Orthogonal Polynomials and Special Functions

SIAM Activity Group on Orthogonal Polynomials and Special Functions

⋆ ⋆ ⋆ ⋆ **Newsletter** ⋆ ⋆ ⋆ ⋆

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Contents

From the Editor

Beginning with this issue of the Newsletter you will see that the *Problems Section* no longer contains all the previously proposed questions that are still unsolved, but only those which were published within the last year. Unsolved problems will be published at least three times. After a problem is solved, it is removed from the list, though. Since back issues of the Newsletter can be obtained by World Wide Web from my home page http://www.zib.de/koepf, everybody who is interested in solving one of the older problems has access to these questions even if one does not have the printed Newsletter around. I am delighted that so many problems have been submitted which makes the above procedure necessary. Please send in your solutions!

Thus far 15 problems have been submitted five of which have been solved (#1, 4, 6, 7, 10), and one of which is new (#15), see page 12. Still unsolved are Problems #2, 3, 5, 8, 9, 11, 12, and 13. A solution of Problem #14 is presented in the current issue on page 12.

Note that the Konrad-Zuse-Zentrum für Informationstechnik Berlin (abbreviated ZIB) has moved (or will move soon). My new postal and email addresses are

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The electronic addresses are already valid. The addresses of the other officiers of our Activity Group can be found on p. 19.

I think that interesting material has been collected, and I hope that you enjoy reading this issue of the Newsletter!

September 30, 1996 Wolfram Koepf

Memorial Note about Waleed Al-Salam

(Editor's Note: Martin Muldoon provided an obituary that was contained in the last issue 6.3; Dick Askey has kindly provided the following additional remarks.)

One of the pleasures in visiting Edmonton was being able to talk with Waleed Al-Salam. The conversations could be on mathematics, but just as frequently would be on other topics. What he did not tell me about one visit is something which shows his character.

It was a cold winter, and he met me at the downtown airport after a flight from Calgary. He never mentioned the sequel. I was wearing a fur hat bought in Moscow, and carrying a bag from the International Congress of Mathematicians in Moscow in 1966. There was a Russian hockey team in Calgary. When this Russian looking man with a Russian hat and bag flew to the downtown airport in Edmonton without the RCMP having been notified, they called to Edmonton to ask that this be checked out. I did not notice them watching, nor do I think Waleed did, but a few days later there were questions asked about him and me at the Mathematics Department. It was only many years later that someone else told me about this. Waleed did not, since he did not want me to feel bad about his being investigated. That was typical. He was a gentleman.

Waleed was also a scholar. On another trip to Edmonton, I talked about some orthogonal polynomials which had first been found by L.J. Rogers, although Rogers was not aware they were orthogonal. These polynomials had been rediscovered twice around 1940, so were known to be orthogonal. However, they were not well known. In discussion after the talk, Waleed said that these polynomials reminded him of some polynomials found by Bill Allaway in his thesis. Waleed had sent me this thesis, but I had not read it carefully, so asked him to bring in a copy the next day. Sure enough, Allaway had rediscovered these polynomials once again, but had been more careful in his work than the two who had found these polynomials about 1940. There was a special case where division by zero occurred unless you were careful, and Allaway had been. These polynomials which had been missed by others are now called sieved polynomials and they have played an important role in a few developments of the general theory of orthogonal polynomials. Both of Waleed's Ph.D. students, Bill Allaway and Mourad Ismail, got a good start in mathematics from the problems which they got from him.

I handled the paper Waleed and Ted Chihara wrote on what are now called the Al-Salam-

Chihara polynomials. This was for SIAM Journal on Mathematical Analysis, so there should be some indication that the work would have applications to problems in science or engineering, or some other applied area. They had none, but the polynomials seemed so natural that there was no question in my mind that they would eventually have many applications. We now know where they live. They are a very important q-extension of Laguerre and Meixner polynomials. The problem Al-Salam and Chihara solved was a very natural extension of the problem solved by Meixner in the middle 1930s when he found Meixner polynomials and what I call Meixner-Pollaczek polynomials. These last arise in the representations of $SU(1,1)$, and so in theoretical physics. Now that quantum groups seem to have appeared in physics, it is very likely that these polynomials of Al-Salam and Chihara will arise in problems in physics, just as the polynomials found earlier by Al-Salam and Carlitz have arisen in the study of q-harmonic oscillators. These last polynomials are natural q-extensions of Charlier polynomials. They also extend the continuous q-Hermite polynomials of L.J. Rogers. The polynomials of Rogers have a very interesting combinatorial structure associated with them, and extensions of this to the Al-Salam-Carlitz polynomials is currently being developed.

Waleed Al-Salam had a remarkable eye for formulas, and the ability and energy to find a number of important ones. The polynomials mentioned above are very appropriately named, and these and some other results found by him will keep his name in the minds of many people who never had the joy of knowing him. Those of us who did will remember him as well as his work, and miss him the more for this.

Richard Askey
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Memorial Note about Marco P. Schützenberger

Professor Marco P. Schützenberger (Paris) died on 30 July 1996 at the age of 75. He is well-known for his work in (algebraic) combinatorics, which touches special functions. He had great influence on Dominique Foata, Doron Zeilberger and others who are working on the interface of combinatorics and special functions. He first published the q-binomial formula and the functional equation for q-exp under relation $xy = qyx$ (C. R. Acad. Sci. Paris 236 (1953), 352-353).

Tom H. Koornwinder
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Memorial Note about Carl Herz

Carl S. Herz, Redpath Professor of Pure Mathematics at McGill University, died on May 1, 1995 at the age of 65. His general area of research was harmonic analysis in a wide sense. His paper *Bessel functions of matrix argument*, Ann. of Math. 61 (1955), 474-523, was a pioneering paper in the field of special functions in several variables associated with Lie groups and with root systems.

See also the obituary *Carl Herz 1930–1995* in Notices AMS, July 1996, which is partly reproduced from Canadian Math. Soc. Notes, December 1995.

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Reports from Meetings and Conferences

1. Chebyshev Memorial Conference: Moscow State University, May 14-19, 1996

The mechanics-mathematics faculty of the *Moscow State University*, the *Chebyshev's Fund* and the *Institute of Nuclear Power Engineering* (Obninsk) held an International Conference *Modern Problems in Mathematics and Mechanics - CHEB96* in honour of the great Russian mathematician Pafnutij L'vovich Chebyshev's 175^{th} anniversary. The conference was held in the main high-rise building of the Moscow State University from 14^{th} to 19^{th} of May 1996 and was partly sponsored by the Moscow State University and the Russian Foundation of Fundamental Investigations. The Organizing Committee of the Conference was chaired by Academician of the Russian Academy of Sciences, Professor N.S. Bahvalov. Among the general sections of the conference we mention the sections on approximation theory and computational mathematics. On Chebyshev's birthday May 16^{th} a visit to his native land, grave and museum was organized. The excursion to the museum was very interesting because it is situated now in a country school. A special performance devoted to mathematics and Chebyshev was prepared by schoolboys and schoolgirls and their teachers. Among the participants at these events were leading and famous Russian mathematicians: the former President of the Russian Academy of Sciences, Academician Professor G.I. Marchuk and Academician Professor S.M. Nikolski. Some relatives of Chebyshev visited the conference also, and it was possible to obtain their autographs. About 280 participants attended the conference. I'd like to mention the following reports which were made to the Conference: Academician Professor N.S. Bahvalov *The use of Chebyshev polynomials in computational mathematics*, Professor Y. Matiasevich *Computer evaluation of generalized Chebyshev polynomials* and the report of the author of this column: *The use of Chebyshev polynomials in the computation of special functions.*

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2. Mini-conference on q-Series: Ohio State University, June 20-22, 1996

A Mini-conference on q-Series (Combinatorics, Classical Number Theory, Special Functions), organized by Gaurav Bhatnagar and Stephen C. Milne, took place at Ohio State University on June 20–22, 1996. Below, we list the speakers and their titles. Further information, including abstracts, is available at the website: http://www.math.ohio-state.edu/~gaurav/conference/conference.html.

- **Richard Askey:** The binomial theorem and extensions through 25 centuries
- **Richard Askey:** How do special functions arise?
- **Bruce Berndt:** The Rogers-Ramanujan Continued Fraction
- **Douglas Bowman:** Analytic continuation of basic hypergeometric series and the inversive closure
- **Heng Huat Chan:** On Ramanujan's Cubic Transformation for ${}_2F_1(1/3, 2/3; 1; z)$
- **Sheldon Degenhardt:** Weighted Inversion Statistics and their Symmetry Groups
- **James Haglund:** Rook Theory and Hypergeometric Series
- **Christian Krattenthaler:** A new bijective proof of Stanley's hook-content formula for semistandard tableaux
- **Christian Krattenthaler:** Advanced determinant calculus
- **Verne Leininger:** Expansions for $(q)_\infty^{n^2-1}$ and basic hypergeometric series in $U(n)$
- **Steve Milne:** New infinite families of exact sums of squares formulas, Jacobi elliptic functions, and Ramanujan's tau function
- **Michael Schlosser:** Multidimensional matrix inversions and multiple basic hypergeometric series.

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3. International Workshop on Orthogonal Polynomials in Mathematical Physics, in Honour of André Ronveaux: Madrid, June 24-26, 1996

A total of 74 participants engaged in friendly discussions and a pleasant atmosphere accompanied the meeting. Thirteen Spanish institutions were represented by 53 people and another thirteen foreign institution by the rest. Each of the five invited speakers gave two one-hour lectures:

- **Natig Atakishiev:** (Instituto de Investigaciones en Matemáticas Aplicadas y en Sistemas. Univ. Nacional Autónoma de México. Cuernavaca. México.) *Difference Equations and Some of their Solutions. Ramanujan-type Continuous Measures for Classical q-Polynomials.*
- **Jesus S. Dehesa:** (Departamento de Física Moderna. Univ. de Granada, Spain.) *Information Theory, Quantum Entropy and Orthogonal Polynomials.*
- **Yuri F. Smirnov:** (Instituto de Física, Univ. Nacional Autónoma de México. México.) *Orthogonal polynomials of a Discrete Variable and Quantum Algebras* $SU_q(2)$ *and* $SU_q(1,1)$*. Hidden* $sl(2)$ *Algebra of the Finite Difference Equations.*
- **H. T. Koelink:** (Univ. of Amsterdam, The Netherlands.) *Addition Formulas for q-Special Functions. Hecke algebras and q-Krawtchouk polynomials.*
- **Alexander Aptekarev:** (Keldysh Institut, Russian Academy of Sciences, Moscow.) *Toda-type Dynamics for the Coefficients of Recurrence Relations.*

The sessions were completed by thirteen half-hour communications. These short communications were delivered by:

- **André Ronveaux:** *Orthogonal Polynomials: Connection and Linearization Coefficients.*
- **Roeloef Koekoek:** *Recent Developments in the Research of Differential Operators for Generalized (Sobolev) Orthogonal Polynomials.*
- **Aldo Tagliani:** *Entropy-convergence, Instability in Stieltjes and Hamburger Moment Problems.*
- **Ramón Orive:** *Rational Approximation to Certain Functions with Branch Points.*
- **Wolfgang Gawronski:** *On the Zeros of Classical Orthogonal Polynomials with Large Complex Parameters.*
- **Lucas Jódar:** *A Matrix Formula for the Generating Function of the Product of Hermite Matrix Polynomials.*

- **Pierpaolo Natalini:** *Some New Sets of Relativistic Orthogonal Polynomials.*
- **Franciszek Szafraniec:** *Orthogonal polynomials in building models of the quantum harmonic oscillator.*
- **Federico Finkel:** *Quasi Exactly Solvable Potentials on the Line and Orthogonal Polynomials.*
- **Franz Peherstorfer:** *Periodic and quasiperiodic Toda lattices.*
- **Jorge Sánchez-Ruiz:** *Position and momentum information entropies of the harmonic oscillator and logarithmic potential of Hermite polynomials.*
- **Jorge Arvesú:** *The classical Laguerre polynomials in a relativistic quantum-statistical model.*
- **Antonio Duran:** *On orthogonal matrix polynomials.*
- **José Carlos Petronilho:** *On Some Polynomial Modifications of Measures. Applications.*

All speakers were kindly invited to submit written versions of their talks for the proceedings of the meeting which will be published in the near future. The Workshop was dedicated in honour of Prof. André Ronveaux on the occasion of his retirement from the University and his fruitful mathematical life.

The organizing Committee was: Manuel Alfaro (Univ. de Zaragoza), Renato Álvarez-Nodarse (Secretary) (Univ. Carlos III), Antonio García García (Univ. Carlos III), Guillermo López Lagomasino (Univ. Carlos III) and Francisco Marcellán (Chairman) (Univ. Carlos III)

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4. SIAM Annual Meeting: Kansas City, July 22-26, 1996

SIAG/OS sponsored a minisymposium on Modern Topics in Orthogonal Systems on Tuesday July 23, at the July 22-26,1996 SIAM Annual Meeting in Kansas City, Missouri. Due to a cancellation by Robert Gustafson, only one major topic was covered: Wavelets. There were three talks.

Gilbert Walter organized the session and led off with *Improving wavelet approximations.* Although discrete wavelets generally have superior convergence properties compared to classical orthogonal systems, they share the Gibbs' phenomenon shortcoming—which causes errors at the edge of a truncated signal or image. It was shown by Shim and Volkmer that this always happens for orthogonal approximations for all continuous wavelets with sufficient decay. Walter presented ways of avoiding this shortcoming for wavelets.

Peter R. Massopust spoke on *Multiwavelets, Multiresolution Schemes, and Hyperbolic Conservation Laws.* He presented multiresolution schemes based on multiwavelets. These schemes employ a combination of interpolation and direct evaluation. He showed how such multiresolution schemes can be used to obtain accurate and computationally efficient numerical weak solutions of partial differential equations arising in computational fluid dynamics.

Truong Nguyen spoke on *Image Coding Using Shift-Invariant Dyadic Wavelet Transform*, joint work with Y. Hui and C.W. Kok. He proposed a new class of wavelet filters, shift-invariant wavelet filters, for the purpose of image compression. The proposed shift-invariant wavelet transform has better shift-invariant properties compared with the conventional dyadic wavelet transform, without changing the structure. He proposed and evaluated two bit allocation schemes, suitable for the proposed shift-invariant wavelet transform coding, and presented experimental results showing that the shift-invariant wavelet transform has better energy compaction properties in image coding than the conventional wavelet transform.

Later in the morning Carl de Boor gave the John von Neumann Lecture on (multivariate) *Polynomial Interpolation.* This was one of the highlights of the meeting.

Two of the SIAG-OS officers, Martin Muldoon and Willard Miller, participated in the weeks events and held an impromptu business meeting to discuss possible minisymposium topics for the SIAG-OS at the 1997 SIAM Annual Meeting (July 14-18, Stanford University). The 1997 meeting themes most relevant to our group are

1. Optimization and linear algebra, and
2. Computer science applicable to large-scale scientific computing (including visualization, and the impact of the World Wide Web.

Among the ideas discussed by your ever vigilant officers was a minisymposium of expository talks on orthogonal polynomials with heavy emphasis on the use of symbolic computation and links to online data bases as an aid to research. An associated session of research talks on the same topics was also proposed. A related idea is a session on handbooks (in a generalized sense, to include the World Wide Web). A session of expository or research talks related to Szegő's work was suggested, appropriate since he was on the Stanford faculty. Another possibility is a session on Orthogonal Polynomials in Signal Processing. In general, Muldoon and Miller thought that a session of expository rather than research talks would be most useful to our membership, though more difficult to organize. They would greatly appreciate suggestions and advice from the SIAG-OS membership. A decision on the minisymposium topic or topics must be reached by late October 1996, for submission to the meeting organizers.

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5. Second International Workshop Transform Methods & Special Functions: Varna, Bulgaria, August 23-30, 1996

This workshop, whose scientific program lasted from August 24 to August 29, 1996, was organized by:

1. Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences;
2. Institute of Applied Mathematics and Computer Sciences of Technical University Sofia;
3. Union of Bulgarian Mathematicians;
4. Union of Scientists in Bulgaria;
5. Istituto per la Ricerca di Base (Monteroduni, Italy).

It took place in the hotels Magnolia and Detelina, north of Varna, with a wide view of the Black Sea. It was dedicated to the 100th anniversary of the birth of the Bulgarian mathematician Academician Nikola Obrechkoff (1896-1963, born in Varna) who did pioneering work in such diverse fields as analysis, algebra, number theory, numerical analysis, summation of divergent series, probability and statistics. The workshop was attended by more than seventy participants from twenty countries, alphabetically from Australia to Vietnam, the largest groups coming from Poland (20), Bulgaria (12) and Japan (7).

After the opening ceremonies (which included a greeting speech from Varna's Mayor and an appreciation of Obrechkoff and his achievements by I. Dimovski (Sofia) the series of lectures commenced with a plenary lecture by Prof. E.R. Love from Melbourne (Australia), the oldest participant at the Conference, on his joint work with M.N. Hunter on "Expansions in series of Legendre functions". In total there were six plenary lectures, presented by E.R. Love (title already mentioned), F. Mainardi (Bologna, Italy) on "Applications of fractional calculus in mechanics", R. Gorenflo (Berlin, Germany) on "The tomato salad problem in spherical stereology, P.G. Rooney (Toronto, Canada) on "The ranges of Mellin multiplier transformations", H.M. Srivastava (Victoria, Canada) on "Some operational techniques in the theory of special functions", and Vu Kim Tuan (Kuwait) on "Integral transforms and supports of functions." Then 63 lectures were presented in (up to three) parallel sessions with headings "Integral Transforms and Special Functions", "Fractional Calculus", "Special Functions", "Geometric Theory of Functions", "Differential Equations", "Integral Transforms & Operational Calculi", "Mathematical Analysis and Applications", "Integral Transforms", "Varia". In addition, in the closing session P. Rusev, I. Dimovski and V. Kiryakova lectured on "Obrechkoff's generalization of Descartes' rule" and "Obrechkoff's integral transforms".

The conference ended with a final ceremony in which P. Antosik (Katowice, Poland) distributed prizes (to the three largest delegations) and praise (to individuals).

In the lectures, besides some educational reviews, many new research results were presented. This reporter, due to the multiple sessions, could not attend all the talks; however he feels it appropriate to mention three of them he considered special highlights in that they gave impressive recent results, obtained just in the few weeks preceding the conference. They were two lectures by P.L. Butzer and S. Jansche (Aachen, Germany) on "The Mellin transform, finite Mellin transform and Mellin-Fourier transform", "The Mellin-Poisson summation formula and the exponential sampling theorem", and the plenary lecture by Vu Kim Tuan. In many lectures the importance of special and of general types of special functions for applications inside and outside of mathematics was stressed. Particular mention should be made of hypergeometric functions and their generalizations, H and G functions, confluent hypergeometric functions, Bessel functions, Mittag-Leffler functions, and various types of orthogonal polynomials.

The program also provided a two-hour plenary discussion on "Physical and geometrical meanings and applications of fractional calculus operators" that was well attended and opened stimulating aspects. Topics of discussion were, among other things, some problems raised in H.M. Srivastava's collection of "Open questions for further researches on fractional calculus and its applications" (pp. 281-284 in K. Nishimoto, ed., Fractional Calculus and its Applications, Nihon University, Tokyo 1990). In particular, arguments were exchanged on the question whether there is a relationship between fractional calculus and fractal geometry or whether the only relationship is the coincidence of the first five characters of the words "fractional" and "fractal". Fortunately, the discussion did not stick to this problem.

The meeting was framed, thanks to the organizers P. Rusev, I. Dimovski, V. Kiryakova, L. Boyadjiev (all from Sofia) and Sh. Kalla (Kuwait), by an appealing social program (reception, on August 27 a whole-day excursion to the old town Shumen, 100 km to the west of Varna, birthplace of Prof. Rusev, and the fascinating surroundings of Shumen, with more than two hours of Bulgarian folklore and after-lunch dancing, and on August 28 a celebration of Prof. Rusev's 65th birthday).

Thanks to St. Peter Black Sea sunrises, beach, sand, sun and clear water swimming could be enjoyed. The organizing committee did, in an admirable manner, their best to smooth all the (in retrospect, few) problems that are bound to arise in such a meeting.

The Workshop's Proceedings (like those of the preceding one held in Bankya/Bulgaria, 12-17 August 1994) will be published by "Science Culture Technology (SCT) Publishing" in Singapore.

Rudolf Gorenflo
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Forthcoming Meetings and Conferences

1. MSRI Program on Combinatorics: Berkeley. Workshops October 14-18, 1996, and April 14-18, 1997

The *MSRI*, Berkeley organizes during 1996–97 a full-year program on Combinatorics. The program focuses on four main areas of approximately half a semester each. During each of these four periods there will be a one-week workshop. See WWW: http://www.msri.org/application/comb.html

Two of the areas may have some relevance for the members of our Activity Group:

1.1. Enumeration and partially ordered sets: September 1-October 25, 1996; Workshop: October 14-18, 1996

Combinatorial identities (especially computer proofs); conjectures on monotone triangles and plane partitions; enumeration and classification of tilings; combinatorial problems arising from statistical mechanics and knot theory; combinatorial properties of Kazhdan-Lusztig polynomials; q-analogues and quantum groups.

The workshop is being organized by Lynne Butler, Ira Gessel, Rodica Simion (chair), and Michelle Wachs. The program of the workshop revolves around enumerative and order-theoretic aspects in the study of combinatorial structures. Included among the workshop topics are:

- q-series
- Partitions
- Plane partitions
- Alternating sign matrices
- Enumerative aspects of group algebras and symmetric functions
- Combinatorics of orthogonal polynomials
- Computer algebra
- Combinatorial, topological, and algebraic aspects of the theory of partially ordered sets

For more information:
WWW: http://www.msri.org/sched/CombPosets.html
email: posets@msri.org

1.2. Symmetric functions and representation theory: March 17-May 30, 1997; Workshop: April 14-18, 1997

Macdonald's two-parameter symmetric functions and the corresponding two variable Kostka polynomial; immanent conjectures of Lieb, Goulden-Jackson, Stembridge, et al.; constant term identities and their connection with nilpotent Lie algebras and cyclic homology; random walks on groups and shuffling problems; internal products of symmetric functions; invariant theory.

The workshop is being organized by Curtis Greene (Chair), Sergey Fomin, Phil Hanlon, and Sheila Sundaram. Scope: In recent years there have been many exciting developments in areas that link combinatorics (especially the theory of symmetric functions) with representation theory and algebraic geometry. This workshop will focus on current problems in these areas, emphasizing the interplay between algebraic and combinatorial methods. The program will include the following topics, as well as others:

- Schur functions and their generalizations
- Combinatorics and representations of finite Coxeter groups
- Quantum groups and Hecke algebras
- Centralizer algebras
- Homology representations

For more information:
WWW: http://www.msri.org/sched/CombSymfns.html
email: symfns@msri.org

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2. Workshop on Special Functions & Differential Equations: Madras, India, January 13-24, 1997

This is a Workshop in the field of Special Functions, Differential Equations and closely related topics, to be held at the *Institute of Mathematical Sciences* (I.M.Sc.), Madras during Jan. 13–24, 1997. Mini-series of lectures by experts will introduce the recent trends and developments in the topics listed. The objective of the Workshop is to bring together experts and active research students, to help strengthening of research activities at the universities and research institutions in this ever green area.

Topics include:

- Special Functions
- Differential Equations
- Orthogonal Polynomials (One/Several Variables)
- Group Theory and Special Functions
- Difference Equations
- q-Special Functions
- Quantum Groups and Special Functions
- Numerical Methods
- Algebraic/Symbolic Computer Packages

Plenary speakers include ([*] means: to be confirmed): R.P. Agarwal (India), B C. Berndt (U.S.A) [*], F. Calogero (Italy), H.D. Doebner (Germany) [*], R.F. Gustafson (U.S.A.) [*], E. Kalnins (New Zealand), T.H. Koornwinder (Netherlands), M. Lakshmanan (India), H.L. Manocha (India), S.C. Milne (U.S.A.) [*], M. Lohe (Australia), T.D.

Palev (Bulgaria), C. Quesne (Belgium), A. Ronveaux (Belgium), T.S. Santhanam (U.S.A.), W. Van Assche (Belgium), G. Vanden Berghe (Belgium), J. Van der Jeugt (Belgium), A. Verma (India), L. Vinet (Canada), M. Waldschmidt (France), P. Winternitz (Canada).

There is an international Advisory Committee and a local Organizing Committee.

Participants should be active research scientists, students, in the field of Special Functions, Differential Equations and related topics. Interested candidates should send by e-mail (or on plain paper) details giving their name, address, age, qualifications, present position and a resume of research work done. (Maximum number of participants: about 60).

Financial support for train travel, board and lodging will be provided to some of the participants. Registration Fee: US$ 100 (Rs. 200 for Indian participants).

Please send your applications soon. Further information: http://www.imsc.ernet.in/~wssf97/, or by sending a message to:

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3. Centenary Conference, including Minisymposium on Special Functions: Madison, Wisconsin, May 22-24, 1997

On May 22–24, 1997 the Department of Mathematics of the University of Wisconsin, Madison will celebrate the 100th anniversary of the awarding of the first PhD. in Mathematics by the University of Wisconsin in 1897.

Invited speakers, who are being asked to discuss early work at Wisconsin and more recent work of former and current Wisconsin students and faculty, include: Richard Arratia, Richard Askey, Carl de Boor, Robert Brown, Joshua Chover, Michael Crandall, George Glauberman, William Jaco, Yiannis Moschavakis, John Nohel, Louis Solomon, and Walter Rudin. There will also be opportunities for contributed papers.

Charles Dunkl will be organizing a minisymposium there on special functions. Participants will have to pay their own transportation, lodging and meals, but there will be no registration fees. Anybody who would like him to consider him/her for the special function session should contact him (cfd5z@virginia.edu) very soon, since he will be forming the program.

All current and former Wisconsin faculty, students, visitors, fellows, and other 'Wisconsin friends' are cordially invited to participate in the conference. Rooms for the nights of May 21, 22, 23, and 24 (1997) have been reserved in the name of "UW-Math PhD. Centennial Conference" at three nearby hotels and will be kept open until two weeks before the conference begins.

A banquet/celebration evening is being planned for Friday, May 23, 1997 in Great Hall of the Memorial Union. Mary Ellen Rudin will be the banquet speaker. Other participants will offer reminiscences. A mailing concerning the conference to all people on the Mathematics Department's newsletter mailing list is planned for the early fall, and this will contain further information about registration and the banquet. Suggestions, proposals for minisymposia, queries can be sent by email to: phdcent@math.wisc.edu or by post to:

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These addresses can also be used to get on the mailing list.

Further information can be obtained on WWW at http://math.wisc.edu/events/cent.html.

Charles F. Dunkl
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4. First ISAAC Conference: University of Delaware, Newark, Delaware, June 2-6, 1997

ISAAC is an abbreviation for *The International Society for Analysis, its Applications and Computation*. Analysis is understood here in the broadest sense, including differential equations, integral equations, functional analysis, and function theory. The first Congress of this newly constituted society will be held at the University of Delaware, between June 2 and June 6, 1997, at the Clayton Hall Conference Center of the University of Delaware. The Conference Secretary is

Pam Irwin
Department of Mathematics
University of Delaware
email: irwin@math.udel.edu

If you have any questions about the conference, please contact her.

The conference has its WWW page at the address http://www.math.udel.edu/isaac/conferen/congr97.htm, and at http://www.math.udel.edu/isaac/cong.html there is an online registration form available.

Session 13 of this conference will be devoted to *Orthogonal Polynomials*, organized by Wolfram Koepf. The emphasis of this session will be on the use of symbolic computation in connection with orthogonal polynomials and special functions. Symbolic computation has the potential to change the daily work of everybody who uses orthogonal polynomials or special functions in research or applications. It is the purpose of the proposed section to bring together developers of symbolic algorithms and implementations which are connected with orthogonal polynomials and special functions with users of computer algebra systems who need this type of software.

Up to now the following have tentatively agreed to present a lecture in this session:

- **Victor Adamchik:** *On Series Involving the Riemann Zeta Function*
- **Renato Álvarez-Nodarse:** *Algebraic and Spectral Properties of Orthogonal Polynomials: A Computer Algebra Approach*
- **Tewodros Amdeberhan:** *Computer Aided Proofs of a Determinant Identity*
- **Richard A. Askey:** *Some Problems on Orthogonal Polynomials*
- **Natig M. Atakishiyev:** *On the Fourier-Gauss Transforms of some q-exponential and q-trigonometric Functions*
- Yang Chen, **Mourad Ismail:** *Asymptotics of the Largest Zeros of Some Orthogonal Polynomials*
- **Charles F. Dunkl:** *Using Maple to Explore Special Functions of Several Variables*
- **Wolfram Koepf**, Dieter Schmersau: *Algorithms for Classical Orthogonal Polynomials*
- **Tom H. Koornwinder**, René Swarttouw: *rec2ortho: An Algorithm for Identifying Orthogonal Polynomials Given by Their Three-Term Recurrence Relation as Special Functions*
- **Kelly Roach:** *Maple and Orthogonal Polynomials*
- **André Ronveaux:** *Recurrence Relations for Connecting Coefficients Between Some Orthogonal Polynomials Families–A Simple Algorithm (Mathematica)*
- **Walter Van Assche:** *Some Examples of Computer Experiments in Research on Orthogonal Systems*
- **Rafael Yanhez:** title open
- **Doron Zeilberger:** *The Super-Holonomic Hierarchy*

The WWW page http://www.zib.de/koepf/isaac.html contains updated informations about the program of this session on *Orthogonal Polynomials.*

Wolfram Koepf
(koepf@zib.de)

5. CRM Workshop on Algebraic Combinatorics

The *Centre de Recherches Mathematiques* (Montreal, Canada) is hosting a year-long program in combinatorics and group theory in 1996-1997. The year will be organized around a certain number of workshops spread throughout the year.

A Workshop on Algebraic Combinatorics will take place during June 9–20, 1997. The purpose of the workshop is to study interactions between Algebraic Combinatorics and Symmetric Functions, with a special emphasis on Descent Algebras of Coxeter groups in relation to quasi-symmetric functions and non-commutative symmetric functions, and on doubly parameterized (Macdonald) (q, t)-symmetric functions, in relation to harmonics of reflection groups.

Organizers: F. Bergeron (UQAM), N. Bergeron (York), C. Reutenauer(UQAM)

Invited Speakers: P. Diaconis (*), A. Garsia, I. Gessel, I. Goulden (*), M. Haiman, I.G. Macdonald, C. Procesi, L. Solomon, R.P. Stanley, J.Y. Thibon, (*), to be confirmed.

Those wishing to participate in the above activities are invited to write to:

Louis Pelletier
CRM, Universié de Montéal
C.P. 6128, Succ. centre-ville
Montreal (Quebec) Canada H3C 3J7

Further information on WWW:
http://www.crm.umontreal.ca/Activites/Thematic_Year_96-97.html

Louis Pelletier
(activites@crm.umontreal.ca)

6. Continued Fractions and Geometric Function Theory: Trondheim, Norway, June 24-28, 1997

Haakon Waadeland celebrates his 70th birthday on 20 May 1997. He is responsible for a long list of valuable contributions to the two fields of continued fractions and geometric function theory, and he is still very active. In recognition of his work we have decided to organize a conference in his honour. The conference will be held in Trondheim, Norway from 24 to 28 June 1997 under the title *Continued Fractions and Geometric Function Theory.* We hope to get together a group of people representing both fields, and we would very much appreciate if you would participate.

You are all welcome to present talks of 25 minutes and 5 minutes for questions. (If somebody has material suited for longer or shorter presentations, we may be able to arrange this.) We plan to publish proceedings from the conference.

The conference has its own world wide web page on http://www.matstat.unit.no/CFGT, where we will put new information when it is available. From this page you can also link to pages about Trondheim and about the university, NTNU.

If you want to receive the second announcement, please contact the conference address: confun@imf.unit.no.

Lisa Lorentzen
(lisa@imf.unit.no)

7. VIII Simposium Sobre Polinomios Ortogonales y Aplicaciones, Sevilla, September 22-26, 1997

You are cordially invited to attend the *VIII Simposium sobre polinomios ortogonales y aplicaciones* (8SPOA, in short) which will be held from September 22–26, 1997 at the Facultad de Matemáticas of the Universidad de Sevilla.

The 8SPOA is on the line of the *Third international symposium on orthogonal polynomials and their applications* or the *VII Simposium sobre polinomios ortogonales y aplicaciones* which were held in Erice (Italy) and Granada (Spain) during June 1990, and September 1991, respectively. The roots of this Symposium are in the *old* spanish meetings (held during the last decade) and which were born to connect and to exchange the knowledge of the Spanish people who use polynomial techniques in any way and/or domain. Year after year the number of non-spanish scientists working in related fields which attended those Simposiums was increasing. It would be a pleasure for all of us that you would be able to attend the Symposium.

The scientific program is currently being elaborated by the scientific committee C. Berg (Copenhagen), A.J. Durán (Sevilla), J.J. Guadalupe (La Rioja), G. López Lagomasino (La Habana), F. Marcellán (Madrid), J.Sánchez Dehesa (Granada) and W. van Assche (Leuven). It consists of some plenary lectures and short communications (20 min). The second circular, to be distributed soon, will give a detailed information about it.

The cost of attendance is expected to be very reasonable. The following estimates are subject to change but it is anticipated that the registration fee will be around 20.000 pesetas (1$ =120 pesetas, approx.), which includes the admission to the Symposium, a copy of the book of abstracts, a copy of the Proceedings, reception and participation in some social events. The price for lodging and meals will total about 6.000 pesetas per person and day.

In order to keep discussions informal, the size of the Symposium must be limited to about 100 invited participants. Owing to space limitations, it may happen that we will not be able to accomodate all those interested in attending it. To be on the safe side with the accomodation of the participants of our Symposium, we have already reserved a number of rooms in the comfortable University Residence "Hernando Colón". If you are interested in being invited to participate or in receiving subsequent circulars, please receive the corresponding form by email, and return it as soon as possible, not later than October 31, 1996, to the Symposium Mailing Address shown below.

The Symposium will be held at the building of the Facultad de Matemáticas of the Universidad de Sevilla. Both the Faculty and the residence are in "Reina Mercedes Campus", 30 minutes on foot from the old center of the city (the biggest in Europe, with the cathedral, Giralda tower, Reales Alcázares, etc.) and 15 minutes from María Luisa Park (España and América Squares, etc.).

Access to Sevilla is easy; it lies along the high speed railroad southwards from Madrid (2 hours and a half). There exist highways from Madrid and Barcelona connecting to Europe, and there is an international airport with several daily flights to Madrid or Barcelona.

A preregistration form can be obtained via email or from the following address.

VIII Simposium Sobre Pol. Orto. y Aplicaciones
Dpto. Análisis Matemático
Facultad de Matemáticas
Universidad de Sevilla
Aptdo. 1160
41080 Sevilla, Spain
fax: 34 5 4557972

The WWW page http://www.wis.kuleuven.ac.be/wis/applied/walter/sevilla.html shows more details about the conference.

Antonio J. Duran
(8spoa@obelix.cica.es)

Books and Journals

Announcements

1. Mathematical Analysis, Wavelets, and Signal Processing
Edited by Mourad E.H. Ismail, M. Zuhair Nashed, Ahmed I. Zayed, and Ahmed F. Ghaleb

Contemporary Mathematics 190, AMS, January 1996, 354 pp., paperback, ISBN 0-8218-0384-0

Contributors include both mathematicians and engineers presenting their ideas on new research trends. This book emphasizes the need for interaction between mathematics and electrical engineering in order to solve signal processing problems using traditional areas of mathematical analysis such as sampling theory, approximation theory, and orthogonal polynomials.

Wolfram Koepf
(koepf@zib.de)

2. Polynomials and Polynomial Inequalities
By P. Borwein and T. Erdélyi

Graduate Texts in Mathematics **161**, Springer, Berlin, 1995, 480 pp., hardcover DM 98, ISBN 0-387-94509-1

Polynomials pervade mathematics, virtually every branch of mathematics from algebraic number theory and algebraic geometry to applied analysis and computer science, has a corpus of theory arising from polynomials. The material explored in this book primarily concerns polynomials as they arise in analysis, focusing on polynomials and rational functions of a single variable. The book is self-contained and assumes at most a senior-undergraduate familiarity with real and complex analysis. After an introduction to the geometry of polynomials and a discussion of refinements of the Fundamental Theorem of Algebra, the book turns to a consideration of various special polynomials. Chebyshev and Descartes systems are then introduced, and Müntz systems and rational systems are examined in detail. Subsequent chapters discuss denseness questions and the inequalities satisfied by polynomials and rational functions. Appendices on algorithms and computational concerns, on the interpolation theorem, and on orthogonality and irrationality conclude the book.

Contents: Introduction and Basic Properties - Some Special Polynomials - Chebyshev and Descartes Systems - Denseness Questions - Basic Inequalities - Inequalities in Müntz Spaces - Inequalities in Rational Function Spaces
Appendices: Algorithms and Computational Concerns - Orthogonality and Irrationality - An Interpolation Theorem - Inequalities for Generalized Polynomials - Inequalities for Polynomials with Constraints.

Wolfram Koepf
(koepf@zib.de)

3. Journal of Symbolic Computation
Volume Editors: Peter Paule, Volker Strehl

Volume 20, Nos. 5 and 6, November/December 1995, Academic Press, London.

This special issue of the Journal of Symbolic Computation covers the Proceedings of a Workshop on *Symbolic Computation in Combinatorics* held during September 21–24, 1993, at Cornell University. Some of the articles are on hypergeometric identities, and might be of interest to the members of the Activity Group.

From the foreword of the volume editors:
"Following an invitation by Moss Sweedler, the director of the Army Center of Excellence for Symbolic Methods in Algorithmic Mathematics (ACSyAM) at Cornell University's Mathematical Sciences Institute (MSI), the editors of this special issue had the pleasure to organize a workshop on "Symbolic Computation in Combinatorics" at MSI, Cornell University, Ithaca, from September 21-24, 1993. It focused on the role of computer algebra in solving problems concerning symbolic manipulation of combinatorial formulae, for example, (q-)binomial sums, (q-)hypergeometric series and recurrences, and on the formal treatment of analytical problems in combinatorics, including nontrivial applications as well as new algorithms and system aspects. Although this special issue is primarily a report on work presented at Cornell, it also contains contributions not discussed at the workshop, but which thematically fit into the volume. Invited talks were given by G.E. Andrews on "AXIOM and the Borwein Conjecture", by P. Flajolet and B. Salvy on "Statistical Combinatorics", and by D. Zeilberger on "The Holonomic Paradigm and beyond". The difference operator Δ_1 is included in the title in order to symbolize the progress which has been made since the appearance of the first JSC special issue with the title "Symbolic Computation in Combinatorics" edited by P. Paule and D. Zeilberger, JSC Vol. 14 (1992)."

Contents

Wolfram Koepf
(koepf@zib.de)

Review

4. Table of Integrals, Series, and Products, CD-ROM Version 1.0
Edited by Alan Jeffrey

Academic Press, San Diego, California, USA, 1996, $ 79.95, ISBN 0-12-294756-8

The venerable Gradshteyn and Ryzhik **Table of Inte-**

grals, Series, and Products was originally planned by Ryzhik, who was later joined by Gradshteyn. The English translation was first published in 1965, and five editions of the volume followed in the next thirty years. In each edition there were corrections of errors and extensions of the material. Ryzhik died in World War II and Gradshteyn died during preparation for the fourth (1980) edition. The subsequent editions were and are edited by Alan Jeffrey.

The fifth edition of the book has been put on CD, which can be viewed from an IBM PC (or compatible) running Microsoft Windows 3.1, or 95, or NT, on a Mac, or on certain UNIX X-windows machines. Of course, there are minimum DRAM contraints, which only the foolhardy would ignore. I ran the CD using Windows 3.1. One can scan the book much as one can read the text of the printed fifth edition. But here one wants more, much more.

The major question is whether one can efficaciously search the CD so as to find integrals and/or integrands involving expressions that are of interest to the user. The manufacturer's guide asserts that the CD-ROM offers desktop access to the 20,000 fomulas for the integrals, sums, etc. The TEX source code for most formulae is obtained by clicking on a nearby icon. To perform the search one need study the TEX code for the expression that occupies your interest, activate the search panel, and fill in some variant of the studied TEX code. Wildcards are allowed.

The sad story here is that the search engine used preempts and prohibits use of the vital TEX characters (,) , < , > , = . It is very difficult to adapt the search program to do something other than find quotations of names of special functions. The TEX used on the CD is AMS-TEX which is at this time not a usual dialect and thus provides a minor nuisance. I believe that the problem of designing a search mechanism to find TEX encoded mathematical expressions is interesting and challenging. I think it would be of interest to check with the experts working with the Latex3 project to see if they have suggestions on how it might be done.

Marvin Rosenblum
(mr1t@virginia.edu)

Problems and Solutions

Thus far 15 problems have been submitted five of which have been solved (#**1**, **4**, **6**, **7**, **10**), and one of which is new (#**15**). Still unsolved are Problems #**2**, **3**, **5**, **8**, **9**, **11**, **12**, and **13**. Please send in your solutions! A solution of the Problem #**14** is presented in the current issue.

13. Product of Chebyshev Polynomials. For any pair of positive even $n, m \in \mathbb{N}$ let

$$F^{(n,m)}(x) = 2^n \prod_{k=0}^{n-1} \cosh\left(\frac{m}{2} \operatorname{arccosh}\left(x - \cos\frac{(2k+1)\pi}{n}\right)\right) = 2^n \prod_{k=0}^{n-1} T_{m/2}\left(x - \cos\left(\frac{(2k+1)\pi}{n}\right)\right),$$

where $T_m(x)$ denotes the Chebyshev polynomials of the first kind. These functions occur in statistical physics. They constitute polynomials in x

$$F^{(n,m)}(x) = \sum_{j=0}^{nm/4} A_j(n,m)\, x^{2j}$$

whose coefficients $A_j(n,m)$ are integers. Show the symmetry

$$F^{(n,m)}(x) = F^{(m,n)}(x),$$

and give a representation of the coefficients $A_j(n,m)$.

Christian Hege
(hege@zib.de)

15. Critical Values of Orthogonal Polynomials. Let P_n be an OP system on $[-1,1]$ with respect to a weight function $w(x)$. Denote $-1 < y_{n,1} < \ldots < y_{n,n-1} < 1$ the set of all critical points, i.e. the set of all zeros of the derivative P_n'. The values $P_n(y_{n,k})$, $k = 1, 2, \ldots n-1$ are known as critical values of P_n. Let $N(P_n)$ be the number of all *different* critical values of P_n.

Problem 15.1. Describe the set of OP systems with the property $N(P_n) = O(1)$, $n \to \infty$.

It is clear that for the first kind Chebyshev polynomials T_n one has $N(T_n) = 2$ for all n.

Problem 15.2. Given $w(x)$, find the value

$$a(w) := \limsup_{n\to\infty} \frac{N(P_n)}{n}.$$

Leonid B. Golinskii
(golinskii@ilt.kharkov.ua)

First Solution of Problem 14
A Trigonometric Integral
by I.J. Zucker
(jz@maxwell.ph.kcl.ac.uk)

The proposed question was to evaluate the integral

$$I = \int_0^{\pi/4} \ln(\sin^{3/2}(x) + \cos^{3/2}(x))\, dx.$$

In the following analysis the Clausen function of order 2 (se [1])

$$\mathrm{Cl}_2(\theta) = \sum_{r=1}^{\infty} \frac{\sin(r\theta)}{r^2},$$

where $-\infty < \theta < \infty$, will play an important role. It is known that $\mathrm{Cl}_2(\pi/2) = G$ where G is Catalan's constant, and

$$\int_0^\theta \ln(\cos x)\mathrm{d}x = -\theta \ln 2 + \frac{1}{2}\mathrm{Cl}_2(\pi - 2\theta)\,,$$

where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$. For the particular case $\theta = \pi/4$ this last equation yields

$$\int_0^{\pi/4} \ln(\cos x)\mathrm{d}x = \frac{1}{4}(2G - \pi \ln 2)\,. \tag{1}$$

We can write I as

$$I = I_1 + I_2,$$

where

$$I_1 = \int_0^{\pi/4} \ln(\cos^{3/2}\theta)\mathrm{d}\theta\,, \tag{2}$$

$$I_2 = \int_0^{\pi/4} \ln(1 + \tan^{3/2}\theta)\mathrm{d}\theta\,. \tag{3}$$

The application of (1) to the integral (2) gives

$$I_1 = \frac{3}{8}(2G - \pi \ln 2)\,. \tag{4}$$

We can evaluate I_2 by first making the substitution $\tan\theta = x^2$. It is found that

$$I_2 = \int_0^1 \frac{2x}{1+x^4} \ln(1+x^3)\mathrm{d}x\,. \tag{5}$$

Now split $2x/(1+x^4)$ into partial fractions and factorise $1+x^3$, then I_2 may be split into four integrals

$$I_2 = I_{21} - I_{22} + I_{23} - I_{24}$$

where

$$I_{21} = \int_0^1 \frac{\sin(\pi/4)\ln(1+x)}{(x-\cos(\pi/4))^2 + \sin^2(\pi/4)}\mathrm{d}x\,,$$

$$I_{22} = \int_0^1 \frac{\sin(\pi/4)\ln(1+x)}{(x+\cos(\pi/4))^2 + \sin^2(\pi/4)}\mathrm{d}x\,,$$

$$I_{23} = \int_0^1 \frac{\sin(\pi/4)\ln(1-x+x^2)}{(x-\cos(\pi/4))^2 + \sin^2(\pi/4)}\mathrm{d}x\,,$$

$$I_{24} = \int_0^1 \frac{\sin(\pi/4)\ln(1-x+x^2)}{(x+\cos(\pi/4))^2 + \sin^2(\pi/4)}\mathrm{d}x\,.$$

Lewin [1, p. 216] has indicated how such integrals might be evaluated. Thus he gives

$$\int_0^t \frac{b\ln(a+x)}{(x+c)^2+b^2}\mathrm{d}x = \frac{1}{2}\arctan\frac{bt}{b^2+c^2+ct}\ln\left(b^2+(a-c)^2\right)$$
$$\frac{1}{2}\Big(-\mathrm{Cl}_2(2\theta+2\phi)+\mathrm{Cl}_2(2\theta_0+2\phi)-\mathrm{Cl}_2(\pi-2\theta)+\mathrm{Cl}_2(\pi-2\theta_0)\Big)\,, \tag{6}$$

where

$$\tan\theta = \frac{t+c}{b}\,,\quad \tan\theta_0 = \frac{c}{b}\,,\quad \tan\phi = \frac{a-c}{b}\,.$$

When $t=1$, $a=1$, $b=\sin\alpha$ and $c=-\cos\alpha$, (6) compacts neatly into

$$\int_0^1 \frac{\sin\alpha\ln(1+x)}{(x-\cos\alpha)^2+\sin^2\alpha}\mathrm{d}x =$$
$$\frac{1}{4}(\pi-\alpha)\ln(2+2\cos\alpha) - \frac{1}{4}\mathrm{Cl}_2(2\alpha)\,. \tag{7}$$

So

$$I_{21} = \frac{3\pi}{16}\ln(2+\sqrt{2}) - \frac{G}{4}$$

and similarly

$$I_{22} = \frac{\pi}{16}\ln(2-\sqrt{2}) + \frac{G}{4}$$

To evaluate integrals such as I_{23} and I_{24} when there are logarithms of quadratics in the numerator leads to results even more complex than (6). After much tedious algebra one can show that

$$I_{23} = \frac{\pi}{48}\ln\left(2-2\cos\frac{\pi}{12}\right) + \frac{17\pi}{48}\ln\left(2-2\cos\frac{7\pi}{12}\right)$$
$$+\frac{1}{2}\left(\mathrm{Cl}_2\left(\frac{\pi}{12}\right) - \mathrm{Cl}_2\left(\frac{7\pi}{12}\right)\right) - \mathrm{Cl}_2\left(\frac{\pi}{4}\right)\,, \tag{8}$$

$$I_{24} = \frac{5\pi}{48}\ln\left(2-2\cos\frac{5\pi}{12}\right) - \frac{11\pi}{48}\ln\left(2-2\cos\frac{11\pi}{12}\right)$$
$$-\frac{1}{2}\left(\mathrm{Cl}_2\left(\frac{5\pi}{12}\right) - \mathrm{Cl}_2\left(\frac{11\pi}{12}\right)\right) - \mathrm{Cl}_2\left(\frac{3\pi}{4}\right)\,. \tag{9}$$

Combining these results together it may be shown that

$$\frac{1}{2}\left(\mathrm{Cl}_2\left(\frac{\pi}{12}\right)+\mathrm{Cl}_2\left(\frac{5\pi}{12}\right)-\mathrm{Cl}_2\left(\frac{7\pi}{12}\right)-\mathrm{Cl}_2\left(\frac{11\pi}{12}\right)\right) = \frac{G}{3} \tag{10}$$

and

$$\mathrm{Cl}_2\left(\frac{\pi}{4}\right) - \mathrm{Cl}_2\left(\frac{3\pi}{4}\right) = \frac{G}{2}\,. \tag{11}$$

After some further elementary algebra using the facts that

$$\cos\frac{\pi}{12} = -\cos\frac{11\pi}{12} = \frac{\sqrt{2}(\sqrt{3}+1)}{4}\,,$$

$$\cos\frac{5\pi}{12} = -\cos\frac{7\pi}{12} = \frac{\sqrt{2}(\sqrt{3}-1)}{4}\,,$$

we eventually obtain

$$I = \frac{G}{12} - \frac{5\pi}{16}\ln 2 + \frac{\pi}{3}\ln(2+\sqrt{3}) - \frac{\pi}{4}\ln(\sqrt{2}+1)\,.$$

Remark: When this problem was set we thought it was a one-off. Since then general solutions in closed form have been found for

$$I = \int_0^{\pi/4} \ln\left(\cos^{m/n}\theta \pm \sin^{m/n}\theta\right) \mathrm{d}\theta$$

where m and n are coprime integers and a paper detailing this has been submitted [2].

References

[1] Lewin, L.: *Dilogarithms and associated functions*. Macdonald, London, 1958.

[2] Zucker, I.J., Joyce, G.S. and Delves, R.T.: On the evaluation of the integrals $\int_0^{\pi/4} \ln\left(\cos^{m/n}\theta \pm \sin^{m/n}\theta\right) \mathrm{d}\theta$. Submitted for publication to the Ramanujan Journal.

The following solution using Mathematica was sent by Victor Adamchik. The calculations require Mathematica Version 3.

Second Solution of Problem 14
A Trigonometric Integral

by Victor Adamchik
(victor@wolfram.com)

In several steps the proposed integral is brought into a form such that Mathematica can evaluate it. This will result in the representation

$$\int_0^{\pi/4} \ln(\sin^{3/2}(x) + \cos^{3/2}(x))\, dx =$$

$$\frac{G}{12} - \frac{5\pi}{16} \ln 2 + \frac{\pi}{8} \ln(2-\sqrt{2}) -$$

$$\frac{\pi}{8} \ln(2+\sqrt{2}) - \frac{\pi}{3} \ln(\sqrt{3}-1) + \frac{\pi}{3} \ln(1+\sqrt{3}) \,.$$

First of all we observe that

$$\int_0^{\pi/4} \ln(\sin^{3/2}(x) + \cos^{3/2}(x))\, dx =$$

$$\frac{1}{2} \int_0^{\pi/2} \ln(\sin^{3/2}(x) + \cos^{3/2}(x))\, dx$$

which is easy to prove by changing the variable $x \to \pi/2 - y$. Then we need to transform that integral to the algebraic form

$$\frac{1}{2} \int_0^{\pi/2} \ln(\sin^{3/2}(x) + \cos^{3/2}(x))\, dx$$

$$= \frac{1}{4} \int_0^1 \frac{\ln((1-z)^{3/4} + z^{3/4})}{\sqrt{z}\,\sqrt{1-z}}\, dz \,.$$

This can be done by $x \to \arcsin\sqrt{z}$. On the next step we divide the integrand into two parts:

$$\frac{\ln((1-z)^{3/4} + z^{3/4})}{\sqrt{z}\,\sqrt{1-z}} = \frac{3\ln z}{4\sqrt{z}\,\sqrt{1-z}} + \frac{\ln(1+(1/z-1)^{3/4})}{\sqrt{z}\,\sqrt{1-z}}$$

so that

$$\frac{1}{4} \int_0^1 \frac{\ln((1-z)^{3/4} + z^{3/4})}{\sqrt{z}\,\sqrt{1-z}}\, dz =$$

$$\frac{3}{16} \int_0^1 \frac{\ln z}{\sqrt{z}\,\sqrt{1-z}}\, dz + \frac{1}{4} \int_0^1 \frac{\ln(1+(1/z-1)^{3/4})}{\sqrt{z}\,\sqrt{1-z}}\, dz =$$

$$\frac{3}{16} \int_0^1 \frac{\ln z}{\sqrt{z}\,\sqrt{1-z}}\, dz + \frac{1}{4} \int_0^\infty \frac{\ln(1+y^{3/4})}{\sqrt{y}\,(1+y)}\, dy \,.$$

In the second integral we changed z to $1/(1+y)$. Now we can use Mathematica Version 3 to evaluate both integrals:

```
In[1]:= Integrate[Log[z]/(Sqrt[z] Sqrt[1-z]),{z,0,1}]

Out[1]= -2 Pi Log[2]

In[2]:= Integrate[Log[1 + y^(3/4)]/(Sqrt[y] (1 + y)),
        {y, 0, Infinity}]/.
        {r1_ Sum[w1_, {e__}] + r2_ Sum[w2_, {e__}] :>
        Sum[r1 w1 + r2 w2 // FullSimplify, {e}]} //
        FullSimplify

Out[2]=(4 Catalan + Pi Log[8] + 6 Pi Log[2 - Sqrt[2]] -

>     6 Pi Log[2 + Sqrt[2]] - 16 Pi Log[-1 + Sqrt[3]] +

>     16 Pi Log[1 + Sqrt[3]]) / 12
```

Finally

```
In[3]:= 3/16 %1 + 1/4 %2 // Simplify

Out[3]= (4 Catalan - 15 Pi Log[2] +

>     6 Pi Log[2 - Sqrt[2]] - 6 Pi Log[2 + Sqrt[2]] -

>     16 Pi Log[-1 + Sqrt[3]] +

>     16 Pi Log[1 + Sqrt[3]]) / 48
```

Editor's Remark: Victor has a WWW Page on the Catalan Constant, and he searches for different representations for this constant, see p. 16.

Miscellaneous

1. Publicity about OP-SF in SIAM News

I had a conversation with Gail Corbett, Editor of SIAM News, during the SIAM meeting in Kansas City. Gail welcomes material from us as from all the Activity Groups.

But not all items from our own media are suitable for SIAM News. In particular, announcements or reports of our own minisymposia are not suitable unless there is something very special about them. These would include important advances in the field, profiles of people, etc. Events surrounding a birthday or anniversary might be suitable especially if they involved a personality or event of interest to several groups.

Expository material is of interest. Basic material that would be "well-known" to members of our own group would be suitable. Graphics and other visual materials are specially welcome. If somebody has written something with a different audience in mind, the Editor would be pleased to indicate how it should be changed to make it suitable for SIAM News.

I agreed to pass on the suggestion that we compile and send to Gail lists of suitable topics and people. Even very tentative suggestions are welcome. There are times when a suggestion might match a similar suggestion from another source and then Gail would be more likely to follow it up. When we send suggestions we need not be sure that the suggested authors would be willing.

Martin Muldoon
(muldoon@mathstat.yorku.ca)

2. Article on Szegő by Askey & Nevai

Richard Askey and Paul Nevai wrote a very readable article *Gabor Szegő: 1895-1985* in the Summer 1996 issue of the Mathematical Intelligencer (vol. 18, no. 3, pp. 10–22). Just one quotation:

> He (Gabor Szegő) said to me (Bob Osserman) something to the effect: Don't you think it's somewhat fraudulent that we claim to teach people how to become research mathematicians? That's like claiming you can teach someone how to become a poet. All you can really do is show by example how research in mathematics is done, and then they either can do it themselves or they can't.

Tom H. Koornwinder
(thk@fwi.uva.nl)

3. Call for contributions to J. Phys. A

As a member of the Editorial Board of *J. Phys. A*, I would like to draw your attention to the fact that Journal of Physics A is in favour of publishing Papers and Letters from the field of Orthogonal Polynomials/Special functions. As an example, see below a (very incomplete) collection of recent titles in fields close to the OP-SF one and published in the first 16 volumes of this year.

You are encouraged to submit some of your manuscripts to the Journal of Physics A. For information on submission visit the URL http://www.iop.org.

- Analytical treatment of the Green function singularity in integral equations of scattering theory (L405-L412) (21 August 1996)
- The measure of the orthogonal polynomials related to Fibonacci chains: the periodic case (4169-4185) (21 July 1996)
- Relativistic orthogonal polynomials are Jacobi polynomials (3199-3202) (21 June 1996)
- Fock-Bargmann representation of the distorted Heisenberg algebra (3281-3288) (21 June 1996)
- Separation of variables for the A_2-Ruijsenaars model and a new integral representation for the A_2-Macdonald polynomials (2779-2804) (7 June 1996)
- Affine Hecke algebra, Macdonald polynomials, and quantum many-body systems (L281-L287) (7 June 1996)
- A new result for Laguerre polynomials (L277-L279) (7 June 1996)
- An operator approach to the construction of generating functions for products of associated Laguerre polynomials (L263-L270) (21 May 1996)
- On a one-parameter family of q-exponential functions (L223-L227) (21 May 1996)
- Relativistic Calogero-Sutherland model: spin generalization, quantum affine symmetry and dynamical correlation functions (L191-L198) (21 April 1996)
- On the coherent states for the q-Hermite polynomials and related Fourier transformation (1659-1664) (21 April 1996)
- The dual Hahn q-polynomials in the lattice (...) and the q-algebras (...) and (...) (1435-1451) (7 April 1996)
- Special representations of (...) at the roots of unity (1201-1214) (21 March 1996)
- Ramanujan-type continuous measures for biorthogonal q-rational functions (329-338) (21 January 1996)
- Minimum uncertainty states for the quantum group and quantum Wigner d-functions (427-435) (21 January 1996)
- Transformation of certain generalized Kampé de Fériet functions (357-363) (21 January 1996)
- Generalization of Weierstrassian elliptic functions to (...) (L17-L22) (7 January 1996)

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4. Question on Bessel Polynomials in Quantum Mechanics

For some time now, I have been attempting to find exactly solvable 1-D quantum mechanical potentials by starting with the second order differential equations solved by orthogonal polynomials and special functions, and through relatively simple variable transformations, converting these to differential equations that match the Schrödinger form. I have recently been exploring Bessel polynomials, and I seem to be finding that for these polynomials, it appears possible to generate eigenfunctions solving a particular Schrödinger form, which have the unique property that on the positive real axis (corresponding to a radial type problem), they have no nodes or argument values where they equal zero (other than at the endpoints $x = 0$ or $x = \infty$). These eigenfunction solutions appear to be square integrable. My question is, can these be valid quantum mechanical wavefunctions, or can these eigenfunctions be used to generate valid quantum mechanical wavefunctions? If they have no nodes, it would seem the original eigenfunctions could not be mutually orthogonal, and yet this seems to be a standard requirement of quantum mechanics, at least as I usually see it dealt with. It has been suggested to me that what I am running into is an example of a "singular Hamiltonian", a term with which I am unfamiliar. Do any readers know anything about this kind of situation or "singular Hamiltonians?" I would appreciate any help or contacts you might be able to suggest.

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5. WWW Page on the Catalan Constant

I created a page regarding all known to me series and integral representations for the Catalan constant. The short Mathematica proof is provided for all of them.

The page is at http://www.wolfram.com/~victor/articles/catalan/catalan.html.

This might be of interest to the members of the Activity Group. I'd highly appreciate if people could submit to me some other representations for the Catalan constant.

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6. (q)-Zeilberger Algorithm

1. The most recent versions of Doron Zeilberger's own Maple implementations are EKHAD and qEKHAD, obtainable by anonymous ftp from math.temple.edu, directory pub/zeilberg/programs or via Doron's home page http://www.math.temple.edu/~zeilberg. An accompanying book, in fact covering much more, appeared this summer:

> M. Petkovsek, H.S. Wilf & D. Zeilberger
> $A = B$
> A.K. Peters, 1996

2. Tom Koornwinder's Maple implementations zeilb and qzeilb dating back from 1992 have just been slightly revised and adapted to Maple V, Release 4. They are obtainable by anonymous ftp from ftp.fwi.uva.nl, directory pub/mathematics/reports/Analysis/koornwinder/zeilbalgo.dir or via Tom's home page http://turing.fwi.uva.nl/~thk/. The accompanying paper

> T.H. Koornwinder
> On Zeilberger's algorithm and its q-analogue
> J. Comput. Appl. Math. **48** (1993), 91–111

has been adapted accordingly. The slightly revised version, with title *On Zeilberger's algorithm and its q-analogue: a rigorous description* is also available from the ftp site just mentioned.

3. Wolfram Koepf implemented Zeilberger's algorithm and certain extensions in Maple V, Release 4. The present implementation is part of the official distribution of Maple V, Release 4. It can be made operational by the two commands

```
> with(sumtools):
> readlib(‘sum/simpcomb‘):
```

possibly followed, to get help, by

```
> ?sumtools
```

The accompanying paper is:

> W. Koepf
> Algorithm for m-fold hypergeometric summation
> J. Symbolic Comput. **20** (1995), 399–417

Wolfram Koepf also wrote a book manuscript about these topics:

> W. Koepf
> Algorithmic Summation and Special Function Identities with Maple
> To appear

The new package "code", to be used for the generation of recurrence and differential equations for sums and integrals, written in connection with this book, can be obtained from his home page http://www.zib.de/koepf/.

4. Peter Paule and Markus Schorn implemented Zeilberger's algorithm in Mathematica, while Peter Paule and Axel Riese similarly implemented the q-Zeilberger algorithm. These implementations are available on email request to Peter Paule (ppaule@risc.uni-linz.ac.at). The accompanying papers are:

> P. Paule & M. Schorn
> A Mathematica version of Zeilberger's algorithm for proving binomial coefficient identities
> J. Symbolic Comput. **20** (1995), 673–698

> P. Paule & A. Riese
> A Mathematica q-analogue of Zeilberger's algorithm based on an algebraically motivated approach to q-hypergeometric telescoping
> To appear in
> *Special Functions, q-Series and Related Topics*
> The Fields Institute Communications Series

5. René Swarttouw used Koepf's Maple implementations of the Zeilberger algorithm for an interactive package on World Wide Web for calculating formulas for orthogonal polynomials belonging to the Askey-scheme. See http://www.can.nl/~demo/CAOP/CAOP.html. You can also approach this via René's home page http://star.cs.vu.nl/~rene/.

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7. Extended Version of Askey-Wilson Scheme Report

Roelof Koekoek and René Swarttouw are currently working on an extended version of their report

> *The Askey-scheme of hypergeometric orthogonal polynomials and its q-analogue*, Report 94-05, Delft University of Technology, Faculty TWI, 1994

This new version will include Rodrigues' type formulae, forward and backward shift-operators, leading coefficients and the monic recurrence relations for all the orthogonal polynomials in the Askey-Wilson-scheme. They are also working on an update of the list of references.

Concerning the latter subject they urge users of the report (or the electronic version, see `http://www.can.nl/~demo/AWscheme/index.html`) to go through the current bibliography and to look if they are missing some important references. If so, please contact René Swarttouw by e-mail or send him a preprint of the missing article(s) to

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We thank everybody for their cooperation.

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8. WWW Pages

The Ramanujan Journal has a home page containing the contents of Vol. 1, No. 1, January 1997, an editorial and further information:
http://www.math.ufl.edu/~frank/ramanujan/vol1/issue1/toc.html.

The contents of J. Math. Analysis and Applications and of J. Symbolic Computation can be found via the Academic Press site http://www.europe.idealibrary.com.

The electronic preprint archive solv-int occasionally contains papers which are relevant for the field of OP & SF. See http://www.msri.org:80/preprints/q-alg.html.

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The Package sumtools in Maple V.4

by René Swarttouw
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Recently Waterloo Maple released a new version of their computer algebra system: Maple V Release 4. A very interesting and powerful new feature is the package `sumtools`, which contains a collection of routines for manipulating indefinite and definite sums. The principal routines are Gosper's algorithm for indefinite summation [1] and Zeilberger's algorithm [4], and extensions to both algorithms by Koepf [3]. Apart from these algorithms the `sum` command, already available in Maple V.3, includes other considerable improvements. Here however, I will only discuss the routines of the package `sumtools` and give some examples.

The routines can be made directly available by loading the package:

```
> with(sumtools);
```

$$[\mathit{Hypersum}, \mathit{Sumtohyper}, \mathit{extended\_gosper}, \mathit{gosper}, \mathit{hyperrecursion}, \mathit{hypersum}, \mathit{hyperterm}, \mathit{simpcomb}, \mathit{sumrecursion}, \mathit{sumtohyper}]$$

Gosper's algorithm deals with the problem of finding an *antidifference* s_k of a given a_k, i.e. finding an s_k such that $a_k = s_{k+1} - s_k$. If there exists such an s_k any sum with summand a_k can be calculated by an evaluation of s_k at the boundary points:

$$\sum_{k=m}^{n} a_k = s_{n+1} - s_m.$$

Gosper's algorithm will find such an antidifference s_k if it is a *hypergeometric term*, i.e. if $\frac{s_{k+1}}{s_k}$ is a rational function in k. In this case also a_k must be a hypergeometric term. The command `extended_gosper` deals with an antidifference s_k for which $\frac{s_{k+j}}{s_k}$ is a hypergeometric term for a certain positive integer j. Some examples:

```
> gosper(k*k!,k);
```

$$k!$$

```
> gosper(k/(k+1)!,k=m..n);
```

$$-\frac{n+2}{(n+2)!} + \frac{m+1}{(m+1)!}$$

```
> extended_gosper((k/3)!*k,k);
```

$$3\left(\frac{1}{3}k\right)! + 3\left(\frac{1}{3}k + \frac{1}{3}\right)! + 3\left(\frac{1}{3}k + \frac{2}{3}\right)!$$

In these examples the function `simpcomb`, loaded with `sumtools`, simplifies (in the background) any factorial, Γ or binomial input. The procedure `sumtohyper`, also loaded with `sumtools`, does the conversion of an infinite sum into hypergeometric notation. The procedure `convert/hypergeom` in Maple V.4 gives similar results.

Zeilberger's algorithm deals with *definite sums*, i.e. sums of the form:

$$s_n = \sum_{k=-\infty}^{\infty} F(n,k).$$

In particular this covers sums of the type $\sum_{k=k_1}^{k_2} F(n,k)$, where $F(n,k) = 0$ for $k < k_1$ and $k > k_2$. Zeilberger's algorithm can be applied when $F(n,k)$ is a hypergeometric term with respect to both n and k. It generates a homogeneous linear recurrence relation with polynomial coefficients for s_n. In Maple V.4 this algorithm can be used directly with the command `sumrecursion`. An example:

```
> sumrecursion(binomial(n,k)^2,k,s(n));
```

$$-2(2n-1)\,\mathrm{s}(n-1) + \mathrm{s}(n)\,n$$

calculates the recurrence relation satisfied by

$$s(n) = \sum_{k=-\infty}^{\infty} \binom{n}{k}^2 = \sum_{k=0}^{n} \binom{n}{k}^2,$$

which can be solved using the initial value $s(0) = 1$. In fact this is what happens when Maple calculates $s(n)$:

```
> sum(binomial(n,k)^2,k=-infinity..infinity);
```

$$\frac{\Gamma(1+2n)}{\Gamma(n+1)^2}$$

Another proof of the strength of the algorithm is shown in the following example. Gessel and Stanton [2] were not able to present a proof for their statement

$$ {}_2F_1\left(\left.\begin{matrix} -n, -n+1/4 \\ 2n+5/4 \end{matrix}\right| \frac{1}{9}\right) = \frac{(5/4)_{2n}}{(2/3)_n(13/12)_n}\left(\frac{2^6}{3^5}\right)^n.$$

The proof is below. What we are actually doing is looking for a recurrence relation for the expression at the left hand side divided by the expression at the right hand side. We used the newly implemented functions `hyperterm`, which is a shorthand for hypergeometric term, and `pochhammer`:

```
> sumrecursion(
> (hyperterm([-n,-n+1/4],[2*n+5/4],1/9,k))/
> (pochhammer(5/4,2*n)/pochhammer(2/3,n)/
> pochhammer(13/12,n)*(2^6/3^5)^n),k,s(n));
```

$$\mathrm{s}(n) - \mathrm{s}(n+1) = 0$$

which means that $s(n)$ is a constant, namely 1, which is found by substituting $n = 0$.

The other functions loaded with `sumtools` are shorthands for the functions already described above. For example `hyperrecursion` has the same effect as `sumrecursion( hyperterm(...))` and `hypersum` is a shorthand for `sum` with hypergeometric-term summands, but works more powerful:

```
> hypersum([-n,b],[c],1,n);
```

$$\frac{\mathrm{pochhammer}(-b+c,n)}{\mathrm{pochhammer}(c,n)}$$

Without any doubt the package `sumtools` can be of great value for anyone working in the field of special functions. Hopefully in the near future this package will be extended with routines that calculate differential/difference equations and that can handle the q-case.

References

[1] Gosper, R.W.: Decision procedure for indefinite hypergeometric summation. Proc. Nat. Acad. Sci. USA **75** (1978), 40–42.

[2] Gessel, I. and Stanton, D.: Strange evaluations of hypergeometric series. SIAM J. Math. Anal. **13** (1982), 295–308.

[3] Koepf, W.: Algorithms for m-fold hypergeometric summation. J. Symbolic Computation **20** (1995), 399–417.

[4] Zeilberger, D.: A fast algorithm for proving terminating hypergeometric identities. J. Comput. Appl. Math. **32** (1990), 207–211.

Note On a Problem of Koornwinder

by Wolfram Koepf
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Zeilberger's algorithm ([5]–[6], see also [2], [1]) determines recurrence equations for hypergeometric functions

$$\begin{aligned} S(n) \quad &:= \quad {}_pF_q\left(\begin{array}{cccc} \alpha_1 & \alpha_2 & \cdots & \alpha_p \\ \beta_1 & \beta_2 & \cdots & \beta_q \end{array} \middle| \, x\right) \\ &= \quad \sum_{k=0}^{\infty} A_k\, x^k = \sum_{k=0}^{\infty} \frac{(\alpha_1)_k \cdot (\alpha_2)_k \cdots (\alpha_p)_k}{(\beta_1)_k \cdot (\beta_2)_k \cdots (\beta_q)_k} \, \frac{x^k}{k!} \qquad (1) \end{aligned}$$

whose upper parameters α_k and lower parameters β_k are rational-linear in a variable n, whenever the term ratio

$$\frac{A_{k+1}}{A_k} = \frac{(k+\alpha_1)\cdot(k+\alpha_2)\cdots(k+\alpha_p)}{(k+\beta_1)\cdot(k+\beta_2)\cdots(k+\beta_q)\cdot(k+1)} \in \mathbb{Q}(k,n)$$

is a rational function in both n and k. As usual $(a)_k = a(a+1)\cdots(a+k-1)$ denotes the *Pochhammer symbol*. We call the summand $A_k x^k$ a *hypergeometric term*. The resulting recurrence equation has polynomial coefficients with respect to n. If it is of first order, the sum has a rational term ratio with respect to n, and hence itself is a hypergeometric term.

In [3] Koornwider asked the question whether an application of Zeilberger's algorithm might generate a hypergeometric term whose upper an lower parameters are not rational assuming the parameters of the input summand are rational:

> **Problem 6.1.** If Zeilberger's algorithm succeeds, can $S(n)/S(n-1)$ then always be factorized as a quotient of products of linear forms over $\mathbb{Z}$ in n and the parameters?

In this note, we will answer Koornwinder's question in the negative, by providing a counterexample.

Note that Koornwinder's question in principle is independent of Zeilberger's algorithm, and asks whether there are hypergeometric sums that can be represented by hypergeometric terms with nonrational parameters. None example of this type can be found in the literature, see in particular the rather extensive mathematical dictionary on hypergeometric function identities [4]. Nevertheless we have used Zeilberger's algorithm to find our counterexamples.

The identity

$$\begin{aligned} &{}_3F_2\left(\begin{array}{c} 3/4, 5/4-k, -k \\ 1/4-k, -1/4 \end{array} \middle| -1\right) \\ &= \sum_{j=0}^{k} \frac{(3/4)_j\,(5/4-k)_j\,(-k)_j}{(1/4-k)_j\,(-1/4)_j} \, \frac{(-1)^j}{j!} \\ &= \frac{(\sqrt{3}/2)_k\,(-\sqrt{3}/2)_k}{(\sqrt{3}/2-1)_k\,(-\sqrt{3}/2-1)_k\,(1-4k)} \, 2^k \end{aligned}$$

constitutes such an example. With the `sumtools` package of Maple V.4 (see René Swarttouw's article on p. 17) this result is deduced by an application of Zeilberger's algorithm using the commands

```
> with(sumtools):
> readlib(`sum/simpcomb`):
> sumrecursion(
  hyperterm([3/4,5/4-k,-k],[1/4-k,-1/4],-1,j),j,S(k));
```

with the result

$$\begin{aligned} -2\,(-5+4\,k)\left(4\,k^2-8\,k+1\right)S(k-1) &+ \\ S(k)\,(-1+4\,k)\left(4\,k^2-16\,k+13\right) &= 0\,. \end{aligned}$$

We finish this note with a rather simple family of examples. If $j \in \mathbb{N}$ then for the polynomials

$${}_2F_1\left(\begin{array}{c} a+j, -k \\ a \end{array} \middle| \, x\right)$$

one gets a recurrence equation of first order with respect to k with polynomial coefficients in k of degree j that have no factorization over $\mathbb{Q}$. As a particular case one has for $j = 2$

```
> sumrecursion(hyperterm([a+2, -k],[a],x,j),j,S(k));
```

$$\begin{aligned} &(x^2\,k + x^2\,a^2 + x^2\,k^2 - 2\,x\,k\,a + a - 2\,x\,k + a^2 + 2\,x^2\,k\,a \\ &\quad - 2\,x\,a + x^2\,a - 2\,x\,a^2)\mathrm{S}(k+1) + \mathrm{S}(k)\,(x-1) \\ &\quad (x^2\,k^2 + x^2\,a^2 + 3\,x^2\,k + 3\,x^2\,a + 2\,x^2\,k\,a + 2\,x^2 \\ &\quad - 4\,x\,a - 2\,x - 2\,x\,k - 2\,x\,k\,a - 2\,x\,a^2 + a^2 + a) = 0\,. \end{aligned}$$

A more detailed discussion will be given in a forthcoming paper.

References

[1] Koepf, W.: Algorithms for m-fold hypergeometric summation. Journal of Symbolic Computation **20**, 1995, 399–417.

[2] Koornwinder, T. H.: On Zeilberger's algorithm and its q-analogue: a rigorous description. J. of Comput. and Appl. Math. **48**, 1993, 91–111.

[3] Koornwinder, T. H.: Hypergeometric series evaluation by Zeilberger's algorithm. In: Open Problems, ed. by Walter van Assche. Journal of Computational and Applied Mathematics **48**, 1993, 225–243.

[4] Prudnikov, A.P., Brychkov, Yu.A. and Marichev, O.I.: *Integrals and Series, Vol. 3: More Special Functions.* Gordon and Breach Science Publ., 1990.

[5] Zeilberger, D.: A fast algorithm for proving terminating hypergeometric identities. Discrete Math. **80**, 1990, 207–211.

[6] Zeilberger, D.: The method of creative telescoping. J. Symbolic Computation **11**, 1991, 195–204.

How to Contribute to the Newsletter

Send your Newsletter contributions directly to the *Editor:*

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preferably by email, and in LaTeX format. Other formats are also acceptable and can be submitted by email, regular mail or fax.

Deadline for submissions to be included in the February issue 1997 is January 15, 1997.

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pub/mathematics/reports/Analysis/koornwinder/opsfnet.dir

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ftp://ftp.fwi.uva.nl/pub/mathematics/reports/Analysis/koornwinder/opsfnet.dir

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- Projects
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- Personal, Obituaries, etc.
- Positions available
- Miscellaneous

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