

1064.05024**Larcombe, Peter J.; Fennessey, Eric J.; Koepf, Wolfram A.****Integral proofs of two alternating sign binomial coefficient identities.** (English)

Util. Math. 66, 93-103 (2004). [ISSN 0315-3681]

The paper evaluates two alternating sign binomial coefficient summations

$$m \binom{m+n}{n} \sum_{k=0}^n \frac{(-1)^k}{(k+m)^2} \binom{n}{k} = \sum_{k=m}^{m+n} \frac{1}{k}$$

and

$$2m \binom{m+n}{n} \sum_{k=0}^n \frac{(-1)^k}{(k+m)^3} \binom{n}{k} = \left(\sum_{k=m}^{m+n} \frac{1}{k} \right)^2 + \sum_{k=m}^{m+n} \frac{1}{k^2}$$

(integer $m \geq 1$, $n \geq 0$) by evaluating the integrals

$$\int_0^\infty x e^{-mx} (1 - e^{-x})^n dx \quad \text{and} \quad \int_0^\infty x^2 e^{-mx} (1 - e^{-x})^n dx,$$

respectively, in different ways.

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*05A19 Combinatorial identities

05A10 Combinatorial functions

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