



## Special Issue on Orthogonal Polynomials and Computer Algebra

### Foreword of the Guest Editors

In the last decade major steps toward an algorithmic treatment of orthogonal polynomials and special functions have been made, notably Zeilberger's brilliant extension of Gosper's algorithm on algorithmic definite hypergeometric summation. By implementations of these and other algorithms, symbolic computation has the potential to change the daily work of anyone who uses orthogonal polynomials or special functions in research or applications. It can be expected that symbolic computation will also play an important role in on-line versions of major revisions of existing formula books in these areas.

Many articles on the algorithmic treatment of orthogonal polynomials and special functions and on applications of such algorithms have been published in the meantime. However, such articles are distributed widely in the literature. To bring together some of these efforts Wolfram Koepf organized a session on orthogonal polynomials at the first ISAAC Congress (<http://www.math.udel.edu/isaac>) with an emphasis on the use of computer algebra. ISAAC is the International Society for Analysis, its Applications and Computation. This congress took place at the University of Delaware, Newark, DE, U.S.A., June 3–7, 1997.

In this special issue we have collected articles about the interaction between computer algebra and orthogonal polynomials and special functions. Some of the participants at the ISAAC Congress have submitted their papers, but this special issue on Orthogonal Polynomials and Computer Algebra was open to all. Rather than a proceedings of a session, the issue is meant as a state of the art account of these topics.

In the call for papers the following was requested:

Contributions should discuss non-trivial usage of symbolic computation which significantly contributes to the theory of Orthogonal Polynomials and Special Functions. Examples of categories in which contributions may fall are:

- New symbolic algorithms for obtaining specific results in OP & SF. This may also be a presentation of a drastically improved implementation of an existing algorithm.
- Proofs aided by symbolic computation of significant new results in OP & SF which are yet intractable by purely human effort.
- New significant results in OP & SF which are finally provable without computer aid, but which would have been hard to find without experimentation by symbolic computation. The description of this experimentation should be illustrative for future efforts by others.
- Aspects of symbolic computation in connection with the production of on-line textbooks and formula dictionaries on OP & SF.

All submissions were refereed according to the usual JSC refereeing process. An anonymous refereeing process was also applied to the contribution of one of the Guest Editors.

The issue contains seven papers. The article *Toward symbolic integration of elliptic integrals* by Carlson is the only one whose emphasis is on the wider range of special functions and does not deal directly with orthogonal polynomials. In this paper, a new algorithm for the representation of elliptic integrals is presented.

The following papers deal with orthogonal polynomials of one variable: in the article *Decomposition of polynomials with respect to the cyclic group of order  $m$*  by Ronveaux, Zarzo, Area and Godoy differential equations for  $m$ -ary parts, in particular ( $m = 2$ ) even and odd parts, of orthogonal polynomials are investigated. *Inversion problems in the  $q$ -Hahn tableau* are solved by Area, Godoy, Ronveaux and Zarzo. A Maple package including *Algorithms for  $q$ -hypergeometric summation in computer algebra*, in particular the  $q$ -analogues of Gosper's, Zeilberger's and Petkovšek's algorithm, is applied by Böing and Koepf to orthogonal polynomials of the  $q$ -Askey–Wilson tableau. Finally, Foupouagnigni, Ronveaux and Hounkonnou compute *The fourth-order difference equation satisfied by the associated orthogonal polynomials of the  $\Delta$ -Laguerre–Hahn class*.

Multivariate orthogonal polynomials are the topic of the papers *Computing with differential-difference operators* by Dunkl and *Partial differential equations and bivariate orthogonal polynomials* by Schwartz. Dunkl presents a computational scheme that can be used to discover and prove operator identities in an algebra of operators including Dunkl operators, and he successfully applies this scheme to obtain new explicit results. Schwartz arrives by computer algebra at a characterization of disk polynomials as the unique bivariate orthogonal polynomials being eigenfunctions of a pair of linear differential operators together with some further conditions.

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