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Koepf, Wolfram
Hypergeometric summation. An algorithmic approach to summation and special function identities. (English)
Advanced Lectures in Mathematics. Wiesbaden: Vieweg. x, 230 p. DM 69.00 (1998). [ISBN 3-528-06950-3/pbk]

The book deals with recently developed algorithmic techniques for hypergeometric summation. Such algorithms depend heavily upon use of a suitable computer algebra system, and the author decided to choose Maple. Some of the algorithms in the book are already included in Release V.4, and further packages are downloadable from the publisher.
The three first chapters contain background material on the functions $\Gamma$, $_pF_q$, $_r\phi_s$, and their special cases; some simple and useful algorithms are presented. Next comes an introductory consideration of holonomic recurrence equations for sums (a development of Sister Celine's idea), again set up as an algorithm. The main part of the book deals with the algorithms by Gosper (indefinite summation), Wilf-Zeilberger (hypergeometric identities), Zeilberger (holonomic recurrence relations), and Petkovsek (polynomial and hypergeometric term solutions of holonomic recurrence equations). Each of these is discussed in great detail, with Maple implementations, and examples. The corresponding $q$-cases are briefly considered.
In the final chapters, the author proceeds from the discrete to the continuous case, from recurrence equations to differential equations. A number of differential and integral analogues are given, including the algorithm by Almkvist and Zeilberger finding a holonomic differential equation for the integral $I(x) = \int_a^b F(x,t)dt$. Each chapter is accompanied by quite a few exercises. While most of them exemplify the algorithms, there are also some that deal more directly with implementations in Maple. Many results (in the exercises as well as in the main text) are of course classical ones whose proofs are well known. It is interesting to see that they are new derivable also by the new approaches. These do not always imply easier proofs than the classical ones, but clearly there is an enormous potential for considering cases that were hitherto beyond reach because of complexity. The book is recommended as a comprehensive and reasonably accessible treatment of the subject.

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*Keywords :* Almkvist; Wilf; Zeilberger; Petkovsek

*Classification:*

- 33-02 Research monographs (special functions)
- 33C20 Generalized hypergeometric series
- 33C45 Orthogonal polynomials and functions of hypergeometric type
- 33D20 Generalized basic hypergeometric series
- 33D15 Basic hypergeometric functions of one variable
- 33D45 Basic hypergeometric functions and integrals in several variables
- 11B65 Binomial coefficients, etc.