

Speaker: Mathias Schulze

Title: Algorithms for Gauss-Manin Systems

Abstract: We consider a local and a global situation:

1. In the local situation, $f \in \mathbb{C}\{x_1, \dots, x_n\}$ is a convergent power series with an isolated critical point at the origin and $f : X \rightarrow T$ with $X \subset \mathbb{C}^n$ and $T \subset \mathbb{C}$ is a good representative.
2. In the global situation, $f \in \mathbb{C}[x_1, \dots, x_n]$ is tame polynomial and $f : X \rightarrow T$ with $X = \mathbf{A}_{\mathbb{C}}^n$ and $T = \mathbf{A}_{\mathbb{C}}^1$ is the corresponding polynomial map. A tame polynomial has only isolated critical points and there is no contribution from infinity to the cohomology of the fibres.

The Gauss-Manin system of f is the left \mathcal{D}_T -module $G = \mathcal{H}^0(f_+ \mathcal{O}_X)$ obtained from the right \mathcal{D}_X -module Ω_X^n by integrating along the fibres of f .

The Brieskorn lattice $H'' = f_* \Omega_X^n / df \wedge d(f_* \Omega_X^{n-2})$ was first defined by Brieskorn in the local situation. Later, it turned out that $H'' \subset G$ defines the Hodge filtration of a limit mixed Hodge structure on the cohomology $H^n(f^{-1}(t), \mathbb{C})$ for $t \rightarrow 0$ in the local and $t \rightarrow \infty$ or $\theta = \partial_t^{-1} \rightarrow \infty$ in the global situation. This is due to Varchenko, Schmid, and Scherk-Steenbink in the local, and to Steenbrink-Zucker and Sabbah in the global situation.

This mixed Hodge structure leads to the existence of good bases of H'' such that the action of f is of the form $A_0 + \theta A_1 + \theta^2 \partial_\theta$. This was shown by M. Saito in the local and C. Sabbah in the global situation. The spectral pairs and Hodge numbers can be read off from the constant matrices A_0 and A_1 immediately.

We present an explicit algorithmic construction of good bases in the local situation using (partial) standard bases for modules over power series rings. These algorithms are implemented in the computer algebra system SINGULAR. As work in progress, we describe the analogous construction in the global situation.