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Orthogonal polynomials and computer algebra. (English)

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Summary: Orthogonal polynomials have a long history, and are still important objects of consideration in mathematical research as well as in applications in Mathematical Physics, Chemistry, and Engineering. Quite a lot is known about them. Particularly well-known are differential equations, recurrence equations, Rodrigues formulas, generating functions and hypergeometric representations for the classical systems of Jacobi, Laguerre and Hermite which can be found in mathematical dictionaries. Less well-known are the corresponding representations for the classical discrete systems of Hahn, Krawtchouk, Meixner and Charlier, as well as addition theorems, connection relations between different systems and other identities for these and other systems of orthogonal polynomials. The ongoing research in this still very active subject of mathematics expands the knowledge database about orthogonal polynomials continuously. In the last few decades the classical families have been extended to a rather large collection of polynomial systems, the so-called Askey-Wilson scheme, and they have been generalized in other ways as well.

Recently new algorithmic approaches have been discovered to compute differential, recurrence and similar equations from series or integral representations. These methods turn out to be quite useful to prove or detect identities for orthogonal polynomial systems. Further algorithms to detect connection coefficients or to identify polynomial systems from given recurrence equations have been developed. Although some algorithmic methods had been known already in the last century, their use was rather limited due to the immense amount of calculations. Only the existence and distribution of computer algebra systems makes their use simple and useful for everybody.

In this plenary lecture an overview is given of how algorithmic methods implemented in computer algebra systems can be used to prove identities about and to detect new knowledge for orthogonal polynomials and other hypergeometric type special functions. Implementations for this type of algorithms exist in Maple, Mathematica and REDUCE, and maybe also in other computer algebra systems. Online demonstrations will be given using Maple V.5.

Keywords : orthogonal polynomials; computer algebra systems

Classification:

- **68W30** Symbolic computation and algebraic computation
- **33C45** Orthogonal polynomials and functions of hypergeometric type