
2002k:33018
Koepf, Wolfram (D-KSSL); **Schmersau, Dieter** (D-FUB)
On a structure formula for classical q-orthogonal polynomials. (English summary)
J. Comput. Appl. Math. **136** (2001), *no. 1-2,* 99–107.
33D45 (33C45)

References: 0	Reference Citations: 0	Review Citations: 0

Summary: "The classical orthogonal polynomials are given as the polynomial solutions $P_n(x)$ of the differential equation

$$\sigma(x)y''(x)+\tau(x)y'(x)+\lambda_n y(x)=0,$$

where $\sigma(x)$ turns out to be a polynomial of at most second degree and $\tau(x)$ is a polynomial of first degree. In a similar way, the classical discrete orthogonal polynomials are the polynomial solutions of the difference equation

$$\sigma(x)\Delta\nabla y(x)+\tau(x)\Delta y(x)+\lambda_n y(x)=0,$$

where $\Delta y(x)=y(x+1)-y(x)$ and $\nabla y(x)=y(x)-y(x-1)$ denote the forward and backward difference operators, respectively. Finally, the classical q-orthogonal polynomials of the Hahn tableau are the polynomial solutions of the q-difference equation

$$\sigma(x)D_qD_{1/q}y(x)+\tau(x)D_qy(x)+\lambda_{q,n}y(x)=0,$$

where

$$D_qf(x)=\frac{f(qx)-f(x)}{(q-1)x},\quad q\neq 1,$$

denotes the q-difference operator. We show by a purely algebraic deduction—without using the orthogonality of the families considered—that a structure formula of the type

$$\sigma(x)D_{1/q}P_n(x)=\alpha_nP_{n+1}(x)+\beta_nP_n(x)+\gamma_nP_{n-1}(x),$$

$n\in\mathbb{N}:=\{1,2,3,\dots\}$, is valid. Moreover, our approach not only proves this assertion, but generates the form of this structure formula. A similar argument applies to the discrete and

continuous cases and yields

$$\sigma(x)\nabla P_n(x) = \alpha_n P_{n+1}(x) + \beta_n P_n(x) + \gamma_n P_{n-1}(x), \quad n \in \mathbb{N},$$

and

$$\sigma(x) P_n'(x) = \alpha_n P_{n+1}(x) + \beta_n P_n(x) + \gamma_n P_{n-1}(x), \quad n \in \mathbb{N}.$$

Whereas the latter formulas are well known, their previous deduction used the orthogonality property. Hence our approach is also of interest in these cases."

Reviewed by *H. S. P. Shrivastava*