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Bieberbach's conjecture, the de Branges and Weinstein functions and the Askey-Gasper inequality. (English)

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This paper gives an excellent description of the history of the Bieberbach conjecture, covering the period from its conception in 1916 [L. Bieberbach, S.-B. Preuss. Akad. Wiss. 38, 940–955 (1916)] to its final proof in 1984 [*L. de Branges*, Acta Math. 154, 137–152 (1985; Zbl 0573.30014)] and a subsequent proof in 1991 [*L. Weinstein*, Internat. Math. Res. Notices 5, 61–64 (1991; Zbl 0743.30021)]. This 27 page paper (including a list of 75 references) touches upon each historically important step of the nearly 70 years between Bieberbach and de Branges and reads like a 'hard-to-put-down detective story'. Discussing the contributions by Loewner, Nevanlinna, Littlewood, Dieudonné, Rogosinski, Paley, Robertson, Schiffer, Grunsky, Hyman, Reade, Garabedian, Schiffer, Charzyński, the story finally reaches Milin and Lebedev and their famous conjecture. Proving this conjecture, de Branges succeeded to prove both the Robertson and Bieberbach conjectures in 1984. At that time a result on special functions that 'was on the shelf' [*R. Askey, G. Gaspar*, Am. J. Math. 98, 709–737 (1976; Zbl 0355.33005)] suddenly played the role of the missing link! The final touch in the application of the Askey-Gaspar identity (representing the de Branges function as a linear combination of ${}_3F_2$ hypergeometric functions) and the Askey-Gaspar inequality (asserting that each of the ${}_3F_2$'s was non-negative) was actually Clausen's identity, expressing the ${}_3F_2$ as the square of a ${}_2F_1$.

Moreover, the paper covers the application of computer algebra (specifically Zeilberger's algorithm) in automated proofs and also gives an extensive description of the proof of the Bieberbach conjecture by Leonard Weinstein in 1991 (cited above), a proof that circumvents the use of the Askey-Gaspar inequality.

The author is to be commended for this lucid description of the history and proof of one of the famous conjectures in mathematics.

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Keywords : univalent functions; Bieberbach conjecture; Robertson conjecture; Milin conjecture; convex functions; close-to-convex functions; Grunsky inequalities; Shiffer variation; support points; extreme points; Loewner differential equation; Loewner theory; Lebedev-Milin inequalities; de Branges theorem; de Branges functions; Weinstein functions; hypergeometric functions; generalized hypergeometric series; Askey-Gaspar inequality; Askey-Gaspar identity; Legendre addition theorem; FPS algorithm; Zeilberger algorithm; Maple; symbolic computation; computer algebra

Classification :

∗30C50 Coefficient problems for univalent and multivalent functions

30C45 Special classes of univalent and multivalent functions
30C80 Maximum principle, etc. (one complex variable)
33C20 Generalized hypergeometric series
33C45 Orthogonal polynomials and functions of hypergeometric type
33F10 Symbolic computation of special functions
68W30 Symbolic computation and algebraic computation