Explizite Differentialgleichungen erster Ordnung

Differentialgleichung, welche durch direkte Integration gelöst werden kann

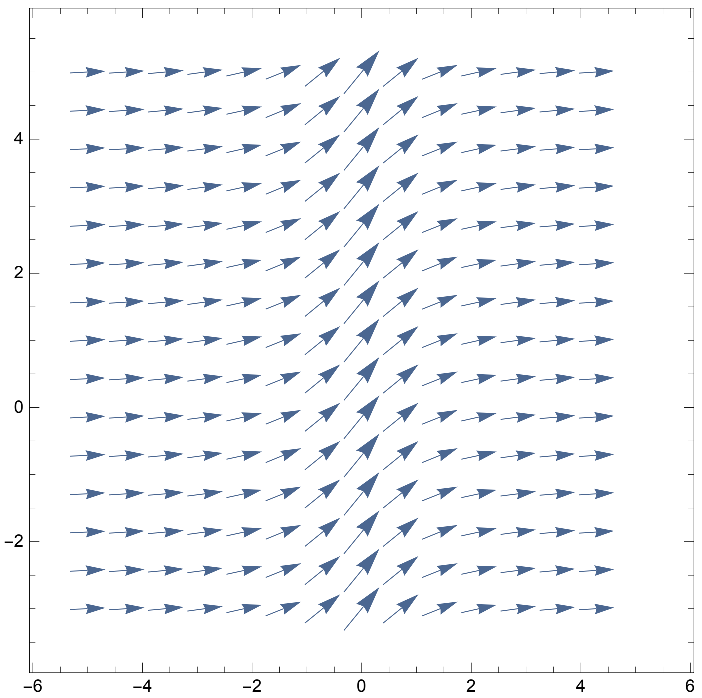
In[1]:=
```
DE = y'[x] == 1/(1 + x^2)
```
Out[1]= $y'(x) = \frac{1}{x^2 + 1}$

Wir zeichnen das Richtungsfeld

In[2]:=
```
DirectionField[DE_, y_[x_], {x_, a_, b_}, {y_, c_, d_}, options___] := Module[{g},
  g = DE[[2]] /. y[x] → y;
  VectorPlot[{1, g}, {x, a, b}, {y, c, d}, options]
 ]
```
Modul

In[3]:=
```
plot1 = DirectionField[DE, y[x], {x, -5, 5}, {y, -3, 5}, Frame → True]
```
Rahmen wahr

Out[3]=



Wir lösen die Differentialgleichung bzw. das zugehörige Anfangswertproblem

In[4]:= $\int \frac{1}{1 + x^2} \, dx$

Out[4]= $\tan^{-1}(x)$

In[5]:=
```
DSolve[DE, y[x], x]
```
löse Differentialgleichung

Out[5]= $\{\{y(x) \to c_1 + \tan^{-1}(x)\}\}$

In[6]:=
$$1 + \int_0^x \frac{1}{1+t^2}\,dt$$

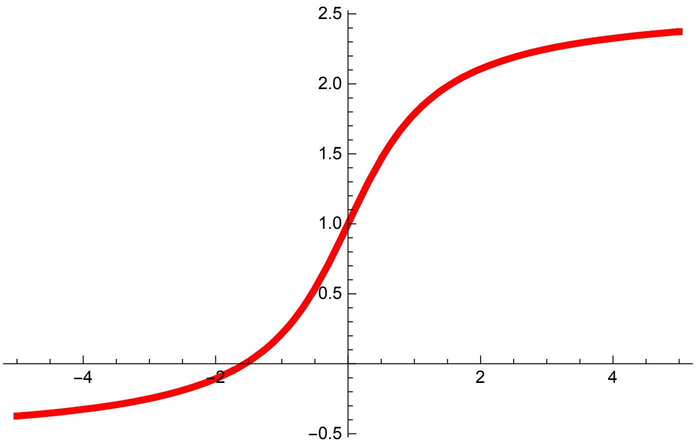
Out[6]= ConditionalExpression$[\tan^{-1}(x) + 1,\ \mathrm{Re}(x) \neq 0 \lor -1 < \mathrm{Im}(x) < 0 \lor 0 < \mathrm{Im}(x) < 1]$

In[7]:=
```
lösung = DSolve[{DE, y[0] == 1}, y[x], x]
```
löse Differentialgleichung

Out[7]= $\{\{y(x) \to \tan^{-1}(x) + 1\}\}$

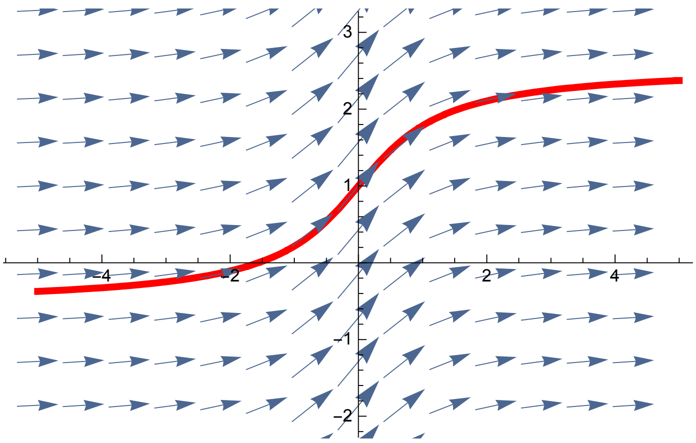
In[8]:=
```
plot2 = Plot[y[x] /. lösung, {x, -5, 5},
  PlotStyle → {Thickness[0.01], RGBColor[1, 0, 0]}]
```
stelle Funktion graphisch dar; Darstellungsstil; Dicke; RGB Farbe

Out[8]=



In[9]:=
```
Show[plot2, plot1, PlotRange → {-2, 3}]
```
zeige an; Koordinatenbereich der Graph

Out[9]=



In[10]:=
```
sol = NDSolve[{DE, y[0] == 0}, y[x], {x, -1 000 000, 1 000 000}]
```
löse Differentialgleichung numerisch

Out[10]= $\{\{y(x) \to$ InterpolatingFunction[Domain: $(-1.\times10^6\ \ 1.\times10^6)$ Output: scalar]$(x)\}\}$

In[11]:=
```
N[y[x] /. sol[[1]] /. x → 1 000 000]
```
numerischer Wert

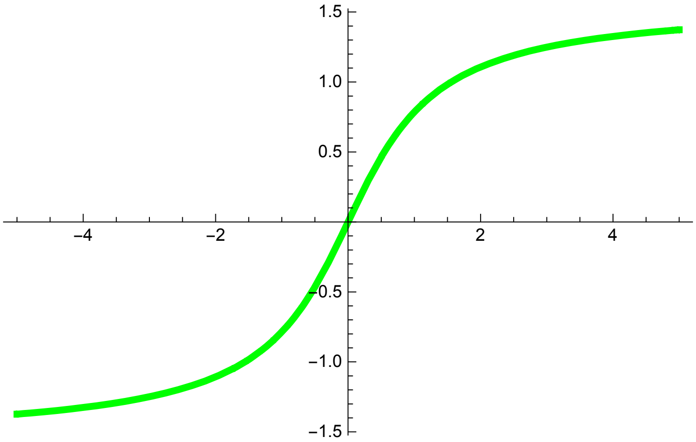
Out[11]= 1.5708

In[12]:=
```
N[π/2]
```
numerischer Wert

Out[12]= 1.5708

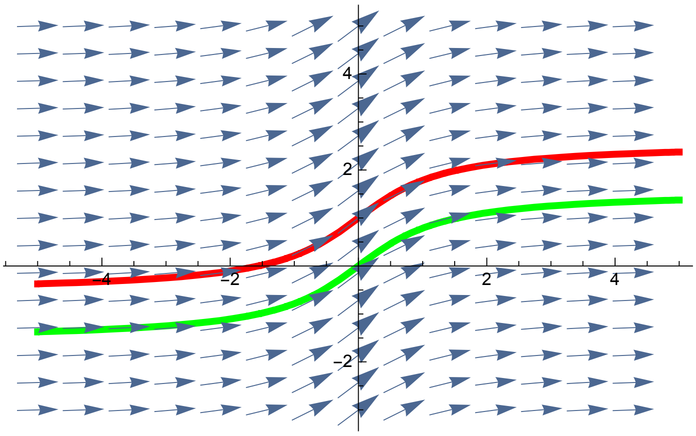
In[13]:=
```
plot3 = Plot[y[x] /. sol, {x, -5, 5},
  PlotStyle → {Thickness[0.01], RGBColor[0, 1, 0]}]
```
stelle Funktion graphisch dar | Darstellungsstil | Dicke | RGB Farbe

Out[13]=



In[14]:=
```
Show[plot3, plot2, plot1, PlotRange → {-3, 5}]
```
zeige an | Koordinatenbereich der Graph

Out[14]=



Die Differentialgleichung des unbegrenzten Wachstums

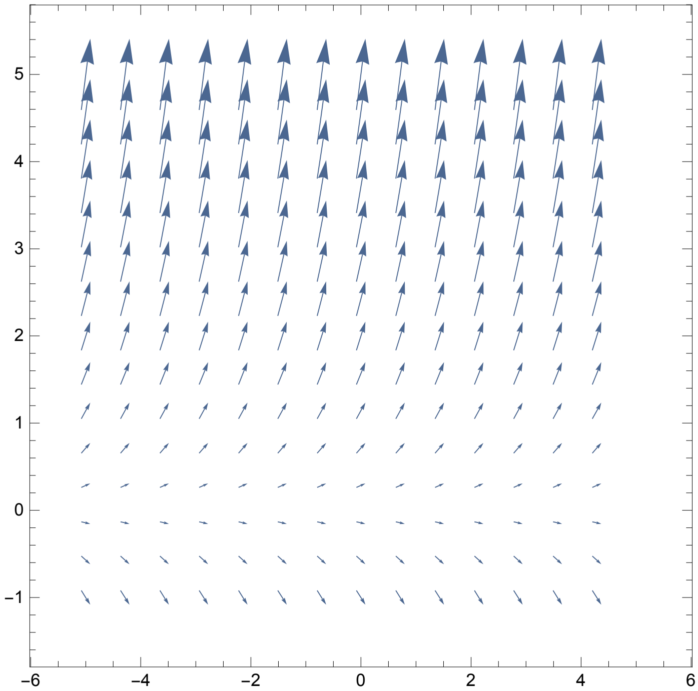
In[15]:=
```
DE = y'[x] == y[x]
```

Out[15]= $y'(x) = y(x)$

Wir zeichnen das Richtungsfeld

In[16]:=
```
plot1 = DirectionField[DE, y[x ], {x, -5, 5}, {y, -1, 5}, Frame → True]
```
Rahmen wahr

Out[16]=



Wir lösen die Differentialgleichung bzw. das zugehörige Anfangswertproblem

In[17]:=
```
DSolve[DE, y [x], x]
```
löse Differentialgleichung

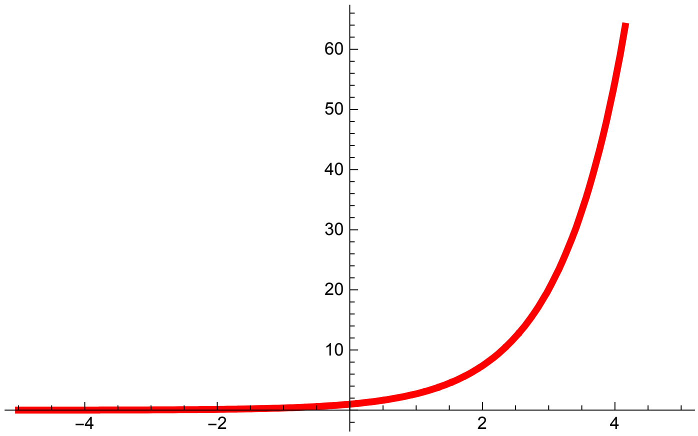
Out[17]= $\{\{y(x) \to c_1 e^x\}\}$

In[18]:=
```
lösung = DSolve[{DE, y[0] == 1}, y [x], x]
```
löse Differentialgleichung

Out[18]= $\{\{y(x) \to e^x\}\}$

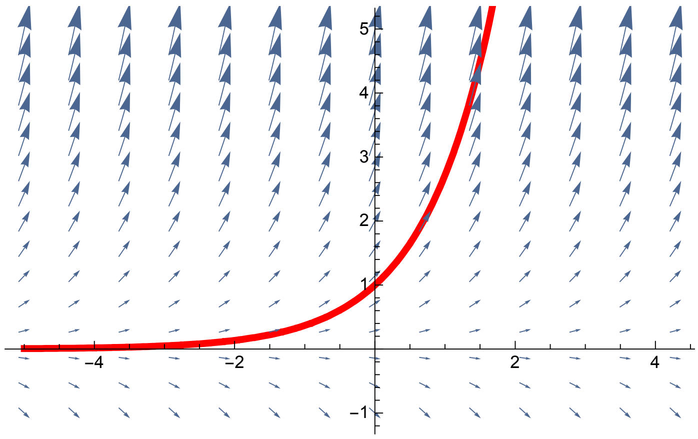
In[19]:=
```
plot2 = Plot[y[x] /. lösung, {x, -5, 5},
  PlotStyle → {Thickness[0.01], RGBColor[1, 0, 0]}]
```
stelle Funktion graphisch dar | Darstellungsstil | Dicke | RGB Farbe

Out[19]=



In[20]:=
```
Show[plot2, plot1, PlotRange → {-1, 5}]
```
zeige an | Koordinatenbereich der Graph

Out[20]=



allgemeineres Problem

In[21]:=
```
lösung = DSolve[{y'[x ] == α y[x], y[0] == P}, y [x], x]
```
löse Differentialgleichung

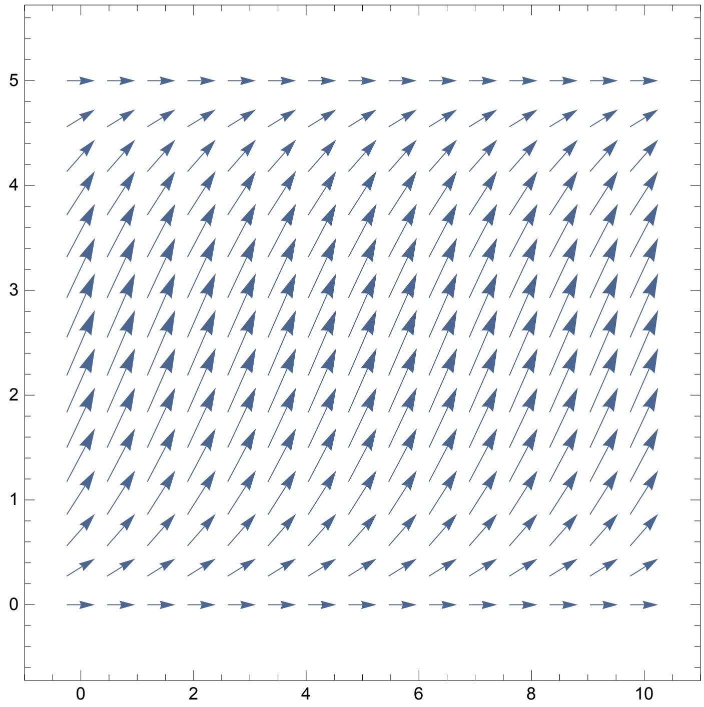
Out[21]= $\{\{y(x) \to P\, e^{\alpha x}\}\}$

Die Differentialgleichung des logistischen Wachstums

In[22]:=
```
DE = y'[x] == α y[x] - β y[x]^2
```

Out[22]= $y'(x) = \alpha\, y(x) - \beta\, y(x)^2$

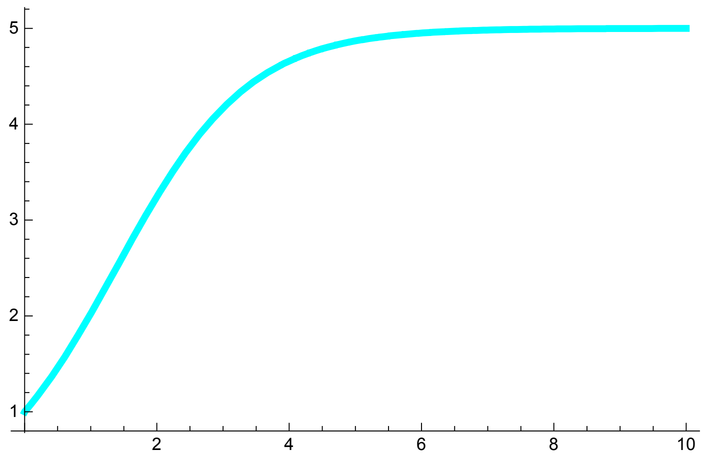
In[23]:=
```
plot1 =
  DirectionField[DE /. {α → 1, β → 1/5}, y[x], {x, 0, 10}, {y, 0, 5}, Frame → True]
```
Rahmen wahr

Out[23]=


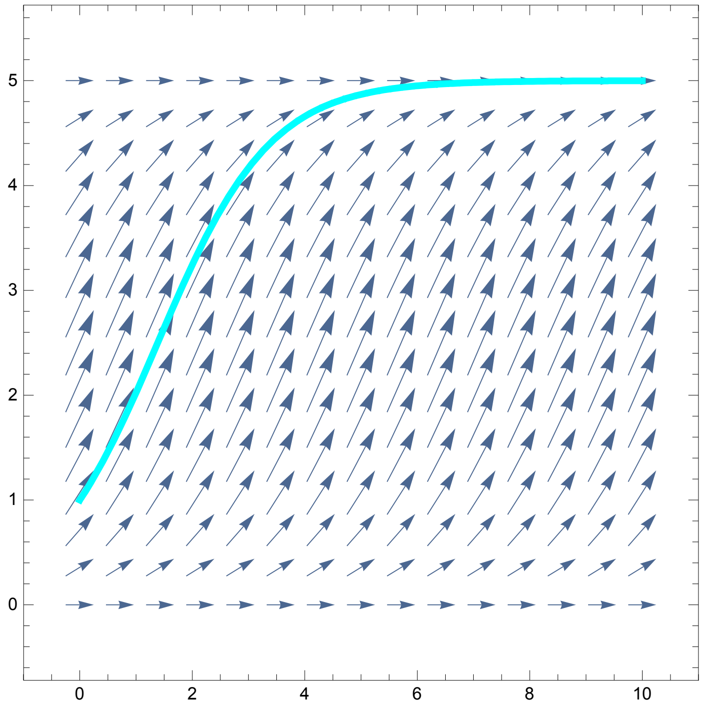
In[24]:=
```
plot2 =
  Plot[Evaluate[y[x] /. DSolve[{DE, y[0] == 1}, y[x], x][[1]] /. {α → 1, β → 1/5}],
   {x, 0, 10}, PlotStyle → {Thickness[0.01], RGBColor[0, 1, 1]}]
```
stell··· werte aus löse Differentialgleichung

Darstellungsstil Dicke RGB Farbe

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

Out[24]=


In[25]:=
```
Show[plot1, plot2]
```
zeige an

Out[25]=


In[26]:=
```
y[x] /. DSolve[{DE, y[0] == P}, y[x], x][[1]]
```
löse Differentialgleichung

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. >>

Out[26]= $\frac{\alpha P e^{\alpha x}}{\alpha - \beta P + \beta P e^{\alpha x}}$

Schrittweise Lösung

In[27]:=
```
DE
```

Out[27]= $y'(x) = \alpha y(x) - \beta y(x)^2$

In[28]:= Gleichung = $\int \frac{1}{\alpha y - \beta y^2} dy == \int 1 \, dx$

Out[28]= $\frac{\log(y)}{\alpha} - \frac{\log(\alpha - \beta y)}{\alpha} = x$

In[29]:=
```
Solve[Gleichung, y]
```
löse

Out[29]= $\left\{\left\{y \to \frac{\alpha e^{\alpha x}}{\beta e^{\alpha x} + 1}\right\}\right\}$

In[30]:= Apart$\left[\frac{1}{\alpha y - \beta y^2}, y\right]$
Partialbruchzerlegung

Out[30]= $\frac{1}{\alpha y} - \frac{\beta}{\alpha (\beta y - \alpha)}$

Zugehöriges Anfangswertproblem

In[31]:= **Gleichung** $= \int_P^y \frac{1}{\alpha\, s - \beta\, s^2}\, ds == \int_0^x 1\, dt$

Out[31]= ConditionalExpression$\Big[\frac{1}{\alpha}(-\log(P) + \log(y) + \log(\alpha - P\beta) - \log(\alpha - y\beta)) = x,$

$\Big(\Big(\mathrm{Re}(y) < 0 \bigwedge \Big(\Big(\mathrm{Im}(y) < 0 \bigwedge \Big(\mathrm{Im}(P) < \mathrm{Im}(y) \bigvee \Big(\mathrm{Im}(P) = \mathrm{Im}(y) \bigwedge \Big(\mathrm{Re}(P) < \big(\mathrm{Im}(y)^2 - \mathrm{Im}(P)\,\mathrm{Im}(y) + \mathrm{Re}(y)^2\big)/$

$\mathrm{Re}(y) \bigvee \mathrm{Re}(P) > \big(\mathrm{Im}(y)^2 - \mathrm{Im}(P)\,\mathrm{Im}(y) + \mathrm{Re}(y)^2\big)/\mathrm{Re}(y)\Big)\Big) \bigvee \mathrm{Im}(y) < \mathrm{Im}(P) <$

$0 \bigvee (P \in \mathbb{R} \wedge (\mathrm{Re}(P) < 0 \vee \mathrm{Re}(P) > 0)) \bigvee \Big(\mathrm{Im}(P) > 0 \bigwedge \mathrm{Re}(P) > \frac{\mathrm{Im}(P)\,\mathrm{Re}(y)}{\mathrm{Im}(y)}\Big)\Big)\Big) \bigvee$

$\Big(y \in \mathbb{R} \bigwedge \Big(\mathrm{Im}(P) < 0 \bigvee \Big(P \in \mathbb{R} \bigwedge \Big(\mathrm{Re}(P) < \mathrm{Re}(y) \bigvee \mathrm{Re}(y) < \mathrm{Re}(P) < \frac{\mathrm{Re}(y)}{2} + \frac{1}{2}\sqrt{\mathrm{Re}(y)^2}\Big)\Big) \bigvee$

$\mathrm{Im}(P) > 0\Big)\Big) \bigvee$

$\Big(\mathrm{Im}(y) > 0 \bigwedge \Big(\Big(\mathrm{Im}(P) < 0 \bigwedge \mathrm{Re}(P) > \frac{\mathrm{Im}(P)\,\mathrm{Re}(y)}{\mathrm{Im}(y)}\Big) \bigvee (P \in \mathbb{R} \wedge (\mathrm{Re}(P) < 0 \vee \mathrm{Re}(P) > 0)) \bigvee$

$0 < \mathrm{Im}(P) < \mathrm{Im}(y) \bigvee \Big(\mathrm{Im}(P) = \mathrm{Im}(y) \bigwedge \Big(\mathrm{Re}(P) < \big(\mathrm{Im}(y)^2 - \mathrm{Im}(P)\,\mathrm{Im}(y) + \mathrm{Re}(y)^2\big)/\mathrm{Re}(y) \bigvee$

$\mathrm{Re}(P) > \big(\mathrm{Im}(y)^2 - \mathrm{Im}(P)\,\mathrm{Im}(y) + \mathrm{Re}(y)^2\big)/\mathrm{Re}(y)\Big)\Big) \bigvee \mathrm{Im}(P) > \mathrm{Im}(y)\Big)\Big)\Big)\Big) \bigvee$

$(\mathrm{Re}(y) = 0 \wedge ((\mathrm{Im}(y) < 0 \wedge (\mathrm{Im}(P) < \mathrm{Im}(y) \vee (\mathrm{Im}(P) = \mathrm{Im}(y) \wedge (\mathrm{Re}(P) < 0 \vee \mathrm{Re}(P) > 0)) \vee$

$\mathrm{Im}(y) < \mathrm{Im}(P) < 0 \vee (P \in \mathbb{R} \wedge (\mathrm{Re}(P) < 0 \vee \mathrm{Re}(P) > 0)) \vee (\mathrm{Im}(P) > 0 \wedge \mathrm{Re}(P) > 0))) \vee$

$(\mathrm{Im}(y) > 0 \wedge ((\mathrm{Im}(P) < 0 \wedge \mathrm{Re}(P) > 0) \vee (P \in \mathbb{R} \wedge (\mathrm{Re}(P) < 0 \vee \mathrm{Re}(P) > 0)) \vee 0 < \mathrm{Im}(P) < \mathrm{Im}(y) \vee$

$(\mathrm{Im}(P) = \mathrm{Im}(y) \wedge (\mathrm{Re}(P) < 0 \vee \mathrm{Re}(P) > 0)) \vee \mathrm{Im}(P) > \mathrm{Im}(y))))) \bigvee \Big(\mathrm{Re}(y) > 0 \bigwedge$

$\Big(\Big(\mathrm{Im}(y) < 0 \bigwedge \Big(\mathrm{Im}(P) < \mathrm{Im}(y) \bigvee \Big(\mathrm{Im}(P) = \mathrm{Im}(y) \bigwedge \Big(\mathrm{Re}(P) < \big(\mathrm{Im}(y)^2 - \mathrm{Im}(P)\,\mathrm{Im}(y) + \mathrm{Re}(y)^2\big)/\mathrm{Re}(y) \bigvee$

$\mathrm{Re}(P) > \big(\mathrm{Im}(y)^2 - \mathrm{Im}(P)\,\mathrm{Im}(y) + \mathrm{Re}(y)^2\big)/\mathrm{Re}(y)\Big)\Big) \bigvee \mathrm{Im}(y) < \mathrm{Im}(P) < 0 \bigvee$

$(P \in \mathbb{R} \wedge (\mathrm{Re}(P) < 0 \vee \mathrm{Re}(P) > 0)) \bigvee \Big(\mathrm{Im}(P) > 0 \bigwedge \mathrm{Re}(P) > \frac{\mathrm{Im}(P)\,\mathrm{Re}(y)}{\mathrm{Im}(y)}\Big)\Big)\Big) \bigvee$

$\Big(y \in \mathbb{R} \bigwedge \Big(\mathrm{Im}(P) < 0 \bigvee \Big(P \in \mathbb{R} \bigwedge \Big(\frac{\mathrm{Re}(y)}{2} - \frac{1}{2}\sqrt{\mathrm{Re}(y)^2} < \mathrm{Re}(P) < \mathrm{Re}(y) \bigvee \mathrm{Re}(P) > \mathrm{Re}(y)\Big)\Big) \bigvee$

$\mathrm{Im}(P) > 0\Big)\Big) \bigvee$

$\Big(\mathrm{Im}(y) > 0 \bigwedge \Big(\Big(\mathrm{Im}(P) < 0 \bigwedge \mathrm{Re}(P) > \frac{\mathrm{Im}(P)\,\mathrm{Re}(y)}{\mathrm{Im}(y)}\Big) \bigvee (P \in \mathbb{R} \wedge (\mathrm{Re}(P) < 0 \vee \mathrm{Re}(P) > 0)) \bigvee$

$0 < \mathrm{Im}(P) < \mathrm{Im}(y) \bigvee \Big(\mathrm{Im}(P) = \mathrm{Im}(y) \bigwedge \Big(\mathrm{Re}(P) < \big(\mathrm{Im}(y)^2 - \mathrm{Im}(P)\,\mathrm{Im}(y) + \mathrm{Re}(y)^2\big)/\mathrm{Re}(y) \bigvee$

$\mathrm{Re}(P) > \big(\mathrm{Im}(y)^2 - \mathrm{Im}(P)\,\mathrm{Im}(y) + \mathrm{Re}(y)^2\big)/\mathrm{Re}(y)\Big)\Big) \bigvee \mathrm{Im}(P) > \mathrm{Im}(y)\Big)\Big)\Big)\Big)\Big) \bigwedge$

$\Big(\Big(\frac{P}{P-y} \neq 0 \bigwedge \mathrm{Re}\Big(\frac{P}{y-P}\Big) \geq 0\Big) \bigvee \mathrm{Im}\Big(\frac{P}{y-P}\Big) \neq 0 \bigvee \mathrm{Re}\Big(\frac{P}{P-y}\Big) >$

$1\Big) \bigwedge$

$\Big(\Big(\mathrm{Im}(P) \geq 0 \bigwedge \mathrm{Re}\Big(\frac{P}{P-y}\Big) \geq 1 \bigwedge \mathrm{Im}(P) \leq \mathrm{Im}(y)\Big) \bigvee$

$$\left(\text{Im}(P) \geq 0 \bigwedge \text{Re}\left(\frac{P}{y-P}\right) \geq 0 \bigwedge \text{Im}(P) \leq \text{Im}(y)\right) \bigvee$$

$$\left(\text{Im}(P) \geq 0 \bigwedge \text{Im}(P) \leq \text{Im}(y) \bigwedge \text{Im}\left(\frac{P}{y-P}\right) \neq 0\right) \bigvee$$

$$\left(\text{Im}(P) \geq \text{Im}(y) \bigwedge \text{Im}(y) \geq 0 \bigwedge \text{Re}\left(\frac{P}{P-y}\right) \geq 1\right) \bigvee$$

$$\left(\text{Im}(P) \geq \text{Im}(y) \bigwedge \text{Im}(y) \geq 0 \bigwedge \text{Re}\left(\frac{P}{y-P}\right) \geq 0\right) \bigvee$$

$$\left(\text{Im}(P) \geq \text{Im}(y) \bigwedge \text{Im}(y) \geq 0 \bigwedge \text{Im}\left(\frac{P}{y-P}\right) \neq 0\right) \bigvee$$

$$\left(\text{Im}(P) \geq \text{Im}(y) \bigwedge \text{Re}\left(\frac{P}{P-y}\right) \geq 1 \bigwedge \text{Im}(P) \leq 0\right) \bigvee$$

$$\left(\text{Im}(P) \geq \text{Im}(y) \bigwedge \text{Re}\left(\frac{P}{P-y}\right) \geq 1 \bigwedge \text{Im}(y)\,\text{Re}(P) \leq \text{Im}(P)\,\text{Re}(y)\right) \bigvee$$

$$\left(\text{Im}(P) \geq \text{Im}(y) \bigwedge \text{Re}\left(\frac{P}{y-P}\right) \geq 0 \bigwedge \text{Im}(P) \leq 0\right) \bigvee$$

$$\left(\text{Im}(P) \geq \text{Im}(y) \bigwedge \text{Re}\left(\frac{P}{y-P}\right) \geq 0 \bigwedge \text{Im}(y)\,\text{Re}(P) \leq \text{Im}(P)\,\text{Re}(y)\right) \bigvee$$

$$\left(\text{Im}(P) \geq \text{Im}(y) \bigwedge \text{Im}(P) \leq 0 \bigwedge \text{Im}\left(\frac{P}{y-P}\right) \neq 0\right) \bigvee$$

$$\left(\text{Im}(P) \geq \text{Im}(y) \bigwedge \text{Im}(y)\,\text{Re}(P) \leq \text{Im}(P)\,\text{Re}(y) \bigwedge \text{Im}\left(\frac{P}{y-P}\right) \neq 0\right) \bigvee$$

$$\left(\text{Im}(y)\,\text{Re}(P) \geq \text{Im}(P)\,\text{Re}(y) \bigwedge \text{Re}\left(\frac{P}{P-y}\right) \geq 1 \bigwedge \text{Im}(P) \leq \text{Im}(y)\right) \bigvee$$

$$\left(\text{Im}(y)\,\text{Re}(P) \geq \text{Im}(P)\,\text{Re}(y) \bigwedge \text{Re}\left(\frac{P}{y-P}\right) \geq 0 \bigwedge \text{Im}(P) \leq \text{Im}(y)\right) \bigvee$$

$$\left(\text{Im}(y)\,\text{Re}(P) \geq \text{Im}(P)\,\text{Re}(y) \bigwedge \text{Im}(P) \leq \text{Im}(y) \bigwedge \text{Im}\left(\frac{P}{y-P}\right) \neq 0\right) \bigvee$$

$$\left(\text{Re}\left(\frac{P}{P-y}\right) \geq 1 \bigwedge \text{Im}(P) \leq \text{Im}(y) \bigwedge \text{Im}(y) \leq 0\right) \bigvee$$

$$\left(\text{Re}\left(\frac{P}{y-P}\right) \geq 0 \bigwedge \text{Im}(P) \leq \text{Im}(y) \bigwedge \text{Im}(y) \leq 0\right) \bigvee$$

$$\left(\text{Im}(P) \leq \text{Im}(y) \bigwedge \text{Im}(y) \leq 0 \bigwedge \text{Im}\left(\frac{P}{y-P}\right) \neq 0\right)\Big)\Big]$$

In[32]:=
```
Gleichung = Integrate[1/(α s - β s^2), {s, P, y}, GenerateConditions → False] == ∫_0^x 1 ⅆt
```
integriere · generiere Bedingungen · falsch

Out[32]= $\frac{1}{\alpha}(\log(\alpha - \beta P) - \log(P) - \log(\alpha - \beta y) + \log(y)) = x$

In[33]:=
```
Solve[Gleichung, y]
```
löse

Out[33]= $\left\{\left\{y \to \frac{\alpha P e^{\alpha x}}{\alpha - \beta P + \beta P e^{\alpha x}}\right\}\right\}$

Wo ist der Wendepunkt? Wir leiten die Differentialgleichung ab und erhalten

In[34]:=
```
D[DE, x]
```
leite ab

Out[34]= $y''(x) = \alpha y'(x) - 2 \beta y(x) y'(x)$

In[35]:=
```
zweiteableitung = D[DE, x] /. {Apply[Rule, DE]}
```
leite ab · wende·· · Regel

Out[35]= $y''(x) = \alpha \left(\alpha y(x) - \beta y(x)^2\right) - 2 \beta y(x) \left(\alpha y(x) - \beta y(x)^2\right)$

In[36]:=
```
Map[Factor, zweiteableitung]
```
w··· · faktorisiere

Out[36]= $y''(x) = y(x) (\alpha - 2 \beta y(x)) (\alpha - \beta y(x))$

In[37]:=
```
sol = Solve[zweiteableitung[[2]] == 0, y[x]]
```
löse

Out[37]= $\left\{\{y(x) \to 0\}, \left\{y(x) \to \frac{\alpha}{2 \beta}\right\}, \left\{y(x) \to \frac{\alpha}{\beta}\right\}\right\}$

Beispiel 1.4

In[38]:=
```
DE = y'[x] == 1/(1 + y[x]^2)
```

Out[38]= $y'(x) = \frac{1}{y(x)^2 + 1}$

In[39]:= `DSolve[DE, y[x], x]`
löse Differentialgleichung

Out[39]= $$\left\{\left\{y(x)\to\left(\left(\sqrt{\left((81\,c_1+81\,x)^2+2916\right)}+81\,c_1+81\,x\right)\hat{}\,(1/3)\right)\Big/\left(3\sqrt[3]{2}\right)-\left(3\sqrt[3]{2}\right)\Big/\left(\left(\sqrt{\left((81\,c_1+81\,x)^2+2916\right)}+81\,c_1+81\,x\right)\hat{}\,(1/3)\right)\right\},\right.$$
$$\left\{y(x)\to\left(3\left(1+i\sqrt{3}\right)\right)\Big/\left(2^{2/3}\left(\sqrt{\left((81\,c_1+81\,x)^2+2916\right)}+81\,c_1+81\,x\right)\hat{}\,(1/3)\right)-\left(\left(1-i\sqrt{3}\right)\left(\sqrt{\left((81\,c_1+81\,x)^2+2916\right)}+81\,c_1+81\,x\right)\hat{}\,(1/3)\right)\Big/\left(6\sqrt[3]{2}\right)\right\},$$
$$\left.\left\{y(x)\to\left(3\left(1-i\sqrt{3}\right)\right)\Big/\left(2^{2/3}\left(\sqrt{\left((81\,c_1+81\,x)^2+2916\right)}+81\,c_1+81\,x\right)\hat{}\,(1/3)\right)-\left(\left(1+i\sqrt{3}\right)\left(\sqrt{\left((81\,c_1+81\,x)^2+2916\right)}+81\,c_1+81\,x\right)\hat{}\,(1/3)\right)\Big/\left(6\sqrt[3]{2}\right)\right\}\right\}$$

Typen expliziter Differentialgleichungen erster Ordnung

rechte Seite hängt nur von x ab:

In[40]:= `DSolve[y'[x] == g[x], y[x], x]`
löse Differentialgleichung

Out[40]= $$\left\{\left\{y(x)\to\int_1^x g(K[1])\,dK[1]+c_1\right\}\right\}$$

In[41]:= `DSolve[{y'[x] == g[x], y[x0] == y0}, y[x], x]`
löse Differentialgleichung

Out[41]= $$\left\{\left\{y(x)\to\int_1^x g(K[1])\,dK[1]-\int_1^{x0} g(K[1])\,dK[1]+y0\right\}\right\}$$

rechte Seite hängt nur von y ab:

In[42]:= `DSolve[y'[x] == h[y[x]], y[x], x]`
löse Differentialgleichung

Out[42]= $$\left\{\left\{y(x)\to\text{InverseFunction}\left[\int_1^{\#1}\frac{1}{h(K[1])}\,dK[1]\,\&\right][c_1+x]\right\}\right\}$$

In[43]:= `DSolve[{y'[x] == h[y[x]], y[x0] == y0}, y[x], x]`
löse Differentialgleichung

Out[43]= $$\left\{\left\{y(x)\to\text{InverseFunction}\left[\int_1^{\#1}\frac{1}{h(K[1])}\,dK[1]\,\&\right]\left[\int_1^{y0}\frac{1}{h(K[1])}\,dK[1]+x-x0\right]\right\}\right\}$$

Separable Differentialgleichung

In[44]:= `DSolve[y'[x] == g[x] h[y[x]], y[x], x]`
löse Differentialgleichung

Out[44]= $$\left\{\left\{y(x)\to\text{InverseFunction}\left[\int_1^{\#1}\frac{1}{h(K[1])}\,dK[1]\,\&\right]\left[\int_1^{x} g(K[2])\,dK[2]+c_1\right]\right\}\right\}$$