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On a structure formula for classical q-orthogonal polynomials. (English)

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A short proof is given for the fact that any solution of the q-difference equation

$$\sigma(x)D_qD_{1/q}y(x)+\tau(x)D_qy(x)+\lambda_{q,n}y(x)=0,$$

where $\sigma(x)=ax^2+bx+c$, $\tau(x)=dx+e$, for some real numbers a,b,c,d,e, D_q is the usual q-difference operator defined by $D_qf(x):=(f(qx)-f(x))/(q-1)x$, and $\lambda_{q,n}$ is (enforcedly) equal to $-a[n]_{1/q}[n-1]_q-d[n]_q$ with $[n]_q:=(1-q^n)/(1-q)$, satisfies the "structure formula"

$$\sigma(x)D_{1/q}P_n(x)=\alpha_nP_{n+1}(x)+\beta_nP_n(x)+\gamma_nP_{n-1}(x),$$

with explicitly known real numbers $\alpha_n,\beta_n,\gamma_n$. The proof is based on the fact that any solution of the q-differene equation satisfies a three-term recurrence

$$P_{n+1}(x)=(A_nx+B_n)P_n(x)-C_nP_{n-1}(x)$$

(and is therefore a family of orthogonal polynomials). The authors mention that an independent proof of the same fact was given by Steffen Häcker in his Ph.D. thesis "Polynomiale Eigenwertprobleme zweiter Ordnung mit Hahnschen q-Operatoren," using a different approach.

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Keywords : classical q-orthogonal polynomials; q-Hahn tableau; structure formula; q-Jacobi polynomials

Classification :

∗33D45 Basic hypergeometric functions and integrals in several variables
33C45 Orthogonal polynomials and functions of hypergeometric type

Cited in ...