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On fourth-order difference equations for orthogonal polynomials of a discrete variable: Derivation, factorization and solutions. (English)

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Let $(P_n)_n$, $deg\ P_n = n$, be a classical discrete orthogonal polynomial sequence. The authors consider all transformations

$$\overline{P}_n(x) = A_n(x)P^{(1)}_{n+k+1} + B_n(x)P_{n+k}, \quad n \geq k', \tag{1}$$

where A_n and B_n are polynomials of degree not depending on n, $P^{(1)}_n$ is the first associated of P_n, and k, k' are non-negative integers. The following result is proved. $(\overline{P}_n)_{n\geq k'}$ satisfy a difference equation

$$F_n(y) = \left(\sum_{i=0}^{4} J_i \mathcal{F}^i\right) y = 0 \tag{2}$$

where J_i are polynomials with degree not depending on n, $\mathcal{F}^i$ are the shift operators defined by $\mathcal{F}^i(P(x)) = P(x+i)$, $i = 0, ..., 4$, on the space of polynomials. The operators F_n can be factored as a product

$$X_n F_n = S_n T_n, \quad n \geq k$$

where S_n and T_n are second-order linear difference operators with polynomial coefficients not depending on k, and X_n is a polynomial of fixed degree. Moreover, the authors obtain four linearly independent solutions of (2) for operators F_n corresponding to the following polynomials $\overline{P}_n$: rth associated, generalized co-recursive, generalized co-recursive associated, generalized co-modified and co-dilated. Recall that all these polynomials related to starting classical discrete $(P_n)_n$ by particular cases of transformations (1). The extensions results to real order association of classical discrete orthogonal polynomials and to semi-classical cases are also presented.

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Classification :

∗33C45 Orthogonal polynomials and functions of hypergeometric type

13P05 Polynomials, factorization

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