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Coefficients of multiplication formulas for classical orthogonal polynomials.
 (English summary)

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In this paper, the authors use both analytic and algorithmic approaches to compute the coefficients of the multiplication and translation formulas for all families of classical orthogonal polynomials of a continuous, a discrete and a q -discrete variable, as well as for other special polynomials. These results are correct and make sense.

The main method for deriving the coefficients of the multiplication formula

$$(1) \quad P_n(ax) = \sum_{m=0}^n D_m(n, a) P_m(x)$$

and the translation formula

$$(2) \quad P_n(a+x) = \sum_{m=0}^n D_m(n, a) P_m(x)$$

makes use of generating functions.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.