$

```
int1 = (int1 /. {Conjugate[t] → t})
```

$\frac{4}{3}+\frac{2i}{3}$

```
int2 = ∫_0^1 (Conjugate[z[t]]^2 z'[t] /. {z → (# ⅈ &)}) ⅆt +
  ∫_0^1 (Conjugate[z[t]]^2 z'[t] /. {z → (# + ⅈ &)}) ⅆt
```

$-\frac{2}{3}-\frac{4i}{3}$

```
int2 = int2 /. {Conjugate[t] → t}
```

$-\frac{2}{3}-\frac{4i}{3}$

```
int1 - int2
```

$2+2i$

■ Beispiel 3.8

```
int = ∫_0^{2π} (f[z[t]] / ((z[t] - 1) (z[t] + 1) (z[t] + 2 ⅈ)) z'[t] /.
        {z → (r ⅇ^(ⅈ #) &)}) ⅆt
```

$$\int_0^{2\pi} \frac{i\, r\, e^{i t} f\left(r\, e^{i t}\right)}{\left(-1 + r\, e^{i t}\right)\left(r\, e^{i t} + 2\, i\right)\left(1 + r\, e^{i t}\right)} \, dt$$

```
1/((z - 1) (z + 1) (z + 2 ⅈ)) // Apart
```

$$-\frac{1}{5\,(z + 2\, i)} + \frac{\frac{1}{10} + \frac{i}{5}}{z + 1} + \frac{\frac{1}{10} - \frac{i}{5}}{z - 1}$$

■ Beispiel 3.9

```
int = ∫_0^{2π} (z + r ⅇ^(ⅈ t))^3 ⅆt
```

$$2\,\pi\, z^3$$

- **Kompliziertere Beispiele:**

```
int = ∫_0^{2π} (z + r ⅇ^(ⅈ t))^n ⅆt
```

$$0$$

- ***Mathematica* macht bei komplizierten Integralen viele derartige Fehler! Die meisten rühren von der Mehrdeutigkeit komplexer Funktionen.**

```
int = ∫_0^{2π} (z + r ⅇ^(ⅈ t))^100 ⅆt
```

$$2\,\pi\, z^{100}$$