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Extremal problems for close-to-convex functions. (English)
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A function $f(z) = z + a_2z^2 + a_3z^3 + ...$ in class S is called close-to-convex of order β, $\beta \geq 0$, if there is a convex function ϕ so that $f'/\phi' = p^\beta$ for some function p, $p(0) = 1$ and $Re(e^{i\delta}p) > 0$, for δ a real constant. The family of such functions is denoted by C(β). The author solves a variety of extremal problems for C(β) and the related classes $C_m(\beta)$ and Sub C(β), where $C_m(\beta)$ is the subclass of C(β) of functions of the form

$$f(z) = z + a_{m+1}z^{m+1} + a_{2m+1}z^{2m+1} + ...$$

with m-fold symmetry and Sub C(β) is the class of functions subordinate to some function in C(β).
For $\beta \geq 1$ it is shown that the extreme points of the closed convex hull of Sub C(β) have the form

$$f(z) = [w/(\beta{+}1)(x{+}y)][((1{+}xz)/(1{-}yz))^{\beta+1}{-}1], \quad |x| = |y| = |w| = 1, \quad x \neq y.$$

The coefficient problem in C(β) is solved for all $\beta \geq 0$. For $f \in C_m(\beta)$, $\beta \geq 0$, it is shown that $|f'(z)| \leq k'(|z|)$ and $|f(z)| \leq k(|z|)$, where k is defined by

$$k'(z) = (1 + z^m)^\beta/(1 - z^m)^{\beta+2/m}, \quad k(0) = 0.$$

The extreme points of the closed convex hull of $C_m(\beta)$ are given for $\beta \geq 1$. The next to last section considers integral means, where it is shown that $M_p(r, f') \leq M_p(r, k')$ for $f \in C_m(\beta)$, $\beta \geq 0$, $p \in$. In the final section it is shown that $f \in C_m(\beta)$ has a quasiconformal extension when $\beta < 1$ and $m > 4/(1 - \beta)$.

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Keywords : close-to-convex; subordinate; extreme points

Classification:

- 30C75 Extremal problems for (quasi-)conformal mappings, other methods
- 30C80 Maximum principle, etc. (one complex variable)
- 30C45 Special classes of univalent and multivalent functions
- 30C50 Coefficient problems for univalent and multivalent functions