An outstanding, difficult problem is that of finding sharp bounds for the coefficients of functions
\( f(z) = \sum_{n=0}^{\infty} a_n z^n \) which are analytic, bounded by 1, and nonvanishing in the unit disc. Trivially, all \( |a_n| \) are bounded by 1. The Krzyż conjecture is that \( |a_n| \leq 2/e \) for all \( n > 0 \). This has been
proved for only \( n = 1, 2, 3, \) and 4. In an earlier paper, S. Scheinberg, L. Zalcman and the reviewer
[J. Analyse Math. 31 (1977), 169–190; MR 58 #11358] considered this problem from a number of
different points of view and since then a number of papers have appeared considering different
aspects of the problem. The authors consider the problem from the point of view of subordination,
oberving that any \( f(z) \) in the class is subordinate to \( \exp\{-t(1+z)/(1-z)\} = \sum_{n=1}^{\infty} F_n(t) z^n \),
and studying bounds on the \( F_n(t) \). (Notice that it is not even immediately evident that all of the
\( F_n(t) \) satisfy the Krzyż conjecture.) These \( F_n(t) \) turn out to be functions studied by Bateman
via an integral representation. Results of Bateman and in this paper give a difference equation, a
differential equation, and residue representations. Examples of results obtained are that \( |F_n(t)| \) is
bounded above by \( (1 - e^{-2t})^{1/2}, 2n/t, \) and \( (4t/(2n - t))^{1/2} \). It is then shown that \( |F_n(t)| \leq 2/e \)
for all \( t \) and all \( n > 21138 \). The paper makes use of methods more usually used in the study of
special functions and contains a great many results about the Bateman functions \( F_n(t) \).

Reviewed by James A. Hummel

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