Orthogonal Polynomials and Special Functions

SIAM Activity Group on Orthogonal Polynomials and Special Functions

⋆ ⋆ ⋆ ⋆ **Newsletter** ⋆ ⋆ ⋆ ⋆

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From the Editor

This is my first issue as editor of the Newsletter, and I hope you are satisfied with its appearance. The current officers of the Activity Group appointed me for an initial one year term, and I accepted this appointment last June. Therefore, beginning with the current issue, we will come back to a normal schedule. The Newsletter will appear three times a year now, in October (this issue), February and June.

Since I was very pleased with the design of the Newsletter by the previous editor Eugene Tomer, who edited the Newsletter in the past few years, I changed as little as possible. I would like to thank Eugene for his contribution. In particular I am very grateful that he gave me the opportunity to use his layout, and sent me all the necessary files so that I could easily adapt his style. Also I thank Eugene for his work on the Newsletter logo.

Furthermore I would like to take the opportunity to thank Peter Deuflhard, the president of the Konrad-Zuse Center, who made my editorship possible.

This issue includes a message from the Chair of the Activity Group, Charles Dunkl, who gives fuller acknowledgement to Eugene for all the work he has done.

Eugene Tomer had resigned after the Winter Newsletter Volume 5, Number 2. In the meantime, all Activity Group members who have not subscribed to the electronic service OP-SF Net, run by Tom Koornwinder, received a hard copy issue with important material taken from several OP-SF Net issues. That issue of the Newsletter was prepared by George Gasper and Tom Koornwinder.

I would like to have the following topics regularly covered in future issues of the Newsletter:

– Reports from Meetings and Conferences
– Forthcoming Meetings and Conferences
– Books and Journals
– Software Announcements
– Problems and Solutions
– Miscellaneous

Any material concerning these topics, and any other announcements and articles that may be of interest to the members of the Activity Group are welcome. In this issue articles by Dick Askey on the work of Gábor Szegő (p. 19), by Amílcar Branquinho on the work of José Vicente Gonçalves (p. 22) and by Walter Van Assche on the INTAS project (p. 22) are included.

I would like to express my thanks to all contributors to the current issue. Contributors are cited by their names and email connections while their mailing addresses can be obtained from the Editor by request.

Note that our Newsletter can be only as good as your cooperation is. It will fulfill its purpose only if you contribute reports on past, and announcement of forthcoming meetings, and if other articles and announcements are submitted. For the submission procedure see *How to Contribute to the Newsletter* on page 23.

As all of you know, there is an election for officers whose terms will begin January 1 scheduled for this fall. Unfortunately the members of our Activity Group were inadvertently included on a mailing list for the distribution of the activity group on dynamical systems' ballot by SIAM Headquarters, and therefore were sent the wrong ballots. The ballots for our Activity Group should be mailed in early October. I would like to remind you to send in your filled-in ballot by the time posted.

September 30, 1995 Wolfram Koepf

Message from the Chair

Tribute to Eugene Tomer

On behalf of the members and officers I express our gratitude and appreciation for the accomplishments and hard work that Eugene performed as editor of the Newsletter of this Activity Group. He first volunteered his services in 1992 and swiftly moved us from a small annual letter that George Gasper and I put together to a beautifully produced quarterly Newsletter. Eugene also designed the logo for the group, a design based on Chebyshev polynomials, and incorporated it into an attractive masthead for the newsletter. He organized the Problems column and attracted interesting submissions from both basic and applied areas of mathematics. The Newsletter played a large part in the growth of the group's membership with sizable delegations from many different countries. Eugene held the newsletter to high professional standards of accuracy and carefully edited material. By the latter half of 1994 he began to feel that he had done his share of the work in getting the group under way and that others should pick up the load. He expressed to me strongly his opinion that the group should do more to get involved with the applications of mathematics, for example, in astrophysics, physics, and the sciences that depend on special function solutions of differential equations. The officers accept this challenge and hope that the slate of candidates for the coming election of officers for the 1996–1998 term is a good beginning. We are all grateful for the contribution that Eugene made toward the functioning and success of the group and we wish him well in his future endeavors.

Welcome to Wolfram Koepf

It is with great pleasure and appreciation that we welcome Wolfram to the Editorship of the Newsletter. Please keep in mind that the Newsletter exists to promote communications among researchers and those who apply special functions to solve problems in applied mathematics. We need news items—conference announcements, informal book reviews, brief descriptions of applications and problems in special functions and so on. Also there are computer-related topics that are of interest to our group: software for computations, systems for symbolic mathematics that can be involved in mathematics education, etc.. Send in an item soon!

Charles Dunkl
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Reports from Meetings and Conferences

1. Density Functionals of Quantum-Mechanical Systems and Constructive Complex Analysis

This workshop was held at the *Universidad de Granada*, February 20–23, 1995, getting together all the participants of this INTAS research project. Detailed information about the topic of this workshop can be found in the article *INTAS Project: Constructive Complex Analysis and Density Functionals* on page 22.

A total of 26 people attended the workshop. The first day was devoted to *classical and Sobolev orthogonal polynomials*, with talks by Francisco Marcellán, Manuel Alfaro, Renato Alvarez, Miguel Piñar, and Jose Carlos Petronilho. During the second day, the talks centered around *operator theory, rational approximation and orthogonal polynomials*, with talks by Valeri Kaliaguine, Guillermo López, and Andrei Martínez. The third day was reserved for the central theme of the project: *entropy, density functionals and complex analysis*. The morning talks were of a theoretical physics flavour, with expositions by Marco Frontini, Juan Carlos Angulo, and José Caro. The afternoon sessions were for the fans of orthogonal polynomials who heard talks by Alexander Aptekarev and myself. The final day attracted the interest of those working with *rational approximation and potential theory*. Pablo Gonzalez-Vera and Sergei Suetin gave talks in the morning, the afternoon was covered by two splendid expositions of Herbert Stahl and Andrei Gonchar.

Walter Van Assche
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2. Lee Lorch Conference

In September 1995, Lee Lorch celebrated his eightieth birthday. For this occasion, a conference was organized at York University (Canada) on June 9 and 10, 1995. Some forty people were present and attended various talks on mathematical topics of interest to Lee Lorch, but also talks related to the societal topics in which Lee always has had active interest.

One of Lee's mathematical interests has been the monotonicity properties of zeros of Bessel functions or functions of a similar nature. A basic method for studying monotonicity properties of zeros of special functions is Sturm's method, for which there is a continuous version (for zeros of the solution of a Sturm-Liouville differential operator) and there is also a discrete version for polynomials which form a Sturm sequence (a three-term recurrence with a particular sign property). Lee Lorch's main contributions can be found in joint papers with Peter Szego [1,7,8] and with Martin Muldoon and Peter Szego [2,9]. Lee also had a continuing interest in Lebesgue constants and various methods of summability [3-6].

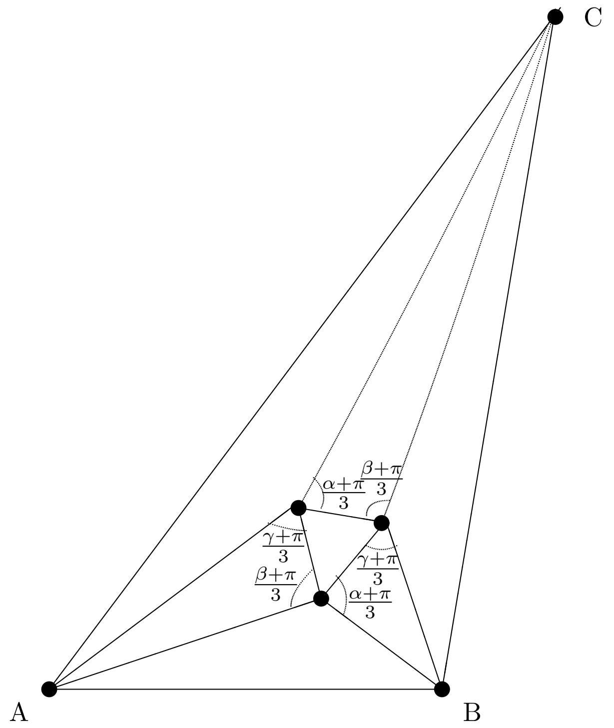
On the first day of the conference (June 9, 1995) there were one hour talks by Richard Askey on *Bessel functions and how to use them when considering more general classes of functions* and Cora Sadosky on *Restricted BMO in products spaces*. There were also some 30 minute talks by P.G. Rooney, Walter Van Assche, Angelo Mingarelli, A. McD. Mercer, Mark Ashbaugh, Mourad Ismail, James A. Donaldson, and Árpád Elbert dealing with Hankel transforms, zeros of orthogonal polynomials, eigenvalues of matrices and differential operators, Bessel functions and isoperimetric inequalities, integral operators and q-Sturm-Liouville problems, and linear shallow water theory.

The morning of the second day of the conference was still devoted to mathematics with one hour talks by Roderick Wong on *Asymptotics and special functions*, Jean-Pierre Kahane on *Summability, order and products of Dirichlet series*, and 30 minute talks by Dennis Russell and Mark Pinsky on *Fourier transforms and Fourier integrals in several variables*. In the afternoon there was a nice talk by

Donald J. Newman with a beautiful proof of the *Morley Triangle Theorem*:

Theorem 1 *Let ABC be an arbitrary triangle with vertices at the points A, B, C. Construct a new triangle (the Morley triangle) by trisecting the angles α, β and γ at respectively A, B and C inside the triangle ABC and by taking the first intersection points as the vertices of the new triangle. Then this Morley triangle is always an equilateral triangle.*

Newman's proof is basically contained in the following picture.



There was also a 30 minute talk by Amram Meir on random trees, and the rest of the afternoon of June 10 was devoted to Lee's societal contributions for which he has become well-known, with talks by Chandler Davis, Mary Gray, and Johnny Houston pointing out the *contributions of Lee Lorch to expanding access to mathematics*, showing that Lee Lorch is *a mathematician who cares*.

Finally, the conference was closed with a dinner during which Lee was awarded the Lifetime Achievement Award of the National Association of Mathematicians by Jack Alexander, president of the Association.

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Walter Van Assche
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3. Workshop on Special Functions, *q*-Series and Related Topics

From June 12 to June 23, 1995, a workshop on special functions, q-series and related topics was held at the University of Toronto, under the auspices of the *Fields Institute For Research In Mathematical Sciences*. As the title already indicated, a great variety of subjects were to be expected, both in the five minicourses during the first week and in the contributed lectures during the second week. Roughly speaking, the minicourses and the lectures could be divided into three groups. The first group was concerned with special functions and q-series as such (mostly in one variable), the second group concentrated on the relationship between special functions, q-series and representation theory (not necessarily in one variable), and the third group treated miscellaneous aspects of special functions, as diverse as, e.g., connections with computer algebra, combinatorics, probability theory and superbly converging algorithms for approximations of π. The topics in the latter category had the virtue of appealing easily to a general uninitiated audience, but it did not go unnoticed that the audiences for the first and second group of topics appeared to be somewhat disjoint. Even though the theory of special functions and the representation theory of (quantum) groups are related, this has apparently not encouraged too many people to be engaged in both fields—although there are of course exceptions. At any rate, anyone interested in special functions must have found something of interest at this workshop—the diversity of the program was a guarantee for that.

Marcel de Jeu
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The weather in Toronto for the Fields Institute session on q-series and special functions was quite nice making attendance quite a pleasure. The first week talks were tutorials which I think were moderately successful, some drifted into the informal talk category, some were quite competent introductory surveys of their subject, while others were aimed at students and were replete with exercises. I understand that one person even handed in solutions! I think that I gained something from all that I attended. I found the atmosphere among the international participants friendly, especially the second week, when I think people had relaxed somewhat. The talks during the second week were in general well presented, and toward the end a disagreement even broke out after one talk livening up the atmosphere. One fellow could be counted on throughout the conference to provide comic relief, but I'm not sure that this was intentional. Another highlight for me was attending planning sessions on a Bateman project revision. I have always loved tables of special functions.

Doug Bowman
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The Toronto workshop was great for me, first of all because I could meet there so many colleagues in my field. Concerning the program, I was in particular excited by the last week, which was more research oriented than the first week and was also more intensive. The regular program (9-12 a.m. and 1.30-5.00 p.m.) was already quite tough in the second week, but moreover there was a busy fringe program: two business meetings on a possible Askey-Bateman book project and two evening sessions on Multivariable Special Functions, Algebraic Structures and Mathematical Physics, organized by Luc Vinet. Some of the lectures I heard there were among the most memorable things of my two weeks in Toronto.

Compared to the Columbus NATO Advanced Study Institute of 1989 there was less emphasis on classical analysis aspects and much more on multi-variable, algebraic, formal and combinatorial aspects. In this respect Toronto may not have given a completely balanced survey of the state of the art in the theory of orthogonal polynomials and special functions, but this will be easily compensated by some other meetings in the past (e.g. Delft 1994) or in the future.

The first week was devoted to introductory minicourses. Then the program was much more relaxed, starting only at 9.30 a.m. and often ending at 3 p.m. It is my feeling that we might have been slightly more effective here if we had defined better for each course what knowledge could be assumed and if some or all of the courses would have comprised four rather than three one-hour lectures. There were some tutorial sessions in connection with the minicourses. For one topic this was given by an advanced PhD student, for the other topics it was done by the lecturers themselves. Since this was kind of an experiment, I am curious to hear from participants whether they appreciated these tutorials. Another event in the first week, which I personally liked very much, was a demonstration by Christian Krattenthaler of his Mathematica package implementing part of the book by Gasper and Rahman on basic hypergeometric series.

The setting of the workshop, at the Oxford and Cambridge modeled St. George campus in central Toronto, was superb. The lectures were in University College, the building where the University of Toronto started in the 19th century. Never before at a conference have I had registration and refreshments service in such a nice room as here. It was surprising that this historic building housed such well-equipped lecture rooms, although better air conditioning would have been welcome when outside temperature rose to 36 degrees Celsius. The lecture room for the main lectures in the second week compensated for this by having all doors open, including a door to the street. Sometimes, a concert on nearby bells enlivened the lectures. This room was very special because of its high balcony and the long stairs coming down to the ground level. There was something very theatrical about this room, and indeed, some of the lectures and subsequent discussions were theater.

Social events included a barbecue in the first week in the pleasant Hart House quadrangle and a banquet in the second week in the splendid Great Hall of Hart House. Both occassions were also an opportunity for fraternizing with the people of the parallel PDE workshop of the Fields Institute. The banquet concluded with a piano recital by Christian Krattenthaler, a man with many-sided talents.

The Fields Institute organisational machinery had some trouble getting off the ground in the first week, but gradually everything went quite smoothly, the congress secretary was kind and efficient, and the refreshments in the breaks were delicious. David Masson, coordinator of the scientific program (and of much more) has really done a great job.

Tom H. Koornwinder
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On June 16th, for submitting the "best" set of solutions to three exercises in the "q-Series" Minicourse by George Gasper, Dennis Eichhorn was awarded an autographed copy of the N.M. Atakishiyev and S.K. Suslov Russian translation of the Gasper and Rahman *Basic Hypergeometric Series* book. It was autographed by the authors, translators, and R. Askey who wrote the Foreword in the book.

George Gasper
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4. Askey-Bateman Project

Everyone interested in special functions probably knows the Bateman Manuscript Project (A. Erdélyi et al.), which was published in the 1950's. This project consists of three volumes of *Higher Transcendental Functions* and two books of *Tables of Integral Transforms*. They have been a rich source of information for about 40 years now.

In the meantime our knowledge of special functions and the applicability of them has grown. Some aspects of special functions which are important in contemporary research are not covered in the Bateman manuscript project. Furthermore the newest technology (internet, WWW) forces us to consider an online version of the Bateman project.

For some years people have started to play with the idea of preparing an upgrade of the Bateman project, worthy for the state-of-the-art at the end of the twentieth century. One of the most active in promoting the idea of a new Bateman project is Richard Askey.

During the Fields Institute mini program *Special Functions, q-Series and Related Topics* in Toronto, we had some planning meetings for a *Askey-Bateman project*. Mourad Ismail and Walter Van Assche are ready to coordinate this project and a list of possible topics has been compiled and includes all the chapters already available in the 1950 Bateman project together with some new topics such as group representations and special functions, q-series, continued fractions, hypergeometric functions of several variables and matrix argument, and computer algebra for special functions. Some consultants have been proposed and for some of the topics a number of people have already been assigned for the preparation of a text. We have not yet been able to approach everybody suggested in Toronto, so we would rather not give a list of names yet.

The SIAM activity group *Orthogonal Polynomials and Special Functions* can play an active role in this Askey-Bateman project and the coordinators will attempt to keep the group informed of the progress. We invite everyone interested in this project to contact the coordinators

Mourad Ismail: ismail@math.usf.edu

Walter Van Assche: walter@wis.kuleuven.ac.be

All help and suggestions are welcome.

Walter Van Assche
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5. Minisymposium at ICIAM '95 (Hamburg)

The ICIAM '95 took place in Hamburg, Germany during July 3–7, 1995.

This quadrennial series was born in Paris in 1987. SIAM and SMAI, together with GAMM and IMA sponsored this first conference.

The second conference was hosted and organized by SIAM and held in Washington DC in 1991 under the sponsorship of a growing group of members of CICIAM (*Committee for the International Conferences on Industrial and Applied Mathematics*).

3000 contributed lectures were attended in Hamburg and about half of them were incorporated in the 230 Minisymposia. Thirty two main lectures covered the most active fields of industrial and applied mathematics.

A Minisymposium on *Orthogonal Polynomials and Spectral Methods* sponsored by SIAM in the framework of the activities of the *Activity Group on Orthogonal Polynomials and Special Functions* was organized by Martin E. Muldoon, André Ronveaux and Francisco Marcellán. This minisymposium was attended by 35 people on Tuesday afternoon July 4.

Spectral methods are techniques for the discretization of PDE's which rely on the approximation by high degree polynomials, in particular partial Fourier sums with respect to orthogonal polynomials related to the geometry of the problem.

Four invited speakers gave thirty minute lectures to form the core of the minisymposium:

A. Ronveaux (Facultés Universitaires Notre Dame de la Paix, Namur, Belgium) presented a short introduction to polynomials orthogonal with respect to Sobolev inner products. A survey of the more recent results in this emerging domain constituted the aim of this lecture with special emphasis in the analytic theory of such polynomials and the properties of their zeros.

C. Bernardi (Université Pierre et Marie Curie, Paris, France) introduced spectral methods for solving axisymmetric partial differential equations, a joint work with Y. Maday. The computation of the discrete solution of 3D equations is always very expensive. However, when the domain is invariant by rotation about an axis, the problem can often be reduced to a family of 2D equations thanks to the use of cylindrical coordinates and Fourier transform with respect to the angular variable. In this lecture was presented the spectral discretization of elliptic problems and its numerical analysis together with some numerical experiments.

The lecture by S.P. Norsett (Norvegian Institute of Technology, Trondheim, Norway) was related to Sobolev-orthogonal polynomials and pseudospectral methods for parabolic PDE's. This joint work with A. Iserles (University of Cambridge, U.K.) and J. Sanz-Serna (Universidad de Valladolid, Spain) continuates their research about the applications of nonstandard polynomials and the so called coherent pairs.

B. D. Shizgal (University of British Columbia, Vancouver, Canada) considered a quadrature discretization method in the solution of a 1D Schrödinger equation. This spectral method is based on the discretization of the wave function on a grid of points that coincides with the zeros of polynomials orthogonal with respect to a weight function determined by the potential function in the Schrödinger equation. In this contribution, the weight functions are derived from the Morse potential that approximates the potential. The rate of convergence of the eigenvalues and eigenfunctions is extremely rapid.

Francisco Marcellán
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Forthcoming Meetings and Conferences

1. Minisymposium at SIAM Annual Meeting in Charlotte

The Minisymposium *Computational Aspects of Special Functions and Orthogonal Polynomials*, sponsored by the *SIAM Activity Group on Orthogonal Polynomials and Special Functions*, will be held during the **1995 SIAM Annual Meeting**, Adam's Mark Hotel, Charlotte, North Carolina, USA, October 23-26, 1995. It is scheduled for Wednesday afternoon, October 25, 1995, from 5.00 to 7.30 p.m. Details concerning the complete meeting can be obtained by WWW:

http://www.siam.org/meetings/an95/an95home.htm

Organizer: Martin E. Muldoon, York University (Program Director, SIAM Activity Group on Orthogonal Polynomials and Special Functions)

Summary: Many problems of applied mathematics lead to approximations which eventually involve the computation of special functions. Such approximations can provide insight as well as computational benefits. The important issues of software availability and reliability are addressed in general terms in the talk by Lozier and in some specific cases in that by Temme. The talks by Dunkl and Boyd address specific problems related to the computation of special functions from surface measures of ellipsoids and in solitary wave theory respectively. Temme and Boyd address the issue of asymptotic behavior (i.e., the situation which arises when a parameter is large). There has been much recent work on classes of orthogonal polynomials and the relations between the orthogonality measures, recurrence relations and zeros of the polynomials. The talk by Gautschi will be concerned with some numerical aspects of these relations.

Program
Wednesday, October 25

5:00 p.m.: **John P. Boyd**, University of Michigan. *Numerical Verification of Exponential Asymptotics via a Chebyshev Polynomial Pseudospectral Algorithm in Multiple Precision: Radiation Coefficient of a Weakly Nonlocal Solitary Wave.*

> *Abstract:* Weakly nonlocal solitary waves are similar to classical solitary waves except for tiny sinusoidal waves of amplitude α which radiate away from the core. Because α is an *exponential* function of the *reciprocal* of $\varepsilon \ll 1$, power series in ε cannot compute it. New perturbation methods known variously as "exponential asymptotics" or "hyperasymptotics" can. We used multiple precision algorithms, both finite differences of fourteenth and twenty-fourth order and Chebyshev polynomial methods, to demonstrate a puzzling discrepancy between the predictions of theory and the actual $\alpha(\varepsilon)$.

5:30 p.m.: **Charles F. Dunkl**, University of Virginia. *Hyperelliptic Integrals, the Surface Measure of Ellipsoids, and Response Surfaces* (with Donald E. Ramirez).

> *Abstract:* The surface measure of an ellipsoid in N-space can be expressed as a Lauricella F_D-type function. An efficient Romberg-type quadrature is used to evaluate the function. The algorithm is then incorporated into a criterion for constructing optimal response surfaces in statistical applications.

6:00 p.m.: **Walter Gautschi**, Purdue University. *Computing Orthogonal Polynomials of Sobolev Type.*

> *Abstract:* The concern here is with polynomials that are orthogonal with respect to an inner product involving not only function values (on the real line) but also derivative values up to some order. Each derivative has associated with it a measure, which may be of a continuous or discrete type. Given modified moments, or Gaussian quadrature rules, for these measures, it is shown how to use them to compute appropriate recurrence coefficients as well as zeros for the desired polynomials.

6:30 p.m.: **Daniel W. Lozier**, National Institute of Standards and Technology.
Software Issues in the Computation of Special Functions.

> *Abstract:* The classical special functions are useful in engineering, scientific and statistical applications because they can provide computational benefits and mathematical insights. Subroutines for their computation are among the earliest examples of computer software; however current li-

braries and interactive systems are neither complete nor uniformly accurate. A review of current software for computing and testing special functions will be given. A suggestion for establishing a Software Testing Service Center will be presented for discussion.

7:00 p.m.: **Nico M. Temme**, CWI — Amsterdam.
Large Parameter Evaluations of Some Classical Distribution Functions.

Abstract: We discuss the numerical evaluation of several classical distribution functions, such as the incomplete gamma and beta functions and the non-central χ^2-distribution. In particular we consider the case that the parameters are large and we mention asymptotic inversion methods for the incomplete gamma and beta functions. For the incomplete gamma function we also consider the evaluation for the case that parameters are large and complex. We give information on the availability of algorithms for this kind of functions in the well-known software collections, in particular on how they perform in the large parameter case.

Martin Muldoon
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2. Minisemester on Quantum Groups and Quantum Spaces

Stefan Banach International Mathematical Center in Warsaw, Poland will host a Minisemester on *Quantum Groups and Quantum Spaces* during November 6 until December 1, 1995. The last (fourth) week will also include applications to the theory of special functions. A more definite First Announcement is now obtainable from Stanislaw Zakrzewski (szakrz@fuw.edu.pl). From this Announcement we extract the following information.

The fourth week (27 November - 1 December) will have the two themes:

- Special Functions.
  Main topics: q-special functions (ζ, Γ, hypergeometric, polynomials, exponentials), polylogarithms, solutions of q-symmetry problems.
- Noncommutative Geometry and Physics (II).
  Main topics: Deformed spacetime (Poincaré symmetry). Deformed harmonic oscillator (rotation group). Field theory on quantum spacetime.

The Advisory Scientific Committee consists of A. Connes, A. Van Daele, K. Gawcedzki, M. Gerstenhaber, M. Flato, T.H. Koornwinder, J.-H. Lu, J. Wess, S.L. Woronowicz.

Participation and registration: Participants are expected to come, as a rule, for one week of their choice (optionally, two weeks). The Banach Center will cover the living expenses of a limited number of participants. Unfortunately, no travel expenses can be covered. The proceedings will be published as a special volume of the Banach Center Publications.

Stanislaw Zakrzewski
(szakrz@fuw.edu.pl)

3. International Conference on Harmonic Analysis, Delhi, India

An *International Conference on Harmonic Analysis* will be organized at the Department of Mathematics, University of Delhi, Delhi-110007, India from 18th to 22nd December, 1995. The programme will include special talks on different aspects of harmonic analysis such as measure algebras, homomorphisms, multipliers, positive definite functions, spectral synthesis and H-spaces. A series of expository lectures on Hypergroups by Professors K.A. Ross, A.L. Schwartz and others will be a special feature of the conference.

For further information please contact:

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A.I. Singh

4. Umbral Calculus Special Session: MIT, April 22-23, 1996

In honor of Gian-Carlo Rota's 64th birthday, the *RotaFest* conference is being organized by Richard Stanley, Neil White and Richard Ehrenborg at *The Massachusetts Institute of Technology*, Cambridge, Massachusetts, USA next year from April 17 to April 20, 1996.

This conference will be followed by a two day workshop devoted to Rota's Umbral Calculus. This special session will be held Monday and Tuesday, April 22 and 23, 1996, and is organized by Alessandro Di Bucchianico, Daniel Loeb, and Nigel Ray. (The session is in place of the previously proposed NATO sponsored workshop in the Netherlands.)

Please let us know whether you are interested in attending. We may organize a poster session if there is sufficient interest.

Note that there will be no special funding associated specifically with the Umbral Calculus workshop.

We look forward to seeing you in Boston next year.

Alessandro Di Bucchianico
(sandro@win.tue.nl)

5. CRM Workshop on the Theory of Special Functions

The *Centre de Recherches Mathematiques* is hosting a year long program in applied and numerical analysis in 1995–1996. The year's activities will be devoted to the following subjects: applications to hydrodynamics, the mathematics of finance, and recent developments and methods (wavelets, neural networks, special functions). Elements of the program are organized jointly with the *Centre de Recherche en Calcul Appliqué* (CERCA) or with the *Centre Interuniversitaire de Recherche en Analyse des Organisations* (CIRANO). The CRM Summer School, which will again be held in Banff, Alberta during the month of August, is part of the theme year. Following is a schedule of events:

Between May 6-26, 1996, the *Workshop on the Theory of Special Functions* will be organized. Two workshops are planned for this period, one entitled *Theory of Nonlinear Special Functions: The Painlevé Transcendents* (May 13-18) and the other *Algebraic Methods and q-Special Functions* (May 20-26). A series of lectures on topics related to the workshops will be organized during the week of May 6-10.

Organizers: L. Vinet (CRM), P. Winternitz (CRM).

The list of invited lecturers includes: M. Ablowitz, G.E. Andrews, R. Askey, D. Bressoud, I. Cherednitz, P.A. Clarkson, C.M. Cosgrove, B. Dubrovin, C.F. Dunkl, P.I. Etingof, H. Flaschka, A.S. Fokas, I.B. Frenkel, B. Grammaticos, M. Ismail, A.R. Its, M. Jimbo, N. Joshi, E.G. Kalnins, A.V. Kitaev, T.H. Koornwinder, V.E. Korepin, I.M. Krichever, M.D. Kruskal, I. Macdonald, V.B. Matveev, F. Nijhoff, M. Noumi, E. Opdam, C. Rogers, F. Smirnov, D. Stanton, S. Suslov, A.P. Veselov.

If you wish to participate in any of these activities, please contact Louis Pelletier for registration a month before the event for accommodation or two weeks before for attendance only.

If you wish to receive any financial support (if available) please write us a letter, to be mailed to

Louis Pelletier
CRM, Université de Montreal
C.P. 6128, Succ. Centreville
Montreal (Quebec), Canada, H3C 3J7

or send an email message specifying your research interests.

Louis Pelletier
(PelletL@CRM.UMontreal.ca)

6. Meeting on Symmetries and Integrability of Difference Equations

A meeting on *Symmetries and Integrability of Difference Equations* (SIDE) will be held at the University of Kent at Canterbury from Monday 1st July to Friday 5th July 1996. This conference is the successor of the meeting on the same topics held in Esterel, Quebec, Canada in May 1994.

The proposed meeting, like its predecessor in Esterel, is planned to bring together researchers who work in the field of symmetries and integrability of difference equations. The subject area is relatively young. In the past few years a great deal of progress has been made on the mathematical aspects of discrete integrable systems, including integrable dynamical mappings, ordinary and partial difference equations, lattice solitons, discrete versions of the Painlevé equations, symmetry approaches and singularity analysis, and applications to numerical analysis, computer science and physics. The meeting in Esterel brought together for the first time many leading experts in the various aspects of the field. As with the previous meeting in Esterel, this meeting will be of an interdisciplinary nature, and a source of contact between the different disciplines.

The conference is being organised by Professor Peter Clarkson, Institute of Mathematics & Statistics, University of Kent (P.A.Clarkson@ukc.ac.uk) and Dr. Frank Nijhoff, Department of Applied Mathematical Studies, Unversity of Leeds (frank@amsta.leeds.ac.uk). The scientific committee consists of Peter Clarkson, Frank Nijhoff, Thanasis Fokas, Loughborough University, U.K. (A.S.Fokas@lut.ac.uk) and Pavel Winternitz, University of Montreal, Canada (wintern@ere.umontreal.ca). The local management of the meeting will be run by Peter Clarkson and Alan Common (A.K.Common@ukc.ac.uk), of the Institute of Mathematics & Statistics.

The conference finances will be run on a minimal basis with delegates charged a small registration fee of 50 pounds and a residential fee of about 40 pounds per day to cover bed, breakfast, lunch, coffee, tea and evening meal. Anyone registering after 31 March 1996 will be charged an additional late registration fee of 50 pounds. Accommodation is limited so early registration is encouraged. We will attempt to attract some supporting grants. We are grateful to the Institute of Mathematics & Statistics for financial support.

The organisers would like to encourage all communications regarding the conference to be carried out by email as far as possible. Please email the organisers as soon as possible if you are interested in participating in this meeting; this is not a definite commitment.

Peter Clarkson
(P.A.Clarkson@ukc.ac.uk)

7. Workshop Transform Methods & Special Functions, II (1996)

The Second International Workshop *Transform Methods*

& Special Functions will take place in the Black Sea resort *Golden Sands* near Varna, Bulgaria in the tentative period 24–31 August 1996.

The First International Workshop *Transform Methods & Special Functions* took place in the resort town of Bankya (near Sofia), Bulgaria in the period 12–17 August, 1994, see the book announcement of the corresponding proceedings on p. 11.

In 1996 the 100th Anniversary of the eminent Bulgarian mathematician Nikola Obrechkoff (1896–1963), whose achievements are closely related to the above topics, will be celebrated in Bulgaria. It is a good occasion to organize the Second International Workshop *TM & SF*. The proposed period of time is the last week of August, 1996 and the place is a Black Sea resort, near Varna (the birthplace of N. Obrechkoff).

More details on the exact period of time, registration fees, accommodation and scientific programme will be given in the *First Announcement* in October 1995. It will be sent to those who confirm their interest in the meeting. If you are interested, please send an email message to Virginia Kiryakova (virginia@bgearn.bitnet) mentioning your name, email address and ordinary address and preliminary subject of your lecture.

The Organizers will appreciate any suggestions and financial supports. Thank you in advance for your commitment and cooperation.

Organizing Committee: Prof. Dr. Petar Rusev, Prof. Dr. Ivan Dimovski, Prof. Dr. Shyam L. Kalla (Kuwait University), Asso. Prof. Dr. Virginia Kiryakova, Asso. Prof. Dr. Lyubomir Boyadjiev.

Virginia Kiryakova
(virginia@bgearn.bitnet)

8. UN/ESA Workshops

In the past four years the United Nations (UN) in cooperation with the European Space Agency (ESA) organized a series of Workshops on *Basic Space Science* for the benefit of Third World countries in four regions on Earth: Asia and the Pacific (India 1991), Latin America and the Caribbean (Costa Rica and Colombia 1992), Africa (Nigeria 1993), and Western Asia (Egypt 1994). Each Workshop programme addressed selected topics of astronomy and brought to life an astronomical follow-up project for the respective region, usually the establishment of an astronomical observatory in a Third World country. This series of Workshops was co-organized by the *German Space Agency* (DARA), the *International Centre for Theoretical Physics Trieste* (ICTP), the *National Aeronautics and Space Administration of the United States of America* (NASA), and *The Planetary Society* (TPS). A full account of the Workshop deliberations is provided in the homepage at

ftp://ecf.hq.eso.org/pub/un/un-homepage.html

The scientific programme in the past four Workshops focused on astronomy and physics, also emphasizing analytical and numerical mathematical techniques applied to problems in astrophysics.

The United Nations has been asked to organize a second circle of Workshops on Basic Space Science of which the 1995 Workshop will be held in Karachi, Pakistan (12-16 November 1995), again for Asia and the Pacific, and the 1996 Workshop will be hosted by Germany, and will take place at September 9.–13., 1996 at the *Max-Planck-Institut für Radioastronomie* and at the *Institut für Astronomie der Universität Bonn* in Bonn, this time for Europe; discussions are progressing well to have the 1997 Workshop again in Latin America and the Caribbean.

In all past Workshops it had been the desire of participating scientists to increase the number of presentations addressing topics in mathematics applied to problems in astrophysics. It is the intention of this note to bring the above homepage to the attention of Members of OP-SF Activity Group and to invite you to advice the United Nations on whether you would like to recommend mathematicians from Third World countries that are working in the field of orthogonal polynomials and special functions and their applications, preferably to problems in astronomy and physics.

The United Nations is currently in contact with Wolfram Research to channel more information on Mathematica into universities and research institutes in the developing world. Mathematica will be part of the programme of the next two UN/ESA Workshops on Basic Space Science.

I would be happy to provide any further information as might be appropriate.

Hans J. Haubold
(haubold@ekpvs2.dnet.tuwien.ac.at)

Books and Journals

1. Heun's Differential Equations
Edited by Professor André Ronveaux
with the contribution of:
F. M. Arscott, S. Yu. Slavyanov, D. Schmidt, G. Wolf, P. Maroni and A. Duval
Oxford University Press, New York, October 1995, 368 pp., ISBN 859695-2, $ 100.00.

Heun's Equation is a linear second-order differential equation which occurs in a wide range of problems in applied mathematics:

$$\frac{d^2y}{dz^2}+\left(\frac{\gamma}{z}+\frac{\delta}{z-1}+\frac{\varepsilon}{z-a}\right)\frac{dy}{dz}+\frac{\alpha\beta z-q}{z(z-1)(z-a)}y=0\,.$$

This differential equation with 4 regular singular points generates by confluence 4 different differential equations.

Areas in which these equations occur include the integral equations of potential theory, wave propagation, electrostatic oscillation, and Schrödinger's equation.

A recent symposium on the topic surveyed both the current theory and also the main areas of application. The book is a commissioned collection of edited papers from contributors to the symposium.

Contents:

Preface · Introduction

A. Heun's Equation
1. General Features of Heun's Equation
2. Transformations of Heun's Equation
3. Solutions of Heun's Equation: General and Power Series
4. Solutions of HE: Hypergeometric Function Series
5. Orthogonality Relations
6. Integral Equations and Integral Relations
7. Appendix
8. References

B. Confluent Heun Equation
1. General Features of the Confluent Heun Equation
2. Solutions of the Confluent Heun Equation
3. Confluent Heun Functions
4. Asymptotic Expansions
5. References

C. Double Confluent Heun Equation
1. General Features of the DCHE
2. The Analytic Theory of the DCHE
3. Special Results
4. References

D. Biconfluent Heun Equation
1. General Features of the BHE
2. Transformations of the BHE
3. Solutions of the BHE
4. Integral Relations
5. Miscellaneous
6. References

E. Triconfluent Heun Equation
1. General Features of the THE
2. Transformations of the THE
3. Solutions of the THE
4. Solutions in the Vicinity of the Singularity
5. Asymptotics with Respect to a Parameter
6. References

Addendum on Classifications · Biblibliography

2. Proceedings of First International Workshop Transform Methods & Special Functions
Editors: P. Rusev, I. Dimovski, V. Kiryakova

Science Culture Technology Publishing, Singapore 1995, 380 pp.

The First International Workshop *Transform Methods & Special Functions* took place in the resort town of Bankya, near Sofia (Bulgaria), 12–17 August 1994.
Organizing Committee: Profs. P. Rusev, I. Dimovski, S. L. Kalla (Chairmen), Asso. Profs. V. Kiryakova, L. Boyadjiev (Secretaries).
Main topics: Integral Transforms, Special Functions, Series Expansions, Fractional Calculus and Generalizations, Algebraical Analysis, Operational Calculus, Univalent Functions Theory and their applications to Complex Analysis, Differential and Integral Equations.
Participants: 46 mathematicians from Australia, Bulgaria, Byelorussia, Canada, Egypt, Germany, Italy, Japan, Kuwait, Poland, Russia, Taiwan, USA, Vietnam and Yugoslavia.
The volume contains invited surveys and papers submitted by the participants, among them many eminent experts in the areas of the Workshop.
The authors are: A. Al-Zamel, P. Antosik - W. Kierat - K. Skornik, I. Dimovski - R. Petrova, D. Dryanov - V. Vatchev, E. M. Elabd, S. Fukui, H.-J. Glaeske, R. Gorenflo - R. Rutman, R. N. Kalia, S. L. Kalla, A. Kilbas - M. Saigo, V. Kiryakova, S. Krasinska, A. Lecko - T. Yaguchi, E. R. Love, F. Mainardi - M. Tomirotti, S. Mincheva, D. Nikolic-Despotovic, K. Nishimoto, S. Owa, J. Paneva - Konovska, I. Podlubny, D. Przeworska, S. Rolewicz, P. G. Rooney, P. Rusev, M. Saigo - A. Kilbas, H. Saitoh, T. Sekine, Shih-Tong Tu - Ding-Kuo Chyan, K. Skornik, J. Sokol, J. Stankiewicz - Z. Stankiewicz - K. Wilczek, B. Stankovic, D. Takaci - A. Takaci, Vu Kim Tuan - R. Gorenflo, R. Yamakawa.

Contents: Preface · List of Participants · Articles · Appendix (Photomaterials of the Workshop).

To order copies of the volume, please write directly to the Publisher:

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3. Applications of Hypergroups and Related Measure Algebras
Edited by: William C. Connett, Marc-Olivier Gebuhrer, and Alan L. Schwartz

American Mathematical Society, Contemporary Mathematics **183**, February 1995, 441 pp., softcover, ISBN 0-8218-0297-6, List Price $69, Institutional Member Price $55, Individual Member Price $41, AMS Corporate Member Price $62.

From the Introduction: "The most important single thing about this conference was that it brought together for the first time representatives of all major groups of users of hypergroups. [They] talked to each other about how they were using hypergroups in fields as diverse as special functions, probability theory, representation theory, measure algebras, Hopf algebras, and Hecke algebras. This led to fireworks."

Hypergroups occur in a wide variety of contexts, and mathematicians the world over have been discovering this same mathematical structure hidden in very different applications. The diverse viewpoints on the subject have led to the need for a common perspective, if not a common theory. Presenting the proceedings of a Joint Summer Research Conference held in Seattle in the summer of 1993, this book will serve as a valuable starting point and reference tool for the wide range of users of hypergroups and make it easier for an even larger audience to use these structures in their work.

Contents:

N. Ben Salem and M.N. Lazhari, Limit theorems for some hypergroup structures on $\mathbb{R}^n \times [0, \infty[$
Yu.M. Berezansky, Nuclear spaces of test functions connected with hypercomplex systems and representations of such systems
Yu.M. Berezansky and A.A. Kalyuzhnyi, Hypercomplex systems and hypergroups: Connections and distinctions
W.R. Bloom and Z. Xu, The Hardy-Littlewood maximal function for Chébli-Trimèche hypergroups
H. Chebli, Sturm-Liouville hypergroups
W.C. Connett and A.L. Schwartz, Continuous 2-variable polynomial hypergroups
P. Eymard, A survey of Fourier algebras
A. Fitouhi and M.M. Hamza, Expansion in series of Laguerre functions for solution of perturbed Laguerre equations
L. Gallardo, Asymptotic behaviour of the paths of random walks on some commutative hypergroups
M.-O. Gebuhrer, Bounded measures algebras: A fixed point approach
H. Heyer, Progress in the theory of probability on hypergroups
T.H. Koornwinder, Discrete hypergroups associated with compact quantum Gelfand pairs
B.M. Levitan, Transmutation operators and the inverse spectral problem
M. Mabrouki, Limit theorems for the spin process
N.H. Mahmoud, Differential operators with matrix coefficients and transmutations
B.P. Osilenker, Generalized product formulas for orthogonal polynomials
G.B. Podkolzin, An infinitesimal algebra of the hypergroup generated by double cosets and nonlinear differential equations
M. Rösler, Convolution algebras which are not necessarily positivity-preserving
K.A. Ross, Signed hypergroups—a survey
V.S. Sunder, On the relation between subfactors and hypergroups
R. Szwarc, Connection coefficients of orthogonal polynomials with applications to classical orthogonal polynomials
K. Trimeche, Generalized transmutation and translation operators associated with partial differential operators
L. Vainerman, Gel′fand pairs of quantum groups, hypergroups and q-special functions
M. Voit, Central limit theorems for Jacobi hypergroups
N.J. Wildberger, Finite commutative hypergroups and applications from group theory to conformal field theory

4. New Journal: The Ramanujan Journal Editor-in-Chief: Krishnaswami Alladi

Kluwer Academic Publishers, Dordrecht–Boston–London

Commencing with publication in 1997 Kluwer plans a new quarterly journal: *The Ramanujan Journal*. It will be an international journal devoted to the areas of mathematics influenced by Ramanujan.

Editor-in-Chief: Krishnaswami Alladi (University of Florida)

Coordinating Editors: Bruce Berndt (University of Illinois), Frank Garvan (University of Florida)

Editorial Board: George Andrews · Richard Askey · Frits Beukers · Jonathan Borwein · Peter Borwein · David Bressoud · Peter Elliott · Paul Erdős · George Gasper · Dorian Goldfeld · Basil Gordon · Andrew Granville · Adolf Hildebrand · Mourad Ismail · Marvin Knopp · James Lepowsky · Lisa Lorentzen · Jean-Louis Nicolas · Alfred van der Poorten · Robert Rankin · Gerald Tenenbaum · Michel Waldschmidt · Don Zagier · Doron Zeilberger

Scope: The Ramanujan Journal will publish original research papers of the highest quality in all areas of mathematics influenced by Srinivasa Ramanujan. His remarkable discoveries have made a great impact on several branches of mathematics, revealing deep and fundamental connections.

The following alphabetical listing of topics of interest to the journal is not intended to be exclusive but to demonstrate the editorial policy of attracting papers which represent a broad range of interests: Circle method and asymptotic formulae · Connections between Lie algebras and q-series · Continued fractions · Diophantine analysis including irrationality and transcendence · Elliptic and theta functions · Fourier analysis with applications to number theory · Hypergeometric and basic hypergeometric series (q-series) · Mock theta functions · Modular forms and automorphic functions · Number theory · Partitions, compositions and combinatory analysis · Special functions and definite integrals.

For further information and for instruction to authors please contact Kelly Kriddle (krkluwer@world.std.com).

Software Announcements

1. Maple Umbral Calculus Package

Anne Bottreau, Alessandro Di Bucchianico and Daniel Loeb have developed a symbolic algebra package written in Maple to perform a variety of calculations related to the umbral calculus. This computer algebra toolkit was officially released at the conference on *Formal Power Series and Algebraic Combinatorics* (Marne-la-Vallée, France, May 29–June 2, 1995).

On request or by WWW

http://www.labri.u-bordeaux.fr/~loeb/umbral.html

you can be sent documentation *A Maple Umbral Calculus Package* (6 pages), or a tar file containing the actual Maple source code include online help, and an example worksheet. A 34 page report is also available in French.

This package has already proven to be a useful research tool. For example, our research on convolution sequences with persistent roots began in April 1991. However, we were only able to classify such sequences thanks to extensive computations with our Maple package.

We hope you find the package equally useful.

Alessandro Di Bucchianico
(sandro@win.tue.nl)

2. Kan: A System for Computational Algebraic Analysis

Kan is a system for doing algebraic analysis by computer based on computations of Groebner bases. We can do computations in the rings of polynomials, differential operators, difference operators and q-difference operators by the Kan system. The integrals (direct images), restrictions (inverse images) and free resolutions of modules can be computed by the system. These abilities can be used for computation of invariants for D-modules and mechanical theorem provings of binomial and special function identities. A tutorial is also provided for the version 1.

Kan is developed and implemented by Nobuki Takayama, Department of Mathematics, Kobe University, Japan. Kan is available for Unix platforms by anonymous ftp at ftp.math.s.kobe-u.ac.jp in the directory pub/kan. For questions, please contact the developer by email at kan@math.s.kobe-u.ac.jp. A first version came out on April 19, 1992 and the latest version on January 7, 1995.

If one is interested in computing free resolutions, it is recommended to use D-Macaulay that is based on Macaulay by D. Bayer and M. Stillman. D-Macaulay is sharing a code with Kan and is more efficient for computing free resolutions of D-modules. An alpha test version of D-Macaulay is available from the same ftp site.

Nobuki Takayama
(taka@math.s.kobe-u.ac.jp)

3. Maple and REDUCE Packages on Summation

Careful implementations of the Gosper and Zeilberger algorithms for indefinite and definite summation for rather general input covering extensions described in [2] are available in REDUCE and Maple and can be obtained by WWW from

ftp://elib.zib-berlin.de/pub/UserHome/Koepf/homepage.html

These packages have been done by Wolfram Koepf and Gregor Stölting on the lines of Tom Koornwinder's Maple program `zeilb`, and they come with test files including many examples.

The REDUCE package `ZEILBERG` is part of the new REDUCE Version 3.6, and the Maple package `sumtools` will be part of the next Maple release. Furthermore, Maple's `sum` command will include enhancements invoking this package by an implementation of Wolfram Koepf.

A forthcoming book, tentatively titled *Algorithmic Summation and Special Function Identities with Maple*, will cover a description of algorithms for the generation of recurrence equations, differential equations and derivative rules for hypergeometric sums and hyperexponential integrals, furthermore implementations of these algorithms in Maple, and many worked examples concerning special functions and orthogonal polynomials.

References

[1] Gosper, Jr., R. W.: Decision procedure for indefinite hypergeometric summation, Proc. Nat. Acad. Sci. USA **75** (1978), 40–42.

[2] Koepf, W.: Algorithms for the indefinite and definite summation. Konrad-Zuse-Zentrum Berlin (ZIB), Preprint SC 94–33 (December 1994).

[3] Koornwinder, T. H.: On Zeilberger's algorithm and its q-analogue: a rigorous description. J. of Comput. and Appl. Math. **48** (1993), 91–111.

[4] Zeilberger, D.: A fast algorithm for proving terminating hypergeometric identities, Discrete Math. **80** (1990), 207–211.

Wolfram Koepf
(koepf@zib-berlin.de)

Problems and Solutions

Thus far ten problems have been submitted while three have been solved (#**1**, **4**, **6**).

Solutions of the problems #**7** and #**10** are presented in the current issue.

2. Is it true that

$$x^2 t^x \, {}_2F_1(x+1,\, x+1;\, 2;\, 1-t)$$

is a convex function of x whenever $-\infty < x < \infty$ and $0 < t < 1$?

Submitted by George Gasper, August 19, 1992.
(george@math.nwu.edu)

3. The following Toeplitz matrix arises in several applications. Define for $i \neq j$

$$A_{ij}(\alpha) = \frac{\sin \alpha\pi(i-j)}{\pi(i-j)},$$

and set $A_{ii} = \alpha$. Conjecture: the matrix

$$M = (I - A)^{-1}$$

has positive entries. A proof is known for $1/2 \le \alpha < 1$. Can one extend this to $0 < \alpha < 1$?

Submitted by Alberto Grünbaum, November 3, 1992.
(grunbaum@math.berkeley.edu)

5. The result of Problem #**4** can be generalized to

$$\begin{aligned} S_m &= \sum_{n=0}^{\infty} \frac{(-1)^n (mn+1/2)!}{\sqrt{\pi}\,(mn+1)!} \\ &= \frac{1}{m} \sum_{k=0}^{m-1} \frac{\sin\left(5(2k+1)\pi/(4m) + \pi/4\right)}{\left[2\sin\left((2k+1)\pi/(2m)\right)\right]^{1/2}} \end{aligned}$$

valid for integral $m \ge 2$.

Submitted by J. Boersma and P.J. de Doelder,
July 12, 1993.
(wstanal@win.tue.nl)

7. The incomplete Airy integral given by[1]

$$I_0(\sigma, \gamma; k) = \int_{\gamma}^{\infty} e^{jk(\sigma z + z^3/3)}\, dz \tag{1}$$

serves as a canonical integral for some sparsely explored diffraction phenomena involving the evaluation of high frequency EM fields[2] near terminated caustics and composite shadow boundaries. In equation (1), k is the wavenumber of the propagation medium and is assumed to be the large parameter. Both the parameters σ and γ are real.

The desired task is to derive a complete asymptotic expansion for I_0 in inverse powers of $k \to \infty$ for the case when the saddle points of the integrand satisfying

$$z^2 + \sigma = 0 \tag{2}$$

$$z_{1,2} = \pm(-\sigma)^{1/2} \tag{3}$$

are real and widely separated ($\sigma \ll -1$). The asymptotic expansion should be of the form

$$I_0(\sigma, \gamma; k) \sim \sum_{n=0}^{\infty} k^{-n} f(\sigma, \gamma, n) \tag{4}$$

in which $f(\sigma, \gamma, n)$ is expressed in terms of known and easily computed functions. The asymptotic expansion in (4) should also hold uniformly as the endpoint γ approaches, or coincides with, one of the saddle points.

Submitted by E.D. Constantinides and R.J. Marhefka, August 11, 1993.
(rjm@tiger.eng.ohio-state.edu)

8. Can the real and imaginary parts of a hypergeometric series of type ${}_pF_q$ with one complex parameter (either in the numerator or the denominator) be expressed by means of multiple hypergeometric series?

Submitted by Ernst D. Krupnikov, July 25, 1993.
(ernst@net.neic.nsk.su)

9. Prove or disprove: The functions

$$H_n(t) = (-1)^n F_n(nt)$$

that are defined in terms of the *Bateman functions*

$$\begin{aligned} F_n(t) &= e^{-t}\Big(L_n(2t) - L_{n-1}(2t)\Big) \\ &= -e^{-t}\,\frac{2t}{n}\,L_{n-1}^{(1)}(2t) \\ &= (-1)^n \frac{2}{\pi} \int_0^{\pi/2} \cos\left(t \tan\theta - 2n\theta\right) d\theta\,, \end{aligned}$$

$L_n^{(\alpha)}(t)$ denoting the *generalized Laguerre polynomials*, have the property that $H_n(t_0)$ is strictly decreasing with increasing n at the point $t_0 = 2$. Note that this is not true for any $t_0 < 2$, but on the other hand at $t_0 = 2$ seems to be numerically evident. Note further that $H_n(t)$ satisfy the simple differential equation

$$t\, H_n''(t) = n^2\,(t-2)\,H_n(t)\,.$$

The differential equation demonstrates the importance of the point $t_0 = 2$.

These functions occur particularly in the study of nonvanishing analytic functions of the unit disk. For more details on these functions, see Koepf, W. and Schmersau, D.: Bounded nonvanishing functions and Bateman functions, Complex Variables **25** (1994), 237–259.

Submitted by Wolfram Koepf, February 10, 1995.
(koepf@zib-berlin.de)

10. Express the partial derivatives with respect to their parameters of any of the Laguerre, Gegenbauer, and Jacobi polynomials in terms of the initial polynomials.

Submitted by Ernst D. Krupnikov, May 17, 1995.
(ernst@net.neic.nsk.su)

[1] Electrical engineers use j for $\sqrt{-1}$, reserving $i = v/r$ for current.
[2] See their brief article on electromagnetic (EM) diffraction in the Fall, 1993 issue of the Newsletter.

Remarks on Problem 7
The Incomplete Airy Function

by Nico M. Temme
(nicot@cwi.nl)

Let

$$I_0(\sigma,\gamma;k) := \int_\gamma^\infty e^{ik(\sigma z+z^3/3)}\,dz$$

where k,σ,γ are real, $\sigma<0$. The proposers ask for an asymptotic expansion for large values of k that holds uniformly for γ in the neighborhood of the saddle points at $\pm\sqrt{-\sigma}$. The saddle points are widely separated because σ is bounded away from 0. It is not possible to construct an expansion as simple as shown in the problem:

$$I_0(\sigma,\gamma;k) \sim \sum_{n=0}^\infty k^{-n} f(\sigma,\gamma,n),$$

in which $f(\sigma,\gamma,n)$ is expressed in terms of known and easily computed functions. To give a proper description of the asymptotics a Fresnel type integral should be used. Standard methods to obtain such expansions are available, and I will give only the main steps for deriving the expansion.

First, assume that $\gamma \ge 0$. When $\gamma<0$ it is better to use the complete integral (indeed, an Airy function, which is exponentially small as $k\to\infty$) plus $I_0(\sigma,-\gamma;-k)$; the latter can be treated in the same way as the original I_0.

Second, transform the phase function into a simpler one: write $z=w\sqrt{-\sigma}$ to obtain

$$I_0(\sigma,\gamma;k) = \sqrt{-\sigma}\int_\delta^\infty e^{i\kappa(-w+w^3/3)}\,dw$$

where $\delta=\gamma/\sqrt{-\sigma},\ \kappa = k\,(-\sigma)^{3/2}$. We assume that κ is large (we see that σ may be small, in some sense).

Third, transform the phase function into a quadratic function by writing $-w+w^3/3 = t^2-2/3$ with $\operatorname{sign}(t) = \operatorname{sign}(w-1)$. This maps the saddle point at $w=1$ to the saddle point at $t=0$ and gives a standard form

$$I_0(\sigma,\gamma;k) = \sqrt{-\sigma}\; e^{-2i\kappa/3}\int_\varepsilon^\infty e^{i\kappa t^2} f(t)\,dt,$$

where

$$\varepsilon^2 = 2/3-\delta+\delta^3/3, \quad \operatorname{sign}(\varepsilon)=\operatorname{sign}(\delta-1),$$

and $f(t) = dw/dt = 2t/(w^2-1)$. The assumption $\gamma\ge 0$ implies $\varepsilon \ge -\sqrt{2/3}$. The function f is singular at $t=-2/\sqrt{3}$ (this point corresponds to $w=-1$, the other saddle point).

Fourth, use integration by parts. The first step is to write $f(t) = [f(t)-f(0)]+f(0)$. We have $f(0)=1$, and this term yields the Fresnel integral $\int_\varepsilon^\infty e^{i\kappa t^2}\,dt$. The other term can be used for integrating by parts:

$$\int_\varepsilon^\infty e^{i\kappa t^2}[f(t)-f(0)]\,dt = \frac{1}{2i\kappa}\int_\varepsilon^\infty \frac{f(t)-f(0)}{t}\,de^{i\kappa t^2}.$$

The integrated term vanishes at infinity, because $f(t) = \mathcal{O}(t^{1/3})$ as $t\to\infty$. The procedure can be repeated and this gives eventually

$$I_0(\sigma,\gamma;k) \sim \sqrt{-\sigma}\; e^{-2i\kappa/3}\times \left[\int_\varepsilon^\infty e^{i\kappa t^2}\,dt\sum_{n=0}^\infty \frac{A_n}{(2i\kappa)^n} + \frac{e^{i\kappa\varepsilon^2}}{2i\kappa}\sum_{n=0}^\infty\frac{B_n(\varepsilon)}{(2i\kappa)^n}\right],$$

as $\kappa\to\infty$, uniformly with respect to $\varepsilon\in[-\sqrt{2/3},\infty)$, with

$$A_n = f_n(0),\ B_n(\varepsilon) = \frac{f_n(\varepsilon)-f_n(0)}{\varepsilon}$$

and

$$f_{n+1}(t) = -\frac{d}{dt}\frac{f_n(t)-f_n(0)}{t},\ n=0,1,2,\ldots,\ f_0(t)=f(t).$$

A similar procedure for non-oscillating integrals is given in my paper in SIAM J. Math. Anal. **13**, Nr. 2 (1982), 239–253.

Solution of Problem 10
Parameter Derivatives of Orthogonal Polynomials

by Wolfram Koepf
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The given problem to represent the derivatives with respect to its parameters α and β of the Jacobi polynomials $P_n^{(\alpha,\beta)}(x)$ was recently treated by Jochen Fröhlich in [3], Theorem 3. It turns out that

$$\frac{\partial}{\partial\alpha}P_n^{(\alpha,\beta)}(x) = \sum_{k=0}^{n-1}\frac{1}{\alpha+\beta+1+k+n}\cdot\Big(P_n^{(\alpha,\beta)}(x) + \frac{\alpha+\beta+1+2\,k}{n-k}\,\frac{(\beta+k+1)_{n-k}}{(\alpha+\beta+k+1)_{n-k}}\,P_k^{(\alpha,\beta)}(x)\Big),$$

and

$$\frac{\partial}{\partial\beta}P_n^{(\alpha,\beta)}(x) = \sum_{k=0}^{n-1}\frac{1}{\alpha+\beta+1+k+n}\cdot\Big(P_n^{(\alpha,\beta)}(x) + (-1)^{n-k}\,\frac{\alpha+\beta+1+2\,k}{n-k}\,\frac{(\alpha+k+1)_{n-k}}{(\alpha+\beta+k+1)_{n-k}}\,P_k^{(\alpha,\beta)}(x)\Big)$$

are the representations requested. Jochen uses these results for a Galerkin method with moving weights.

Note that these results implicitly are also contained in work of Gábor Szegő ([5], (9.4.4), compare [1], (13), and [2],(2.7)–(2.8))

$$P_n^{(a,\beta)}(x) = \sum_{k=0}^n \frac{\Gamma(n+\beta+1)(2k+\alpha+\beta+1)}{\Gamma(a-\alpha)\Gamma(n+a+\beta+1)}\cdot \frac{\Gamma(n+k+a+\beta+1)\Gamma(n-k+a-\alpha)\Gamma(k+\alpha+\beta+1)}{\Gamma(n+k+\alpha+\beta+2)\Gamma(n-k+1)\Gamma(k+\beta+1)}P_k^{(\alpha,\beta)}(x).$$

The limit $a \to \alpha$ yields the first result; similarly the other case can be treated.

In [4], I used these results to deduce the parameter derivatives for the Laguerre and Gegenbauer polynomials (compare also [1], (8)):

$$\frac{\partial}{\partial \alpha} L_n^{(\alpha)}(x) = \sum_{k=0}^{n-1} \frac{1}{n-k} L_k^{(\alpha)}(x) = \sum_{k=1}^{n} \frac{1}{k} L_{n-k}^{(\alpha)}(x)$$

and

$$\frac{\partial}{\partial \lambda} C_n^{\lambda}(x) = \sum_{k=0}^{n-1} \left(\frac{2\,(1+k)}{(2\lambda+k)\,(2\lambda+1+2k)} + \frac{2}{2\,\lambda+k+n} \right)$$

$$\cdot\, C_n^{\lambda}(x) + \sum_{k=0}^{n-1} \frac{2\,(1+(-1)^{n-k})\,(\lambda+k)}{(2\lambda+k+n)\,(n-k)} C_k^{\lambda}(x)\;.$$

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[4] Koepf, Wolfram: Identities for families of orthogonal polynomials and special functions. Integral Transforms and Special Functions, 1995, to appear. Konrad-Zuse-Zentrum Berlin (ZIB), Preprint SC 95–1 (January 1995).

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Miscellaneous

1. Macdonald-Stanley Conjecture for Jack Polynomials Solved in Weak Form

(This item is partially based on email communications by Doron Zeilberger (zeilberg@euclid.math.temple.edu) and by Luc Vinet (vinet@ere.umontreal.ca))

Many conjectures involving the Jack polynomials have been formulated. A famous one is due to I.G. Macdonald and is reproduced in R.P. Stanley's reference article *Some combinatorial properties of Jack symmetric functions*, Adv. in Math. 77 (1988), 76–115. It states the following. Let $J_\lambda(x_1,\ldots,x_n;\alpha)$ be the Jack polynomial with parameter α. Let the $v_{\lambda,\mu}$ be its expansion coefficients in terms of the symmetric monomials $m_\mu(x)$, i.e., $J_\lambda = \sum_{\mu\le\lambda} v_{\lambda\mu}(\alpha) m_\mu$. Let the partition μ have $m_i(\mu)$ parts equal to i. Put $\widetilde{v}_{\lambda\mu}(\alpha) := \dfrac{v_{\lambda\mu}(\alpha)}{\prod_{i\ge1} m_i(\mu)!}$. It is conjectured that the $\widetilde{v}_{\lambda\mu}(\alpha)$ are polynomials in α with nonnegative integer coefficients.

Recently, Luc Lapointe and Luc Vinet proved a weak form of the Macdonald-Stanley conjecture: The coefficients $\widetilde{v}_{\lambda\mu}(\alpha)$ are polynomials in α with integer coefficients. They obtained this result by first deriving a Rodrigues type formula for the Jack polynomials. This Rodrigues type formula involves so-called creation operators which are built from Dunkl operators.

The full proof of the Rodrigues type formula is given in:

L. Lapointe and L. Vinet: Exact operator solution of the Calogero-Sutherland model. CRM preprint 2272, electronically available from WWW: http://www.crm.umontreal.ca (under L. Vinet only).

A brief announcement of the Rodrigues type formula together with a proof of the weak form of the Macdonald-Stanley conjecture is given in:

L. Lapointe and L. Vinet: A Rodrigues formula for the Jack polynomials and the Macdonald-Stanley conjecture. CRM preprint 2294, electronically available from WWW: http://www.crm.umontreal.ca.

This paper will appear soon in the International Mathematics Research Notices.

To see the Lapointe-Vinet proof in action see Doron Zeilberger's Maple program LUC, obtainable from

WWW: http://www.math.temple.edu/~zeilberg

Doron Zeilberger adds that he has further news from Adriano Garsia: Garsia can prove a similar polynomiality conjecture for the Macdonald polynomials associated with root system A_n.

Tom H. Koornwinder
(thk@fwi.uva.nl)

2. Early History of Bessel Functions

Here's a reference on the early history of Bessel functions that may be of interest to the members of the Activity Group:

Jacques Dutka: On the early history of Bessel functions, Archive for the History of Exact Sciences **49** (1995), No.2, 105–134.

Walter Van Assche
(Walter.VanAssche@wis.kuleuven.ac.be)

3. Tenure Track Position in Bogota (Columbia)

The Mathematics Department at the National University of Colombia in Bogota is trying to push forward its Ph.D. Program in Mathematics. In order to attract competent new faculty members, it announces an international competition for regular faculty positions. The selection is based only on the evaluation of the curriculum vitae, on publications, on letters of recommendation of known mathematicians and on a proposal of the activity he plans to carry out in Colombia. It is open to a Ph.D. aged 25 to 45 with very good promise or experience in research represented by papers published in known journals and strong commitment of teaching at the Ph.D. level. Willingness to help students with their dissertations is very much desirable. The selected candidates will be granted an initial contract of

one year at the associate professor level. The salary is close to US $ 1,600 a month (after taxes) but payment is for 15 months a year (equivalent to about US $ 2,200 a month stipend in the USA). The nature of the position is tenure track, but there is a one year probation period and a faculty evaluation of performance at the end of that period. The candidate does not have to know Spanish. However, Spanish is the official language in Colombia and it is expected the winners of the contest will learn some Spanish during their first year in Colombia. Some work at the Department is being done in Special Functions, Orthogonal Polynomials, Approximation Theory and related areas. However, Partial Differential Equations, Logic, Category Theory, Lie Algebras and Lie Groups also belong to its priorities.
For further information please contact

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Santafe de Bogota
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4. ftp and WWW Addresses

A list of ftp and WWW addresses relevant for our field is available by anonymous ftp on `ftp.fwi.uva.nl` in the directory
pub/mathematics/reports/Analysis/koornwinder/opsfnet.dir,
file `WWWaddresses`.

This file will be regularly updated. Please mail corrections and additions for this list to me.

Tom H. Koornwinder
(thk@fwi.uva.nl)

5. Changes of Address

The address of Willard Miller at the University of Minnesota changed into:

Willard Miller, Jr.
Professor and Acting Dean
Institute of Technology, University of Minnesota
105 Walter Library, 117 Pleasant Street S.E.
Minneapolis, Minnesota 55455
Tel: +01-612-624-2006 Fax: +01-612-624-2841
email: miller@ima.umn.edu

Marcel de Jeu, who is working in the area of special functions associated with root systems, will stay in Paris (Université Paris 7, to start with) until February 1, 1996. His address there is:

Marcel de Jeu
56 rue Oberkampf
75011 Paris
France
Tel.: +33-1 4338 7735
email: dejeu@mathp7.jussieu.fr

Tom Koornwinder will stay during the period October 2 - November 23, 1995 at the Mittag Leffler Institute near Stockholm, Sweden. Mathijs Dijkhuizen (working on special functions in connection with quantum groups), has returned from Kobe (Japan) and will also stay at the Mittag Leffler Institute during October and November. The address for both will be:

Institut Mittag Leffler
Auravägen 17
S-182 62 Djursholm
Sweden
Tel: +46-8-755 1809 Fax: +46-8-755 9971
email: thk@fwi.uva.nl (Koornwinder),
thijs@cwi.nl (Dijkhuizen)

Memorial Notes about Subrahmanyan Chandrasekhar

(With thanks to Dick Askey for providing information)

Prof. S. Chandrasekhar died on August 22, 1995 at the age of 84. He was born in Lahore, India and he studied at the University of Madras, India and at Trinity College, Cambridge, England. He worked at the University of Chicago. He is well known for his work in theoretical astronomy, see for instance his book *The Mathematical Theory of Black Holes*, Oxford University Press, 1983.

This summer Chandrasekhar's last book appeared, *Newton's 'Principia' for the Common Reader*, Oxford University Press, 1995. Further information on his life can be found in the biography *Chandra* by Kameshwar C. Wali, University of Chicago Press, 1991.

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The distinguished theoretical astronomer Subrahmanyan Chandrasekhar died on August 22, 1995 at the age of 84. Among his many accomplishments was a separation of variables method to find explicit solutions of the spinor equations of mathematical physics in the curved background of a rotating black hole (Kerr space time). In this problem the so called Teukolsky functions play a crucial role. Teukolsky functions are confluent forms of the solutions of the most general linear homogeneous ordinary differential equation of second order with four regular singularities, viz. Heun's equation (see the book announcement on p. 10). E.G. Kalnins, G.C. Williams and I were fascinated with Chandrasekhar's results and corresponded with him over a period of about eight years.

Most of Chandrasekhar's work in this field is contained in his book *The Mathematical Theory of Black Holes*, Oxford University Press, 1984. A crucial advance reported in the book was the solution of the Dirac equation in the space time of a Kerr black hole, already achieved by Chandrasekhar in 1976. Dirac's equation for spin 1/2 particles with mass m_e has the spinor form

$$\nabla^B{}_{B'}\varphi_B = \frac{im_e}{\sqrt{2}}\chi_{B'},\ \nabla_B{}^{B'}\chi_{B'} = -\frac{im_e}{\sqrt{2}}\varphi_B.$$

Restricted to a black hole space time (mass M and angular momentum a) and expressed in terms of spherical coordinates and the modified field components $\varphi_0 = \phi_0 e^{i(m\varphi+\sigma t)}$, $\overline{\rho}^*\varphi_1 = \phi_1 e^{i(m\varphi+\sigma t)}$, $\chi_{0'} = X_0\ e^{i(m\varphi+\sigma t))}$, $\overline{\rho}\chi_{1'} = X_1 e^{i(m\varphi+\sigma t)}$, where $\Delta = r^2 + a^2 - 2Mr$, $\overline{\rho} = r + ia\cos\theta$, $\rho^2 = \overline{\rho}\ \overline{\rho}^*$, these equations can be solved by the separation ansatz

$$\phi_0 = R_{1/2}S_{1/2},\ \phi_1 = R_{-1/2}S_{1/2},$$

$$X_0 = -R_{1/2}S_{-1/2},\ X_1 = R_{-1/2}S_{-1/2}$$

to give the coupled equations

$$L_{1/2}S_{1/2} = (\lambda - am_e\cos\theta)S_{-1/2},$$

$$D_0R_{-1/2} = (\lambda + im_e r)R_{1/2},$$

$$L^\dagger_{1/2}S_{-1/2} = -(\lambda + am_e\cos\theta)S_{1/2},$$

$$\Delta D^\dagger_{1/2}R_{1/2} = (\lambda - im_e r)R_{-1/2}$$

where

$$L_n = \frac{\partial}{\partial\theta} + Q + n\cot\theta,$$

$$L_n^\dagger = \frac{\partial}{\partial\theta} - Q + n\cot\theta,$$

$$Q = \sigma a\sin\theta + m\csc\theta,$$

$$D_n = \frac{\partial}{\partial r} + i\frac{K}{\Delta} + \frac{2n(r-M)}{\Delta},$$

$$D_n^\dagger = \frac{\partial}{\partial r} - i\frac{K}{\Delta} + \frac{2n(r-M)}{\Delta},$$

$$K = (r^2 + a^2)\sigma + am.$$

Here λ is a separation constant and $R_{\pm s}(r), S_{\pm s}(\theta)$ are *Teukolsky functions*, i.e., they satisfy the second order ordinary differential equations

$$(\Delta D_1 D_s^\dagger - 2(2s-1)i\sigma r - \lambda)R_{+s} = 0,$$

$$(\Delta D^\dagger_{1-s}D_0 + 2(2s-1)i\sigma r - \lambda)R_{-s} = 0$$

$$(L^\dagger_{1-s}L_s + 2(2s-1)\sigma a\cos\theta + \lambda)S_{+s} = 0,$$

$$(L_{1-s}L_s^\dagger - 2(2s-1)\sigma a\cos\theta + \lambda)S_{-s} = 0$$

for $s = 1/2$. Similar results hold for solutions in Kerr geometry of Maxwell's equations ($s = 1$), gravitational perturbation equations ($s = 2$) and the Rarita-Schwinger equations ($s = 3/2$). The main difference in these other equations is that only the components of the spinor fields associated with the opposite spins $\pm s$ separate; the remaining $2s-1$ spinor components do not separate but can be expressed in terms of the separable pieces.

Teukolsky functions have fascinating properties. For example, for any integer $2s$ it can be shown that, for suitably normalized $R_{\pm s}$,

$$\Delta^s D_0^{2s} R_{-s} = C_s \Delta^s R_{+s}, \quad \Delta^s D_0^{\dagger 2s}\Delta^s R_{+s} = C_s^* R_{-s}$$

where C_s is a *Starobinsky constant*, with similar properties for $S_{\pm s}$.

A basic question left open by Chandrasekhar's work was how the separation parameter λ and the Starobinsky constants C_s can be characterized intrinsically. Kalnins, Williams and I showed in a number of cases that, in analogy with the scalar theory of variable separation, these constants arose as eigenvalues of pairs of commuting differential symmetry operators, i.e., operators that also commuted with the Dirac or other Hamiltonian associated with the particles. We also developed a formal theory of Teukolsky-Starobinsky identities for arbitrary spin s, including the unphysical cases where $2s > 4$. Chandrasekhar responded with a determinantal expression for the Starobinsky constant for arbitrary spin. A review, solicited by Chandrasekhar, appeared in the paper *Phil. Trans. R. Soc. Lond.* A (1992) **340**, 337-352.

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The following memorial note appeared in the weekly German magazine *Der Spiegel* **35/1995**, August 28, 1995:

In einem Alter, in dem sich Teenager sonst noch am Dreisatz versuchen, löste er schon die Rätsel des Universums. Sterbende Sonnen waren sein Thema, jene unvorstellbar dichten Plasmakugeln, die als Weiße Zwerge am Ende eines Sternenlebens stehen. Mit 19 Jahren bereits führte der Sproß einer angesehenen indischen Wissenschaftlerfamilie den mathematischen Nachweis, der die Weißen Zwerge von anderen Himmelsexoten abgrenzt: Kollabierende größere Sterne werden zu Neutronensternen oder Schwarzen Löchern mit so großer Schwerkraft, daß nicht einmal Licht ihnen entweichen kann. An der Entwicklung der amerikanischen Atombombe während des Zweiten Weltkriegs mochte der Astrophysiker, der 1936 in die USA ging, nicht mitarbeiten; eine entsprechende Anfrage aus Los Alamos lehnte er ab. Für seine Forschungen erhielt er, mit einem halben Jahrhundert Verspätung, 1983 den Nobelpreis für Physik. Subrahmanyan Chandrasekhar starb vergangenen Montag in Chicago.

Memorial Note about Joseph L. Ullman (1923–1995)

Joe L. Ullman, Professor Emeritus of the University of Michigan, died on Monday, September 11, 1995 at the age of 72. Ullman is known for his work in logarithmic potential theory and orthogonal polynomials, and he has made important contributions to the theory of orthogonal polynomials on the infinite interval and Chebyshev quadrature (with equal weights at every node). Joe was a student of Gabor Szegő under whose supervision he prepared his Ph.D. thesis on *Studies of Faber polynomials* at Stanford University in 1949.

Ullman was born January 30, 1923 in Buffalo, New York. He received his B.A. from the University of Buffalo in 1942. He fought in World War II, receiving a purple heart, and later served as an instructor of mathematics at army schools in Czechoslovakia and France. After his Ph.D. he joined the faculty of the Department of Mathematics at the University of Michigan, where he taught and did his research for 44 years.

Ullman's name will forever be connected with Ullman's criterion for regular asymptotic behaviour of orthogonal polynomials and regular zero behaviour [3]. Ullman's criterion for orthogonal polynomials with respect to a positive measure μ on $[-1,1]$ is that the minimal carrier capacity of μ is equal to the capacity of the support of μ, which is $1/2$ if the support is $[-1,1]$. The asymptotic distribution of the contracted zeros of Freud-type orthogonal polynomials (with weight $w(x) = \exp(-|x|^\alpha)$ on $\mathbb{R}$) is given by the measure with density

$$v_\alpha(x) = \frac{1}{\pi} \int_{|x|}^1 \frac{1}{\sqrt{t^2 - x^2}}\, dt^\alpha, \qquad -1 \le x \le 1,$$

which is known as the Ullman measure in view of Ullman's results in [4]. He also showed that equal weight quadrature (Chebyshev quadrature) is possible on an infinite interval [2], which is a rather surprising result.

At home he was always ready to help his wife Barbara at their little farm (with a dozen of sheep and some goats), but most of us will remember him for his love of classical analysis and his interesting research, as can be judged from the following quote from the book by Stahl and Totik [1, Preface]: *'It was especially J. Ullman who systematically studied different bounds and asymptotics on orthogonal polynomials with respect to arbitrary measures μ on* $[-1,1]$*, and we owe a lot to his research and personally to him for initiating and keeping alive the subject'.*

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[4] Ullman, J.L.: Orthogonal polynomials associated with an infinite interval. Michigan J. Math. **27** (1980), 353–363.

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Celebration of the 100-th Anniversary of Gábor Szegő's Birth

A memorial celebration of the 100-th anniversary of Gábor Szegő's birth (on January 20, 1895 in Kunhegyes, Hungary) was held on January 21, 1995 in Kunhegyes. According to report by Paul Nevai several mathematicians and over one hundred local people came to the celebration which was co-organized by the János Bolyai Society and the Hungarian Academy Sciences. Akos Csaszar was the master of ceremonies and the introduction was made by Mr. Baranya. There were four speeches about Szegő's life, so every aspect of it was repeated and analyzed four times from at least four (entirely) different points of views. The ailing Barna Szenassy's essay was read by Erzsebet Daroczy, nee Kotora. Essays written by Dick Askey and Lee Lorch, which were translated into Hungarian, were read by Gyuri Petruska. Paul Nevai presented a talk that was a spiced up and updated version of his obituary article "Gábor Szegő", Magyar Tudomany 8-9 (1986), 728-736 (see also Nyelvunk es Kulturank 65 (1986), 57-63). Lorch's essay and Paul's report on his visit to the celebration can be obtained from the Editor (koepf@zib-berlin.de) by request. A copy of Dick's essay is given below, followed by two reports on the Szegő bust dedication which took place on Wednesday, August 23, 1995, in Kunhegyes, Hungary.

George Gasper
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Gábor Szegő - One Hundred Years

by Richard Askey
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It is 100 years since Gábor Szegő was born and 80 years since the publication of his first paper. Before this publication he had already shown a strong talent in mathematics by winning the Eötvös competition in 1912. His first paper contained the solution of a problem of George Pólya. However, he had earlier, in 1913, published the solution of another problem of Pólya. For the nonmathematicians it should be remarked that there are problems at various levels. Some, like those you did in school, are ones which everyone should learn how to do. Then there are contest problems, like those in the Eötvös competition. These are harder and frequently require deeper insight than seems so at first reading. The problem of Pólya which Szegő solved and published in 1913 is an example of a harder type, which has long attracted prospective mathematicians. Hungary has long specialized in the use of problems to attract young students to mathematics, and other countries have learned from you and have contests of mathematics problems to encourage students to think harder than is needed for the typical school problem. The problem of Pólya which made up Szegő's first published paper was an open problem and there is still great interest in extensions of Szegő's solution to more complicated problems of this nature.

Since Szegő had been a mathematical prodigy himself, he was an ideal person to be asked to tutor one of the great mathematical minds of this century, John von Neumann. Here is what Norman Macrae wrote in his book "John von Neumann".

"Professor Joseph Kürschák [of the Lutheran School in Budapest] soon wrote to a university tutor, Gabriel Szegő, saying that the Lutheran School had a young boy of quite extraordinary talent. Would Szegő, as was the Hungarian tradition with infant prodigies, give some university teaching to the lad?

"Szegő's own account of what happened was modest. He wrote that he went to the von Neumann house once or twice a week, had tea, discussed set theory, the theory of measurement, and some other subjects with Jancsi, and set him some problems. Other accounts in Budapest were more dramatic. Mrs. Szegő recalled that her husband came home with tears in his eyes from his first encounter with the young prodigy. The brilliant solutions to the problems posed by Szegő, written by Johnny on the stationery of his father's bank, can still be seen in the von Neumann archives in Budapest."

Macrae was wrong in saying Kürschák was at the Lutheran School. He was a professor at the university. He was the appropriate contact between the mathematics teacher at the Lutheran School and Szegő.

Szegő served in the army in the First World War, but continued to do mathematics, and received his Ph.D. in 1918 in Vienna. Fifty years later he returned to Vienna for a celebration of this, and I still remember how pleased he was with this when he described it to me a few years later. After temporary positions in Hungary, Szegő went to Berlin in 1921. Pólya was in Zürich and they started to work on a problem book. It turned out to contain too much material for one volume, so was published in two volumes. Pólya wrote the following about their work together on these books.

"It was a wonderful time; we worked with enthusiasm and concentration. We had similar backgrounds. We were both influenced, like all other young Hungarian mathematicians of that time, by Leopold Fejér. We were both readers of the same well directed Hungarian Mathematical Journal for high school students that stressed problem solving. We were interested in the same kind of questions, in the same topics; but one of us knew more about one topic and the other more about some other topic. It was a fine collaboration. The book 'Aufgaben und Lehrsätze aus der Analysis, I, II', the result of our cooperation, is my best work and also the best work of Gábor Szegő."

It is hard to argue with Pólya's assessment about the quality of the problem books. They set a standard for others who would later write books of problems and no one has come close to the level achieved by Pólya and Szegő. Not only are their problems interesting and important, they build on each other, so that working the problems in a section allows the reader to grow and learn new mathematics and techniques.

Szegő spent more than ten years in Germany, first in Berlin as privatdocent and then in Königsberg as professor. His first two Ph.D. students were in Königsberg. He was a beloved teacher, and when the situation in Germany became hard and then impossible for Jewish mathematicians, and Jews in general, Szegő was one of the last to suffer because he was so highly respected by students and colleagues. While in Berlin, he was awarded the Jules König prize in 1924. F. Riesz gave the report for the prize committee, and this is reprinted in Riesz's "Oeuvres completes".

In 1934, Gábor Szegő moved to the United States, first to St. Louis, Missouri, where he taught for four years at Washington University, and then to Stanford, California, where he was chairman of the mathematics department for 15 years, building it into an excellent department. At Washington University, Szegő wrote the other great book of his, the first and still the best book on orthogonal polynomials. The study of these polynomials started in the 19th century, and continues to the present. In the 1920's, Szegő found a variant on the earlier work, and one of facts he discovered was eventually used in speech synthesisers.

A Ph.D. student from Stanford, Paul Rosenbloom, wrote about life as a student under Pólya and Szegő. In addition to the mathematical education he received, Szegő looked out for his cultural education, giving Rosenbloom a ticket to Bartók's concert at Stanford.

In 1952, Szegő published an extension of his first paper. About this paper Barry McCoy wrote: "It is easily arguable that, of all Szegő's papers, 'On Certain Hermitian Forms Associated with the Fouries Series of a Positive Function' has had the most applications outside of mathematics. In the first place, the problem which inspired the theorem was propounded by a chemist working on magnetism. Extensions of this work made by physicists have led to surprising connections with integrable systems of nonlinear partial difference and differential equations. ... In addition Szegő's theorem has recently been used by physicists investigating quantum field theory."

One way mathematicians are honored is to have something they discovered named after them. There is now the Szegő kernel function, the Szegő limit theorem and the strong Szegő limit theorem, Szegő polynomials orthogonal on the unit circle, the Szegő class of polynomials. Another way we show that the work of a mathematician is deep enough to last is to publish their selected or collected works. Szegő's "Collected Works" were published in 1982.

I first met Gábor Szegő in the 1950's when he returned to St. Louis to visit old friends, and I was an instructor at Washington University. Earlier, when I was an undergraduate there, I had used a result found by Hsu in his Ph.D. thesis at Washington University under Szegő. This was in the first paper I wrote. While at the Univ. of Chicago in the early 1960's, Szegő visited. I still remember seeing him at one end of the hall and a graduate student, Stephen Vagi, at the other end of the same hall. They walked toward each other and both started to speak in Hungarian. I am certain they had not met before, and I have always wondered how Szegő recognized another former Hungarian. In 1972 I spent a month in Budapest and Szegő was there. We talked most days, and though his health was poor and his memory was not as good as it had been a few years earlier, we had some very useful discussions. Three years earlier, also in Budapest, Szegő had mentioned two papers of his which he said should be studied. I did not do it immediately, but three months later did. One contained the solution of a problem I had been trying to solve for three years. His paper had been written 40 years earlier. I learned from this that when a great mathematician tells you to look at a paper of theirs which they think has been unjustly neglected, one should do it rapidly.

Szegő left a memorial for us, his mathematical work. It continues to live and lead to new work. One of his main areas was orthogonal polynomials and this is now a very active field. I often regret that he is not here to appreciate all of the work being done on problems he started.

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The unveiling of the bust of Gábor Szegő, 1895-1985, commemorating the life of this great mathematician, took place in front of the Town Library in his place of birth of Kunhegyes, Hungary, at 3 p.m. on August 23, 1995. The bust was made by the artist Lajos Győrfi from the nearby town of Püspökladány. He is well known in Kunhegyes and his artwork can be seen in various places in the town. Financial support was provided by more than a hundred friends and admirers of Szegő.

Participants at AFS95 (International Conference on Approximation Theory and Function Series), the ongoing conference in Budapest, were treated to an almost three hour bus ride, organized [and partially paid (the editor)] by the János Bolyai Mathematical Society, complete with a packed lunch. Rumours that the lunch boxes were filled with bananas proved to be groundless and all [junk food loving (the editor)] culinary tastes were accomodated as we travelled through the fields of [partially burned out (the editor)] maize and sunflower fields that adorn the Hungarian countryside.

Upon arrival at the small town of Kunhegyes, we inspected an exhibit from the works of Lajos Győrfi inside the Town Library before the ceremony took place on the lawn outside.

Several curious villagers joined the gathering of mathematicians and town dignitaries to listen to the proceedings in the blazing afternoon sun. The ceremony was both solemn and informal with speeches by the Mayor of Kunhegyes, Veronica Tincher (the daughter of Gabor Szegő), Vilmos Totik, Paul Nevai, and the town historian, Lajos Szabó. Steven Tincher, Szegő's grandson, and the sculptor of the bust were also present at the unveiling. The speeches by the locals were in Hungarian (with partial English translation provided by the [very pretty but somewhat inexperienced (the editor)] English teacher of the local high school), whereas Totik and Nevai gave bilingual presentations. Veronica Tincher spoke in English and delivered a short, moving tribute to her father. She dwelt not on his impressive contributions to mathematics but on the ideals that he cherished, saying that he was a warm and caring man, committed to the values of equality and justice for all. Altogether there were approximately a hundred people (including about thirty mathematicians) watching the dedication ceremony.

The bust itself is impressive and appropriate, a solid head-and-shoulders cast made of bronze, mounted on a rectangular block. There is a shiny brass plaque on the back of the pedestal engraved in Hungarian and English:

Gábor Szegő
Mathematician
Born on January 20, 1895, in Kunhegyes, Hungary
Died on August 7, 1985, in Palo Alto, California, U.S.A.

Placed here by friends and admirers
from Kunhegyes and around the world

Two copies of the sculpture will be displayed at Stanford University and Washington University (St. Louis) in the United States.

The proceedings were concluded with a reception in the Town Hall at which presentations were made to Veronica Tincher and to the people whose efforts had facilitated the creation of this memorial to the life and work of Gabor Szegő, mathematician extraordinaire. [The ever hungry mathematicians emptied the buffet tables in no time (the editor).]

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A blue veil fell to the ground, and one of the greatest analyst of the 20th century Gabor Szegő (more precisely, his bronze bust on top of a rectangular pedestal) appeared in front of a hundred or so people who had gathered that sunny afternoon on the lawn outside the Town Library in his place of birth of Kunhegyes, Hungary.

PUZZLE 1. When was the last time that Kunhegyes had a company of nearly 30 scientists from at least 8 countries (Hungary, USA, Ukraine, Russia, Germany, South Africa, Israel, and Taiwan)?

In my opinion Szegő looks somewhat resolute, even stern, especially when compared to his last photography. Veronica Tincher–Szegő agreed with me. She told me that her father was a kind and gentle man and a faint smile on his face would much more conform to his character.

Before that dedication we visited the exhibition of the sculptor's works in the basement of the library and listened to several speeches: a short introductory word by the Mayor of Kunhegyes and a long, difficult to understand (even for our pretty interpreter) talk by the local historian; a few cordial words by Veronica Tincher about her father and a majestic ode by Paul Nevai, the main organizer of the event, who fluently passed from one language to another so that I could hardly make out familiar English phrases among totally incomprehensible and charming music of Hungarian.

Before that there was an almost three hour trip from Budapest by a comfortable bus (which as much resembled Latvija minibus as a modern train resembles the first Stephenson's locomotive). We were heading the south-east of Hungary towards the small town of Kunhegyes. I never heard about it up to February 5, 1995 (see AT-NET Bulletin at http://www.math.ohio-state.edu/JAT, devoted to Szegő's centennial). The flat green fields were floating past the window. The landscape seemed so familiar that it made me feel as if I were in my native Ukraine. Each passenger was given a bag with food equivalent to "Macdonalds" (joy for the junk food lovers!). Wise and forethoughtful enough, I constrained myself with the only small roll and a bottle of water. [Leonid is about 1.75 m and weighs about 60 kg (the editor).]

PUZZLE 2 (for the former soviet citizens only [and for every other sensible person as well (the editor)]). How can you make use of empty boxes from hamburgers? [I can't wait for a good answer (the editor)!]

After that a reception was held in the Town Hall and we were treated to some delicious Hungarian meals (where are you, junk food lovers?). The buffet tables emptied in a jiffy. I picked up a lot of free souvenirs for my colleagues and friends in Kharkov. An hour later the full and content mathematicians embarked into the vehicle and in less than 2 and a half hours the bus with sleepy passengers pulled up at the Police Academy building in a picturesque district of Buda.

PUZZLE 3. Who is the next?

Well, whoever he/she will be, I definitely would like to participate in the ceremony...

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Both reports were edited by Paul Nevai (nevai@math.ohio-state.edu).

Two Unknown Works of José Vicente Gonçalves (1896-1985)

by Amílcar Branquinho
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In 1942/43 José Vicente Gonçalves published two works in Portugaliæ Mathematicæ about the classical orthogonal polynomials (Hermite, Laguerre, Jacobi and Bessel). In [4] he proved a very important result:

- The polynomials $y_n = x^n + \dots$ satisfy

$$(a_0x^2 + a_1x + a_2)y'' + (b_0x + b_1)y' + \lambda_n y = 0 \qquad (1)$$

if and only if they satisfy a three term recurrence relation

$$xy_n = y_{n+1} + \beta_n y_n + \gamma_n y_{n-1},\ n \geq 1 \qquad (2)$$

with $(\beta_n), (\gamma_n) \subset \mathbb{R}$ given in terms of the constants $a_0, a_1, a_2, b_0, b_1, \lambda_n$.

This work is very important because he treats all cases that can occur in this equation. So he has anticipated the work of Krall and Frink [6] about the Bessel poynomials. Also, he has proved that the polynomial solutions of (1) satisfy

$$\begin{aligned} 2(a_0x^2 + a_1x + a_2)y_n' + (b_0x + b_1)y_n - \\ (2a_0n + b_0)(x - \beta_n)y_n = (\lambda_{n-1} - \lambda_{n+1})\gamma_n y_{n-1} \end{aligned} \qquad (3)$$

for $n \geq 1$. So he got the formula usually attributed to Tricomi [7], cf. Chihara's book [3].

From this work Branquinho and Marcellán [2] gave conditions for the regularity (in the Chihara sense [3]) of the solutions of the *Pearson Equation*

$$\Big((a_0x^2 + a_1x + a_2)w(x)\Big)' = (b_0x + b_1)w(x) \qquad (4)$$

In [5] he gives a nice representation for the polynomial solutions (even if they are not orthogonal), $\{P_n\}$, of (1):

$$P_n(x) = w^{-1}(x)D^n\left(w(x)\phi^n(x)\left(c_1 + \int_{x_0}^{x} \frac{N(t)}{w(t)\phi^{n-1}(t)}dt\right)\right) \qquad (5)$$

where w is a solution of the equation (4), $\phi(x) = a_0x^2 + a_1x + a_2$, $\psi(x) = b_0x + b_1$, c_1 is a real constant and N is a polynomial of degree $\leq n$.

Remarks:

- The first result is more or less the Bochner theorem [1] for the classical orthogonal polynomials, but the proof of the author is very clear and is not made by exhaustion. He also shows that (2) is in some sense the discretization of (1).
- The second result gives us a characterization theorem for the classical orthogonal polynomials.
- If we take $N \equiv 0$ and $c_1 = 0$ in (5) we get the Rodrigues formula for the classical orthogonal polynomials:

$$P_n(x) \quad = \quad w^{-1}(x)D^n\Big(w(x)\phi^n(x)\Big)\ .$$

We intend to organize a meeting in Coimbra in August or September of 1996 about the work of this Professor.

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INTAS Project: Constructive Complex Analysis and Density Functionals

by Walter Van Assche
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Some years ago, the European Union initiated a scientific program INTAS for the promotion of international cooperation between scientists from the European Union and the independent states of the former Soviet Union. In 1993 a group of scientists decided to apply for a research project in which mathematical physics, special functions, and constructive approximation theory are the main objectives. The team consists of Jesús Sanchez-Dehesa (Universidad de Granada, Spain), who is the coordinator of the project, together with Walter Van Assche (Katholieke Universiteit Leuven, Belgium) and Francisco (Paco) Marcellán (Universidad Carlos III de Madrid, Spain) as the European partners. The partners from the former Soviet Union are Andrei A. Gonchar (Steklov Institute of the Russian Academy of Sciences, Moscow), Alexander (Sasha) I. Aptekarev (Keldysh Institute of Applied Mathematics, Moscow), and Vladimir S. Buyarov (Moscow State University).

One of the main objectives is the exact evaluation of fermionic density functional quantities such as information entropies, kinetic and exchange energies, average densities and information energies. This problem can, at least for some simple quantum mechanical systems, be reduced to a study of the asymptotic behaviour of certain orthogonal polynomials. Two simple physical systems can be described in detail, namely the D-dimensional harmonic oscillator and the hydrogen atom. In 1975, Bialynicki-Birula and Mycielski (BBM) formulated a stronger version of the Heisenberg uncertainty relation expressed as an inequality $S_\rho + S_\gamma \geq D(1+\log\pi)$, where

$$S_\rho = -\int |\psi(\vec{r})|^2 \log|\psi(\vec{r})|^2 \, d\vec{r}$$

is the Boltzmann-Shannon entropy in the position space and

$$S_\gamma = -\int |\hat{\psi}(\vec{p})|^2 \log|\hat{\psi}(\vec{p})|^2 \, d\vec{p}$$

the entropy in the momentum space. The wave functions ψ and their Fourier transform $\hat{\psi}$ for these physical systems are known and are in terms of the classical orthogonal polynomials of Gegenbauer, Laguerre and Hermite. Our first research activity consists in determining these information entropies as accurate as possible and to check the BBM inequality.

Using the explicit form of the wave functions, one ends up with integrals of the form

$$E_n = -\int p_n^2(x) \log p_n^2(x) \, d\mu(x), \qquad (1)$$

where p_n $(n = 0, 1, 2, \ldots)$ are polynomials orthonormal on the real line with respect to the positive measure μ. The orthogonal polynomials of Laguerre, Hermite, and Gegenbauer are the relevant systems for the two quantum mechanical systems in consideration. A first problem then is to evaluate, as accurate as possible, entropy integrals for Gegenbauer polynomials. Using a heuristic argument, we were able to conjecture—and then prove—that (see [5], [1], [2], [3])

$$\frac{1}{\pi}\int_{-1}^{1} [C_n^\lambda(x)]^2 \log[C_n^\lambda(x)]^2 (1-x^2)^{\lambda-1/2} \, dx$$
$$= \frac{2^{1-2\lambda}\Gamma(n+2\lambda)}{\Gamma^2(\lambda)(n+\lambda)n!}\left(\frac{n}{n+\lambda} + 2\log\frac{\Gamma(n+\lambda)}{\Gamma(\lambda)n!}[1+o(1)]\right).$$

For Chebyshev polynomials we have simple expressions, namely [5]

$$\frac{2}{\pi}\int_{-1}^{1} T_n^2(x)\log T_n^2(x) \frac{dx}{\sqrt{1-x^2}} = 1 - 2\log 2$$

and

$$\frac{2}{\pi}\int_{-1}^{1} U_n^2(x)\log U_n^2(x)\sqrt{1-x^2}\, dx = 1 - \frac{1}{n+1}.$$

For general orthogonal polynomials on the real line, we observed that the entropy (1) is in fact a mutual energy

$$E_n = -2\log\gamma_n + 2nI(\mu_n, \nu_n),$$

where

$$I(\mu,\nu) = \iint \log\frac{1}{|x-y|}\, d\mu(x) d\nu(y),$$

with $d\mu_n(x) = p_n^2(x)\, d\mu(x)$ and $\nu_n = \frac{1}{n}\sum_{k=1}^{n}\delta_{x_{j,n}}$ the zero distribution for the polynomial p_n. This mutual energy is therefore in terms of two measures closely related to the zeros of p_n, namely the measure ν_n with jumps of size $1/n$ at each zero of p_n, and the measure μ_n which is absolutely continuous with respect to μ with a Radon-Nikodym derivative that vanishes at the zeros of p_n. A precise knowledge of the weak convergence of both measures is therefore essential for studying entropies as in (1).

Progress has also been made for entropies dealing with Hermite and Laguerre polynomials (see [1], [2], [4].) For (orthonormal) Hermite polynomials we found

$$E_n = -n + \log\sqrt{2n} - \frac{3}{2} + \log\pi + o(1)$$

and for (orthonormal) Laguerre polynomials with weight $x^\alpha e^{-x}$ on $[0,\infty)$ we found

$$E_n = -2n + (\alpha+1)\log n - \alpha - 2 + \log 2\pi + o(1).$$

This year, February 20–23, a workshop *Density Functionals of Quantum-Mechanical Systems and Constructive Complex Analysis* was held at the Universidad de Granada, getting together all the participants of this INTAS research project, but also open to other interested scientists, see the report on page 3.

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preferably by email, and in LaTeX format. Other formats are also acceptable and can be submitted by email, regular mail or fax.

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