

# Adaptive ECG beat classification by ordinal pattern based entropies

Jean Bertin Bidias à Mougoufan<sup>1</sup>, J. S. Armand Eyebe Fouda<sup>b,c,\*</sup>, Maurice Tchuenta<sup>1</sup>, Wolfram Koepf<sup>c</sup>

<sup>a</sup>Department of Informatics and Computer Science, Faculty of Science, University of Yaoundé I, P.O. Box 812, Yaoundé, Cameroon

<sup>b</sup>Department of Physics, Faculty of Science, University of Yaoundé I, P.O. Box 812, Yaoundé, Cameroon

<sup>c</sup>Institute of Mathematics, University of Kassel, Heinrich-Plett Str. 40, 34132 Kassel, Germany

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## Abstract

In this paper, we investigate the applicability of the permutation entropy (PE) and the conditional entropy of ordinal patterns (CEOP) to Electrocardiogram (ECG) data analysis. We define a signal dependent threshold based on the PE and the CEOP for the detection of abnormal ECG beats. Parameters of the proposed threshold formula are calibrated using the MIT-BIH Arrhythmia and the European Society of Cardiology ST-T (ESC) databases. The experimental results show that the difference between CEOP and PE is marginal, and that the algorithm is less sensitive to the parameter setting. We achieved a classification rate of 93.62% in the case of the MIT-BIH database, and 99.57% in the case of the ESC database. Although these algorithms still need to be improved, the above results for the ESC database confirm that ordinal pattern based entropies are promising for ECG beat classification.

*Keywords:* ECG classification, Ordinal patterns, Permutation entropy, Conditional entropy

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## 1. Introduction

In the literature several methods for detecting and classifying abnormalities in ECG data are mentioned. Among these methods, one can quote time domain based approaches that present shortcomings, due to random changes in amplitude and the heart rate variability (HRV) [1]. Techniques such as Fourier Transform and the Power Spectral Density also have been shown unsuitable for the analysis of the temporal variation of ECG data, due to their non-stationarity. Considering its spatiotemporal resolution properties, the Discrete Wavelet Transform is preferred for the analysis of ECG signals [2]. Recent methods such as higher order statistical features and independent component analysis were also introduced for ECG signals analysis [3]. These methods listed are linear, while ECG data are assumed to be nonlinear. The analysis of nonlinear data by linear methods as those above described has been shown inefficient while dealing with nonlinear data [4]. This situation justifies the use of approaches such as Correlation Dimension, Lyapunov Exponents and entropy-like methods for ECG analysis [5–7]. Some of these methods are robust against noise and present important features that are resistant to important artifacts. They can also be associated with classification techniques including support vector machines and others.

Recently new nonlinear time series analysis algorithms based on an ordinal pattern approach were introduced [8]. Ordinal pattern based algorithms are used for measuring complexity from time series. They identify the inherent patterns or dynamics changes in a time series, and take into account the order

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\*Corresponding author

Email addresses: bbidias@yahoo.fr, Tel: +237690573020 (Jean Bertin Bidias à Mougoufan),  
efoudajsa@yahoo.fr, Tel: +237691896001 (J. S. Armand Eyebe Fouda), maurice.tchuenta@gmail.com, Tel:  
+237696890165 (Maurice Tchuenta), koepf@mathematik.uni-kassel.de, Tel: +49 561 804 4207 (Wolfram Koepf)  
URL: <http://www.mathematik.uni-kassel.de/~fouda/> (J. S. Armand Eyebe Fouda),  
<http://www.mathematik.uni-kassel.de/~koepf/> (Wolfram Koepf)

relation between samples of the time series instead of the values themselves [9]. The complexity of a dynamical system thus depends on the variety of ordinal patterns. An example is given with the permutation entropy (PE) which evaluates the diversity of ordinal patterns in a time series, for a finite pattern length. Indeed, there are various entropy measures based on ordinal patterns, ranged from the PE that does not well approximate the Kolmogorov-Sinai entropy (KSE) [10], to the recent ordinal matrix based complexity (OMC) measure which has been shown to be a good approximate of the KSE [11]. The first element of the OMC series is known as the conditional entropy of ordinal patterns (CEOP) that has been introduced by Keller *et al* [12].

Ordinal pattern based algorithms are known to be fast and easy to implement. According to these two particular properties, we investigate in this paper their applicability to ECG data. We consider two different approaches, namely the PE and CEOP, and compare their performances. Our purpose is to discriminate between normal and abnormal ECG beats. As the objective is to evaluate the discriminative property of the above algorithms for real-world ECG data, we used annotated databases for their validation. For this purpose, we propose a signal-dependent threshold that is supposed to adapt the decision made to the signal and the acquisition system properties. The threshold formula is calibrated using the MIT-BIH arrhythmia database which is known to be complex, as it contains various types of abnormalities. The rest of the paper is organized as follows: Sect. 2 presents an overview of ordinal pattern based algorithms, Sect. 3 is devoted to the proposed classification approach, Sect. 4 presents and discusses the results obtained with real-world ECG data, while Sect. 5 gives some concluding remarks.

## 2. Overview of the ordinal pattern based entropy algorithms

### 2.1. Permutation entropy (PE)

The permutation entropy of a series of data is the Shannon entropy corresponding to the set of ordinal symbols  $S$ . Indeed, for a given time series  $\{x_t\}_{t=0,1,\dots,T-1}$  of length  $T$ , the permutation entropy of order  $n$  is expressed by

$$H(n) = -\sum p(\theta) \cdot \ln(p(\theta)), \quad (1)$$

where

$$p(\theta) = \frac{\#\{k | k \leq T - n\tau, P_k = \theta\}}{T - n\tau + 1}, \quad (2)$$

is the probability of the permutation  $\theta$  and  $\#$  denotes the cardinality [13]. In that case, the ordinal symbol  $S$  is equivalent to the permutation  $\theta$ . More details about the PE can be found in [10].

### 2.2. Conditional entropy of ordinal patterns (CEOP)

The need for reducing the gap between the PE and the KSE led to the definition of the CEOP. It characterizes the diversity of successors of a given ordinal pattern, whereas PE characterizes the diversity of ordinal patterns themselves [12]. Eyebe *et al.* in [11] defined the ordinal array complexity as a generalized approximation of the ordinal KSE by considering permutation arrays as patterns. They showed that the possible number of ordinal arrays increases as a power of  $n!$ . Thus, for ordinal arrays containing  $L$  permutations of order  $n$ , the maximum number of ordinal arrays that can be obtained from the data series is  $\Lambda_0 = (n!)^L$  and the CEOP is obtained by setting  $L = 2$ . Applying the definition of the Shannon entropy  $H$  in Eq. (1) to the set  $P_k$  of permutations and the set of ordinal matrices  $S_l$ , the CEOP can be represented as

$$h(n) = H(s) - H(n). \quad (3)$$

where  $n$  is the permutation length and  $s = n \times 2$  the size of the ordinal matrices [11].

### 3. Classification approach

The method we are proposing combines two main steps, namely the segmentation and the classification steps. The segmentation phase is simplified by considering built-in defined functions available in physionet [14], and that included with each database. These functions allow for example to detect RR intervals, that are considered as ECG beats and also allow to reconstitute QRS complexes.

The classification phase consists of assigning a class to each ECG beat. We first consider two classes, namely the normal and abnormal class. Details on the definition of these classes are given in Table 1 [15, 16]. The classification algorithm is efficient if it allows to assign the right class to a particular beat in the ECG record. To achieve this goal, a signal-dependent threshold is determined.

#### 3.1. Definition of classification quantifiers

Table 1: Proposed classification and corresponding AAMI annotation for each beat type.

Class	Beat type	Annotation
Normal	Left Bundle Branch Block	L
	Right Bundle Branch Block	R
	Normal Beat	N
	Atrial Escape Beat	e
	Nodal (junctional) Escape Beat	j
Abnormal	Atrial Premature Contraction	A
	Premature Ventricular Contraction	V
	Paced Beat	P
	Aberrated Atrial Premature Beat	a
	Supraventricular Premature Beat	S
	Fusion of Ventricular and Normal Beat	F
	Unclassifiable Beat	U
	Fusion of Paced and Normal Beat	f
	Ventricular Escape Beat	E
	Nodal (junctional) Premature Beat	J

For a good classification, we need specific quantifiers that better describe differences to be discriminated. For ECG data for example, abnormalities are related to the form of QRS complexes and the relative length of RR intervals, to cite a few. Some specific abnormalities have been reported in the AAMI classification as shown in Table 1. Some relevant studies have also established that abrupt changes in the map of RR intervals are useful for discriminating between beats of type S (abnormal beats) and beats of type N (normal beats) [17]. In this section, we consider the relative RR intervals map [18] for defining classification rules. Our purpose is to learn from each record, assuming that we have no prior knowledge on the nature of beats to be analyzed. As a consequence of this matter, rules are defined in a more general way, by including all the beats of the record, regardless their nature. We therefore expect the method to finally take a right decision on the nature of each beat by using properties of the underlying record solely, as the gap between relative RR intervals of abnormal and normal beats varies with patients [19].

We thus define two quantifiers  $r_1$  and  $r_2$  that can be combined to determine rules allowing to discriminate beats. These quantifiers are pre-RR ( $RR_{i-1}$ ) and post-RR ( $RR_i$ ) intervals dependent and the corresponding mathematical definitions are the following [18]:

$$r_1(i) = \frac{RR_i - RR_{i-1}}{0.5(RR_{i-1} + RR_i)}; \quad (4)$$

and

$$r_2(i) = \frac{\overline{RR} - RR_i}{0.5(RR_{i-1} + RR_i)}. \quad (5)$$

These quantifiers are to be compared to a threshold value  $t_r$  to be defined. As  $r_j < t_r$  or  $r_j > t_r$ , beats are set as either normal or abnormal.

We assume that all the beats of the same type ( $N$  for example) do not present the same values for  $r_j$ . Therefore, setting a fixed threshold  $t_r$  may reduce the classification performance. Our idea is to consider an adaptive threshold for the method to take care of variations that may occur while passing from a patient to another one. Another motivation of this idea is that even normal beats from the same patients are not rigorously similar. We thus suggest that the threshold is another quantifier to be compared to  $r_j$ .

### 3.2. Definition of the signal-dependent threshold $t_r$

We assume that there exists a particular parameter of the ECG that can allow to distinguish between normal and abnormal beats. The exploitation of this parameter thus requires the knowledge of the particular value that makes beats different, that is the threshold. We assume that the threshold depends on the record under investigation (patient under investigation) and the acquisition system. For this purpose, we first define the fluctuation ratio of a time series or record  $\{x(k)\}_{k \in \mathbb{N}}$  as

$$f(k) = \frac{x(k) - \bar{x}}{x(k) + \sigma_x}, \quad (6)$$

where  $\bar{x}$  is the mean value of  $x$  and  $\sigma_x$  its standard deviation. The fluctuation ratio allows to highlight changes occurring in the time series, thus to make them easily detectable. Using  $f(k)$ , the signal-dependent threshold  $t_r$  is defined in terms of the mean value of  $f(k)$ , hence

$$t_r = \alpha \cdot |\overline{f(k)}|. \quad (7)$$

$0 < \alpha \leq 1$  is a scaling factor that depends on the acquisition system, hence to the database under investigation. For a given database, there is a single value of  $\alpha$  that should be set. Nonetheless,  $\alpha$  may also be considered to depend on the patient (taking into account the age for example).

### 3.3. Ordinal pattern based classification of ECG beats

For the classification purpose, we consider the ordinal pattern based fluctuation ratio. We thus evaluate the fluctuation ratio value of the highpass filtered series of PE and CEOP obtained by applying each of these methods to the series of RR segments. Indeed, each series of ordinal pattern quantifiers is filtered using a difference filter of order  $m$ , whose frequency response is given by

$$H(z) = \sum_{i=0}^m (-1)^i \binom{i}{m} z^{-i} \quad (8)$$

and the fluctuation ratio in Eq. (6) is computed with the output  $y$  of the filter as  $x = |y|$ . The highpass filter allows to emphasize hidden details in the series of ordinal patterns. Thus,  $x(k)$  is the entropy detail of the  $k$ -th beat while  $f(k)$  is the corresponding fluctuation ratio value. The threshold  $t_r$  is then the weighted mean value of the ordinal pattern fluctuation ratio, and beats can be classified by comparing ratios  $r_j(k)$  to  $t_r$ . Thus, by evaluating the threshold related to each record, we are able to assign a class to each beat. Indeed, we consider that the class of abnormal beats is related to  $r_1(k) > t_r$  and  $r_2(k) > t_r$ , while normal beats are those for which this previous relation is not verified. An example is given in Fig. 1 where the fluctuation function of record 100 of the MIT database and the corresponding threshold are given for  $\alpha = 0.4$ .

The main steps of the algorithm can be summarized as follows:

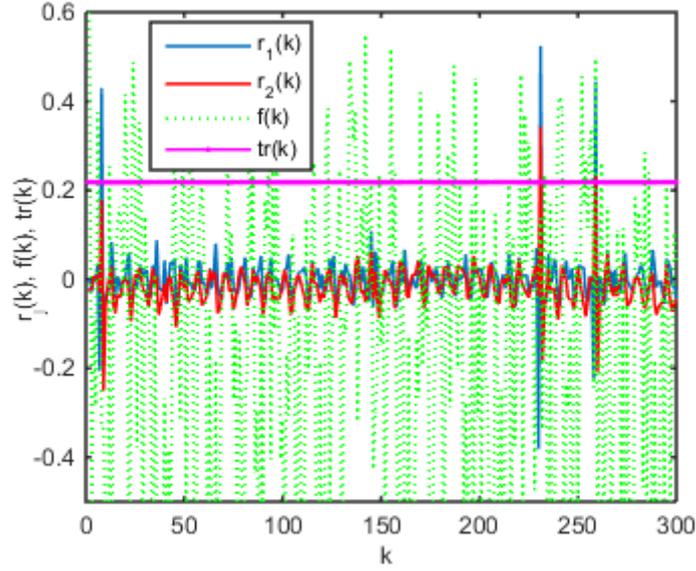


Figure 1: Example of detection for signal 100 of the MIT-BIH database.  $r_1$  and  $r_2$  are to be compared to the threshold  $t_r$ . For this example, there are three abnormal beats represented by the peaks above  $t_r$ . The threshold in this figure corresponds to  $\alpha = 0.4$ .

*Algorithm 1*

1. split the signal into RR segments
2. consider the  $k_{th}$  RR segment and compute its PE or CEOP to obtain the entropy value  $h(k)$
3. apply  $H(z)$  to the series of  $\{h_k\}_{k=1,2,\dots,N}$  and obtain  $\{y(k)\}$
4. determine the fluctuation ratio series  $\{f(k)\}$  by considering  $x(k) = |y(k)|$  in Eq. (6)
5. determine  $t_r$  as defined in Eq. 7.

**4. Results and discussion**

*4.1. Description of experimental data*

We used the MIT-BIH Arrhythmia and European Society of Cardiology databases for our experiments.

*4.1.1. MIT-BIH Arrhythmia database*

The MIT-BIH Arrhythmia database contains 48 records of 30-minutes each, sampled at 360 Hz with 11-bit resolution over a 10 mV range. Data are acquired from 47 different individuals (25 men of 89 years old and 22 women aged from 23 to 83), and records 201 and 202 belong to the same individual. In total, 18 types of heartbeats were labeled and each record contains two channels. In our simulation, only the first lead corresponding to the modified limb lead II is considered. Records 102, 104, 107 and 217 containing stimulated beats that are not coming from the sinusoidal node of the heart are discarded [16, 20]. Finally, the data set investigated contains 90125 normal beats and 10593 abnormal beats, according to the repartition in Table 1.

#### 4.1.2. European Society of Cardiology database (ESC)

The ESC database consists of 90 records from 78 patients sampled at 250 Hz with 12-bit resolution. These records are extracted from 70 men aged between 30 and 84 years and 8 women aged between 55 and 71 years. All these patients have a specific heart disease (i.e., myocardial ischaemia). At the beginning, the database was set up for the analysis of the ST segments and the T wave. The records have a duration of 2 hours and each record contains two channels and we consider only the first one in our simulations. Two cardiologists in accordance with the AAMI standard annotated the records.

Most heartbeats in these databases are annotated. AAMI standard specifies how annotations should be done. It recommends grouping beats into five classes including normal beats (N), supraventricular ectopic beats (S), ventricular ectopic beats (V), fusion of a V and an N beats (F), and the unknown beat type (Q). In this work, we will group heartbeats into two classes as presented in Table 1, namely the class of normal (N) beats and the class of abnormal (A) beats, in order to verify the effectiveness of ordinal pattern based algorithms for ECG data analysis.

#### 4.2. Classification Results of the MIT-BIH database

##### 4.2.1. Impact of $\alpha$

We apply the PE and CEOP algorithms to the MIT-BIH database for various values of  $\alpha$  ranged from 0 to 1. The order of ordinal patterns is set to  $n = 4$ . The corresponding detection rates are depicted in Fig. 2. The detection rate or accuracy  $\beta$  is computed as

$$\beta(\%) = 100 \cdot \frac{TN + TP}{TN + TP + FP + FN}, \quad (9)$$

where True Positive ( $TP$ ) is the cardinality of abnormal beats detected as belonging to the set of abnormal beats  $\{A\}$ , True Negative ( $TN$ ) the cardinality of normal beats detected as belonging to the set of normal beats  $\{N\}$ , False Positive ( $FP$ ) the number of normal beats detected as abnormal, and False Negative ( $FN$ ) the number of abnormal beats detected as normal.

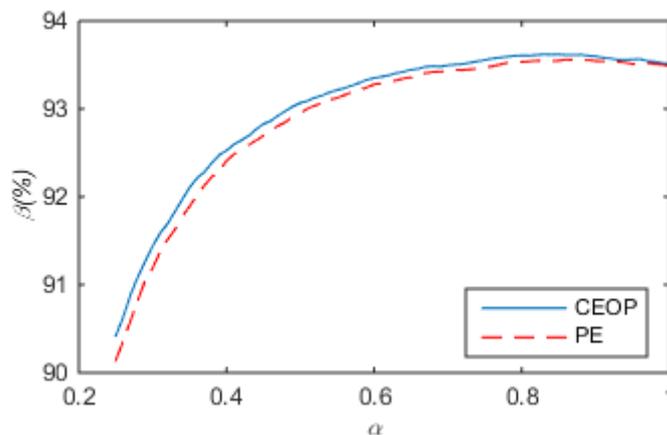


Figure 2: Behavior of the detection rate  $\beta$  in terms of  $\alpha$  for the MIT-BIH database. The filter order is set to  $m = 4$ .

According to this figure, the detection rate of the CEOP is slightly greater than that of the PE, but this difference is marginal. Therefore, one can indifferently use one of the two entropy approaches. The maximum detection rate  $\beta = 93.62\%$  for the CEOP is obtained for  $\alpha = 0.83$ , while that of the PE  $\beta = 93.56\%$  is obtained for  $\alpha = 0.86$ . Nevertheless, one can observe that for a wide range of  $\alpha$  values

( $0.5 \leq \alpha \leq 1$ ), the detection rate remains close to its maximum value, thus attesting the robustness of the detection approach with respect to  $\alpha$ . Fig. 3 shows the variation of the threshold values in terms of

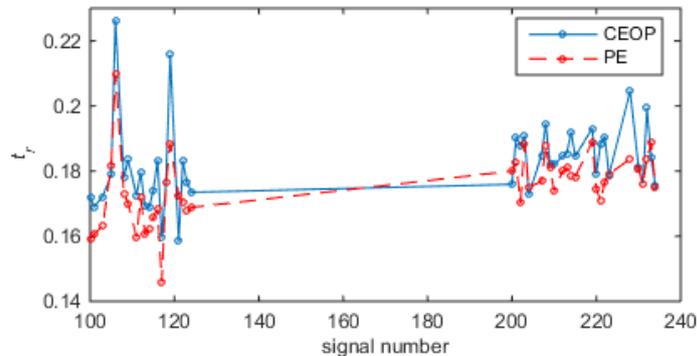


Figure 3: Behavior of the signal dependent threshold for the MIT-BIH database. The filter order is set to  $m = 4$  and the scaling factor to  $\alpha = 1$ .

records. From this figure, we can appreciate the adaptability of the threshold whose evolution rule is ordinal pattern entropy controlled. Such an adaptive threshold detection approach was presented in [21], but the set of thresholds was evaluated by using some records of the database with prior knowledge on the beats types for training. In our approach, we assumed that there is no prior knowledge of beats types and each record has its own threshold which only depends on its distribution of ordinal patterns.

We can also evaluate the sensitivity  $\xi$ , the specificity  $\mu$ , the positive predictive value  $\gamma_+$  and the negative predictive value  $\gamma_-$  respectively defined as

$$\xi(\%) = 100 \cdot \frac{TP}{TP + FN}, \quad (10)$$

$$\mu(\%) = 100 \cdot \frac{TN}{TN + FP}, \quad (11)$$

$$\gamma_+(\%) = 100 \cdot \frac{TP}{TP + FP}, \quad (12)$$

$$\gamma_-(\%) = 100 \cdot \frac{TN}{TN + FN}. \quad (13)$$

The corresponding behaviors are shown in Fig. 4. It appears that the sensitivity of the algorithm decreases as  $\alpha$  increases. The figure shows a good performance of the algorithm. In the case of the CEOP for example, for  $\alpha = 0.25$  the accuracy of the classifier is  $\beta = 90.42\%$ , which corresponds to a false positive rate ( $FPR = 1 - \mu$ ) of  $FPR = 6.91\%$  and a false negative rate ( $FNR = 1 - \xi$ ) of  $FNR = 32.26\%$ . While considering the case of  $\alpha = 0.83$ , the performance indices become  $\beta = 93.62\%$ ,  $FPR = 1.91\%$  and  $FNR = 44.44\%$ . Thus,  $\alpha$  can be seen as the control parameter of the sensitivity of the classification approach. Indeed, the sensitivity gets smaller as  $\alpha$  increases. In other terms, the false positive rate reduces as  $\alpha$  increases, while the rate of false negative increases with  $\alpha$ .

#### 4.2.2. Impact of the ordinal pattern order $n$

We evaluate the effect of the order  $n$  of CEOP on the classification result. We set  $n = 2, 3, 4, 5$  and apply the corresponding series of CEOP to the detection of abnormal beats. We evaluate the impact of the change of  $n$  on the accuracy of the classification algorithm for various values of  $\alpha$ . The corresponding results are depicted in Fig. 5, where it appears that the choice of  $n$  does not significantly affect the

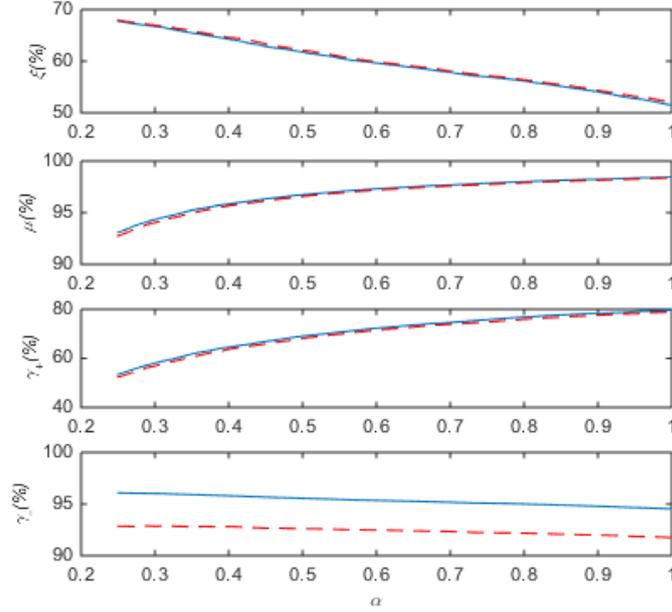


Figure 4: Performance of the classification approach in terms of  $\alpha$ , for the MIT-BIH database. The filter order is set to  $m = 4$ . The solid lines describe behaviors related to the CEOP, while dashed lines describe those related to the PE.

classification result, even for  $n = 5$ . Indeed, the case  $n = 5$  does not meet the condition  $L \gg n!$  that is necessary for an efficient evaluation of the CEOP and PE.  $L$  is the length of the data series whose ordinal pattern entropy is evaluated, the RR interval in our case.

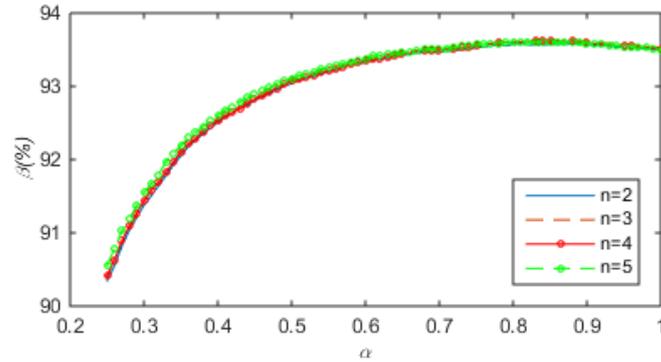


Figure 5: Impact of  $n$  on the accuracy of the CEOP based classification in terms of  $\alpha$ , for the MIT-BIH database. The filter order is set to  $m = 4$ .

#### 4.2.3. Impact of the filter order $m$

For simplification purposes, we used a difference filter for determining the threshold value. Another type of highpass filter may be used to perform similar results, depending on its frequency response. The

order of the chosen filter is then another parameter that may change the output of the classifier. The order of the filter should not be too large to avoid large delays, although the subsequent transition bandwidth is large. We evaluated the performance of the classifier for  $1 \leq m \leq 4$ . The corresponding behaviors of  $\beta$ ,  $\xi$  and  $\mu$  in terms of  $\alpha$  are shown in Fig. 6.

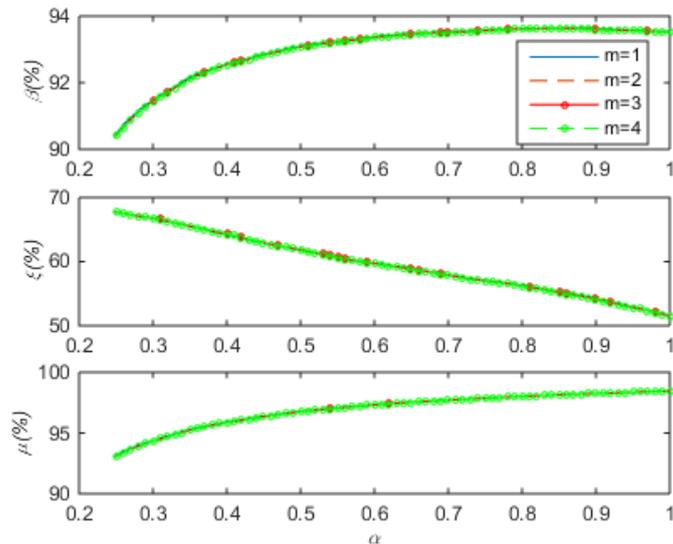


Figure 6: Impact of the filter order  $m$  on the accuracy of the CEOP based classification in terms of  $\alpha$ , for the MIT-BIH database. The ordinal pattern order is set to  $n = 4$ .

As in the case of the CEOP of order  $n$ , the impact of the filter on the classification result is not really significant. The maximum detection rate for  $m = 1$  is  $\beta = 93.65\%$ , thus corresponding to  $\alpha = 0.88$ . The corresponding false negative and false positive rates are respectively  $FNR = 45.27\%$  and  $FPR = 1.78\%$ . In the case of  $m = 4$ , these metrics are respectively  $\beta = 93.62\%$ ,  $FNR = 44.44\%$  and  $FPR = 1.91\%$ , for  $\alpha = 0.83$ . We can observe that the small improvement in  $\beta$  for  $m = 1$  is balanced by the small improvement in  $\beta$  for  $m = 4$ . However,  $m = 4$  is preferred to  $m = 1$  as we would like both  $FNR$  and  $FPR$  to be as small as possible.

#### 4.3. Classification results of the ESC database

We analysed a set of 275648 normal beats and 2624 abnormal beats. Using the CEOP and PE of order  $n = 4$  and a difference filter of order  $m = 4$ , we obtained the results in Fig. 7. We can observe that for the same range of  $0.25 \leq \alpha \leq 1$ , the maximum detection rate occurs at  $\alpha = 0.86$  and is equal to  $\beta = 99.57\%$ , which is large enough compared to the result of the MIT database. The corresponding other metrics are respectively  $FNR = 37.42\%$  and  $FPR = 0.07\%$ . These values are also much better than those obtained with the MIT-BIH database. The  $FPR$  is almost zero, thus attesting that almost all the normal beats detected as normal were well classified. The  $FNR$  is quite large, although it is smaller than that obtained with the MIT-BIH database. This value can be readjusted by reducing  $\alpha$ . For  $\alpha = 0.25$  for example, we found  $FNR = 27.10\%$  and  $FPR = 1.94\%$ ,  $\beta = 97.82\%$ . The results obtained for the PE in that case were too close to those of the CEOP.

In the case of the CEOP, the maximum value of  $\beta$  was obtained with  $\alpha = 0.83$  for the MIT-BIH database and  $\alpha = 0.86$  for the ESC database. This result proves the sensitivity of  $\alpha$  on the database. Indeed,  $\alpha$  is a sensitivity parameter that allows to adjust the threshold on the acquisition system and

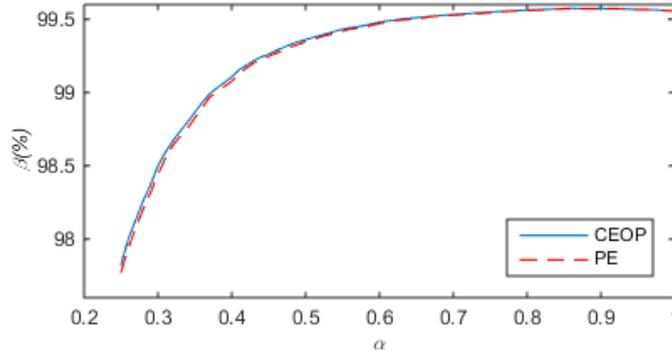


Figure 7: Detection result for the ESC database in terms of  $\alpha$ . The filter order is set to  $m = 4$  and the order of ordinal pattern entropies to  $n = 4$ .

also on patients. The patient dependency of  $\alpha$  is proven by the fact that for two different patients in the same database (same acquisition system), there can be two different values of  $\alpha$  giving the optimal classification for each patient. However, as it is difficult to adjust  $\alpha$  for each patient, it is better to make it only system dependent. Therefore, the values of  $\alpha$  we presented were depending only on the database under investigation, hence they were system dependent. We can also observe that the maximum detection rate occurs for two very close values of  $\alpha$  for the two databases, which clearly proves the robustness of our algorithm against system change. This result is justified by the fact that the computation of the threshold is signal dependent. Therefore,  $\alpha$  may be easily used for just balancing the  $FNR$  and  $FPR$ . Nevertheless, it should be pointed out that at the present stage, the  $FNR$  is still large and needs to be improved.

#### 4.4. Comparison with existing methods

Comparing the actual results with existing classification algorithms is quite difficult as most of them consider more than two classes. Therefore, a rigorous comparison cannot be performed. Nonetheless, we compared our results with four other algorithms. Some of the results obtained with these algorithms are given in Table 2. In these algorithms, the database is divided into two sets, one is used for training and the other one for testing.

Table 2: Results on MIT-BIH comparing our method against state-of-art methods

Method	Accuracy (%)	Sensitivity (%)	Specificity (%)
Khoshnoud et al. [22]	92.9	93.17	-
Inan et. al. [23]	95.16	89,22	-
Khazaei et al. [24]	89.67	99.97	99.98
Ravelo et al. [25](cepstrum)	86.00	63.80	89.20
Our method	93.67	55.5731	98.23

We compared our results with non-ordinal methods [22–24] for evaluating the effect of the ordinal recipe on the classification performance. In Ref. [22], Khoshnoud et al. adopted linear predictive coefficients (LPC) as beat features for classifying beats into normal and abnormal. Their classification approach which is based on probabilistic neural networks, has achieved 92.90 % accuracy with the MIT-BIH data base. Inan et. al. [23] defined as classification feature the combination of ECG waves wavelet-transform with the heart rate variability (HRV). Including these features into the neural network, they derived three

types of beats namely, normal, premature ventricular contraction (PVC) and others. The method was applied to 40 ECG records of the MIT-BIH database with an accuracy of 95,16%. Khazaee et al. [24] used support vector machines (SVM) and genetic algorithms (GA) to detect arrhythmia, which is referred to as identification of PVC. The method was applied to nine records of the MIT-BIH database with an overall classification accuracy of 99.81%. In [25], Ravelo et al. applied the PE to the HRV. The PE series is analyzed in a statistical model integrating electrocardiogram derived respiratory (EDR) features and cepstrum coefficients in order to detect obstructive sleep apnea (OSA) events. 70 ECG records from a database provided for the Computers in Cardiology Challenge 2000 are divided into learning and test sets of equal size. The logistic regression (LR) is applied to the classification of sleep apnea epochs. The EDR presents a sensitivity of 64.30% and a specificity of 86.50% with an accuracy of 83.90%; while the Cepstrum presents a sensitivity of 63.80% and specificity of 89.20% with 86% accuracy.

The above table reveals that the proposed method provides satisfactory results as compared to the others. Indeed, our approach considers all the 44 records for testing, instead of few of them as its the case while performing both training and testing. For example, our accuracy is much better than that in Ref. [24], although only nine records were suitably selected in their paper. While considering Ref. [25], it appears that the results are in the same range. Moreover, although in our case all the records are used for testing, our classification rate is close to that of the other algorithms in Table 2, which confirms that ordinal pattern based methods are promising.

## 5. Conclusion

In this paper, we investigated the applicability of PE and CEOP to ECG data. The results obtained with the ESC database confirm that ordinal pattern based algorithms are promising for ECG beat classification. Our approach does not need prior knowledge on the ECG database for training, and directly determine a threshold value for a binary classification that solely depends on the record (or patient) under consideration. The simulation results show that the algorithm is less sensitive to the filter order and the order of CEOP or PE. This high robustness of the algorithm against the choice of these parameters is important as one may choose any of the values tested in this paper without considerably affecting the classification result. We also observed that  $\alpha$  may be easily used for controlling the sensitivity  $\xi$  and selectivity  $\mu$  of the algorithm, depending on the required output: weak *FNR* and high *FPR*, or high *FNR* and weak *FPR*. Indeed,  $\xi$  decreases (*FNR* increases) when  $\alpha$  increases, while  $\mu$  increases (decrease of the *FPR*) with  $\alpha$ . Depending on the database and the acquisition system, the detection rate can reach  $\beta = 99.57\%$ . We thus intend to improve this algorithm so as to use it as a pre-classifier in a more extended algorithm performing a multi-class classification.

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