

**Koepf, Wolfram**

**Hypergeometric summation. An algorithmic approach to summation and special function identities. 2nd ed.** (English) [Zbl 1296.33002](#)

Universitext. London: Springer (ISBN 978-1-4471-6463-0/pbk; 978-1-4471-6464-7/ebook). xvii, 279 p. (2014).

The book is the second edition of the book published in 1998. There is the review of P. W. Karlsson ([Zbl 0909.33001](#)) of the first edition.

The book treats the method of hypergeometric summation and its  $q$ -analogue. It covers many publications on this topic. The author himself is an expert on problems of such type.

The book is very useful. The reader who assimilates the book can resolve such problems for example.

1. Prove the identity

$$\sum_{k=-n}^n (-1)^k \binom{n+b}{n+k} \binom{n+c}{c+k} \binom{b+c}{b+k} = \frac{\Gamma(b+c+n+1)}{n!\Gamma(b+1)\Gamma(c+1)},$$

where  $\binom{z}{k} = \frac{\Gamma(z+1)}{k!\Gamma(z+1-k)}$  (equation 2.6).

2. Prove the identity

$$\sum_{k=0}^m \frac{1}{2^k} \binom{m+k}{k} = 2^m$$

(exercise 4.16).

3. Prove the identity

$$\sum_{k=1}^n k \binom{n}{k} \binom{s}{k} = s \binom{n+s-1}{n-1}$$

(exercise 6.7).

Concerning problems 1–3 see [*G. P. Egorychev*, Integral representation and the computation of combinatorial sums. Transl. from the Russian by H. H. McFaden, ed. by Lev J. Leifman. Providence, R.I.: American Mathematical Society (AMS) (1984; [Zbl 0524.05001](#))] also.

4. Calculate

$$\sum_{n=1}^{\infty} \frac{(-1)^n (4n+1)(2n-1)!!}{2^n (2n-1)(n+1)!}$$

(exercise 5.11).

5. Evaluate the integral

$$\int_0^1 e^{-\frac{1}{t}} t^{-3-n} (1-t)^n dt$$

(exercise 12.3).

The author presents the algorithms by Gosper, Zeilberger, van Hoeij. These algorithms are necessary to solve several problems.

Decisions of some problems from the book is not possible without the use of a computer. Maple is the system to keep the book self-contained.

Reviewer: [Anatoly Filip Grishin \(Khar'kov\)](#)

**MSC:**

- 33-02 Research monographs (special functions)
- 33C05 Classical hypergeometric functions,  ${}_2F_1$
- 33C20 Generalized hypergeometric series,  ${}_pF_q$
- 33D70 Basic hypergeometric functions and integrals in several variables
- 33D90 Applications of basic hypergeometric functions
- 40C99 General summability methods

Cited in **5** Documents**Keywords:**

[hypergeometric term](#); [hypergeometric identities](#); [hypergeometric database](#); [antidifference](#); [Gosper's algorithm](#); [van Hoeff's algorithm](#); [Wilf-Zeilberger method](#)

**Software:**

[Maple](#)

**Full Text:** [DOI](#)