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Identities for families of orthogonal polynomials and special functions. (English)

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An algorithmic approach is decribed for working with orthogonal polynomials and special functions. This approach is based on the notion of an holonomic system $f_n(x)$ which satisfies a linear homogeneous recurrence relation with coefficients that are polynomials in n and x, and furthermore also satisfies a linear homogeneous differential equation with polynomial coefficients. The author introduces the notion of an admissible family which satisfies a holonomic recurrence relation and a differentiation rule with rational coefficients in n and x. It is shown that every admissible family is also a holonomic system and various properties of admissible families are given. Quite a few hypergeometric functions (and thus also a lot of classical orthogonal polynomials) turn out to be admissible families. Algorithms are given to generate shifts $f_{n\pm k}(x)$, derivatives $f_n^{(k)}(x)$, compositions $f_n(r(x))$ for rational functions r, sums and products of admissible families. Other algorithms generate identities and the derivative rule. Several examples are given, including Whittaker functions, Laguerre polynomials, Jacobi polynomials, Krawtchouk polynomials, Meixner polynomials, and Charlier polynomials. Finally the author shows how one can find identities involving parameter derivatives of Gegenbauer and Laguerre polynomials.

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Keywords : orthogonal polynomials; computer algebra; Whittaker functions; Gegenbauer polynomials; Jacobi polynomials; Krawtchouk polynomials; Meixner polynomials; Charlier polynomials; Laguerre polynomials

Classification:

- 33C25 Orthogonal polynomials and functions
- 33C45 Orthogonal polynomials and functions of hypergeometric type