

# Orthogonal Polynomials and Special Functions

*SIAM Activity Group on Orthogonal Polynomials and Special Functions*



## Newsletter



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Once again, a great deal of material comes from the OP-SF Net. We thank all the authors for sending their items. As usual, we hope you find this issue interesting and useful, and remind you that you can send items for future issues to either of us.

October 1, 1999

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### Reports from Meetings and Conferences

#### 1. Workshop on Special Functions – Asymptotics, Harmonic Analysis and Mathematical Physics. City University of Hong Kong, Hong Kong, June 21-25, 1999.

The International Workshop on Special Functions – Asymptotics, Harmonic Analysis and Mathematical Physics was held June 21-25, 1999 at the City University of Hong Kong. The organizing committee was Charles Dunkl, Mourad Ismail, and Roderick Wong. The scientific program was outstanding, with a very full schedule of plenary lectures and workshops being presented from 9:00 am until 6:00 pm every day. I do not normally attend conferences devoted to asymptotics, so one of the most exciting parts of the program for me were the many fine presentations by the specialists in this area. The talks of Frank

### From the Editors

This is the last issue of the Newsletter for 1999. Once more there is information on many meetings related to our areas of research. Of special interest is the report on the Panel Discussion held during the Hong Kong workshop in June.

**SIAM Activity Group**  
 on  
*Orthogonal Polynomials and Special Functions*

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*THE PURPOSE of the Activity Group is*

*—to promote basic research in orthogonal polynomials and special functions; to further the application of this subject in other parts of mathematics, and in science and industry; and to encourage and support the exchange of information, ideas, and techniques between workers in this field, and other mathematicians and scientists.*

Olver, Roderick Wong, Nico Temme, Mark Dunster, and Walter Gautschi were very helpful to me, and suggested a number of new approaches to problems of long standing concern. In another direction, the community of users of hypergroups were quite active, and hypergroups appeared in at least five talks.

In his paper in the proceedings of the Madison conference of 1975, Tom Koornwinder said "It does not yet seem to be the time to make a final decision, which systems should be called two-variable analogues of the classical orthogonal polynomials." It is now 25 years later, and after many fits and starts the time seems to be at hand. I was astonished to see that the effort to answer this question was seriously addressed by a number of speakers, including Lance Littlejohn, Yuan Xu, Hjalmar Rosengren, Kil Kwon, Charles Dunkl, and your reporter. It was exciting to see the variety of approaches and results that were obtained. The final answer is not there, but this is a problem whose time has come. I also feel obliged to comment on the very energetic presentation of Adam McBride. The pride of Scotland managed to inspire an exhausted audience on the last day of the conference with a celebration of fractional integration. A delight.

The physical surroundings were magnificent. The city is quite beautiful, and the streets were electric with the

excitement of the new political order. Despite the recent financial collapse of the Asian Tigers, there was nothing depressed about the economy in Hong Kong. Certainly one of the most popular attractions to Hong Kong is the food, and the conferees were treated to many epic and delicious meals. The final banquet featured a wonderful selection of dishes including traditional standards like Chinese bar-b-que, and special delights like crispy fish maws, and sweet bean soup for desert. A special treat at the banquet was an informal concert by Jojo Wong who played the erhu (a Chinese version of the violin) and Moody Lam who played the pipa (a Chinese guitar). The traditional music was a delight, even this untrained ear heard hints of Celtic fiddle tunes in the erhu, and wonderful Andalusian overtones in the pipa. The international audience gave the talented musicians a well deserved standing ovation. Finally the conference secretary, Colette Lam was one of the most competent and helpful people that I have ever dealt with. She solved all problems, and did so unobtrusively.

The 120 participants from 26 countries were edified, charmed, and delighted by the conference, the city, and the people. A heartfelt thank-you to the organizers, and especially to Roderick Wong, who made this all possible.

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From June 21 to June 25 an International Workshop on Special Functions, with an emphasis in asymptotics, harmonic analysis and mathematical physics was held at the City University of Hong Kong. The organizing committee, consisting of Charles Dunkl, Mourad Ismail and Roderick S. C. Wong, were in charge of a high level program: 18 one-hour invited lectures by the most relevant specialists in these subjects, 51 half-hour invited speakers and 42 short communications in three parallel sessions gave the possibility to experience the modern building of this young university. In fact, the workshop was one of the main activities of the university's 15th anniversary.

All participants must thank the organizers for the friendly atmosphere as well as the computer facilities and the opportunity to taste authentic Chinese food not only in the university restaurant but in (at least) two places where we enjoyed the quality and quantity of a delicious buffet. Everyone agreed that a wonderful job was done by Colette Lam, an excellent secretary. (I think she is one of the most efficient people that I ever met as an administrative support in a scientific event.)

On the other hand, from the scientific point of view, we had the occasion to hear from people working on many non-overlapping topics. What was very instructive for me was the set of wonderful lectures in orthogonal polynomials in several variables, a subject too often neglected in conferences on orthogonal polynomials, as well as the lectures on the role of special functions in integrable systems, soli-

tons and algebraic combinatorics which open new fields of applications and increase the value of the research in our domain. The  $q$ -world was also present with many contributions, in particular I was very impressed by the work of Suslov on basic Fourier series.

Finally, I would like to remark that the workshop justified the round trip from the dilapidated airport in Madrid airport to the modern and quiet one in Hong Kong. If you travel for 14 hours in each direction, you can imagine how it is possible to read and do mathematics away from your office!

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The participants came from all disciplines, from applied to theoretical areas, from the classical to basic group theoretical  $q$ -aspects of special functions. Lectures on “Asymptotics” were given by Wong (an overview of methods from uniform asymptotics, in particular for orthogonal polynomials), Olver (a new method for constructing error bounds for the asymptotic expansion of solutions of a class of difference equations, with application to Legendre functions), Huntley (on asymptotics of  $Gl(3, R)$  Whittaker functions), Dunster (on Airy-type uniform asymptotic expansions of generalised Bessel polynomials by using methods for differential equations), Temme (on asymptotic relations between the hypergeometric polynomials of the Askey scheme), O’Malley (on several problems from singular perturbations that arise in boundary value problems), Nakano (on the WKB method for third order linear differential equations, with classification and discussion on the Stokes phenomenon), Qiu (on a new method to give uniform Airy-type asymptotics for orthogonal polynomials governed by an exponential weight function), Gerardin (on asymptotics of the eigenfunctions on type  $A$  affine buildings with rank  $\geq 2$ ), Martínez-Finkelshtein (on the distribution of the zeros of certain classical families of polynomials with parameters coupled with the large degree of the polynomials), López (on asymptotic expansions of the symmetric Carlson-type elliptic integrals, complete with error bounds), Zarzo (on the zero distribution of solutions of oscillatory second order differential equations) and Zhao (on smoothing the Stokes discontinuity for a generalised Bessel function).

I learned about several new problems, had many interesting discussions with new and old workers in the field of asymptotics and special functions, also in connection with a new effort to prove the Riemann hypothesis. We hope to meet again, and are preparing another meeting in this series of workshops in Amsterdam, 2002.

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The International Workshop on Special Functions-Asymptotics, Harmonic Analysis and Mathematical

Physics was held at the City University of Hong Kong (in Kowloon, Hong Kong), June 21-25, 1999. The organizers were Professors Charles Dunkl (U. Virginia), Mourad Ismail (U. South Florida), Roderick Wong (City U. Hong Kong).

**Participants:** There were 119 registered attendees from 29 different countries. Here is a list of countries with 4 or more participants (with actual number in parentheses): U.S. (30), Japan (18), Hong Kong (10), China (6), Spain (6), Canada (5), France (5), Germany (5), Netherlands (5), S. Korea (4). **Lectures:** There were 18 plenary lectures (50 mins.), 48 invited lectures (30 mins.) and 41 contributed talks (20 mins.). The plenary speakers were: K. Aomoto (Nagoya U., Japan), R. Askey (U. Wisconsin), T. Baker (Kyoto U., Japan), C. Berg (U. Copenhagen, Denmark), C. Dunkl (U. Virginia), G. Gasper (Northwestern U.), W. Gautschi (Purdue U.), E. Koelink (T.U. Delft, NL), A. McBride (U. Strathclyde, UK), F. Olver (U. Maryland), R. O’Malley (U. Washington), E. Opdam (Leiden U., NL), D. Stanton (U. Minnesota), N. Temme (CWI, NL), A. Terras (UCSD), L. Vinet (U. Montreal), R. Wong (City U., HK), Y. Xu (U. Oregon).

**Themes:** In asymptotics, the talks concerned the new methods of uniform asymptotic expansions and hyperasymptotics. These methods allow greater regions of validity and exponential rates of decay of the error as the number of used terms from the asymptotic expansion increases. The special functions to which these methods are applied come mostly from differential equations. The workshop themes of mathematical physics and harmonic analysis became closely intertwined. Techniques from group theory and algebraic combinatorics are blended with the classical theories of hypergeometric functions and polynomials to produce important new structures, which allow more detailed study of the integrable models of quantum mechanics. In particular, systems of many identical particles, which allow symmetry group actions, such as Calogero-Moser-Sutherland models, occur in this context. In turn, these ideas from physics cross-fertilize research in special functions. Of course this includes functions of one or several variables.

**Future Directions:** A panel discussion was part of the program. The plenary speakers and some others put forth their ideas about important problems. There was a strong sense that the interaction between modern mathematical physics and special functions (including group theory and algebraic combinatorics) has been very fruitful and beneficial. In more detail, some of the mentioned topics were: extend the idea of hyper-asymptotic expansions to parametrized integrals and solutions of difference equations, discover better asymptotics for parabolic partial differential equations, the theory of expansions of functions in series of orthogonal polynomials of several variables, the study of multi-variable special functions and math-

ematical physics (including the hypergeometric functions of basic( $q$ -) and elliptic types). As well, important connections between solvable quantum models and algebraic combinatorics (examples: the Rogers-Ramanujan identities, the  $n$ -factorial conjecture) were mentioned.

**Support:** Contributions were made by Epson Foundation, K.C. Wong Education Foundation, Lee Hysan Foundation Ltd., Liu Bie Ju Centre for Mathematical Sciences at City U. HK, NSF, and the Royal Plaza Hotel (Kowloon).

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## Hong Kong Panel Discussion

This is a report on the Panel discussion held on Thursday, June 24, 1999, 5.00-6.30 p.m. during the International Workshop on Special Functions, Asymptotics, Harmonic Analysis, and Mathematical Physics at the City University of Hong Kong.

The basic account was written by Tom Koornwinder modified by input from the participants and from Daniel Lozier, Martin Muldoon and André Ronveaux.

The session was chaired by Charles Dunkl. He stressed the importance of a discussion of important directions so that the rest of the world would know what we were up to. This could be useful both to funding agencies and to inform young people of directions in research. He proposed a discussion of research perspectives for, successively:

- 1. Asymptotics
- 2. Harmonic analysis
- 3. Classical special functions
- 4. Mathematical physics

In the ensuing discussion, classical special functions were skipped as a separate item, but they were covered by the item on harmonic analysis.

Mourad Ismail raised the issue of future meetings in the series Fields-Toronto (1995) - CRM-Montreal (1996) - Mount Holyoke (1998) - Hong Kong (1999) - Arizona (2000) ... with the suggestion that there should be more than one year between meetings and that perhaps there should be a joint meeting with the (mainly European) series on Orthogonal Polynomials. There was unanimous agreement that the series should continue and support for the idea of a meeting in the Netherlands in 2002 to be organized by Tom Koornwinder, Nico Temme and Erik Koelink. Luc Vinet proposed that there should be some kind of coordinating group for these meetings. There was general agreement that the SIAM Activity Group on Orthogonal Polynomials and Special Functions might provide

this coordination. It was stressed that this did not include the actual organization of meetings.

### 1. Asymptotics

#### 1.1. Frank Olver

For ordinary differential equations it is time to write another book to replace W. Wasow's "Asymptotic expansions for ordinary differential equations", Wiley, 1965. The new book should be more practical, giving simpler proofs, examples, error bounds (where possible), increased regions of validity in the complex plane, and a description of the different types of asymptotic solutions (explicit and implicit). Some of the recent work on re-expansions of the remainder terms (hyper-asymptotics) should also be included.

For difference equations, the whole asymptotic theory needs reworking in a much simpler and more readily applicable form than in the classic (and almost impenetrable) papers of G. D. Birkhoff and W.J. Trjitzinsky. Proofs can be given (by use of boundary-value type methods) that obviate the need to pass from the set of integers on which the solutions actually live, into the complex plane—with all its attendant problems of analyticity. The two 1992 papers of R. Wong and H. Li are a good beginning. Expansions should be sought in inverse factorial series as well as, or in place of, conventional expansions in inverse powers.

#### 1.2. Nico Temme

For asymptotics of integrals, the theory and construction of error bounds lags behind the corresponding results for differential equations.

#### 1.3. Roderick Wong

He agreed with Temme. In addition to the problem of error bounds, for asymptotics of integrals, regions of validity are usually smaller than the corresponding ones that can be established by using the differential equation theory. This situation needs to be improved. For asymptotic solutions of difference equations, Airy-type expansions should be considered. He referred to the paper by O. and R. Costin "Rigorous WKB for finite-order linear recurrence relations with smooth coefficients", *SIAM J. Math. Anal.* **27** (1996), 110–134. In singular perturbations, derivations of asymptotics are mostly formal; rigorous proofs may give more insight. For the nonlinear Klein-Gordon equation, even to show that the solution is uniformly bounded for all time is not a simple matter. For specific problems even with two terms, it is difficult to show that the remainder is of the right order.

#### 1.4. Robert O'Malley

Parabolic p.d.e.'s sometimes have traveling wave solutions (like hyperbolic tangent in Burgers' equation). To obtain such solutions one has to go back to solutions of very special o.d.e.'s and their asymptotics. Dumb computing won't work there. Applied people have to learn

about the powerful tools of solutions by special functions and their asymptotics.

### 1.5. Eric Opdam

People in this workshop include, on the one hand, researchers with very detailed knowledge of one-variable theory and, on the other hand, workers in higher rank theory (i.e., more variable theory associated with root systems). They should join forces in order to do asymptotics in the higher rank case. Olver commented that in asymptotics the difficulty goes up by an order of magnitude as each new variable is added. Moreover, because of the increasing complexity a point of diminishing returns is soon reached. Koelink responded that the symmetry available in the higher rank case may help in doing asymptotics.

## 2. Harmonic analysis

### 2.1. George Gasper

A big problem is that of convolution structures. He suggested a study of nonnegative kernels, as done for  $q$ -Racah polynomials in the paper G. Gasper and M. Rahman, "Nonnegative kernels in product formulas for  $q$ -Racah polynomials", J. Math. Anal. Appl. 95 (1983), 304-318. They were able to handle only the case with symmetry; the non-symmetric case is still open. Many other cases (of convolution structures) are still unknown. Dunkl remarked that all positivity results are important.

### 2.2. Erik Koelink

Be aware of possible applications of special functions in signal analysis, stochastic processes, financial mathematics. Furthermore, a lot of inspiration for further work can be obtained from the work by Percy Deift on Riemann-Hilbert problems and random matrix theory, see for instance P. Deift, "Orthogonal polynomials and random matrices: a Riemann-Hilbert approach", Courant Lecture Notes in Math., Vol. 3, Courant Institute of Math. Sciences, 1999. See also the topics covered in this fall's special semester on Foundations of Computational Mathematics in Hong Kong (<http://www.damtp.cam.ac.uk/user/na/FoCM/HK.html>), in particular the workshop on Minimal Energy Problems (<http://www.math.usf.edu/FoCM99/>). Finally, KZ equations and elliptic hypergeometric functions are promising fields.

### 2.3. Richard Askey

Terwilliger's results on Leonard pairs (see Terwilliger's lecture in this workshop) gives a new approach to Leonard's characterization of  $q$ -Racah polynomials and some related polynomials in D.A. Leonard, "Orthogonal polynomials, duality and association schemes", SIAM J. Math. Anal. 13 (1982), 656-663. Terwilliger's approach works for any field. What are the implications for special functions for other fields than the reals? Next, Askey sug-

gests finding an integral equation for the so-called Barnes G-function, which satisfies  $G(x+1) = \Gamma(x)G(x)$ ,  $G(1) = 1$ . There is a problem on this function in Whittaker and Watson with a reference to late 19th century work by a Russian. Barnes has a number of papers on this function, in the early part of this century. There might be an infinite dimensional integral which represents this function, based on the form Selberg's integral takes. The G-function shows up in a number of places. Andrew Lenard used it in a paper (J. Math. Phys. 5 (1964), 930-943) on the strong Szegő limit theorem. Szegő pointed out to him how some of his results could be stated using the G-function. It also shows up in K-theory. Finally, for 9-j-symbols, a representation as a double sum should be found. Such a representation can be expected, since they have a two variable orthogonality.

### 2.4. Christian Berg

1. Concerning the indeterminate moment problem one should try to find a closer relationship between the growth rate of the coefficients of the three term recurrence relation for the orthonormal polynomials and the growth properties of the entire functions in the Nevanlinna matrix. In particular one should relate the order of these functions to the growth rate of the coefficients. In a special case of birth and death rates being a specific polynomial in  $n$  of degree four, it turned out that the entire functions have order  $1/4$ , see C. Berg and G. Valent, "The Nevanlinna parametrization for some indeterminate Stieltjes moment problems associated with birth and death processes", Meth. Appl. Anal 1 (1994), 169-209. In a recent manuscript by G. Valent, "Indeterminate moment problems and a conjecture on the growth of the entire functions of the Nevanlinna parametrization", there is an example of birth and death rates being polynomials of degree 3 and the order of the entire functions are  $1/3$ . The paper also formulates a conjecture.

2. It was proved by C. Berg and W. Thill, "Rotation invariant moment problems", Acta Math. 167 (1991), 207-227, that a moment problem in  $\mathbb{R}^n$ ,  $n > 1$ , can have a unique solution  $\mu$  (i.e.,  $\mu$  is a determinate measure on  $\mathbb{R}^n$ ) and yet the polynomials are not dense in the Hilbert space  $L^2(\mathbb{R}^n, \mu)$ . Apart from the rotationally invariant case no criterion seems to be known for this phenomenon to happen, and this kind of question needs further study.

### 2.5. Eric Opdam

Special function theory reflects deep properties in group theory, mathematical physics and number theory. For advances in special functions one should better understand these three fields. In mathematical physics the Calogero-Moser system is a deformation of the boson gas. Find correlation functions for the Calogero-Moser system as generalizations of such functions for the boson gas. Macdonald theory corresponds to the boson gas with periodic con-

straints. In getting rid of these constraints one would arrive at Macdonald functions for the noncompact case (see lectures by Koelink and Stokman at this workshop for the rank one case), which might be studied from the point of view of double affine Hecke algebras. In representation theory one should not restrict oneself to the spherical case. Askey added that we need to get past page 2 of Zygmund and start solving hard problems with real applications.

## 2.6. Yuan Xu

Most of the  $\mathbb{L}^p$  theory for orthogonal polynomials in several variables is not yet understood. There is the question of finding explicit formula for orthogonal polynomials associated to a weight function that is invariant under an octahedral group. For example, Dunkl's h-harmonics associated to the type B weight. Such a basis may be useful in studying cubature formulae (numerical integration formulae). There is a possible connection between common zeros of invariant orthogonal polynomials for the weight function  $(x_1^2 - x_2^2)^2(x_2^2 - x_3^2)^2(x_3^2 - x_1^2)^2$  on the sphere  $\mathbb{S}^2$  and a family of cubature formulae on  $\mathbb{S}^2$  conjectured by V. I. Lebedev.

## 2.7. Dennis Stanton

Macdonald's conjecture about the positivity of the coefficients in the polynomial expansion of the Macdonald-Kostka coefficients and the Garsia-Haiman n factorial conjecture are at present the most important conjectures in algebraic combinatorics, see for instance A.M. Garsia and M. Haiman, Some natural bigraded  $S_n$ -modules and  $q, t$ -Kostka coefficients. The Foata Festschrift. Electron. J. Combin. 3 (1996), no. 2, Research Paper 24, [http://www.combinatorics.org/Volume\\_3/Abstracts/v3i2r24.html](http://www.combinatorics.org/Volume_3/Abstracts/v3i2r24.html) Furthermore, problems associated with graph spectra for finite matrix groups (A. Terras, lecture at this workshop) are very important.

## 2.8. Audrey Terras

Graph spectra could lead to some interesting special functions.

## 2.9. Charles Dunkl

1. I suggest the study of orthogonal polynomials (special functions, transforms) associated with the non-crystallographic reflection groups, that is, H3 and H4; of icosahedral type. These groups are related to quasicrystals; an area of research in both physics and mathematics.

2. Alberto Grünbaum (referring to his work with Duijstermaat on bispectral problems) suggested to me earlier, the idea of finding bispectral differential-difference operators (eigenfunctions of one operator satisfy another d-d equation with respect to the eigenvalue). Alberto and I worked out a small example (a new way of looking at a known situation) which involved a perturbed one-variable differential-difference operator related to the group  $Z^2$ .

Thus I speculate there may be perturbations for a limited set of parameter values (more speculation: those called singular by de Jeu, Opdam and myself).

3. I speculate that the phenomenon called superintegrability (e.g., Konstein) may apply to algebras of differential-difference operators with integer parameter values.

## 2.10. Mourad Ismail

Study asymptotics of

$$P_n(x) := c_n \int_a^b \dots \int_a^b \prod_{j=1}^n (x - \lambda_j) \times$$

$$\prod_{1 \leq i < k \leq n} |\lambda_i - \lambda_k|^{2\beta} d\alpha(\lambda_1) \dots d\alpha(\lambda_n),$$

first for  $\beta = 1$ , next for general  $\beta$ . In the case  $\beta = 1$ , it is a formula valid for all orthogonal polynomials. In the case  $\beta \neq 1$ , the polynomials are no longer orthogonal.

## 2.11. Adam McBride

One should not generalize without further motivation, for instance working on a transformation in  $n$  variables that reduces to the Laplace transform if  $n - 1$  variables are taken zero. McBride also observed that there are two kinds of generalization. (i) Thesis-type problems that are often somewhat contrived; (ii) Real problems. We should resist generalisation for its own sake.

## 2.12. Tom Koornwinder

The positivity result by Margit Roesler (her lecture at this workshop) should be extended to the positivity proof for the kernel in the integral representation for a Heckman-Opdam hypergeometric function (Jacobi function) associated with a root system. Similarly, the positivity of the kernel for the product formula of such functions, both in the compact and in the non-compact case, should be proved (i.e., their hypergroup property). For parameter values admitting an interpretation as spherical functions these positivity results are clear. As for quantum groups and related q-special functions, the spherical theory for quantum analogues of compact symmetric spaces is already much advanced by work of Noumi and coworkers, including Dijkhuizen and Stokman. However, the work on the spherical theory for quantum analogues of non-compact Riemannian symmetric spaces has just started (see lectures by Koelink and Stokman in this workshop), while even less work has been done on the case that  $q$  is on the unit circle, but not necessarily a root of 1 (analytically very interesting and challenging).

## 2.13. Norman Wildberger

Give explicit descriptions of representations of groups. Koelink adds that a lot in this direction can already be found in the 3-volume work "Representation of Lie groups

and special functions" by N.J. Vilenkin and A.U. Klimyk.

### 3. Mathematical physics

#### 3.1. Luc Vinet

Solvable models are important and lead to special functions. In physics it is very important to explain phenomena by symmetry. More people should be working on the factorial  $n$  problem. Find a physical interpretation for the generalized Rogers-Ramanujan identity pointed out by Dennis Stanton (his lecture at this workshop), see also the preprint "Variants of the Rogers-Ramanujan identities" by T. Garrett, M.E.H. Ismail and D. Stanton, <http://www.math.umn.edu/~stanton/pap.html>. There are many open problems for special functions in black hole physics and in string theory. There is lots to explain in the interplay between mathematical physics and algebraic combinatorics. Finally study special function solutions of non-linear equations (Painlevé theory).

#### 3.2. Frank Olver

The project "Digital Library of Mathematical Functions" (see Lozier's lecture in this workshop) will be very important for bringing special functions to physicists and vice versa, and also for drawing attention to research needs of practical importance. There have been an enormous, and increasing, number of citations to the "Handbook of mathematical functions", Abramowitz and Stegun (eds.) in the Science Citation Index, and the majority are in physics papers. Koelink asked if a similar citation analysis has been made for the Bateman project. Olver is not aware of such an analysis.

#### 3.3. Jesus Sánchez Dehesa

Study entropy integrals (see Dehesa's lecture in this workshop). These are important in physical applications and much of the theory still has to be developed. Orthogonal polynomials in several variables are also relevant for physics and in connection with numerical problems. Study the asymptotics of  $\mathbb{L}^p$  norms of sequences of orthogonal polynomials. We know only the dominant terms. Finally there is an important connection between Information theory and Special functions.

In closing the discussion, Dunkl suggested that people send e-mail to the organizers or to opsftalk with other suggestions.

In correspondence arising in connection with this Report, André Ronveaux mentions that "Differential Equations problems coming from Separation of variables in classical linear PDE of Mathematical Physics are not yet completely investigated, even in dimension 3. (I am thinking mainly on Spheroidal and Ellipsoidal coordinate systems involving many eigenvalue problems, connection between

solutions...)".

Martin Muldoon  
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## Research Perspectives

As a follow-up of the Honk Kong panel discussion, the SIAM activity group will maintain a list of "research perspectives" on the web. The activity group homepage will soon contain a link to a list of possible directions in research relevant for (young) people interested in our field. This link will be coordinated by Walter Van Assche. Please send possible suggestions and items for inclusions to [walter@wis.kuleuven.ac.be](mailto:walter@wis.kuleuven.ac.be)

Walter Van Assche  
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## Forthcoming Meetings and Conferences

### 1. Workshop on Contemporary Problems in Mathematical Physics, Institut de Mathématiques et de Sciences Physique (IMSP), Cotonou, République du Benin (Africa), October 31st - November 7th, 1999.

A Workshop on Contemporary Problems in Mathematical Physics will take place at The Institut de Mathématiques et de Sciences Physiques (Cotonou, République du Benin), from October 31st to November 7th, 1999. L'Institut de Mathématiques et de Sciences Physiques (L'IMSP), as one of the Graduate Schools of Université Nationale de Benin, has organized a third Cycle in Mathematics and Physics since 1989. L'IMSP was the first African Institution initiated with the support of the Third World Academy of Science, and affiliated in 1988 to the ICTP (International Center for Theoretical Physics, Trieste, Italy) created by the late Nobel laureate Abdus Salam. PhD students coming from many countries around Benin and having a bachelor's degree in Mathematics or Physics, are selected every two years, in order to start a DEA (Diplôme d'Études Approfondies), followed by the preparation during at least two more years of a PhD Dissertation in IMSP and abroad (mainly in Europe and North America). One of the orientations, within Mathematical Physics covers the topic "Special Functions and Orthogonal Polynomials".

The main objective of the workshop is to contribute to the development of a **critical mass** of researchers in Africa in such a dynamic area as mathematical physics. The main **aim** of the workshop is to bring together specialists from diverse topics in Theoretical and Mathematical Physics. It will offer to the international scientific community the possibility of exchanging useful informa-

tion on contemporary problems in these fields through direct interaction. The workshop will also provide young African researchers with an opportunity to know each other and to initiate scientific cooperation. Therefore, the workshop will help to strengthen research capacity and revitalize activities in mathematical physics in African universities. The workshop will include invited review talks and contributed communications. The invited review talks will be presented in plenary sessions whereas the contributed papers will be presented in three working groups corresponding to the three main topics.

#### Main topics:

1. Coherent States, Wavelets and Geometric Methods in Theoretical Physics
2. Quantum Field Theory, Atomic and Molecular Physics
3. Operator Theory and Orthogonal Polynomials

The official languages of the workshop are English and French.

The activity is open to scientists from all countries. Travel and subsistence expenses of the participants should be covered by the home institution. However, limited funds will be made available for scientists from African countries. Participants interested in presenting a paper are encouraged to send a camera ready copy. The deadline for submission of papers and/or of requests for financial support is July 15th, 1999. Applications should be submitted to:

Prof. M. N. Hounkonnou (Workshop in COPROMAPH)  
 Institut de Mathématiques et de Sciences Physiques (IMSF)  
 B. P. 613, Porto-Novo, République du Benin  
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The local **Organizing Committee** consist of: Prof. M. N. Hounkonnou, (Unité de Recherche en Physique Théorique), Institut de Mathématiques et de Sciences Physiques, Porto-Novo-Benin (E-mail: hounkon@syfed.bj.refer.org) and Prof. W. A. Lester, Department of Chemistry, University of California, Berkeley (E-mail: walester@cchem.berkeley.edu)

The **Scientific Advisory Committee** consist of: S. T. Ali (Canada), J.P. Antoine (Belgium), A. Banyaga (USA), A. Bellemans (Belgium), S. Belmehdi (France), X. Chapuisat (France), J. M. Combes (France), J. P. Gazeau (France), G. Ciccotti (Italy), C. S. Diatta (Sénégal), J. P. Ezin (Benin), C. Goudjo (Benin), M. N. Hounkonnou (Benin), R. Kerner (France), W. Koepf (Germany), W. A. Lester (USA), M. Mareschal (CECAM), A. Msezane (USA), R. Murenzi (USA), A. Ronveaux (Belgium), J. P.

Ryckaert (Belgium), J. Shabani (UNESCO), B. Torrésani (France).

André Ronveaux  
 (Andre.Ronveaux@fundp.ac.be)

#### 2. Workshop on the Minimal Energy Problems, City University of Hong Kong, Hong Kong, November 8-12, 1999.

A workshop on Minimal Energy Problems will be held from November 8-12, 1999, at the City University of Hong Kong, as part of a series of Foundations of Computational Mathematics workshops to be held this fall.

The meeting will emphasize research work related to distributing points on a sphere and on general Riemann surfaces, discrepancy results (e.g. for Fekete points), potential theoretic tools and applications to orthogonal polynomials, random matrices, integrable systems, etc.

For further information, contact Ed Saff (esaff@math.usf.edu) or Arno Kuijlaars (arno@wis.kuleuven.ac.be), or consult the workshop home page <http://www.math.usf.edu/FoCM99>

Arno Kuijlaars  
 (arno@wis.kuleuven.ac.be)

#### 3. Gainesville Conference on Symbolic Computation, Number Theory, Special Functions, Physics and Combinatorics, University of Florida, Gainesville, USA, November 11 - 13, 1999.

This is taken from the URL: <http://www.math.ufl.edu/~frank/qsconf.html> with additional material from a poster circulated by the organizers

**THEME:** The main emphasis of the conference will be Computer Algebra and how it relates to the fields of Number Theory, Special Functions, Physics and Combinatorics. A subject that is common to all of these fields is  $q$ -series. We hope to bring together those who do symbolic computation with  $q$ -series and those that need  $q$ -series including workers in Physics and Combinatorics. The conference should inform mathematicians and physicists that use  $q$ -series of the latest developments in the field of  $q$ -series and especially how symbolic computation has aided these developments. It should also inform participants of new computer algebra software and algorithms used in the study of  $q$ -series.

#### MAIN SPEAKERS:

1. George Andrews, Pennsylvania State University

2. Jon Borwein, Simon Fraser University
3. Barry McCoy, Stony Brook
4. Dennis Stanton, University of Minnesota
5. Sergei Suslov, Arizona State University
6. Doron Zeilberger, Temple University

The following have confirmed that they will give half-hour talks: A. Berkovich, B. C. Berndt, S. Bradley, S. Milne, Murata, K. Muttalib, K. Ono, Quine.

**FUNDING:** University of Florida, NSA and The Number Theory Foundation

**PROCEEDINGS:** The conference proceedings will be published by Kluwer and will be combined with one of the number theory workshops to be held at the University of Illinois as part of their program "Special Year in Number Theory".

**ORGANIZERS:** Frank Garvan, University of Florida (frank@math.ufl.edu) Mourad Ismail, University of South Florida

Martin Muldoon  
(muldoon@yorku.ca)

**4. Berkeley Workshop on Computational Algebraic Analysis, Mathematical Sciences Research Institute, Berkeley, January 5-7, 2000.**

**Organizers:** Bernd Sturmfels, Nobuki Takayama and Uli Walther

Linear partial differential equations can be regarded as left ideals in the Weyl algebra  $D$ . This enables one to study differential equations as  $D$ -modules and was the starting point for the fundamental work of Bernstein, Deligne, Kashiwara, Malgrange, Sato, and others in the 1970's. The term "Algebraic Analysis" was chosen by Sato to emphasize affinity with "Algebraic Geometry".  $D$ -modules now sit at the crossroads of many areas: algebraic geometry, representation theory, PDE's, combinatorics, and hypergeometric functions.

A new trend in  $D$ -module theory is the appearance of algorithms and explicit computations. This lead to an interplay of algebraic analysis and computational and combinatorial algebraic geometry. For example, the computation of de Rham cohomology groups, restriction functors and local cohomology modules has been reduced to computing Grobner bases over  $D$ .

This workshop will focus on current developments in this area. Topics include:

- Non-commutative Grobner bases

- Combinatorics and  $D$ -modules,
- Cohomological computations,
- $D$ -modules and tight closure,
- Hypergeometric functions,
- Multidimensional residues,
- Regular and irregular singularities.

This is an informal workshop, and there is no financial support through MSRI. Senior participants are expected to pay their own expenses. Participating graduate students and postdocs will be accommodated with locals. If you are interested in attending, please write to Uli Walther (walther@math.umn.edu).

Bernd Sturmfels  
(bernd@math.berkeley.edu)

**5. San Diego Symposium on Asymptotics and Applied Analysis, San Diego State University, San Diego, U.S.A., January 10 - 14, 2000**

This announcement is from the Symposium web site:  
[http://www.sci.sdsu.edu/math\\_cs/asymptotics2000](http://www.sci.sdsu.edu/math_cs/asymptotics2000)

**Organizers:** T. M. Dunster and D. A. Lutz (Department of Mathematical and Computer Sciences, San Diego State University)

This symposium will compose of talks in the following three categories of asymptotics and applied analysis: difference equations, ordinary differential equations, and functions defined by integrals. The first of these would emphasize representations for solutions in large sectors and asymptotic methods involving special functions. Included in the second and third categories would be new results related to exponentially-improved asymptotics and hyper-asymptotics, summation of divergent solutions, connection problems, error analysis, and applications to special functions. In all of the above three areas physical applications would be covered under the scope of the symposium: for example new results in wave physics (where asymptotics describes the high-frequency limit and separation of variables leads to ordinary differential equations with a large parameter), and tunneling in quantum mechanics (where asymptotics of certain special functions could be used in the determination of exponentially small widths of energy bands in one-dimensional potentials).

The following eleven leading experts in the above fields have indicated a willingness to participate as principal speakers in the proposed symposium: F. W. J. Olver (Maryland), R. Askey (Wisconsin), W. Balser (Ulm, Germany), C. Bender (St. Louis), M. Berry (Bristol, UK),

B. Braaksma (Netherlands), F. Pham (Nice, France), R. Schaeafke (Strasbourg, France), Y. Sibuya (Minneapolis), N. Temme (Netherlands), and R. Wong (Hong Kong). Each will give a 50 minute talk focusing on significant, new developments. In addition, there will be a series of shorter 30 minutes talks from other participants. Post-doctoral and graduate students, and especially those from under-represented groups in mathematics, are encouraged to attend.

Martin Muldoon  
(muldoon@yorku.ca)

## 6. Workshop on Quantum Groups, Morelia, Mexico, March 27-31, 2000

This information is taken from the web site:  
<http://msri.org/activities/events/9900/qgroups/>

**Organizing Committee:** Susan Montgomery (USC), Jose Antonio de la Pena (UNAM), Claudio Procesi (U. of Roma), and Nicolai Reshetikhin (UCB).

Quantum groups emerged from mathematical physics in mid 80's as an algebraic structure hidden behind quantum integrable systems. Algebraically quantum groups are Hopf algebras which are noncommutative deformations of functions on Lie groups, or dualizing, non-commutative deformations of universal enveloping algebras of Lie algebras. Immediately after these structures were discovered they were used to construct new invariants of knots and 3-manifolds.

One of the most important discoveries in representation theory in the 90's was the universal (crystal) basis discovered by Kashiwara and Lusztig, discovered using quantum groups, and more recently, Nakagima and others constructed representations of affine Lie algebras and corresponding quantum groups using geometry of certain moduli spaces. Another area where quantum groups clarified a lot the existing results and made possible fast progress in the theory of special functions ( $q$ -special functions). Conceptually, this direction can be regarded as harmonic analysis on quantum groups. Yet another direction emerged from study of the study of Hopf algebras with real structure by means of functional analysis. This direction, is well represented in the community of people working in  $C^*$ -algebras.

**Topics to be covered in the conference are as follows:**

- Finite dimensional Hopf algebras
- Geometric realizations of quantized universal enveloping algebras
- Applications of quantum groups
- Representation theory of quantum groups

This workshop will be held March 27-31, 2000 in Morelia, Mexico. A proposal for funding has been submitted jointly to the NSF and CONACyT. It is anticipated that there will be approximately 70 participants, half of whom are NSF supported, and the others supported by CONACyT. Preference will be given to recent PhD's and graduate students.

To apply for financial support: To apply for funding, send a letter explaining your interest in the workshop together with a vita or bibliography and a budget for travel/living expenses. If you are a student, also solicit a letter from a faculty advisor. Those coming from North America should apply through MSRI; those coming from Latin America should apply through the Instituto de Matematicas, UNAM (contact Jose Antonio de la Pena, [jap@penelope.matem.unam.mx](mailto:jap@penelope.matem.unam.mx)). All information should be received by December 1, 1999.

**For more information:** Communications about this workshop should be sent either by email to [qgroups@msri.org](mailto:qgroups@msri.org) or by regular mail to:

Quantum Groups Mathematical Sciences Research Institute 1000 Centennial Drive Berkeley, CA 94720-5070.

Martin Muldoon  
(muldoon@yorku.ca)

## 7. Third Workshop Orthogonal Polynomials: Approximation and Harmonic Analysis Inzell, Germany, April 14–18, 2000

The third Workshop on Orthogonal Polynomials will focus on approximation theoretic methods and the relationship to abstract harmonic analysis. The Workshop will take place at Inzell located in the Alps southeast of Munich. For more information consult the homepage of the workshop.

**Language:** English

**Organizing Committee:** S. Ehrich, F. Filbir, R. Girgensohn, R. Lasser, J. Obermaier, J. Prestin

**Mailing Address:**

Dr. J. Prestin  
Institute of Biomathematics and Biometry  
GSF - National Center for Environment and Health  
D - 85764 Neuherberg  
WWW:

<http://www.gsf.de/institute/ibb/prestin/work3.html>

Jürgen Prestin  
(prestin@gsf.de)

## 8. Symposium on Trends in Approximation Theory. Vanderbilt University, Nashville, Tennessee

**on May 17-20, 2000.**

An International Symposium Celebrating the 60th Birthday of Larry L. Schumaker will be held in connection with the 15th annual Shanks Lecture at Vanderbilt University, Nashville, Tennessee on May 17-20, 2000.

**The Plenary Speakers are:**

1. Charles Chui (Stanford, USA)
2. Zbigniew Ciesielski (Sopot, Poland)
3. Ron DeVore (Columbia, USA)
4. Nira Dyn (Tel-Aviv, Israel)
5. Manfred von Golitschek (Wuerzburg, Germany)
6. Jacob Korevaar (Amsterdam, The Netherlands)
7. George G. Lorentz (Chico, USA) - Shanks Lecturer
8. Sergej Mikhajlovich Nikol'skii (Moscow, Russia)
9. Richard Varga (Kent, USA)

**Contributed Talks:** We invite you to contribute a talk in any area of approximation theory and its applications. The duration of contributed talks will depend on the number of participants and will be announced later.

**Symposium Topics:** The topics of interest include, but are not limited to:

1. Abstract approximation
2. Approximation with constraints
3. Classical approximation
4. Complex approximation
5. Extremal problems
6. Interpolation and smoothing
7. Curves and surfaces
8. Multiresolution analysis
9. Nonlinear approximation
10. Orthogonal polynomials
11. Radial basis functions
12. Shift-invariant spaces
13. Splines
14. Subdivision and refinable functions

15. Image and signal processing

16. Wavelets

**Proceedings:** We expect to publish a proceedings containing survey papers by the invited speakers and refereed contributed papers.

**Financial Support:** We are currently applying for funding to be able to partially support the expenses of graduate students and other mathematicians without support.

**Organizing Committee:** Kirill Kopotun (Vanderbilt University, USA) Tom Lyche (University of Oslo, Norway) Mike Neamtu (Vanderbilt University, USA)

**Address:**

e-mail: [at@math.vanderbilt.edu](mailto:at@math.vanderbilt.edu)

Symposium Address

Trends in Approximation Theory 2000  
Department of Mathematics  
Vanderbilt University  
1326 Stevenson Center  
Nashville, TN 37240  
USA

More information is available at the Symposium Web site: <http://www.math.vanderbilt.edu/at/>

Martin Muldoon  
[muldoon@yorku.ca](mailto:muldoon@yorku.ca)

**9. NATO Advanced Study Institute "Special Functions 2000: Current Perspective and Future Directions", Arizona State University, Tempe, Arizona, U.S.A., May 29 to June 9, 2000**

This is preliminary information from the ASI web site:  
<http://math.la.asu.edu/~sf2000/>

**Objective of the ASI:** to summarize results in special functions and their diverse applications obtained over the last 3 decades, and to discuss future directions.

**International Organizing Committee:** Sergei Suslov, Director from NATO country, Arizona State University, U.S.A.; Vyacheslav Spiridonov, Director from Partner country, Joint Institute for Nuclear Research, Dubna, Russia; Tom Koornwinder, KdV Institute, University of Amsterdam, The Netherlands; Luc Vinet, McGill University, Montreal, Canada.

**Local Organizing Committee:** Joaquin Bustoz, Chair, Arizona State University; Mourad Ismail, University of

South Florida; Sergei Suslov, Arizona State University.

**Lecturers:** G. Andrews, Pennsylvania State University, U.S.A.; R. Askey, University of Wisconsin, Madison, U.S.A.; P. Deift, Courant Institute, U.S.A.; C. Dunkl, University of Virginia, U.S.A.; A. Grunbaum, University of California, Berkeley, U.S.A.; M.E.H. Ismail, University of South Florida, Tampa, U.S.A.; A. Its, Indiana University - Purdue University, Indianapolis, U.S.A.; E. Koelink, Technische Universiteit Delft, The Netherlands; T. Koornwinder, KdV Institute, University of Amsterdam, The Netherlands; I. Macdonald, Queen Mary College, London, England (not confirmed); S. Milne, The Ohio State University, U.S.A.; O. Njastad, Norwegian University of Science & Technology, Norway; M. Rahman, Carleton University, Ottawa, Canada; V. Spiridonov, Joint Institute of Nuclear Research, Dubna, Russia; D. Stanton, University of Minnesota, U.S.A.; S. K. Suslov, Arizona State University, U.S.A.; N. Temme, CWI, Amsterdam, The Netherlands; V. N. Tolstoi, Moscow State University, Russia; L. Vinet, McGill University, Montreal, Canada; A. Zhedanov, Donetsk Institute for Physics and Technology, Ukraine.

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 Web page: <http://math.la.asu.edu/~sf2000/>

Sergei Suslov  
 (sks@asu.edu)

#### 10. Alhambra 2000: A joint Mathematical European-Arabic Conference. Granada, July 3rd to 7th, 2000

#### PRELIMINARY ANNOUNCEMENT

The ALHAMBRA 2000 JOINT MATHEMATICAL EUROPEAN-ARABIC CONFERENCE arises within European Mathematical Society (EMS) as an opportunity of a meeting between mathematicians from every European-Arabic culture. Granada, with the Alhambra as an ensign, accepts this challenge of promoting a meeting between scientists of the three Mediterranean cultures, which is to be held at the same time to commemorate the year 2000 as the World Year of Mathematics.

The Spanish Royal Mathematical Society, integrated in the EMS since 1998, offers support and heads the organization of this encounter in Granada. ALHAMBRA 2000

is already going to become a reality.

ALHAMBRA 2000 is an acknowledged satellite activity of the 3rd European Congress of Mathematics, Barcelona, July 10th to 14th.

**STRUCTURE:** The ALHAMBRA 2000 Conference includes:

- The ALHAMBRA 2000 European-Arabic Congress of Mathematics that will deal with historical perspectives on contributions of both cultures to the present mathematical knowledge, and also will discuss about the state of the more relevant mathematical concepts over the centuries and the way they have evolved. Plenary lectures of the above subjects will be held in morning sessions, and short communications will be held.
- Eight ALHAMBRA 2000 Symposia on current mathematical subjects to be held in afternoon parallel sessions. The Symposia serve as platforms for the presentation and discussion of the state of the art of topics in the list of scientific topics below, and they include lectures as well as short communications, either spoken or in the form of posters.

#### TENTATIVE LIST OF SCIENTIFIC TOPICS:

1. Computational mathematics
2. Geometry of submanifolds
3. Mathematical demography
4. Non-linear problems
5. Orthogonal polynomials
6. Public mathematics
7. Representation theory of algebras
8. Symmetry

**PRE-REGISTRATION:** If you wish to receive the Second Announcement and information about the ALHAMBRA 2000 by e-mail, please pre-register. Pre-registration is free and implies no obligation whatsoever. In order to become a participant in the ALHAMBRA 2000, full registration and payment of the registration fee are necessary.

Pre-registration can be made online, by accessing the Conference web site <http://www.ugr.es/local/alhambra2000>, or via e-mail, by filling the pre-registration form and sending it to the Secretariat Conference e-mail address: [eurocongres@mx3.redestb.es](mailto:eurocongres@mx3.redestb.es)

For further information please contact with the conference secretariat:

ALHAMBRA 2000 Conference  
 eurocongres Avda. Constitución, 18 - Blq.4 E-18012 -

GRANADA, SPAIN Voice: +34 958 209 361 Fax: +34 958 209 400 E-mail: eurocongres@mx3.redestb.es

#### **Local organisation committee:**

E-mail: alhambra2000@ugr.es

Web: <http://www.ugr.es/local/alhambra2000>

Francisco Marcellán  
(pacomarc@ing.uc3m.es)

#### **11. I Colloquium on Lie Theory and Applications. University of Vigo, Vigo, Spain, July 17–22, 2000.**

I am pleased to inform you that at the University of Vigo we are organizing the "I Colloquium on Lie Theory and Applications", which will be held in Vigo (Spain) from July 17 to July 22, 2000.

Some information on the colloquium and a pre-registration form are available in the following web page: <http://www.dma.uvigo.es/clieta/index>

(The following is extracted from the web page - Ed)

#### **FIRST ANNOUNCEMENT**

The University of Vigo is pleased to announce the I Colloquium on Lie theory and Applications which will be held at Vigo (Spain) from July 17 to July 22, 2000.

The Colloquium will be the first of a series of conferences devoted to all aspects of Lie Theory and to be held in different locations biennially.

**SCIENTIFIC PROGRAMME AND PRE-REGISTRATION:** The Colloquium will include three courses, of three hours each, delivered by Professors D.V. Alekseevsky, A.T. Fomenko and M. Scheunert, eleven invited lectures and several short communications (fifteen minutes each).

All participants should fill, as soon as possible, a pre-registration form. Those intending to present a short communication must submit by ordinary mail a printed abridged version to the Organizing Committee NOT LATER THAN NOVEMBER, 30, 1999. Instructions on the dimensions and style of the abridged version are detailed in the call for communications.

The languages of the colloquium will be English, Spanish and French; the organizing committee, however, encouragingly recommends the use of English specially in abstracts and in the written version of the communications.

#### **FURTHER INFORMATION:**

##### **1. List of Invited Speakers and Titles:**

###### **(a) COURSES: D. V. Alekseevsky: Semisimple Lie**

algebras, Dynkin diagrammes and geometry of flag manifolds.

A.T. Fomenko: Lie groups and integrable Hamiltonian systems.

M. Scheunert: (Title not provided)

**(b) LECTURES:** S. Benayadi, M. Bordemann, V. Cortes, A. Gonzalez-Lopez, Yu. B. Hakimjanov, E. Koelink, M. de Leon, E. Macias-Virgos, A. Medina, C. Moreno, and K-H. Neeb.

**2. Scientific Committee:** D.V. Alekseevsky (Max-Planck Institut für Mathematik, Germany), S. Benayadi (Université de Metz, France), M. Bordemann (Universität Freiburg, Germany), V. Cortes (Universität Bonn, Germany), A.T. Fomenko (Moscow State University, Russia), A. Gonzalez-Lopez (Universidad Complutense de Madrid, Spain), Yu.B. Hakimjanov (Université d'Haute Alsace, France), K.H. Hofmann (Tulane University, USA), E. Koelink (Technische Universität Delft, The Netherlands), M. de Leon (CSIC, Spain), E. Macias-Virgos (Universidad de Santiago, Spain), A. Medina (Université de Montpellier II, France), C. Moreno (Université de Bourgogne, France / Universidad Complutense de Madrid, Spain), K-H. Neeb (Technische Universität Darmstadt, Germany), M. Scheunert (Universität Bonn, Germany).

**3. Registration Fees:** Registration in the colloquium will be possible from February 1 to April 30, 2000. The registration fees are the following:

- General Fee: 22,000 pesetas (132 Euro)
- Reduced Fees: Participants registered in the 3ecm 18,000 pesetas (108 Euro), Individual members of SEMA 18,000 pesetas (108 Euro), Young Researchers<sup>1</sup> 16,000 pesetas (96 Euro)

**4. Organizing Committee:** N. Alonso, I. Bajo, R. Gonzalez, A. Martin and E. Sanmartín (Universidad de Vigo, Spain)

**5. Coordination:** Ignacio Bajo (Dept. Matemática Aplicada, Universidad de Vigo, Spain) Esperanza Sanmartín (Dept. Matemáticas, Universidad de Vigo, Spain)

#### **Address:**

I Colloquium on Lie Theory and Applications  
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<sup>1</sup>The "Young researchers" fee is applicable to researchers under 30 who declare to be unemployed or whose net income does not exceed 125,000 pesetas (750 Euro) per month.

**Sponsors and Collaborating Entities:** Universidad de Vigo, Xunta de Galicia, Ministerio de Educación y Cultura, Real Sociedad Matemática Española (RSME), Sociedad Española de Matemática Aplicada (SEMA), European Mathematical Society (EMS).

**Social Programme:** The Social Programme of the colloquium will include a closing dinner and a visit to the city of Santiago de Compostela. The city of Santiago de Compostela has been proclaimed one of the Cultural Capitals of Europe 2000.

**How to reach Vigo:** The city of Vigo, with 300,000 inhabitants, is located in Galicia, a region in the northwest of Spain. The airport of Vigo connects the city with Madrid, Barcelona, Bilbao, Valladolid, Frankfurt and Paris daily. The airport of Santiago de Compostela is the largest airport in Galicia; it is only 90 kilometres from Vigo and has flights to many other international destinations. The city of Porto (Portugal) is approximately 150 kilometres from Vigo and a large number of destinations are possible through its airport. There are regular bus and train services to almost all important towns in Spain and Portugal.

Ignacio Bajo Palacio  
(ibajo@dma.uvigo.es)

## 12. SIAG Summer School on Orthogonal Polynomials and Special Functions. Laredo, Spain, July 24-29, 2000.

### Preliminary Announcement

The SIAG on Orthogonal Polynomials and Special Functions intends to organize a series of summer schools starting next year. The first of such meetings will take place in Laredo, Spain. Laredo is a "small" village located at the Cantabria coast, in the Atlantic seaside of Spain near Santander. The goal of the Summer School is to give 5 introductory courses in advanced research topics on Orthogonal Polynomials and Special Functions. Some free discussions as well as some informal seminars will also be available. The expected audience are graduate and recent postgraduate students (around 25 people who will receive grants for their living expenses and accommodation) and active researchers (around 35 people).

**Organizing Committee:** F. Marcellán (Program Director SIAM Activity Group of Orthogonal Polynomials and Special Functions, Universidad Carlos III de Madrid, Spain), W. Van Assche (Vice-Chair SIAG, Katholieke Universiteit Leuven, Belgium), R. Álvarez-Nodarse (Universidad de Sevilla, Spain) and R. Yáñez (Universidad de

Granada, Spain) SIAG Newsletter Co-Editors.

### Invited Lecturers

1. H. T. Koelink (Technische Universiteit Delft, The Netherlands) "Spectral Theory and Special Functions"
2. A. J. Durán (Universidad de Sevilla, Spain) "Matrix Orthogonal Polynomials"
3. J. Prestin (Institute of Biomathematics and Biometry, Neuherberg, Germany) "Polynomial Wavelets"
4. J. Stokman, "Multivariable Orthogonal Polynomials"
5. K. T-R. McLaughlin (University of Arizona, USA) "Asymptotic Analysis of Riemann-Hilbert Problems and Orthogonal Polynomials"

For more information contact F. Marcellán ([pacomarc@ing.uc3m.es](mailto:pacomarc@ing.uc3m.es)) and/or R. Álvarez-Nodarse ([ranc@cica.es](mailto:ranc@cica.es)). Further information will be appear in the WWW page <http://merlin.us.es/~renato/laredo/>

Francisco Marcellán  
([pacomarc@ing.uc3m.es](mailto:pacomarc@ing.uc3m.es))

### Future Planning

At the Hong Kong panel of discussion there was a proposal that the SIAM activity group might provide coordination for the future planning of meetings on orthogonal polynomials and special functions. From now on the activity group's newsletter will keep a list of dates and plans for meetings, with contact addresses of local responsible people.

If you are planning to organize a workshop, summer school, conference, special session, etc., we suggest that you inform one of the officers of the SIAM activity group and we will keep the activity group informed by publishing the intended date in the newsletter, so as to avoid the coincidence of several meeting. Please contact the local organizers in case of conflict or for finding a solution for possible overlap. The SIAM activity group will not be involved in the actual organization.

- 2000. A summer school will be organized in Laredo, Spain from July 24 to July 29. Contact Francisco Marcellán for more details ([pacomarc@ing.uc3m.es](mailto:pacomarc@ing.uc3m.es)) [see also the item 12 of the Forthcoming Meetings and Conferences section of this issue]
- 2001.
  - Andrea Laforgia expressed an intention to organize the next conference on orthogonal polynomials and their applications in

Italy in September. Please contact him at [laforgia@dma.uniroma3.it](mailto:laforgia@dma.uniroma3.it).

- There are plans for a summer school in Munich. The contact person is Jürgen Prestin ([prestin@gsf.de](mailto:prestin@gsf.de))
- 2002. The next meeting on special functions following the Arizona NATO ASI in 2000, will be organized by Tom Koornwinder, Nico Temme and Eric Koelink in Amsterdam, probably early summer. Please contact [thk@wins.uva.nl](mailto:thk@wins.uva.nl), [nicot@cwi.nl](mailto:nicot@cwi.nl) or [koelink@twi.tudelft.nl](mailto:koelink@twi.tudelft.nl) for coordination purposes. There is also a plan to organize a summer school just before the conference. Contact Erik Koelink ([koelink@twi.tudelft.nl](mailto:koelink@twi.tudelft.nl)) for information.

Walter Van Assche  
([walter@wis.kuleuven.ac.be](mailto:walter@wis.kuleuven.ac.be))

## Books and Journals

### Book Announcements

**1. Applications and Computation of Orthogonal Polynomials. Conference at the Mathematical Research Institute Oberwolfach**  
Edited by W. Gautschi, G. H. Golub, and G. Opfer

Conference at the Mathematical Research Institute Oberwolfach, Germany, March 22-28, 1998.

This is the official text from the publisher.

This volume contains a collection of papers dealing with applications of orthogonal polynomials and methods for their computation.

The applications address problems in applied mathematics as well as problems in engineering and the sciences. Prominent among the former are least-squares approximations, Gauss and related quadrature, iterative methods in linear algebra, the detection of singularities, and integral equations. Applications of the latter kind include the use of wavelets in medical diagnostics and the relevance of orthogonal polynomials in optimal control, dynamical systems, and gas dynamics. Computational methods relate to numerical and symbolic computation and include, in particular, matrix interpretation and convergence, perturbation, and stability analyses of relevant algorithms. Generalizations of orthogonal polynomials are also considered, for example, s-orthogonal, matrix- and tensor-valued, Müntz type, and complex orthogonal polynomials.

Wolfram Koepf  
([koepf@imn.htwk-leipzig.de](mailto:koepf@imn.htwk-leipzig.de))

**2. Algebraic Methods and  $q$ -Special Functions**  
Edited by Jan Felipe van Diejen and Luc Vinet

CRM Proceedings and Lecture Notes 22, American Mathematical Society, Providence, R.I., 1999. ISBN: 0-8218-2026-5

In May 1996, the Centre de Recherches Mathématiques at Montreal organized the workshop "Algebraic Methods and  $q$ -Special Functions". The idea of this workshop was to bring together a diverse group of people working in the areas of combinatorics, special functions, orthogonal polynomials and representation-theoretic methods, with the purpose of getting some kind of an overview of the current developments in the field. Much of the reported research progress was inspired by the seminal contributions of R. A. Askey and colleagues on one-variable basic hypergeometric series and of I. G. Macdonald on multivariate orthogonal polynomials related to root systems.

Recently, the proceedings of this workshop appeared in the CRM Proceedings and Lecture Notes Series published by the AMS

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16. A. Strasburger, *On algebras of creation and annihilation operators.*

Further information on these proceedings can be obtained via the AMS Bookstore WEB page: [www.ams.org/cgi-bin/bookstore/bookpromo/crmpseries](http://www.ams.org/cgi-bin/bookstore/bookpromo/crmpseries)

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**3. Continued Fractions: from analytic number theory to constructive approximation. A volume in honor of L.J. Lange**

**Edited by Bruce C. Berndt and Fritz Gesztesy**

Contemporary Mathematics, volume 236, Amer. Math. Soc., 1999

This volume contains contributions by

1. R. Askey: Continued fractions and orthogonal polynomials.
2. B.C. Berndt, Y.-S. Choi, S.-Y. Kang: The problems submitted by Ramanujan to the Journal of the Indian Mathematical Society.
3. B. Bojanov, A. Sri Ranga: Some examples of moment preserving approximation.
4. C.F. Bracciali: Relations between certain symmetric strong Stieltjes distributions.
5. A. Bultheel, C. Diaz-Mendoza, P. Gonzalez-Vera, R. Orive: Estimates of the rate of convergence for certain quadrature formulas on the half line.
6. A. Bultheel, P. Gonzalez-Vera: Wavelets by orthogonal rational kernels.
7. H.H. Chan, V. Tan: On the explicit evaluations of the Rogers-Ramanujan continued fraction.
8. D. Chelst: Absence of phase transitions in modified two-component plasmas: the analytic theory of continued fractions in statistical mechanics.
9. M.E.H. Ismail, D.R. Masson: Some continued fractions related to elliptic functions.
10. W.B. Jones, G. Shen: Asymptotics of Stieltjes continued fraction coefficients and applications to Whittaker functions.
11. L.J. Lange: A generalization of Van Vleck's theorem and more on complex continued fractions.

12. X. Li: Convergence of interpolation Laurent polynomials on an annulus.
13. L. Lorentzen: Convergence criteria for continued fractions  $K(a_n/1)$  based on value sets.
14. O. Njåstad: Strong Stieltjes moment problems.
15. F. Peherstorfer, R. Steinbauer: Weak asymptotics of orthogonal polynomials on the support of the measure of orthogonality and considerations on functions of the second kind.
16. S. Perrine: Trees of approximation constants.
17. I. Rodnianski: Continued fractions and Schrödinger evolution.
18. W. Van Assche: Multiple orthogonal polynomials, irrationality and transcendence.
19. A.J. van der Poorten: Reduction of continued fractions of formal power series.
20. H. Waadeland: Some observations in frequency analysis.
21. F. Wielonsky: Some properties of Hermite-Padé approximants to  $e^z$ .

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## Book Reviews

### Special Functions

**By George E. Andrews, Richard Askey, Ranjan Roy**

Encyclopedia of Mathematics and its Applications Vol. 71. Cambridge University Press, Cambridge, 1999. \$ 85.00, xvi, 664 pp., ISBN 0-521-62321-9.

This book covers a wealth of material on special functions, notably knowledge which was developed by Richard Askey and his co-authors during the several decades of his contributions to this subject, but also material which connects special functions with combinatorial questions collected by George Andrews. These two researchers are well-known for their efforts to support and demand the use of hypergeometric functions in their respective fields, hence hypergeometric functions and  $q$ -hypergeometric functions (basic hypergeometric functions) play a prominent role in the book under review. The book covers 12 chapters and 6 appendices. Furthermore, it contains a rich collection of 444 (!) exercises that are distributed among the different chapters. Here are the details:

**Chapter 1: The Gamma and Beta Functions.** In this chapter the usual material about the Gamma and Beta functions is covered. Moreover, results for the logarithmic derivative  $\psi(x) = \Gamma'(x)/\Gamma(x)$  of the Gamma function

and for the Hurwitz and Riemann zeta function are developed; in particular, several integral representations are given. The Gamma function is characterized by the Bohr-Mollerup theorem, and finally the  $p$ -adic Gamma function is introduced.

**Chapter 2: The Hypergeometric Functions.** The generalized hypergeometric function is introduced, and elementary examples are given. Euler's integral representation, and the usual summation theorems (Gauss, Chu-Vandermonde, Pfaff-Saalschütz, Dixon) come next. Then the hypergeometric differential equation is treated from the Riemannian point of view that analytic functions are determined to a large extent by their singularities. Next, Barnes type integrals, contiguous relations, and continued fractions of ratios of hypergeometric functions are covered. The Jacobi polynomials as specific hypergeometric polynomials are introduced. Finally, dilogarithms, binomial sums, and fractional integration by parts are treated.

**Chapter 3: Hypergeometric Transformations and Identities.** This chapter starts with quadratic transformations. Then elliptic integrals are considered as hypergeometric functions, and arithmetic-geometric mean sequences are introduced. Next, transformations for balanced series, Whipple's transformation and Dougall's formula are given. Integral analogs of hypergeometric sums lead to the Wilson polynomials. The Riemannian point of view is reconsidered in connection with quadratic transformations. Gosper's algorithm on indefinite hypergeometric summation is given, and the Wilf-Zeilberger method for proving hypergeometric identities is compared with Pfaff's method, and the question of how these methods are related to contiguous relations is analyzed.

**Chapter 4: Bessel Functions and Confluent Hypergeometric Functions.** Here, the confluent hypergeometric function is introduced. Then a Barnes type integral is given. As special cases, the Whittaker and the Bessel functions are covered. Recurrence equations, integral representations, and asymptotic expansions are treated. A two-dimensional Fourier transform leads to a generating function of the Bessel functions. Addition theorems and integrals of Bessel functions come next. Finally zeros and monotonicity properties of Bessel functions are discussed.

**Chapter 5: Orthogonal Polynomials.** The elementary properties of general orthogonal polynomials are derived. Next, Gauss quadrature is examined. Then zeros of orthogonal polynomials are discussed, and the connection of orthogonal polynomials with continued fractions is treated. After Parseval's formula, the moment-generating function is introduced.

**Chapter 6: Special Orthogonal Polynomials.** Under this heading comes a discussion of the classical hypergeometric type orthogonal polynomials. The Hermite, Laguerre and Jacobi polynomials and their properties are

discussed in detail. Then linearization coefficients are considered, and combinatorial interpretations of the classical systems are given. The Wilson polynomials and their properties come next. Finally a  $q$ -generalization of the ultraspherical polynomials is deduced.

**Chapter 7: Topics in Orthogonal Polynomials.** Connection coefficients are introduced, and for the classical systems these coefficients are explicitly determined. Non-negativity results for hypergeometric functions and positive polynomial sums come next. In particular, the Askey-Gasper inequality which was used by de Branges in his proof of the Bieberbach conjecture [1] is deduced using results about connection coefficients. Theorems by Vietoris and Turán are covered. Finally, Apéry's irrationality proof of  $\zeta(3)$  is given.

**Chapter 8: The Selberg Integral and Its Applications.** Here, Selberg's and Aomoto's integrals and extensions of these formulas are given. A two-dimensional electrostatic problem studied by Stieltjes connects the zeros of the Jacobi polynomials with Selberg's integral in an interesting way. Siegel's inequality, which is a refinement of the arithmetic-geometric mean inequality, is studied next, and a connection to the Laguerre polynomials is considered. Applications of Selberg's integral to constant-term identities and nearly-poised  ${}_3F_2$  identities are given. The Hasse-Davenport relation and a finite-field analog of Selberg's integral finish this chapter.

**Chapter 9: Spherical Harmonics.** Harmonic polynomials and the Laplace equation in three dimensions provide an introduction to the topic of this chapter. Then the harmonic polynomials of degree  $k$  and their orthogonality are studied. Their addition theorem yields an addition theorem for ultraspherical polynomials which was used by Weinstein [3] in his proof of the Bieberbach conjecture. It is shown that Fourier transforms of higher order are still expressible in terms of Bessel functions. Next, finite-dimensional representations of compact groups are studied. Finally, Koornwinder's product formula for Jacobi polynomials is given.

**Chapter 10: Introduction to  $q$ -series.** In this chapter, the theory of  $q$ -hypergeometric series (basic hypergeometric series) is motivated by considering non-commutative  $q$ -algebra, related with the rule  $yx = qx y$ . Using this approach, the definition of the  $q$ -binomial coefficients and their connection with the standard binomial coefficients are straightforward. The  $q$ -integral is defined, and the  $q$ -binomial theorem is proved by two different approaches both based on recurrence equations. The  $q$ -Gamma function, and Jacobi's triple product identity are next. Ramanujan's summation formula is used to give results about the representations of numbers as sums of squares. Elliptic and theta functions are covered, and  $q$ -beta integrals are used to find a  $q$ -analog of the Wil-

son polynomials. Finally, the basic hypergeometric series is studied. Basic hypergeometric identities, the  $q$ -ultraspherical polynomials and the Mellin transform finish this chapter.

**Chapter 11: Partitions.** Partitions are defined, and the connection of partition analysis with  $q$ -series is studied. Generating functions, and other results on partitions are obtained by this method. Next, graphical methods are discussed, and congruence properties of partitions are covered.

**Chapter 12: Bailey Chains.** Rogers's second proof of the Rogers-Ramanujan identities is given. Then, Bailey's lemma and Watson's transformation formula are treated. Finally, some applications are given.

Appendices on *Infinite Products, Summability and Fractional Integration, Asymptotic Expansions, Euler-Maclaurin Summation Formula, Lagrange Inversion Formula, and Series Solutions of Differential Equation* follow, and a bibliography, an index, a subject index and a symbol index complete the book.

To begin with these last items: For a book of this size, the subject index is rather small (3 pages). Hence, obviously not every subject can be found here. Just to mention a few, one finds neither *addition theorem*, nor *Bieberbach conjecture*, nor *irrationality of  $\zeta(3)$* , nor *indicial equation* (notation defined on p. 640 in Appendix F, and used on p. 74). Many other topics cannot be found in the subject index either. In my opinion, a book covering such a wealth of information needs a better index. Similarly, the bibliography (on purpose) contains only the articles that are explicitly mentioned in the text, and by no means covers the topic of the book encyclopedically. Another minor irritation is the fact that the notations  $[x]$  (e.g. on pp. 203, 314) and  $\lfloor x \rfloor$  (e.g. on pp. 279, 340) for the greatest integer in  $x$  are used synonymously, but only the latter is defined on p. 15.

On the other hand, the material is written in an excellent manner, and it gives the reader very interesting insights to special functions. On many occasions, theorems are proved by several alternative methods. This gives the reader a much better feeling for what is going on, indicating that *Special Functions* is not a topic which can be taught deductively. Furthermore, the book contains very few typos.

But a book of this size covers thousands of formulas, and by Murphy's law, a few of them should be incorrect. I tried to find such misprints, in particular in the sections 3.11 and 3.12 about summation methods, since there I could use my Maple software for purposes of detection [2]. Not surprisingly, this search was successful: Formula (3.11.10) is incorrect by a factor  $-n$ ; both identities in the middle of p. 175 are incorrect restatements of the corresponding contiguous relations (3.11.12) and (3.11.15) on p. 173; fur-

thermore in formula (3.12.1) the upper parameter  $z+n-1$  should read  $z+n+1$ .<sup>2</sup>

In spite of these minor shortcomings, I recommend this book warmly as a rich source of information to everybody who is interested in *Special Function*.

## References

- [1] de Branges, L.: A proof of the Bieberbach conjecture. *Acta Math.* 154, 1985, 137–152.
- [2] Koepf, Wolfram: *Hypergeometric Summation. An Algorithmic Approach to Summation and Special Function Identities*. Vieweg, Braunschweig/Wiesbaden, 1998.
- [3] Weinstein, L.: The Bieberbach conjecture. *Int. Math. Res. Not.* 5, 1991, 61–64.

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## Problems and Solutions

Thus far 20 problems have been submitted seven of which have been solved in previous issues. Still unsolved are Problems #3, 5, 8, 9, 11, 12, 13, 15, 17, 18, 19 and 20. This time one new problem has been submitted.

**19. Uniform Bounds for Shifted Jacobi Multiplier Sequences.** For Fourier series the following is immediate: Suppose the real or complex sequence  $\{m_k\}$  generates a bounded operator on  $L^p(\mathbf{T})$ ,  $1 \leq p \leq \infty$ , i.e., for polynomial  $f$

$$\left\| \sum m_k \hat{f}_k e^{ik\varphi} \right\|_{L^p(\mathbf{T})} \leq \|m\|_{M^p(\mathbf{T})} \left\| \sum \hat{f}_k e^{ik\varphi} \right\|_{L^p(\mathbf{T})},$$

then one has for the shifted sequence  $\{m_{k+j}\}_{k \in \mathbf{Z}}$  that

$$\sup_{j \in \mathbf{N}_0} \|\{m_{k+j}\}\|_{M^p(\mathbf{T})} \leq C \|m\|_{M^p(\mathbf{T})}, \quad 1 \leq p \leq \infty. \quad (1)$$

Looking at cosine expansions on  $L^p(0, \pi)$  one easily derives the analog of (1) via the addition formula

$$\cos(k \pm j)\theta = \cos k\theta \cos j\theta \mp \sin k\theta \sin j\theta$$

provided the periodic Hilbert transform is bounded, i.e., for  $1 < p < \infty$ . More generally, by Muckenhoupt's transplantation theorem [2, Theorem 1.6],

$$\begin{aligned} & \left( \int_0^\pi \left| \sum m_{k+j} a_k P_k^{(\alpha, \beta)}(\cos \theta) \right|^p \sin^{2\alpha+1} \frac{\theta}{2} \cos^{2\beta+1} \frac{\theta}{2} d\theta \right)^{1/p} \\ & \equiv \left( \int_0^\pi \left| \sum m_{k+j} b_k \phi_k^{(\alpha, \beta)}(\cos \theta) \right|^p w_{\alpha, \beta, p}(\theta) d\theta \right)^{1/p} \end{aligned}$$

<sup>2</sup>I would like to thank George Andrews for sending me the corrected formula.

$$\approx \left( \int_0^\pi \left| \sum m_{k+j} b_k \cos k\theta \right|^p w_{\alpha,\beta,p}(\theta) d\theta \right)^{1/p},$$

where  $P_k^{(\alpha,\beta)}$  are the Jacobi polynomials,  $\phi_k^{(\alpha,\beta)}(\cos \theta)$  are the orthonormalized Jacobi functions with respect to  $d\theta$ , and

$$w_{\alpha,\beta,p}(\theta) = \sin^{(2-p)(\alpha+1/2)} \frac{\theta}{2} \cos^{(2-p)(\beta+1/2)} \frac{\theta}{2}.$$

Therefore, the above argument for cosine expansions also applies to Jacobi expansions provided the periodic Hilbert transform is bounded with respect to the weight function  $w_{\alpha,\beta,p}$ ; hence, the analog of (1) holds for Jacobi expansions when

$$\frac{2\alpha + 2}{\alpha + 3/2} < p < \frac{2\alpha + 2}{\alpha + 1/2}, \quad \alpha \geq \beta \geq -\frac{1}{2}.$$

(i) Can the above  $p$ -range be extended? By Muckenhoupt [2, (1.3)], a fixed shift is bounded for all  $p$ ,  $1 < p < \infty$ .

(ii) Consider the corresponding problem for Laguerre expansions (for the appropriate setting see [1]); a fixed shift is easily seen to be bounded for all  $p \geq 1$ .

Both questions are of course trivial for  $p = 2$  since  $\ell^\infty = M^2$  by Parseval's formula.

## References

- [1] Gasper, G. and W. Trebels: On necessary multiplier conditions for Laguerre expansions, *Canad. J. Math.* 43 (1991), 1228 – 1242.
- [2] Muckenhoupt, B.: Transplantation Theorems and Multiplier Theorems for Jacobi Series, *Memoirs Amer. Math. Soc.*, Vol. 64, No. 356, Providence, R.I., 1986.

(Submitted on May 19, 1998)

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**20. Question about Elliot's formula** Generalization of Legendre's identity for complete elliptic integrals

Let  $E, K$  be the complete elliptic integrals. Then

$$K'E + KE' - KK' = \pi/2 \quad (*)$$

This is the special case  $p = r = -a + 1/2$ ,  $q = c + a - 3/2$  in Elliott's identity (see Erdelyi e.a., Higher Transcendental Functions, Vol. 1, p. 85):

$$\begin{aligned} & F(p + \tfrac{1}{2}, -r - \tfrac{1}{2}, 1 + p + q; z)F(-p + \tfrac{1}{2}, r + \tfrac{1}{2}; 1 + q + r; 1 - z) \\ & + F(p + \tfrac{1}{2}, -r + \tfrac{1}{2}, 1 + p + q; z)F(-p - \tfrac{1}{2}, r + \tfrac{1}{2}; 1 + q + r; 1 - z) \\ & - F(p + \tfrac{1}{2}, -r + \tfrac{1}{2}, 1 + p + q; z)F(-p + \tfrac{1}{2}, r + \tfrac{1}{2}; 1 + q + r; 1 - z) \\ & = \frac{\Gamma(p + q + 1)\Gamma(q + r + 1)}{\Gamma(p + q + r + \tfrac{3}{2})\Gamma(q + \tfrac{1}{2})} \end{aligned} \quad (**)$$

Question 1. Is there a counterpart of Legendre's identity (\*) for incomplete elliptic integrals?

Question 2. The Elliott identity (\*\*) provides a generalization of the identity (\*) to hypergeometric functions. The only handbook where I have seen this identity is Bateman vol. I. Has Elliott's identity been used or mentioned elsewhere in papers/books?

Question 3. Are there generalizations of the Elliott identity (\*\*) to the  $pF_q$  case or to other generalizations of hypergeometric functions?

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## 21. Question on an exact solvable Schrödinger equation

What are all the Schrödinger equations that have exact solutions expressible in terms of the Kampé de Fériet function?

(Submitted on June 3, 1999)

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**Note:** B. A. Aničin, "Circuit theory of video display lines", *IEE Proc.- Micro. Antennas Propag.*, **146**(1), February 1999, pp. 65-69, acknowledges the help of Professors J. Boersma and M. L. Glasser with certain integral relations which arose in the study in question. Questions concerning these integrals were posed by Aničin in OP-SF NET 5.1. (see the Newsletter volume 8, number 2, February 1998)

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## Miscellaneous

### 1. SIAM Student Paper Prizes

The annual SIAM Student Paper Prizes will be awarded during the 2000 SIAM Annual Meeting, July 10-14, at the Westin Rio Mar Beach Resort in Rio Grande, Puerto Rico.

If you are a student or know of a student who would like to take part in the competition, here are the details:

The authors of the three best papers in applied and computational mathematics written by students and submitted to SIAM will receive a \$1,000 cash prize and a framed calligraphed certificate as well as gratis registration for the meeting. There is no provision for travel expenses associated with the prize.

Papers must be singly authored and not previously published or submitted for publication to be eligible for consideration. To qualify, authors must be students in good standing who have not received their PhDs at the time of submission.

In submitting their work for publication, authors are asked to consider SIAM journals. However, student paper prize winners are not guaranteed publication in any SIAM journal; all papers submitted to SIAM journals are subject to the same refereeing process and standards.

Submissions must be received in the SIAM office before February 15, 2000.

Submissions, which must be in English, can be sent by regular mail or fax. Each submission must include (1) an extended abstract NOT LONGER THAN 5 PAGES (including bibliography); (2) the complete paper, which will be used solely for clarification of any questions; (3) a statement by the student's faculty advisor that the paper has been prepared by the author indicated and that the author is a student in good standing; (4) a letter by the student's faculty advisor describing and evaluating the paper's contribution; and (5) a short biography of the student.

Submissions will be judged on originality, significance, and quality of exposition.

The winners will be notified by April 15, 2000.

Please direct your submission and any questions you may have to A. Bogardo at SIAM, 3600 University City Science Center, Philadelphia, PA 19104-2688; telephone (215) 382-9800; e-mail to [bogardo@siam.org](mailto:bogardo@siam.org).

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([bogardo@siam.org](mailto:bogardo@siam.org))

## 2. SIAM Student Travel Awards

During 2000, SIAM will make a number of awards for \$300 to support student travel to each of the following SIAM conferences:

1. Eleventh ACM-SIAM Symposium on Discrete Algorithms, San Francisco, California, January 9-11.
2. Eighth International Conference on Numerical Combustion, Amelia Island, Florida, March 5-8.
3. Third SIAM Conference on Mathematical Aspects of Materials Science, Philadelphia, Pennsylvania, May 21-24.
4. Tenth SIAM Conference on Discrete Mathematics (SIAG/DM), Minneapolis, Minnesota, June 12-15.
5. 2000 SIAM Annual Meeting, Rio Grande, Puerto Rico, July 10-14.
6. Pacific Rim Dynamical Systems Conference (SIAG/DS), Maui, Hawaii, August 10-12
7. First SIAM Conference on Computational Science and Engineering, Washington, DC, September 21-23.
8. Seventh SIAM Conference on Applied Linear Algebra, Raleigh, North Carolina, October 23-26.

The awards are to be made from the SIAM Student Travel Fund, created in 1991 and maintained through book royalties donated by generous SIAM authors.

Any full-time student in good standing is eligible to receive an award plus gratis meeting registration. Top priority will

be given to students presenting papers at the meeting, with second priority to students who are co-authors of papers to be presented at the meetings. Only students traveling more than 100 miles to the meetings are eligible for the awards.

An application for a travel award must include:

1. A letter from the student describing his/her academic standing and interests, his/her expected graduation date and degree, advisor's name, and, if available, a URL for a working Web page.
2. A one-page vita that includes the student's research interests, projects, and papers published.
3. A detailed letter from the student's faculty advisor indicating why the student is deserving of receiving a travel award and any special circumstances.
4. If applicable, the title(s) of the paper(s) to be presented (co-authored) by the student at the meeting.

Applications should be sent to the SIAM office (Attention: SIAM Student Travel Awards), 3600 University City Science Center, Philadelphia, PA 19104-2688. Students also may apply by e-mail to [bogardo@siam.org](mailto:bogardo@siam.org) or by fax to 215-386-7999.

Complete applications must be received at the SIAM office no later than TWO MONTHS before the first day of the meeting for which support is requested.

Winners will be notified FIVE WEEKS before the first day of the meeting. Checks for the awards will be given to the student awardees when they arrive at the given meeting and pick up their registration packet at the SIAM Registration Desk.

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## 3. Call for Nominations for Polya Prize

**The Polya Prize:** SIAM will present the award at the 2000 SIAM Annual Meeting in Rio Grande, Puerto Rico, July 10-14. The award honors the memory of George Polya and will be given for a notable contribution in combinatorial theory.

**Eligibility:** There are no restrictions.

**Description of Award:** The award will consist of an engraved medal and a \$20,000 cash prize. Travel to the SIAM meeting to receive the award will be paid by the prize fund.

**Nominations:** A letter of nomination, including a description of achievement(s), should be sent by December 31, 1999, to:

Professor Jeffry N. Kahn  
Chair, Polya Prize Selection Committee  
c/o A. G. Bogardo  
SIAM  
3600 University City Science Center  
Philadelphia, PA 19104-2688  
E-mail: [bogardo@siam.org](mailto:bogardo@siam.org)  
FAX: 215-386-7999

**Selection Committee:** The members of the selection committee for the award are Jeffry N. Kahn (Rutgers University), chair; Louis J. Billera (Cornell University); Joel Spencer (Courant Institute, New York University); and Richard P. Stanley (Massachusetts Institute of Technology).

Allison Bogardo  
(bogardo@siam.org)

#### 4. Call for Nominations for Di Prima Prize

**The Di Prima Prize:** SIAM will present the award at the 2000 SIAM Annual Meeting in Rio Grande, Puerto Rico, July 10-14. The award honors the memory of Richard C. DiPrima, long-time Chair of the Department of Mathematical Sciences at Rensselaer Polytechnic Institute and former President and energetic supporter of SIAM. The award will be based on an outstanding doctoral dissertation in applied mathematics.

**Eligibility:** The award, based on Ph.D. research in applied mathematics (defined as those topics covered in SIAM journals or series) is made to a young scientist. The Ph.D. thesis and all other Ph.D. requirements should have been completed in the time period from July 1, 1997 to June 30, 1999. The Ph.D. degree must be awarded by December 31, 1999.

**Description of Award:** The award will consist of a certificate and a cash prize of \$1,000. The SIAM President will notify the recipient of the award in advance of the award date and invite the recipient to attend the annual meeting to receive the award. Travel expenses will be paid by the prize fund.

**Nominations:** A letter of nomination, including a description of achievement(s), should be sent by December 31, 1999, to:

Professor Ronald A. DeVore  
Chair, DiPrima Prize Selection Committee  
c/o A. G. Bogardo  
SIAM  
3600 University City Science Center  
Philadelphia, PA 19104-2688  
E-mail: bogardo@siam.org  
FAX: 215-386-7999

**Selection Committee:** The members of the selection committee for the award are Russel Caflisch (UCLA), Andrew J. Wathen (University of Oxford, UK), and Ronald A. DeVore, Chair (University of South Carolina).

Allison Bogardo  
(bogardo@siam.org)

#### 5. Call for Nominations for Dantzig Prize

Nominations are solicited for the George B. Dantzig Prize, administered jointly by the Mathematical Programming Society (MPS) and the Society for Industrial and Applied Mathematics (SIAM). This prize is awarded to one or more individuals for original research which by its originality, breadth and depth, is having a major impact on the field of mathematical program-

ming. The contributions(s) for which the award is made must be publicly available and may belong to any aspect of mathematical programming in its broadest sense. Strong preference will be given to candidates that have not reached their 50th birthday in the year of the award.

The prize will be presented at the Mathematical Programming Society's triennial symposium, to be held 7-11 August 2000 in Atlanta, Georgia, USA. Past prize recipients are listed on the MPS Web site <http://www.caam.rice.edu/~mathprog/>). The members of the prize committee are William H. Cunningham, Claude Lemarechal, Stephen M. Robinson (Chair), and Laurence A. Wolsey.

Nominations should consist of a letter describing the nominee's qualifications for the prize, and a current curriculum vitae of the nominee including a list of publications. They should be sent to

Stephen M. Robinson  
Department of Industrial Engineering  
University of Wisconsin-Madison  
1513 University Avenue  
Madison, WI 53706-1572, USA  
E-mail: smrobin@facstaff.wisc.edu

Nominations must be received by 15 October 1999. Any nominations received after that date will not be considered. Submission of nomination materials in electronic form (e-mail with attachments as needed) is strongly encouraged.

Allison Bogardo  
(bogardo@siam.org)

#### 6. NSF Funding Received for NIST Digital Library Project

The National Science Foundation recently awarded the National Institute of Standards and Technology \$ 1.3 million for the NIST Digital Library of Mathematical Functions. The money will be used to support contracts with expert non-NIST authors and validators to write chapters for the DLMF.

The DLMF was conceived as the successor for the Handbook of Mathematical Functions, edited by Abramowitz and Stegun and published in 1964 by the National Bureau of Standards (now known as NIST). The style and approach follows Abramowitz and Stegun, but the new reference work will be disseminated from a Web site now under construction at NIST as well as in book form (probably with included CD-ROM). The project was introduced at the 1997 SIAM annual meeting in the Minisymposium on Handbooks for Special Functions and the World Wide Web, organized by our activity group. Articles appeared in the OPSF Newsletter, vol. 8, no. 1, Oct. 1997, pp. 22-23, and in the March 1998 issue of SIAM News.

The NSF award is under the Knowledge and Distributed Intelligence program, which seeks to foster the development of advanced research tools using the emerging capabilities of information technology. The DLMF will satisfy diverse user requirements such as simple lookup, complex search and retrieval, formula validation, interactive visualization, and pointers to software and evaluated numerical methodology. The award pe-

riod is three years.

The current table of contents includes 39 chapters. Most of these address classical special functions and orthogonal polynomials but included also are chapters on relevant methodology (e.g. asymptotic approximations, numerical methods, computer algebra) and recently developed topics (e.g. generalized and basic hypergeometric functions, discrete orthogonal polynomials, Painleve transcedents). The project is being supervised by an editorial board consisting of four NIST editors and ten non-NIST associate editors. For further details, see the project Web site: <http://math.nist.gov/DigitalMathLib>.

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### How to Contribute to the Newsletter

Send your Newsletter contributions directly to one of the Co-editors:

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preferably by e-mail, and in L<sup>A</sup>T<sub>E</sub>X format. Other formats are also acceptable and can be submitted by e-mail, regular mail or fax.

**Deadline for submissions to be included in the February issue 2000 is January 15, 2000.**

Back issues of the Newsletter can be obtained from <http://www.imn.htwk-leipzig.de/~koepf/siam.html>.

The Activity Group also sponsors an electronic news net, called the **OP-SF Net**, which is transmitted periodically by SIAM. The Net provides a rather fast turnaround compared to the Newsletter. To receive transmissions, just send your name and e-mail address to [poly-request@siam.org](mailto:poly-request@siam.org) (as with other nets, nonmembers can also receive the transmissions). Your OP-SF Net *contributions* should be sent to [poly@siam.org](mailto:poly@siam.org). Please note that submissions to OP-SF Net are automatically considered for publication in the Newsletter, and vice versa, unless the writer requests otherwise.

The Net is organized by Martin Muldoon ([muldoon@yorku.ca](mailto:muldoon@yorku.ca)). Back issues of OP-SF Net can be obtained by anonymous ftp from [ftp.wins.uva.nl](ftp://ftp.wins.uva.nl), in the directory <pub/mathematics/reports/Analysis/koornwinder/opsfnet.dir>

or by WWW at the addresses

<http://turing.wins.uva.nl/~thk/opsfnet/>  
<http://www.math.ohio-state.edu/JAT>  
<http://math.nist.gov/opsf/archive>

Martin Muldoon also manages our home page

<http://www.math.yorku.ca/siamopsf/>

on World Wide Web. Here you will find also a WWW version of the OP-SF Net. It currently covers the topics

- Conference Calendar
- Books, Conference Proceedings, etc.
- Compendia, tools, etc.
- Compiled booklist on OP-SF
- Meeting Reports
- Projects
- Problems
- Personal, Obituaries, etc.
- History
- Positions available
- Miscellaneous
- Memberlist
- Links to WWW pages of interest to members

### Activity Group: Addresses

The *SIAM Activity Group on Orthogonal Polynomials and Special Functions* consists of a broad set of mathematicians, both pure and applied. The Group also includes engineers and scientists, students as well as experts. We now have around 140 members scattered about in more than 20 countries. Whatever your specialty might be, we welcome your participation in this classical, and yet modern, topic. Our WWW home page <http://www.math.yorku.ca/siamopsf/> is managed by Martin Muldoon ([muldoon@yorku.ca](mailto:muldoon@yorku.ca)).

The **Newsletter** is a publication of the *SIAM Activity Group on Orthogonal Polynomials and Special Functions*, published three times a year. To receive the Newsletter, you must first be a member of SIAM so that you can join the Activity Group. The annual dues are \$96 for SIAM plus \$10 for the Group; students pay \$20/year with free membership in one activity group; postgraduates can become members of SIAM for \$45/year. To join, contact:

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