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Koepf, Wolfram [Koepf, Wolfram A.] (D-UKSL-NDM)

★**Hypergeometric summation.**
An algorithmic approach to summation and special function identities.
Second edition.
Universitext.
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The book under review deals with the modern algorithmic techniques for hypergeometric summation, most of which were introduced in the 1990's. The first edition of this book appeared in 1998 [W. A. Koepf, *Hypergeometric summation*, Adv. Lectures Math., Friedr. Vieweg, Braunschweig, 1998; MR1644447].

The first three chapters give notations and show how to transform (q-)binomial coefficient summations into hypergeometric functions and then use the known database to find their sums. In contrast to the first edition of the book, the material is presented by giving more detailed advice on implementation. Furthermore, the aim is to cover material not only about recurrence equations but also about differential equations, not only about sums but also about integrals, and finally not only the hypergeometric case but also its q-analogue. The situation is quite different concerning the following parts of the book (Chapters 4–13). Multivariate hypergeometric summation was still unfeasible when the first edition was written. In the meantime, ideas by Wegschaider cleared the way. These newer developments are incorporated and illustrated in Chapter 4, and the corresponding `multsum`-package is introduced. Furthermore, van Hoeij's algorithm has dramatically improved the efficiency of finding hypergeometric term solutions of holonomic recurrence equations over Petkovšek's original approach. Therefore, his ideas have been incorporated in Chapter 8 so that the reader gets a clear impression of where the new efficiency comes from. Nevertheless, the presentation of Petkovšek's original algorithm has not been withdrawn since it is still interesting from a historical point of view. More decisively, the efficiency of van Hoeij's algorithm can only be understood by comparison with Petkovšek's approach. The chapter finishes with the Maple package `qFPS`, which contains the q-case of van Hoeij's algorithm. More details about operator factorization are given in Chapters 9 and 12. Finally, there are some new developments on discrete Rodrigues formulas, which have been incorporated in Chapter 13.

The last six chapters of the book are thus devoted to the "six basic algorithms" by Sister Celine, Gosper, Wilf and Zeilberger, Zeilberger, Koepf and Petkovšek, respectively. Finally, Chapters 10–13 deal with differential equations for sums, hyperexponential antiderivatives, holonomic equations for integrals, and Rodrigues formulas and generating functions, respectively. The author provides many worked out sessions with Maple and a wealth of exercises at the end of each chapter. The book covers many algorithms for summation and integration, most of which have not changed much in the meantime and are still up-to-date. Fasenmyer's algorithm for definite summation (Chapter 4) is very old; nevertheless, it is so easy to describe that it must be included for didactical reasons. Gosper's algorithm (Chapter 5) solves the problem of how to find a hypergeometric antidifference, and it is the starting point of Zeilberger's celebrated algorithm for definite summation (Chapter 7). The book also covers the differential counterpart of Zeilberger's summation algorithm (Chapter 10) as well as its integration counterparts (Chapters 11 and 12), and Gosper's algorithm is the driving force for all these algorithms. Therefore, its description remained unchanged. The other mentioned algorithms are also still up-to-date. Therefore, the above chapters have been updated only cautiously. However, in most chapters, new developments are cited and suggestions for further reading are given. As in the first edition, in all chapters an introduction to the corresponding q-theory is also given.

For the first edition the author selected Maple as the computer algebra system in which the algorithms were programmed and demonstrated. Moreover, these algorithms (and some more) were incorporated in the packages `hsum` (and `qsum` for the q-case). This selection has proven successful, and since the other packages mentioned (`multsum` and `qFPS`) are also written in Maple, it is still the best system to keep the book self-contained.

In summary, the present book is designed for use in the framework of an (undergraduate) seminar, where every participating student is asked to present a lecture about a certain topic. The arrangement of the book makes the division into lectures easy. Each chapter covers a certain subtopic which can be presented by one or two students. Obviously, the book is also suitable for a lecture course in this area, since it was written in connection with such a course presented by the author. The book contains many worked-out examples of the algorithms considered, and Maple implementations of them are presented. Furthermore, a lot of exercises encourage the readers to do their own implementations in Maple, and to study the topics included in detail (exercises that demand Maple implementations are marked by a diamond $\diamondsuit$).

This well-written book should be recommended for anybody who is interested in binomial summations and special functions. It should also prove useful to researchers in mathematics and/or quantum physics working in topics which associate combinatorics of special (q-)functions (e.g., computational mathematics and an algorithmic approach to summation and special function identities) with current quantum mechanics issues.

Christian Lavault