Partial Differential Equations
Higher Analysis II, summer term 2012
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Exercise sheet 3

Solve the following Cauchy problems and verify the solution:

Exercise 3.1: $uu_x + u_y = 2,\ \ u(y,y) = 1 + y.$

Exercise 3.2: $xu_t + tu_x = u,\ \ u(0,x) = h(x)$ for arbitrary, differentiable $h : \mathbb{R} \to \mathbb{R}$.
Hint: In order to solve the characteristic equations, introduce a suitable transformation.

Exercise 3.3: $xu_x + yu_y + u_t = u,\ \ u(0,x,y) = h(x,y)$ for arbitrary, differentiable $h : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$.

Exercise 3.4: $u_t + xu_x = -x^3,\ \ u(0,x) = f(x)$ for arbitrary, differentiable $f : \mathbb{R} \to \mathbb{R}$.

Exercise 3.5 (written):

a) $u_t + u_x = -\frac{1}{3}u^7,\ \ u(0,x) = \mathrm{e}^x(x^2 + \cos x)$.

b) $(1+x)u_x - (1+y)u_y = (y-x),\ \ u(x,x) = x^2$.
Hints: Set up the characteristic equations

$$\frac{\mathrm{d}x(s,\tilde{s})}{\mathrm{d}s} = ...,\quad \frac{\mathrm{d}y(s,\tilde{s})}{\mathrm{d}s} = ...,\quad \frac{\mathrm{d}z(s,\tilde{s})}{\mathrm{d}s} = ...$$

with suitable Cauchy data. Use variation of constants to solve the first two differential equations. To solve the third one, exploit the first two. Moreover, to construct $u(x,y)$, it is not necessary to write $s = s(x,y)$, $\tilde{s} = \tilde{s}(x,y)$ explicitely. Exploit the form of $z(s,\tilde{s})$, instead.

c) Check the assumptions of the local existence theorem for the Cauchy problems in a) and b).

Ex. 3.5 is to be delivered in written form by teams of two persons each in the exercise lesson on 30/04/2012. It will be discussed in the subsequent week.

Exam dates: July 24+25, 2012, September 27+28, 2012.