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Koepf, Wolfram (D-KOZU)

Identities for families of orthogonal polynomials and special functions. (English summary)

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Summary: "In this article we present new results for families of orthogonal polynomials and special functions, that are determined by algorithmic approaches. In the first section, we present new results, especially for discrete families of orthogonal polynomials, obtained by an application of the celebrated Zeilberger algorithm. Next, we present algorithms for holonomic families $f(n,x)$ of special functions which possess a derivative rule. We call those families admissible. A family $f(n,x)$ is holonomic if it satisfies a holonomic recurrence equation with respect to n, and a holonomic differential equation with respect to x, i.e. linear homogeneous equations with polynomial coefficients. The rather rigid property of admissibility has many interesting consequences, that can be used to generate and verify identities for these functions by linear algebra techniques. On the other hand, many families of special functions, in particular families of orthogonal polynomials, are admissible. We moreover present a method that generates the derivative rule from the holonomic representation of a holonomic family. As examples, we find new identities for the Jacobi polynomials and for the Whittaker functions, and for families of discrete orthogonal polynomials by the given approach. Finally, we present representations for the parameter derivatives of the Gegenbauer and the generalized Laguerre polynomials."