MR2035209 (2004m:68293)

Koepf, Wolfram (D-KSSL-MI)

Power series, Bieberbach conjecture and the de Branges and Weinstein functions. (English summary)


68W30 (30B10 30C50 40C15)

Summary: “It is well known that de Branges’ original proof of the Bieberbach and Milin conjectures on the coefficients $a_n$ of univalent functions $f(z) = \sum_{k=1}^{\infty} a_n z^n$ of the unit disk and Weinstein’s later proof deal with the same special function system that de Branges had introduced in his work.

“These hypergeometric polynomials had been already studied by Askey and Gasper who had realized their positiveness. This fact was the essential tool in de Branges’ proof.

“In this article, we show that many identities, e.g. the representation of their generating function with respect to $n$, for these polynomials, which are intimately related to the Koebe function $K(z) = \sum_{k=1}^{\infty} n z^n$ and therefore to univalent functions, can be automatically detected from power series computations by a method developed by the author and accessible in several computer algebra systems.

“In other words, in this paper a new and interesting application of the FPS (formal power series) algorithm is given. As working engine we use a Maple implementation by Dominik Gruntz and the author. In particular, the hypergeometric representations of both the de Branges and the Weinstein functions are determined by successive power series computations from their generating functions.

“The new idea behind this algorithm is the observation that hypergeometric function coefficients of double series can be automatically detected by an iteration of the FPS procedure.

“In a final section we show how algebraic computation enables the fast verification of Askey-Gasper’s positivity results for specific (not too large) $n$ using Sturm sequences or similar approaches.”

{For the entire collection see MR2035188 (2004j:68018)}