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Koepf, Wolfram (D-KSSL); **Masjed-Jamei, Mohammad**
Two classes of special functions using Fourier transforms of some finite classes of classical orthogonal polynomials. (English summary)
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Some new finite families of orthogonal polynomials which are eigenfunctions of a second-order linear differential operator with polynomial coefficients have been analyzed in [M. Masjed-Jamei, Integral Transforms Spec. Funct. **15** (2004), no. 2, 137–153; MR2053407 (2005b:33011); Integral Transforms Spec. Funct. **13** (2002), no. 2, 169–191; MR1915513 (2003i:33011)]. They are orthogonal with respect to the generalized T, inverse Gamma, and F distributions, respectively. Taking into account the Parseval identity of Fourier transform theory, the authors of the paper under review define specific functions associated with two of the above families and apply the Fourier transform in such a way that the resulting functions are rational functions denoted by $A_n(ix;a,b,c,d)$ and $B_n(ix;a,b)$, orthogonal in the Hermitian sense with respect to the complex weights $\Gamma(a+ix)\Gamma(b-ix)\Gamma(c+ix)\Gamma(d-ix)$ and $\Gamma(a+ix)\Gamma(b-ix)$, supported on the real line, respectively. Note that the above rational functions are hypergeometric functions of orders $(3,2)$ and $(2,1)$, respectively. Finally, a conjecture concerning the orthogonality of the family of rational functions $A_n(x;a,b,c,d)$ with respect to the Ramanujan real weight function [see S. Ramanujan, Quart. J. Pure Appl. Math. **48** (1920), no. 4, 294–310]

$$\frac{1}{\Gamma(1-a+x)\Gamma(1-b+x)\Gamma(1-c+x)\Gamma(1-d+x)}$$

is stated. Note that such an orthogonality relation would complement a result in the paper [R. A. Askey, J. Phys. A **18** (1985), no. 16, L1017–L1019; MR0812420 (87d:33021)], concerning a family of polynomials orthogonal with respect to the above Ramanujan weight function.

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Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.