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Algorithms for m-fold hypergeometric summation. (English)
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Zeilberger's algorithm which finds holonomic recurrence equations for definite sums of hypergeometric terms $F(n,k)$ is extended to certain nonhypergeometric terms. An expression $F(n,k)$ is called hypergeometric term if both $F(n+1,k)/F(n,k)$ and $F(n,k+1)/F(n,k)$ are rational functions. Typical examples are ratios of products of exponentials, factorials, Γ function terms, binomial coefficients, and Pochhammer symbols that are integer-linear with respect to n and k in their arguments.
We consider the more general case of such ratios that are rational-linear with respect to n and k in their arguments, and present an extended version of Zeilberger's algorithm for this case, using an extended version of Gosper's algorithm for indefinite summation. In a similar way the Wilf-Zeilberger method of rational function certification of integer-linear hypergeometric identities is extended to rational-linear hypergeometric identities.
The given algorithm on definite summation apply to many cases in the literature to which neither the Zeilberger approach nor the Wilf-Zeilberger method is applicable. Examples of this type are given by theorems of Watson and Whipple, and a large list of identities ("Strange evaluations of hypergeometric series") that were studied by Gessel and Stanton. Finally, we show how the algorithms can be used to generate new identities.

Keywords : Zeilberger's algorithm; hypergeometric identities

Classification:

- 68Q40 Symbolic computation, algebraic computation
- 68W30 Symbolic computation and algebraic computation
- 68Q20 Nonnumerical algorithms
- 68W10 Parallel algorithms