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Parallel accessible domains and domains that are convex in some direction. (English)

Begehr, H. (ed.) et al., Partial differential equations with complex analysis. Dedicated to Robert Pertsch Gilbert on the occasion of his 60th birthday. Harlow: Longman Scientific & Technical. Pitman Res. Notes Math. Ser. 262, 93-105 (1992). [ISBN 0-582-09640-5/pbk]

Let D denote the unit disk. A domain F is convex in the direction ζ ($\zeta \in \partial D$) if for all $z \in F$ and $w = z + u\zeta \in F$ (u real) the segment

$$s(z,w) = \{tz + (1-t)w \mid 0 \le t \le 1\}$$

is in F. A domain F is convex in some direction if there is some $\zeta \in \partial D$ such that F is convex in the direction ζ. A domain F is parallel accessible if there is some direction $\zeta \in \partial D$ such that the complement of F is the union $C\backslash F = \bigcup_{t\in T} \gamma_t \cup \bigcup_{u\in U} \ell_u$, where the γ_t are rays having direction ζ or $-\zeta$, the ℓ_u are lines with direction ζ, and T and U are appropriate parameter sets. The author proves that a domain is convex in some direction if and only if it is parallel accessible.

Let $\widetilde{P}$ denote the set of functions p that are analytic in D, normalized by the condition $p(0) = 1$, and for which there is some real number α such that $R(e^{i\alpha}p(z)) > 0$. The author finds analytic representations for several classes of functions. For example, if f is univalent in D with $f'(0) = 1$, then $f(D)$ is parallel accessible if and only if there is a representation of the form $(1-xz)(1-yz)f' = p$ for some $x, y \in \partial D$ and some $p \in \widetilde{P}$.

Finally, the author introduces parallel accessible domains of order β, whose complement consists of sectors of angle $(1-\beta)\pi$ and with bisector γ_t. A representation is then derived for normalized univalent functions for which $f(D)$ is strongly and weakly accessible of order β.

Most results have been known, but the proof here is different. The method used was developed by the author in an earlier paper [Complex Variables, Theory Appl. 11, No. 3/4, 269-279 (1989; Zbl. 679.30007)].

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- 30C45 Special classes of univalent and multivalent functions
- 30C20 Conformal mappings of special domains