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Koepf, Wolfram (D-KOZU)
Algorithms for m-fold hypergeometric summation. (English summary)
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The problem of indefinite summation is that of finding an anti-difference s_k for a given sequence a_k, i.e., a sequence s_k satisfying $a_k = s_k - s_{k-1}$. We say that s_k is a hypergeometric term if s_k/s_{k-1} is a rational function of k. Gosper's algorithm for indefinite summation [R. W. Gosper, Jr., Proc. Nat. Acad. Sci. U.S.A. **75** (1978), no. 1, 40–42; MR **58** #5497] determines whether a given sequence a_k has a hypergeometric antidifference, and finds such an anti-difference if it exists.

The author considers a more general problem, that of finding a hypergeometric "m-fold antidifference" s_k for a_k, satisfying $a_k = s_k - s_{k-m}$. He shows how this more general problem can be solved by Gosper's algorithm. He then applies this extended version of Gosper's algorithm to an extension of the WZ method [H. S. Wilf and D. Zeilberger, J. Amer. Math. Soc. **3** (1990), no. 1, 147–158; MR 91a:05006] and to an extension of Zeilberger's algorithm [D. Zeilberger, Discrete Math. **80** (1990), no. 2, 207–211; MR 91d:33006], both of which can be used to prove identities for definite sums.

The author also includes proofs using the extended WZ method, of the terminating hypergeometric series summation formulas in W. N. Bailey's book [*Generalized hypergeometric series*, Cambridge Univ. Press, London, 1935; Zbl 011.02303; Stechert-Hafner, New York, 1964; MR **32** #2625], and in the paper of the reviewer and D. Stanton [SIAM J. Math. Anal. **13** (1982), no. 2, 295–308; MR 83c:33002].

Reviewed by *Ira Gessel*