Weitere Beispiele zur Separation der Variablen

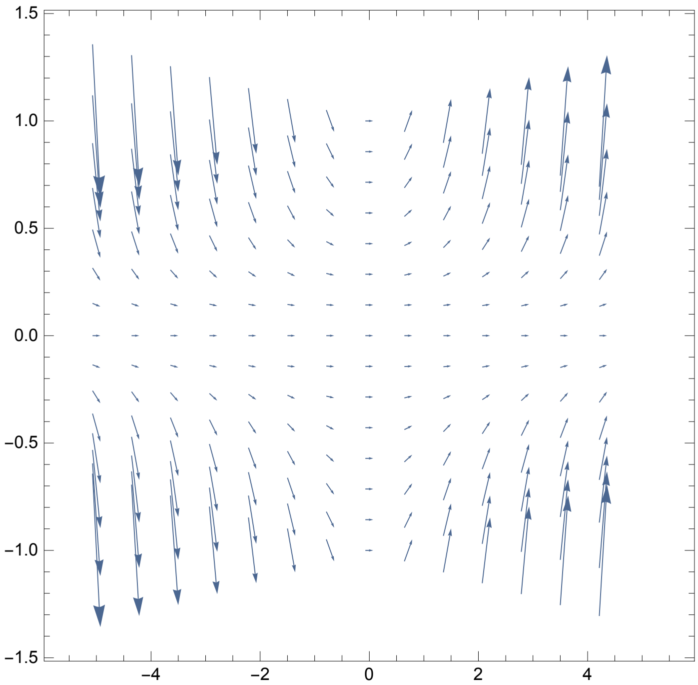
```
DirectionField[DE_, y_[x_], {x_, a_, b_}, {y_, c_, d_}, options___] := Module[{g},
  g = DE[[2 ]] /. y[x] → y;
  VectorPlot[{1, g}, {x, a, b}, {y, c, d}, options]
 ]
```
Modul

Beispiel 1.21

```
DE = y'[x] == x y[x]^2
```

$y'(x) = x\,y(x)^2$

```
plot1 = DirectionField[DE, y[x ], {x, -5, 5}, {y, -1, 1}, Frame → True]
```
Rahmen wahr



Einige Lösungen

```
DSolve[{DE, y[0] == 0}, y[x], x]
```
löse Differentialgleichung

DSolve::bvnul : For some branches of the general solution, the given boundary conditions lead to an empty solution. >>

{}

```
DSolve[{DE, y[x0] == y0}, y[x], x]
```
löse Differentialgleichung

$$\left\{\left\{y(x)\to -\frac{2\,\text{y0}}{x^2\,\text{y0}-\text{x0}^2\,\text{y0}-2}\right\}\right\}$$

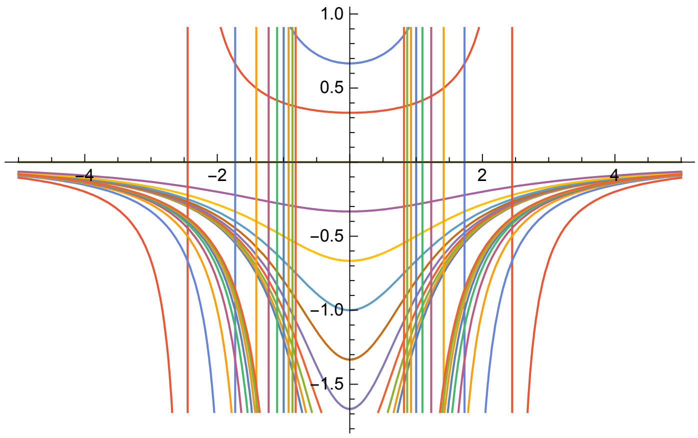
```
lösung = y[x] /. DSolve[{DE, y[0] == k}, y[x], x][[1]]
```
löse Differentialgleichung

$$-\frac{2\,k}{k\,x^2-2}$$

```
liste = Table[lösung, {k, -3, 3, 1/3}]
```
Tabelle

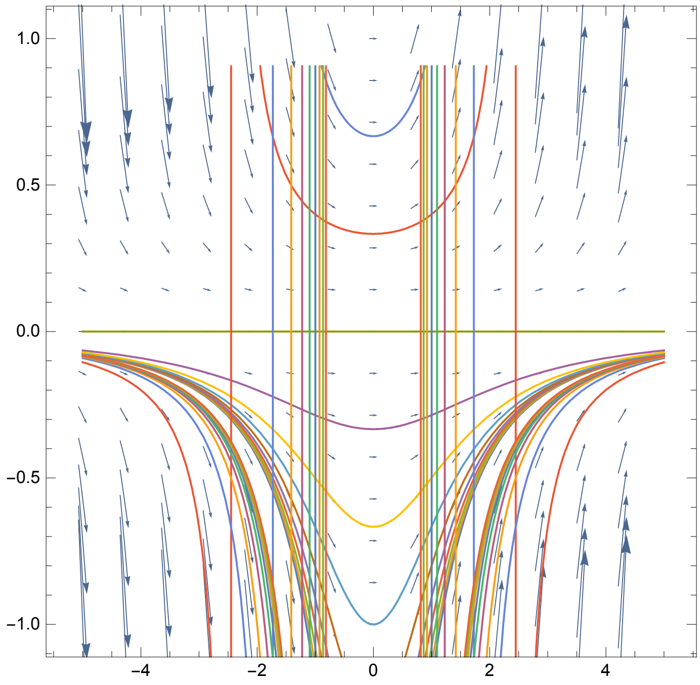
$$\left\{\frac{6}{-3\,x^2-2},\ \frac{16}{3\left(-\frac{8\,x^2}{3}-2\right)},\ \frac{14}{3\left(-\frac{7\,x^2}{3}-2\right)},\ \frac{4}{-2\,x^2-2},\ \frac{10}{3\left(-\frac{5\,x^2}{3}-2\right)},\ \frac{8}{3\left(-\frac{4\,x^2}{3}-2\right)},\ \frac{2}{-x^2-2},\ \frac{4}{3\left(-\frac{2\,x^2}{3}-2\right)},\ \frac{2}{3\left(-\frac{x^2}{3}-2\right)},\ 0,\ -\frac{2}{3\left(\frac{x^2}{3}-2\right)},\ -\frac{4}{3\left(\frac{2\,x^2}{3}-2\right)},\ -\frac{2}{x^2-2},\ -\frac{8}{3\left(\frac{4\,x^2}{3}-2\right)},\ -\frac{10}{3\left(\frac{5\,x^2}{3}-2\right)},\ -\frac{4}{2\,x^2-2},\ -\frac{14}{3\left(\frac{7\,x^2}{3}-2\right)},\ -\frac{16}{3\left(\frac{8\,x^2}{3}-2\right)},\ -\frac{6}{3\,x^2-2}\right\}$$

```
plot2 = Plot[liste // Evaluate, {x, -5, 5}, PlotStyle → Thickness[0.003]]
```
stelle Funktion gr··· | werte aus | Darstellungsstil | Dicke



```
Show[plot1, plot2, PlotRange → {-1, 1}]
```

zeige an Koordinatenbereich der Graph



Beispiel 1.22

```
DE = y'[x] == y[x]^2 - a
```

$$y'(x) = y(x)^2 - a$$

```
DSolve[DE, y[x], x]
```

löse Differentialgleichung

$$\left\{\left\{y(x) \to -\sqrt{a}\, \tanh\left(\sqrt{a}\, x - \sqrt{a}\, c_1\right)\right\}\right\}$$

```
DSolve[DE /. {a → -1}, y[x], x]
```

löse Differentialgleichung

$$\{\{y(x) \to \tan(c_1 + x)\}\}$$

```
DSolve[DE /. {a → 1}, y[x], x]
```

löse Differentialgleichung

$$\left\{\left\{y(x) \to \frac{1 - e^{2 c_1 + 2 x}}{e^{2 c_1 + 2 x} + 1}\right\}\right\}$$

```
DSolve[DE /. {a → 0}, y[x], x]
```

löse Differentialgleichung

$$\left\{\left\{y(x) \to \frac{1}{-c_1 - x}\right\}\right\}$$

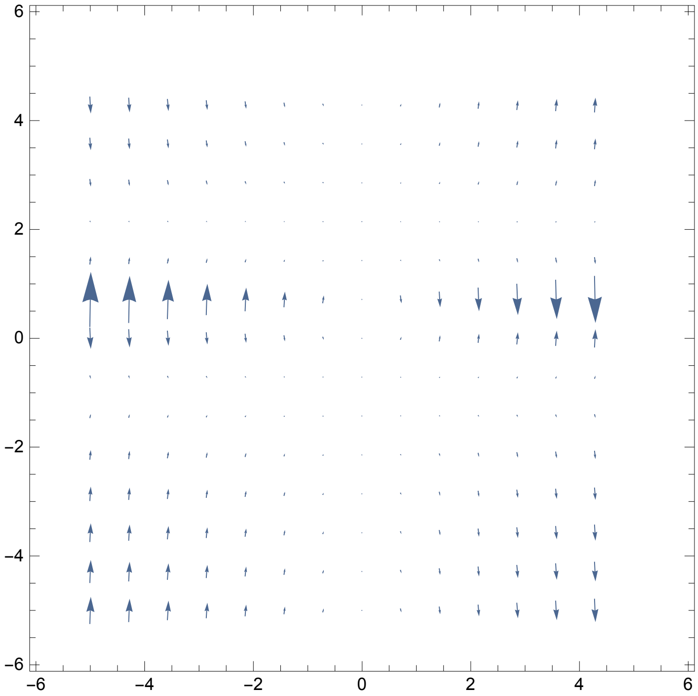
Beispiel 1.23

```
DE = y'[x] == x (2 y[x]^2 - 2 y[x] - 4)/(2 y[x] - 1)
```

$$y'(x) = \frac{x\left(2\,y(x)^2 - 2\,y(x) - 4\right)}{2\,y(x) - 1}$$

```
plot1 = DirectionField[DE, y[x ], {x, -5, 5}, {y, -5, 5}, Frame → True]
```

Rahmen wahr



Einige Lösungen

```
lösung = y[x] /. DSolve[{DE, y[0] == k}, y[x], x]
```

löse Differentialgleichung

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. ≫

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$$\left\{\frac{1}{2}\left(1 - \sqrt{9 - \left(-4\,k^2 + 4\,k + 8\right) e^{x^2}}\right), \frac{1}{2}\left(\sqrt{9 - \left(-4\,k^2 + 4\,k + 8\right) e^{x^2}} + 1\right)\right\}$$

```
y[x] /. DSolve[{DE}, y[x], x]
```

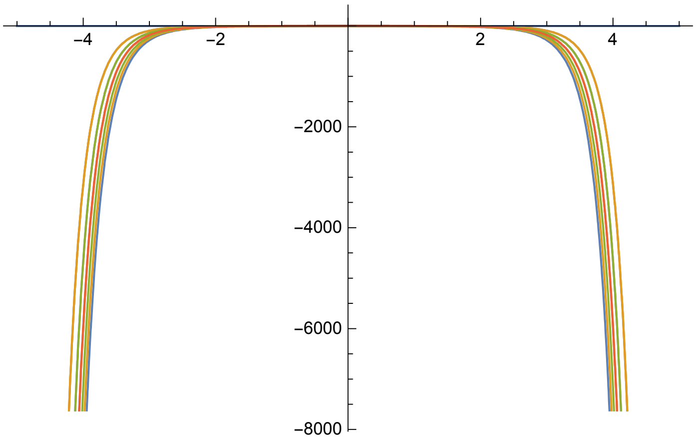
löse Differentialgleichung

$$\left\{\frac{1}{2}\left(1 - \sqrt{9 - 4\,e^{c_1 + x^2}}\right), \frac{1}{2}\left(\sqrt{9 - 4\,e^{c_1 + x^2}} + 1\right)\right\}$$

```
liste1 = Table[lösung[[1]], {k, -3, 3, 1/3}];
```

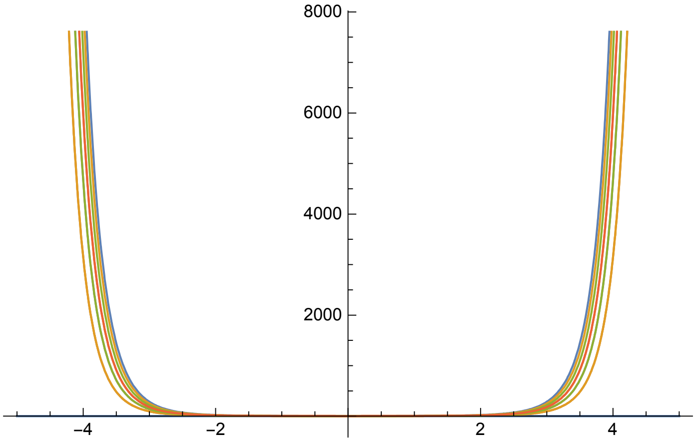
Tabelle

```
plot2 = Plot[liste1 // Evaluate, {x, -5, 5}, PlotStyle → Thickness[0.003]]
```
stelle Funktion grap·· werte aus Darstellungsstil Dicke

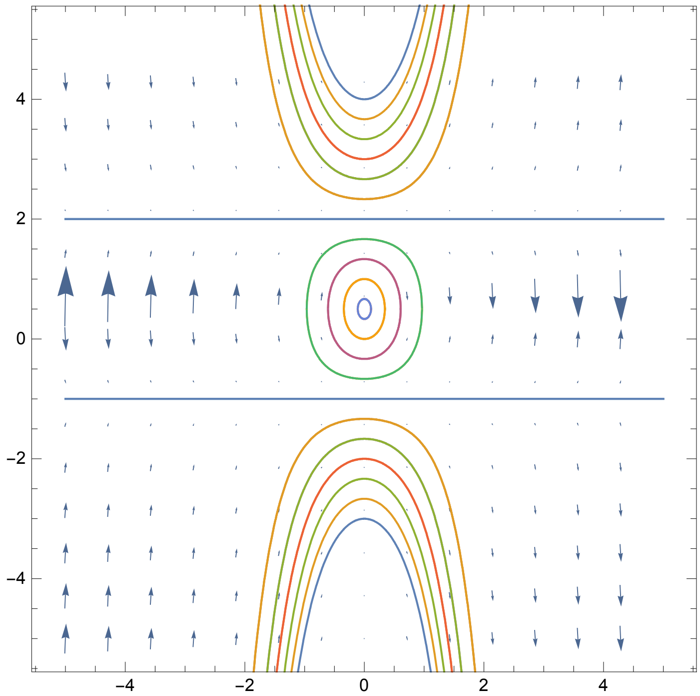



```
liste2 = Table[lösung[[2]], {k, -3, 3, 1/3}];
```
Tabelle

```
plot3 = Plot[liste2 // Evaluate, {x, -5, 5}, PlotStyle → Thickness[0.003]]
```
stelle Funktion grap·· werte aus Darstellungsstil Dicke



```
Show[plot1, plot2, plot3, PlotRange → {-5, 5}]
```
zeige an   Koordinatenbereich der Graph

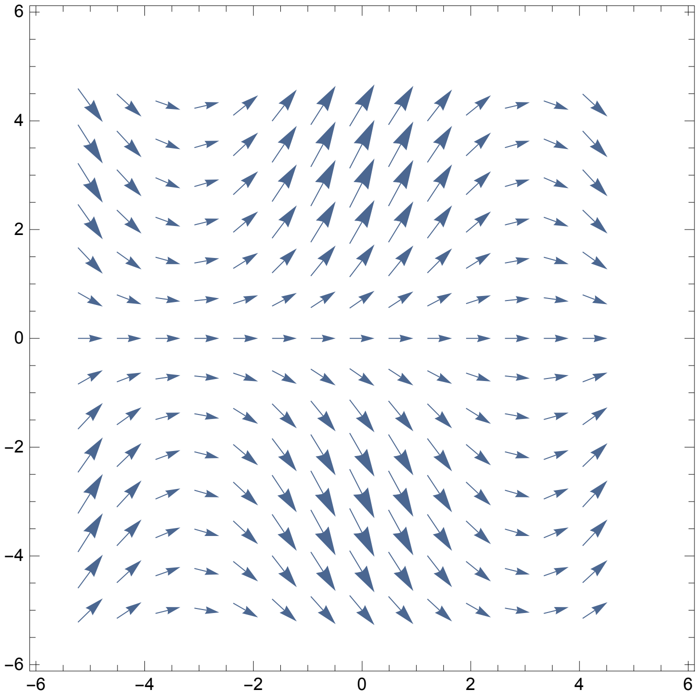


Beispiel 1.24

```
DE = y'[x] == Sin[(x + y[x])/2] - Sin[(x - y[x])/2]
```
Sinus   Sinus

$$y'(x) = \sin\left(\frac{1}{2}(y(x) + x)\right) - \sin\left(\frac{1}{2}(x - y(x))\right)$$

```
plot1 = DirectionField[DE, y[x ], {x, -5, 5}, {y, -5, 5}, Frame → True]
```
Rahmen wahr



Lösung

```
lösung = y[x] /. DSolve[{DE, y[0] == k}, y[x], x]
```
löse Differentialgleichung

Solve::ifun : Inverse functions are being used by Solve, so
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$$\left\{4 \cot^{-1}\left(\frac{e^{-2 \sin\left(\frac{x}{2}\right)}}{\sqrt{\tan^2\left(\frac{k}{4}\right)}}\right)\right\}$$

Mathematica kann auch den Integranden vereinfachen:

```
DE = Map[TrigExpand, DE]
```
wende an

$$y'(x) = 2 \cos\left(\frac{x}{2}\right) \sin\left(\frac{y(x)}{2}\right)$$

```
DSolve[DE, y[x], x]
```

löse Differentialgleichung

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. >>

$$\left\{\left\{y(x)\to 4\cot^{-1}\left(e^{-\frac{c_1}{2}-2\sin\left(\frac{x}{2}\right)}\right)\right\}\right\}$$

Wir lösen die Differentialgleichung schrittweise:

$$\texttt{gleichung} = \int \frac{1}{\texttt{Sin}\left[\frac{y}{2}\right]}\,dy == \int \texttt{Cos}\left[\frac{x}{2}\right]\,dx$$

Kosinus

$$2\log\left(\sin\left(\frac{y}{4}\right)\right)-2\log\left(\cos\left(\frac{y}{4}\right)\right)=2\sin\left(\frac{x}{2}\right)$$

```
Solve[gleichung, y]
```

löse

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. >>

$$\left\{\left\{y\to 4\cot^{-1}\left(e^{-\sin\left(\frac{x}{2}\right)}\right)\right\}\right\}$$