Orthogonal Polynomials and Special Functions

SIAM Activity Group on Orthogonal Polynomials and Special Functions

⋆ ⋆ ⋆ ⋆ **Newsletter** ⋆ ⋆ ⋆ ⋆

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Contents

From the Editor

I received rather positive feedback on the *Compiled Booklist* which was published in the last issue. In particular, the member who had made this suggestion wrote a letter to me. You will find this letter reprinted on p. 12. At the same place there is an enlarged booklist, since some members found that important items were missing.

Hans Haubold who is managing the *Preprint Archive on Orthogonal Polynomials and Special Functions* designed an excellent World Wide Web page for this archive: ftp://unvie6.un.or.at/siam/opsf_new/00index.html from where the archive can be accessed. Furthermore an alphabetical list of authors with links to the papers and a submission form are available.

It is now possible to submit an abstract with a hyperlink rather than the actual paper. This new facility makes it possible that the ftp site might give a rather comprehensive picture of recent preprints in OP & SF. Hence I invite you to submit the data of your articles.

To update the archive, Hans asks the colleagues whose articles reside there to propose changes that should be made. In particular, articles that have appeared in the meantime, should get the appropriate citation. You will find his message on p. 15.

At the end of 1996 the first issue of a new Russian Newsletter on *Integral Transforms and Special Functions* appeared. This Newsletter is generally in Russian language with some English material though. I am pleased that the Russian group around A. P. Prudnikov found our lay-out promising, and adapted it accordingly. You will find the first page of their newsletter reprinted on the back cover of this issue. I will try to reprint articles that might be of interest to the members of our group in English language.

I thank Manfred Braun for his article about the interesting connection between the frequencies of a multiple pendulum and the zeros of the Laguerre polynomials.

May 31, 1997 Wolfram Koepf

Memorial Note about Louis Auslander

Louis Auslander died at 68 on February 25, 1997. His early work was on differential geometry. Later he worked on nilpotent Lie groups, and this led to the study of theta functions. Two of his books are "Abelian Harmonic Analysis, Theta Functions and Function Algebras on a Nilmanifold", Springer, 1975, and "Lecture Notes on Nil-Theta Functions", CBMS lectures, published by American Mathematical Society, 1977. His later work was on finite Fourier transforms, which also involves some special functions. His expository paper with Tolimieri: "Is computing the finite Fourier transform pure or applied mathematics?", Bulletin AMS (New Series), 1 (1979) 847-897, has been cited frequently, and is worth reading. The first of the books mentioned above is also joint with Richard Tolimieri. Auslander attended the Oberwolfach meeting on special functions and group theory run by Askey, Koornwinder and Schempp in 1983. He was fascinated by some of the work described there, as were some of us by his work.

Richard Askey
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Reports from Meetings and Conferences

1. VI International Krawtchouk Conference: Kiev, Ukraine, May 14-17, 1997

The number of classical orthogonal polynomials systems of a discrete variable is highly restricted, hence each discoverer of such an OPS deserves to be known to the scientific community not only as a mathematician but also as an individual. That's why it is very strange to find only some morsels of information about the author of *Sur une généralisation des polynômes d'Hermite* published in 1929 which initiated the new stage in the theory of orthogonal polynomials.

The reason is both simple and tragic.

Mykhailo Pilipovich (in Ukrainian; in Russian his name is sounded Mikhail Philippovich) Krawtchouk was born on September 27, 1892 in the small village of Chovnitsy (Western Ukraine). After graduating from gymnasium he entered Kiev St. Vladimir University, obtaining his first diploma degree in 1914—on the eve of the First World War. Thus the young mathematician had to move to Moscow because of the University evacuation. On September 5, 1917 (80 years ago) he gave his first lecture.

After the 1917 revolution, M. Krawtchouk worked in various Kiev universities, institutes, gymnasia, then for two years of the civil war (1919-1921) he was the head of a rural school near Kiev.

When the situation in the then USSR became relatively stabilized, Krawtchouk got the opportunity for fruitful scientific work. The title of his doctoral thesis was *On Quadratic Forms and Linear Transform* (1924). He took part in the International Mathematical Congresses in Toronto (1924) and Bologna (1928), had close contacts with Hadamard, Hilbert, Courant, Tricomi, i.a. In 1929 he became a full member of the All-Ukrainian Academy of Sciences.

The list of M. Krawtchouk's scientific works contains about 180 titles including such branches of mathematics as the theory of permutation matrices, theory of algebraic, transcendental, differential and integral equations, introduction and use of polynomials associated with the binomial distribution (Krawtchouk polynomials), etc. Moreover his efforts were applied in the fields of philosophy, history of mathematics and mathematical education. It is especially important for the independent Ukraine that it was M. Krawtchouk who was in charge of editing the first three-volume dictionary of Ukrainian mathematical terminology. (Having a knowledge of French, German, Italian, Polish and, of course, Russian, he delivered lectures and wrote articles mostly in Ukrainian).

Anyone knowing even a little Soviet history of the thirties can conclude that Krawtchouk could not avoid the Great Terror. During the Orwellian "hours of hatred" in 1937 he was being denounced as a "Polish spy", "bourgeois nationalist" etc. In 1938 he was arrested and sentenced to 20 years of confinement and 5 years of exile.

Academician Krawtchouk, the author of the results which became part of the world's mathematical knowledge, the brilliant lecturer who inspired many outstanding followers (e.g., Sergey Korolev, the future leader of the Soviet space programme), the member of the French, German and other Mathematical Societies died on March 9, 1942 in Kolyma branch of GULAG (North-Eastern Siberia) more than 6 months up short of his 50th birthday.

M. Krawtchouk was officially rehabilitated in 1956 and restored as a member of the Academy of Sciences only in March 1992, 50 years after his death.

Tempora mutantur, the times are changing, and in September 1992 the *First International M. Krawtchouk Conference* was held in the Ukraine. Since then such conferences have been yearly.

The 6th International Scientific Krawtchouk Conference took place in Kiev from May 15th to 17th 1997. The organizers were the *Ukrainian Ministry of Education, Institute of Mathematics* (*National Academy of Sciences of Ukraine*) and *National Technical University of Ukraine* (formerly *Kiev Polytechnical Institute* where M. Krawtchouk in his time was the head of the Mathematical Chair).

The opening ceremony included speeches from members of the Organizing Committee, addresses from M. Krawtchouk's relatives and followers and some splendid Ukrainian songs executed by a chorus.

The scientific agenda consisted of about 380 reports divided into four sections according to Krawtchouk's scientific interests: Differential and Integral Equations, their Application; Algebra, Geometry, Mathematical and Numerical Analysis; Probability Theory and Mathematical Statistics; History and Teaching Methods of Mathematics. Most of the participants represented the Ukraine, 19 reports were from Russia, 4 from Belarus and 1 from the Czech Republic. Many participants were postgraduates and young scientists presenting their first results.

Let us mention some of the titles which could be of interest to the readers of the Newsletter:

- V. Gaidei: *Generalised m-Lommel-Bessel-Maitland Functions. Properties and Application.*
- V. Zakharov: *New Special Function ${}_4F_1^4$ of Four Arguments.*
- V. Savva, V. Zelenkov: *Orthogonal Krawtchouk Polynomials and Exact Solutions for the Dynamics of Multilevel Systems in Radiation Field.*
- V. Zelenkov, V. Savva: *Peculiarities of the Dynamics of Krawtchouk Quantum Systems with Degenerate Levels.*
- O. Kuzhel: *Recursion Relations for the Euler-Bernoulli Numbers.*
- D. Leykin: *Addition Theorems for the Hyperelliptic Kleinian Functions.*
- Ya. Mamteev, V. Stukalina, T. Huchraeva: *Modified Struve Function.*
- L. Ostrovetskyi: *On Some Analytic Functions Approximation with Algebraic Polynomials.*
- O. Papanova: *On the Zeroes of $P_\nu^\mu(x)$.*

Most of the conference materials have been published in Ukrainian; some of them are in Russian and some in English.

An additional circumstance which made the conference especially attractive was the pleasant weather and the famous Kiev chestnut trees blooming on schedule in May. We were impressed by ancient Kiev though we had visited it repeatedly before (Belarus is not so far from the Ukraine!).

The participants are grateful to the Organizing Committee led by Academician M. Zgurovskyi and especially to Professor Nina Virchenko who carried out enormous work. Moreover, for about thirty years she has studied the biography of M. Krawtchouk and the authors of this report would like to express their personal thanks for the permission to use some of her articles while preparing this text.

The details of the 7th Krawtchouk Conference (1998) are now under discussion. You can send your inquiries to the address:

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2. Centenary Conference, including Minisymposium on Special Functions: Madison, Wisconsin, May 22-24, 1997

This conference was organized to celebrate the hundredth anniversary of the first granting of the Ph. D. in Mathematics by the university. Announcements were sent to the former students and friends of the department and produced a good turn-out for the meeting. It was estimated that 250 attended the opening reception, which was held in the wonderful conference room on the ninth floor of Van Vleck Hall, overlooking the lakes of Madison; and 375 attended the banquet. Current (or recently retired) faculty members who are well-known for their work in special functions, or the use of special functions in harmonic analysis, include Dick Askey, Walter Rudin and Steve Wainger. The format of the meeting consisted of 45-60 minute talks in the mornings mostly with historical emphasis and many parallel minisymposia in the afternoons. Walter Rudin gave the first lecture, titled "Harmonic Analysis at Wisconsin"; he discussed the original motivating examples of vibrating string problems, expansion problems which led to theories of integration, and then the modern era of Calderón-Zygmund singular integrals, Eberlein's almost periodic functions, Askey and Wainger's transplantation theorems, and analytic functions of several variables.

Alan Schwartz and the undersigned are two of Rudin's students who are working in harmonic analysis and special functions. Dick Askey lectured on "Special Functions in Wisconsin" the second morning. He defined special functions as those which occur often enough to need a name (he has previously called them "useful functions"). The first Ph. D. thesis in this area may have been the one by F. T. H'Doubler in 1910, concerning functional equations and theta functions. More recently Jan Korevaar (now in Amsterdam) wrote a paper on Fourier transforms and Hermite polynomials and directed Gil Walter's thesis in 1962. Loyal Durand of the Wisconsin physics department also studied Bessel functions. H. S. Wall was a Ph. D. student of E. Van Vleck and studied $q-$Laguerre polynomials (1927). Other significant results in the field which have Wisconsin connections include Schönberg's paper on positive-definite functions on spheres, the Askey-Gasper inequality instrumental in de Branges' proof of the Bieberbach conjecture, and Berndt's commentaries on Ramanujan's notebooks. Askey discussed work by some of his students - Jim Fitch, Dennis Stanton, Jim Wilson, Warren Johnson and Walter "Chip" Morris.

There were ten lectures in the special function minisymposium (organized by Dunkl and Askey), four on Friday and the remainder on Saturday; here is a list of speakers and brief descriptions of the talks:

- Gilbert Walter: *A comparison between series expansions and other properties of special functions and wavelets*;
- Steve Milne: *A new result on counting solutions of the Diophantine equation* $x_1^2+x_2^2+\cdots+x_s^2=m$ *for given* m *when* $s=4n^2$ *or* $s=4n(n+1)$ *for some integer* n;
- Alan Schwartz: *Classification of polynomial hypergroups in one or two variables, with examples coming from Jacobi and disk polynomials*;
- Charles Dunkl: *An overview of orthogonal polynomials in* N *variables for weight functions which are invariant under the symmetric or hyperoctahedral groups*;
- George Gasper: *Summation formulas for a type of q-Kampé de Férret function, related to a q-version of 9-j symbols*;
- Paul Terwilliger: *A new and improved approach to the structure theorem for Leonard systems and Askey-Wilson polynomials, in the context of type* P *and* Q *association schemes*;
- Anatol Kirillov: *Quantum algebra versions of Schur functions and Schubert polynomials (q-alg/9701005)*;
- Mourad Ismail: *Asymptotics for zeros and recurrence coefficients for the family of orthogonal polynomials for weights of the form* $\exp(-u(x)), x\in\mathbb{R}$, *where* $u'(x)$ *is convex and* $u(x)\to\infty$ *as* $x\to\pm\infty$; *this includes* $u(x)=x^{2n}$;
- Dennis Stanton: *A self-contained and conceptual proof of some identities of the form* ${}_5F_4(*)=0$ *which were originally stated and proven by George Andrews*;
- Sergei Suslov: *A basic analog of Fourier series; joint work with Joaquin Bustoz.*

The attendance at the minisymposium varied from 15 to 25, a consequence of the fact that there were as many as 16 parallel sessions. This report of course has singled out the special functions influence of the faculty and students (in a broad sense: past and present, postdocs, visitors as well as senior professors) of the University of Wisconsin. The organizers estimate that 900 doctorates have been granted by the department in the period 1897-1997 with about 370 in the peak years 1962-1977. Many workers in special functions have some connection with Wisconsin and it would be a major project to describe completely the impact that has been made on the field by these people.

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Forthcoming Meetings and Conferences

1. First ISAAC Conference: University of Delaware, Newark, Delaware, June 3-7, 1997

Session 13 at the First ISAAC Conference, University of Delaware, Newark, Delaware, June 3–7, 1997, is devoted to *Orthogonal Polynomials*, and organized by Wolfram Koepf. This session is scheduled on June 6 and June 7. The emphasis of this session will be on the use of symbolic computation in connection with orthogonal polynomials and special functions. It is the purpose of the proposed section to bring together developers of symbolic algorithms and implementations which are connected with orthogonal polynomials and special functions with users of computer algebra systems who need this type of software.

Via the conference WWW page at the address http://www.math.udel.edu/isaac/conferen/congr97.htm the full program of the conference can be accessed.

Note that on June 6 Wolfram Koepf will present a plenary lecture at the Conference on *Orthogonal Polynomials and Computer Algebra*.

The WWW page http://www.zib.de/koepf/isaac.html contains updated informations about the program of Session 13 on *Orthogonal Polynomials*.

A Special Issue of the *Journal of Symbolic Computation* on *Orthogonal Polynomials and Computer Algebra* will be dedicated to the subject of this session, and include some of the presented lectures, see p. 10.

Wolfram Koepf
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2. CRM Workshop on Algebraic Combinatorics: Montréal, June 9-20, 1997

The *Centre de Recherches Mathématiques* (Montréal, Canada) is hosting a year-long program in combinatorics and group theory in 1996-1997. The year will be organized around a certain number of workshops spread throughout the year.

A Workshop on Algebraic Combinatorics will take place during June 9–20, 1997. The purpose of the workshop is to study interactions between Algebraic Combinatorics and Symmetric Functions, with a special emphasis on Descent Algebras of Coxeter groups in relation to quasi-symmetric functions and non-commutative symmetric functions, and on doubly parameterized (Macdonald) (q, t)-symmetric functions, in relation to harmonics of reflection groups.

Organizers: F. Bergeron (UQAM), N. Bergeron (York), C. Reutenauer (UQAM)

Invited Speakers: P. Diaconis (to be confirmed), A. Garsia, I. Gessel, I. Goulden (to be confirmed), M. Haiman, I.G. Macdonald, C. Procesi, L. Solomon, R.P. Stanley, J.Y. Thibon.

Further information on WWW:
http://www.crm.umontreal.ca/Activites/Thematique_96-97_Eng.html
or
http://www.crm.umontreal.ca/Activites/Thematique_96-97_Frn.html

Louis Pelletier
(activites@crm.umontreal.ca)

3. Continued Fractions and Geometric Function Theory: Trondheim, Norway, June 24-28, 1997

Haakon Waadeland celebrates his 70th birthday on 20 May 1997. He is responsible for a long list of valuable contributions to the two fields of continued fractions and geometric function theory, and he is still very active. In recognition of his work we have decided to organize a conference in his honour. The conference will be held in Trondheim, Norway from 24 to 28 June 1997 under the title *Continued Fractions and Geometric Function Theory*. We hope to get together a group of people representing both fields, and we would very much appreciate if you would participate. We plan to publish proceedings from the conference.

The plenary speakers that so far have been scheduled are Roger W. Barnard, Bruce Berndt, Annie Cuyt, Peter L. Duren, William B. Jones, Thomas MacGregor, Francisco Marcellan and Walter Van Assche.

The conference has its own world wide web page on http://www.matstat.unit.no/CFGT, where we will put new information when it is available. From this page you can also link to pages about Trondheim and about the university, NTNU.

Lisa Lorentzen
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4. SIAM Annual Meeting: Stanford, July 14-18, 1997
Minisymposium: Handbooks for Special Functions and the World Wide Web

The minisymposium will be held at the July 14–18 1997 SIAM Annual Meeting in Stanford as an initiative of the *SIAM Activity Group on Orthogonal Polynomials and Special Functions*. Dick Askey and Willard Miller are co-organizers. The principal handbooks on special functions, the *Bateman Project* and the NIST *Handbook of Mathematical Functions* are among the most useful, widely consulted technical volumes ever published, but they are now out of date, due to rapid research progress and to revolutionary changes in technology. The Minisymposium will feature talks by representatives of the groups that are proposing to update the Bateman Project and Abramowitz & Stegun, respectively, and talks with critiques of those CD-ROM and WWW handbook projects that are already available. The Minisymposium will conclude with a general discussion concerning the appropriate format and structure for handbook projects, and funding possibilities. It will take place Monday, July 14, 3:15 p.m.-5:15 p.m., Law School, Room 290. Confirmed talks are the following.

Daniel W. Lozier
Mathematical and Computational Sciences Division
National Institute of Standards and Technology
Gaithersburg, MD 20899

Toward a Revised NBS Handbook of Mathematical Functions

> *Abstract:* A modernized and updated revision of Abramowitz and Stegun, Handbook of Mathematical Functions, first published in 1964 by the National Bureau of Standards, is being planned for publication on the World Wide Web. The authoritative status of the original will be preserved by enlisting the aid of qualified mathematicians and scientists. The practical emphasis on formulas, graphs, and numerical evaluation will be extended by providing an interactive capability to permit generation of tables and graphs on demand.

Mourad E. H. Ismail
Department of Mathematics
University of South Florida
Tampa, Florida, 33620

The Askey-Bateman Project

> *Abstract:* We hope to update the Bateman Project to reflect the developments in the subject over the last fifty years and cover topics of importance that were not covered in the initial project. A presentation will be made as to the current state of this project, the need for it, and a sketch of the contents and the personnel to be involved. Suggestions, recommendations, criticisms and any useful input will be welcome and greatly appreciated.

Richard Askey
Department of Mathematics
University of Wisconsin
480 Lincoln Drive
Madison, WI 53706

Handbooks of Special Functions Through the Decades

> *Abstract:* Handbooks of special functions have been some of the most widely used mathematics books. Features of some of the better ones will be described and some uses will be illustrated.

Willard Miller, Jr.
(miller@ima.umn.edu)

5. VIII International Conference on Symmetry Methods in Physics, Dubna, Russia, July 28-August 2, 1997

The VIII International Conference on Symmetry Methods in Physics will be held in Dubna, Russia during July 28–August 2, 1997. This Conference, organized by the *Bogoliubov Laboratory of Theoretical Physics* of the *Joint Institute for Nuclear Research*, is dedicated to the 80th anniversary of Professor Smorodinsky's birth.

One of the topics of the conference will be "Quantum groups and q-special functions". See the web page mentioned below for further topics.

Plenary speakers are ((*) means to be confirmed): M. Charlton (London), V.K. Dobrev (Sofia), H.D. Doebner (*) (Clausthal), J.P. Draayer (Baton Rouge), F. Iachello (New Haven), A.U. Klimyk (*) (Kiev), P. Kulish (St.Petersburg), V.B. Kuznetsov (Leeds), F.J. Lambert (*) (Brussels), I. Meshkov (Dubna), W. Miller Jr. (Minneapolis), P. Van Moerbeke (Louvain-la-Neuve), A.Yu. Morozov (Moscow), A.M. Perelomov (Zaragoza), L. O'Raifeartaigh (Dublin), N. Reshetikhin (*) (Berkeley), A.B. Shabat (Moscow), D.V. Shirkov (Dubna), G. Sudarshan (*) (Austin).

For further information and for the application form see the Web page http://thsun1.jinr.dubna.su:80/~symphys8/ or send an email to the chairman of the local organizing committee Dr. George Pogosyan.

G. Pogosyan
(symphys8@thsun1.jinr.dubna.su)

6. VIII Simposium Sobre Polinomios Ortogonales y Aplicaciones, Sevilla, September 22-26, 1997

You are cordially invited to attend the *VIII Simposium sobre polinomios ortogonales y aplicaciones* (8SPOA, in short) which will be held from September 22–26, 1997 at the Facultad de Matemáticas of the Universidad de Sevilla.

The scientific program is currently being elaborated by the scientific committee C. Berg (Copenhagen), A.J. Durán (Sevilla), J.J. Guadalupe (La Rioja), G. López Lagomasino (La Habana), F. Marcellán (Madrid), J. Sánchez Dehesa (Granada) and W. Van Assche (Leuven). It consists of some plenary lectures and short communications (20 min). The invited speakers are Alexander I. Aptekarevi, Richard Askey, Christian Berg, Doron Lubinsky, Andrei Martinez, Paul Nevai, Evgeni Rakhmanov, Edward B. Saff, Herbert Stahl, and Vilmos Totik.

The WWW page http://www.wis.kuleuven.ac.be/wis/applied/walter/sevilla.html shows more details about the conference.

Antonio J. Duran
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In Sevilla, during the the VIII Symposium on orthogonal polynomials and their applications, there will be a special session dedicated to Computer Algebra. The session will be organized by SCAGOP *Spanish Computer Algebra Group on Orthogonal Polynomials*. The main aim of the session will be the use of symbolic computation in connection with orthogonal polynomials and related topics (special functions, approximation theory, etc).

The invited speaker of the session will be Professor Tom H. Koornwinder (University of Amsterdam). A tentative title of his lecture is: *A survey of symbolic computation for orthogonal polynomials and special functions*.

People interested in presenting a communication in the session should indicate it when they fill the official registration form. We expect that will be possible to provide "on line" demostrations during the session (Maple and Mathematica). For more information about the session please contact us (SCAGOP) at renato@dulcinea.uc3m.es .

The session will be partially supported by *Addlink Software Científico* and *Junta de Andalucífa*.

More information about this session can be obtained in http://dulcinea.uc3m.es/users/scagop/scagop.html (forthcoming events).

R. Álvarez-Nodarse
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7. Applications and Computation of Orthogonal Polynomials, Oberwolfach, Germany, March 22-28, 1998

In the Newsletter of the European Mathematical Society I found the following announcement.

A meeting on Applications and Computation of Orthogonal Polynomials will be held at Mathematisches Forschungsinstitut Oberwolfach, Germany from March 22 to March 28, 1998.

The organizers are

Walter Gautschi, West Lafayette
Gene H. Golub, Stanford
Gerhard Opfer, Hamburg

Participants of the meetings at Oberwolfach are invited personally by the director of the institute. The participation is subject to such an invitation. The e-mail address for the administration at Oberwolfach is admin@mfo.de.

Interested researchers, in particular young mathematicians, can contact the administration of the institute. Since the number of participants is restricted, not all enquiries can be considered.

URL of Oberwolfach: http://www.mfo.de/

Tom H. Koornwinder
(thk@wins.uva.nl)

8. SIAM Annual Meeting 1998: Toronto, Canada, July 13-17, 1998

The 1998 SIAM Annual Meeting is scheduled for July 13–17 and will take place in Toronto. The meeting is co-chaired by Max Gunzburger (Iowa State University), Kenneth R. Jackson (University of Toronto) and Roy Nicolaides (Carnegie Mellon University).

Martin Muldoon
(muldoon@yorku.ca)

Books and Journals

1. Asymptotics and Special Functions
By Frank W. J. Olver
AK Peters, Ltd., 1997, $69, ISBN 1-56881-069-5.

Readers of the Newsletter may be interested to learn that my book *Asymptotics and Special Functions*, originally published by Academic Press in 1974, has just been reprinted by AK Peters, Ltd. It is again in hardback form and it lists at $69. Copies can be ordered through booksellers or directly from the publisher:

AK Peters, Ltd.
289 Linden Street
Wellesley, MA 02181
e-mail: akpeters@tiac.net

Frank W. J. Olver
(olver@ipst.umd.edu)

2. New Journal: Inequalities and Applications
Editor-in-chief: Ravi P. Agarwal
The Gordon and Breach Publishing Group, Singapore, 4 issues per volume, approximately 100 pages per issue, ISSN 1025-5834

The aim of this journal is to provide a multi-disciplinary forum of discussion in mathematics and its applications in which the essentiality of inequalities is hightlighted.

This journal accepts high quality papers containing original research results and survey articles of exceptional merit. Subject matters should be strongly related to inequalities, such as, but not restricted to, the following list.

- Inequalities in Analysis
- Inequalities in Approximation Theory
- Inequalities in Calculus of Vatiations
- Inequalities in Combinatorics

- Inequalities in Economics
- Inequalities in Geometry
- Inequalities in Mechanics
- Inequalities in Optimization
- Inequalities in Probability Theory

If you are interested in submitting a paper for publication please contact

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The World Wide Web home page of the journal is http://www.gbhap.com/journals/290/290-top.htm

Wolfram Koepf
(koepf@zib.de)

3. Probabilistic and Analytical Aspects of the Umbral Calculus
By A. Di Bucchianico

CWI Tract 119, ISBN 90 6196 471 7, 1997, 148 pp., CWI, Amsterdam, The Netherlands, Price: NLG 35.00

The subject of this tract is a class of sequences of polynomials $(q_n)_{n\in\mathbb{N}}$ defined by the following functional equations

$$q_n(x+y) = \sum_{k=0}^{n} q_k(x)\, q_{n-k}(y) \qquad (n = 0, 1, \ldots) \tag{1}$$

A sequence of polynomials that satisfies (1) is called a sequence of polynomials of convolution type. These sequences are closely related to the sequences of polynomials of binomial type introduced by Rota, i.e., sequences of polynomials $(p_n)_{n\in\mathbb{N}}$ satisfying

$$q_n(x+y) = \sum_{k=0}^{n} \binom{n}{k} q_k(x)\, q_{n-k}(y) \quad (n = 0, 1, \ldots) \tag{2}$$

The sequence $(x^n)_{n\in\mathbb{N}}$ is of binomial type by the Binomial Theorem, which explains the name binomial type. In this tract sequences of polynomials of convolution type are studied instead of sequences of polynomials of binomial type because convolution is a fundamental operation in analysis and probability theory. The binomial convolution appearing in (2) has advantages when dealing with certain combinatorial problems.

An extension of the class of sequences of polynomials of binomial/convolution type is the class of Sheffer sequences $(s_n)_{n\in\mathbb{N}}$, whose convolution type version is defined by

$$s_n(x+y) = \sum_{k=0}^{n} s_k(x)\, q_{n-k}(y) \quad (n = 0, 1, \ldots) \tag{3}$$

for some fixed sequence $(q_n)_{n\in\mathbb{N}}$ of convolution type. The class of Sheffer sequences includes (amongst others) the Hermite, Bernoulli and Laguerre polynomials.

The history of Sheffer sequences goes back to 1880 when Appell studied sequences $(a_n)_{n\in\mathbb{N}}$ of polynomials satisfying $Da_n = n\, a_{n-1}$ (D is the differentiation operator). Appell showed that these sequences satisfy

$$a_n(x+y) = \sum_{k=0}^{n} \binom{n}{k} a_k(x)\, y^{n-k} \quad (n = 0, 1, \ldots) \tag{4}$$

These sequences are called Appell sequences nowadays. The Hermite polynomials form an Appell sequence.

A survey of the Umbral Calculus with over 400 references can be obtained in electronic form through the Electronic Journal of Combinatorics:
http://ejc.math.gatech.edu:8080/Journal/Surveys/index.html

Contents:

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4. New Book Series: Analytical Methods and Special Functions
Ed.: A. P. Prudnikov, C. F. Dunkl, H.-J. Glaeske, M. Saigo

Gordon and Breach Science Publ. for Mathematics, Singapore

The aims of this series is the presentation of research activities in analytical methods of analysis, including integral transforms, special functions, series expansions, approximation theory, asymptotic analysis, operational calculus, integral equations, ordinary and partial differential equations, perturbation methods and other special analytical methods in problems of pure and applied mathematics, biological and physical sciences and engineering.

The series will be a companion series to the existing journal *Integral Transforms and Special Functions*.

The first book of this series appeared 1996:

Volume 1. *Series of Faber Polynomials.*
By P. K. Suetin, Technical University of Communication and Informatics, Moscow, Russia
December 1996, 320 pp., ISBN 90-5699-058-6

The book contains some of the most important classical and modern results on the series of Faber polynomials and their applications.

Interest in this subject area has rapidly increased over the last decade, yet the presentation of research has been confined mainly to journal articles. Analysis of recent results concerning the theory and application of Faber series shows that these are, at present, a very important object of study in the theory of functions of complex variables, and a convenient investigative tool in the theory of analytic function approximation, as well as in some questions of numerical analysis.

Contents: Some results of approximation theory • The elementary properties of Faber polynomials • Faber series with the simplest conditions • Asymptotic properties of Faber polynomials • Convergence of Faber series inside a domain • Series of Faber polynomials • Some properties of Faber operators • Fabier series in a closed domain • Faber polynomials and the theory of univalent functions • Faber series in Canonical domains • Faber series and the Riemann boundary problem • The summation formula of Dzyadyk • Generalization of Faber polynomials and series • Some recent result.

Forthcoming volumes:

B.G. Korenev. Bessel Functions and Their Applications.

S.G. Samko. Hypersingular Integrals and Their Applications.

A.M. Sedletskii. Fourier Transforms and Approximations.

P.K. Suetin. Orthogonal Polynomials in Two Variables.

V.A. Yurko. Inverse Problems for Differential Operators.

A. P. Prudnikov
(prudnik@ccas.ru)

5. From Polynomials to Sums of Squares
By T. H. Jackson
Institute of Physics Publishing, Bristol, 1995, 184 pp., ISBN 0-7503-0329-8

Contents: Preface • Software copyright and site licence • Polynomials in one variable • Summary and exercises for chapter 1 • Using polynomials to make new number fields • Summary and exercises for chapter 2 • Quadratic integers in general and Gaussian integers in general and Gaussian integers in particular • Summary and exercises for chapter 3 • Arithmetic in quadratic domains • Summary and execises for chapter 4 • Composite rational integers and sums of squares • Summary and exercises for chapter 5 • Appendices • References • Index

A. P. Prudnikov
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4. The Ramanujan Journal
Editor-in-Chief: Krishnaswami Alladi
Kluwer Academic Publishers, Dordrecht–Boston–London

The Ramanujan Journal, an international journal devoted to the areas of mathematics influenced by Ramanujan, was announced in the Newsletter issue 6.1. Here is the *Table of Contents* of Volume 1, Number 1, January 1997:

The Home Page of The Ramanujan Journal is at the URL: http://www.math.ufl.edu/ frank/ramanujan.html

Frank G. Garvan
(frank@math.ufl.edu)

6. Leganes IWOP '96 Proceedings
Ed.: M. Alfaro, R. Álvarez-Nodarse, G. López Lagomasino, F. Marcellán
Servicio de Publicaciones de la Universidad Carlos III de Madrid. Leganés, Madrid, 1997.

The Proceedings of the *International Workshop on Orthogonal Polynomials*, June 1996, have appeared but they are available also (free of charge) in the URL sites http://dulcinea.uc3m.es/users/workshop/proceedings.html and
http://dulcinea.uc3m.es/users/workshop/workshop.html . The proceedings include some of the talks given during the workshop.

Here are the titles of the articles:

- R. Álvarez-Nodarse, F.Marcellán and J.Petronilho: On Some Polynomial Mappings for Measures. Applications.
- J. Arvesú: Laguerre Polynomials in a Quantum Statistical Model

- Natig M. Atakishiyev: Continuous Solutions of Difference Equations and their Orthogonality Relations
- A. Duran: On Orthogonal Matrix Polynomials
- F. Finkel, A. González-López and M. A. Rodríguez: Orthogonal Polynomials and Quasi-Exactly Solvable Potentials on the Line
- M. Frontini and A. Tagliani: Entropy-Convergence, Inestability in Stieltjes and Hamburger Moment Problem
- Lucas Jódar and Emilio Defez: Some New Matrix Formulas Related to Hermite Matrix Polynomials
- J. Koekoek and R. Koekoek: Finding Differential Equations for Symmetric Generalized Ultraspherical Polynomials by Using Inversion Methods
- E. Koelink: Convolution and Addition Theorems for q-orthogonal Polynomials
- M.X. He, P. Natalini, and P.E. Ricci: A Class of Jacobi Polynomials Orthogonal with Respect to Varying Weights
- A. Ronveaux: Orthogonal Polynomials: Connection and Linearization coefficients
- Jorge Sánchez-Ruiz: Position and Momentum Information Entropies of the Harmonic Oscillator and Logaritmic Potential of Hermite Polynomials
- Yu. F. Smirnov: On Factorization and Algebraization of Difference Equations of Hypergeometric Type

R. Álvarez-Nodarse
(renato@dulcinea.uc3m.es)

7. Computation of Special Functions
By S. Zhang and J. Jin

Wiley, 1996, ISBN 0-471-11963-6

From Professor Jin's web site
http://iris-lee3.ece.uiuc.edu/~jjin/specfunc.html :

"The computation of special functions is a fundamental aspect of numerical analysis in virtually all areas of engineering and the physical sciences.

"Because most of these special functions are in the form of infinite series or infinite integrals, their solutions are quite complicated. Fortunately, the difficulty has been lessened considerably by the advent of powerful personal computer hardware and software. Yet, until now, there has been no single-source reference offering comprehensive coverage of the automated computation of special functions.

"*Computation of Special Functions* is a valuable book/ software package containing more than 100 original computer programs for the computation of most special functions currently in use. These include many functions commonly omitted from available software packages, such as the Bessel and modified Bessel functions, the Mathieu and modified Mathieu functions, parabolic cylinder functions, and various prolate and oblate spheroidal wave functions. Also, unlike most software packages, this book/disk set gives readers the latitude to modify programs according to special demands of the sophisticated problems they are working on. The authors provide detailed descriptions of the programs algorithms as well as specific information about each program's internal structure.

"To facilitate quick reference, each chapter follows the same, three-part format. The first part covers major properties and important formulas needed for computation. The second part of each chapter contains a description of the algorithm or algorithms at the heart of the FORTRAN-77 program in question, and in the third part, the authors tabulate representative results to provide an at-a-glance understanding of the function's behavior as well as a valuable data-check for their computed results.

"*Computation of Special Functions* is an indispensable tool for engineers and physical scientists as well as students involved in advanced research in these fields.

"To order, contact John Wiley and Sons, Inc., One Wiley Drive, Somerset, NJ 08875. 1-800-CALL-WILEY"

Tom H. Koornwinder
(thk@wins.uva.nl)

Call For Papers

1. Orthogonal Polynomials and Computer Algebra
Journal of Symbolic Computation: Special Issue

Academic Press, London

Some of the speakers of Session 13 at the First ISAAC Conference on *Orthogonal Polynomials*—with emphasis on the use of symbolic computation—(see p. 5) had been asking me whether the proceedings of this session will be published. I decided that if such a publication should take place then the articles should be published through an official refereeing procedure. I got in touch with Bob Caviness, the Managing Editor of the *Journal of Symbolic Computation*, and proposed a Special Issue of this journal on *Orthogonal Polynomials and Computer Algebra*.

I am happy to inform you that Bob Caviness supports my proposal, and that Dick Askey has agreed to serve as co-editor for this issue. Note that this special issue of the *Journal of Symbolic Computation* will be open for everybody, but the speakers of the above mentioned session are particularly invited to submit their papers. The submission deadline will be November 15, 1997. A *Call for Papers* with details on the submission procedure will be published soon, and will be available from my homepage http://www.zib.de/koepf .

Wolfram Koepf
(koepf@zib.de)

Software Announcements

1. Maple and REDUCE Packages on q-Hypergeometric Summation

Careful implementations of the q-versions of Gosper's and Zeilberger's algorithms for indefinite and definite summation [2] are available in REDUCE [1] and Maple. These implementations allow input in the usual notation, using q-Pochhammer symbols, q-binomial coefficients, q-hypergeometric terms, etc., and they support a posteriori proofs of the resulting terms and recurrence equations, respectively.

These packages have been implemented by Harald Böing under the supervision of Wolfram Koepf and can be obtained by request (boeing@zib.de, koepf@zib.de).

The REDUCE package `QSUM` will part of the next REDUCE Version 3.7, and the Maple package `qsum.maple` (working with version V.3 and V.4) will be submitted to Maple's `share` library.

References

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Wolfram Koepf
(koepf@zib.de)

Problems and Solutions

Thus far 18 problems have been submitted six of which have been solved (#**1**, **4**, **6**, **7**, **10**, **14**), and one of which is new (#**18**). Still unsolved are Problems #**2**, **3**, **5**, **8**, **9**, **11**, **12**, **13**, **15**, **16**, **17** and **18**. Please send in your solutions!

15. Critical Values of Orthogonal Polynomials. Let P_n be an OP system on $[-1, 1]$ with respect to a weight function $w(x)$. Denote $-1 < y_{n,1} < \ldots < y_{n,n-1} < 1$ the set of all critical points, i.e. the set of all zeros of the derivative P_n'. The values $P_n(y_{n,k})$, $k = 1, 2, \ldots n-1$ are known as critical values of P_n. Let $N(P_n)$ be the number of all *different* critical values of P_n.

Problem 15.1. Describe the set of OP systems with the property $N(P_n) = O(1)$, $n \to \infty$.

It is clear that for the first kind Chebyshev polynomials T_n one has $N(T_n) = 2$ for all n.

Problem 15.2. Given $w(x)$, find the value

$$a(w) := \limsup_{n\to\infty} \frac{N(P_n)}{n} .$$

(Submitted on August 16, 1996)

Leonid B. Golinskii
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16. A Definite Integral. Prove that

$$\int_0^1 \frac{\log(\pi^2 + (\log x)^2)}{1 + x^2}\, dx = \pi \log \frac{\sqrt{\pi/2}\,\Gamma(1/4)}{2\,\Gamma(3/4)} .$$

Remark: The integral is related to the Dirichlet L-function. The right side of this identity can be rewritten as

$$\frac{\pi\,(\gamma + 2\,\log(\pi/2))}{2} - 2\,L'(1)$$

where $L(s)$ is the Dirichlet L-function

$$L(s) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^s} .$$

(Submitted on October 10, 1996)

Victor Adamchik
(victor@wolfram.com)

17. Canonical Leibniz' Formula for Difference Operators. Let Δ and ∇ be the usual forward and backward operators. The action on a product $f(x)g(x)$ is usually written in a nonsymmetrical (and nonunique) form, for instance:

$$\begin{aligned}\Delta[f(x)g(x)] &= f(x)\Delta g(x) + g(x+1)\Delta f(x)\\ &= g(x)\Delta f(x) + f(x+1)\Delta g(x) .\end{aligned}$$

A (unique) canonical form, without any shift on argument x, is sometime preferable to the nonsymmetrical formula, like

$$\Delta[f(x)g(x)] = f(x)\Delta g(x) + g(x)\Delta f(x) + \Delta f(x)\Delta g(x) ,$$

or

$$\nabla[f(x)g(x)] = f(x)\nabla g(x) + g(x)\nabla f(x) - \nabla f(x)\nabla g(x) .$$

Iteration of Δ or ∇ acting on a product of r functions $f_i(x)$ can obviously be written in the canonical form

$$\Delta^n \left[\prod_{i=1}^{r} f_i(x)\right] = \sum_{j_1\cdots j_r=0}^{n} R_n(j_1\cdots j_r) \prod_{i=1}^{r} \Delta^{j_i} f_i(x) ,$$

where the coefficients $R_n(j_1\cdots j_n)$ are nonnegative integers, invariant under the group of permutation P_r;

($S_n(j_1 \cdots j_r)$ appears when using the operator ∇^n). Using two times the link with the shift operator E ($\Delta^n = (E-1)^n, E^k = (\Delta+1)^k$), the Δ^n canonical formula can be written:

$$\Delta^n\left[\prod_{i=1}^{r} f_i(x)\right] = \sum_{k=0}^{n}(-1)^k\binom{n}{k}\prod_{i=1}^{r}\left[\sum_{j=0}^{n-k}\binom{n-k}{j}\Delta^j[f_i(x)]\right].$$

Is it possible, using this representation, to obtain coefficients R_n (and S_n) in a closed form?

It is obvious that $R_n(0,\cdots,0) = R_n(s,0,\cdots,0) = R_n(0,s,0,\cdots,0) = \cdots = R_n(0,\cdots,0,s) = 0$, $(s = 1,\cdots,r-1)$.
(Submitted on January 7, 1997)

André Ronveaux
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18. Maclaurin Expansion. For $a, b \in (0,1)$ let

$$Q(a,b,r) = \frac{B(a,b)}{\log\left(\frac{c}{1-r}\right)}\, {}_2F_1\left(\left.\begin{matrix} a, b \\ a+b \end{matrix}\right| r\right)$$

where $B(a,b)$ denotes the Beta function, and

$$c = e^{R(a,b)}\ , \quad R(a,b) = -\Psi(a) - \Psi(b) - 2\gamma\ ,$$

γ is Euler's constant, and

$$\Psi(z) = \frac{\Gamma'(z)}{\Gamma(z)}\ .$$

Let

$$G(a,b,r) = \frac{Q(a,b,r)-1}{1-r} = \sum_{j=0}^{\infty} d_j\, r^j\ .$$

Is it true that all $d_j > 0$?

This question arose in connection with Theorem 1.4 in Trans. Amer. Math. Soc. 347 (1995), 1713–1723, which is a refinement of Ramanujan's asymptotic formula for the zero-balanced hypergeometric function ${}_2F_1$.
(Submitted on March 24, 1997)

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Compiled Booklist

Shortly after the last issue of the Newsletter was distributed, I received the following letter.

Dear Dr. Koepf:

I just received the February issue of the OP-SF Newsletter. First, I would like to congratulate you for the excellent job you are doing in putting together the newsletter. I can just imagine that each issue represents a lot of efforts. Second, I would like to thank you very much for putting together the Compiled Booklist and Electronic Services that is, at least for me, very useful in improving by self study my knowledge about OP-SF. Keeping it "low-key", I just want to say that I am the group member that made that suggestion about the program of self study.

I have to admit that most of the topics covered in OP-SF are "way-above-my-head" in the sense that as an engineer, the questions "why research on that topic, where does it apply and how can one use it in applications" come to my mind. However, I have to say that I find this field of study very exciting, very interesting and of course challenging. That is why I told myself I must find a way to understand more about the research going on in OP-SF to answer the "why-where-how questions". Your Compiled Booklist and Electronic Services will help me tremendously to reach that goal. In closing, I just want to mention that I use OP-SF in my work in Hydrodynamic Stability.

Again thank you for your efforts and help.

René Girard
(rgirard@nbnet.nb.ca)

I am very happy that my efforts in compiling this booklist help you, René, and I hope that this service will help other members as well.

Furthermore, intermediately I received other announcements of books that some members found missing in the booklist. Hence at this place I would like to publish a more complete list. For the convenience of our readers, the following list contains both the old list and the newly inserted items.

[1] Artin, E.: *The Gamma Function*. Holt, Rinehart and Winston, New York, 1964.

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Miscellaneous

1. OP-SF ftp Site

Hans Haubold's ftp archive for preprints in the area of *Orthogonal Polynomials and Special Functions* is the continuation of Waleed Al-Salam's preprint archive. One can approach the archive by anonymous ftp to `unvie6.un.or.at`, directory `siam`. Very recently, Hans Haubold has constructed a convenient WWW interface for this ftp site, at the address ftp://unvie6.un.or.at/siam/opsf_new/00index.html

This home page links to pages

- Index by Author (with links to the actual manuscript files)
- Abstracts listed by Author(s) (mostly with abstracts of papers for which the full manuscript file resides elsewhere; hyperlinks are provided)
- Submission form

You can also move from the home page to the ftp interface and to the the submissions directory, where the most recent contributions reside.

The WWW interface is beautifully done. We recommend that you visit it.

There are two new features in connection with submitting manuscripts to the ftp site:

- If you submit a file with a full manuscript, it is recommended (though not obligatory) to also supply an abstract file (plain ASCII text, no TEX).
- It is also possible to submit only an abstract file together with a hyperlink to the actual full manuscript. In this case, please supply the hyperlink in the "Comments" Text-Area, when filling out the submission form.

Hans Haubold is sending regular information about new submissions to a large mailing list. Please contact him (haubold@Relay1.Austria.EU.net) if you want to be added to this mailing list or if your email address on the list is no longer correct.

Tom H. Koornwinder
(thk@wins.uva.nl)

Editor's Remark: As an update service we would like to ask the colleagues whose articles reside in the archive

- to tell whether some of their manuscripts should be erased,
- and to give the bibliographical data of articles which appeared in the meantime, for inclusion in the file `00contents.ftpsite`.

In the next issue of the Newsletter, an updated list of the archive will be published.

2. Special Functions in Astrophysics and Cosmology

I would like to announce that I have still some free copies of the following books

Gottloeber, S., Haubold, H. J., Muecket, J.-P. and Mueller, V.: *Early Evolution of the Universe and Formation of Structure.* Akademie-Verlag, Berlin, 1990.

Mathai, A. M. and Haubold, H. J.: *Modern Problems in Nuclear and Neutrino Astrophysics.* Akademie-Verlag, Berlin, 1988.

in which special functions are applied to problems in astrophysics and cosmology. Members of the Activity Group who would like to receive one of these books (or both) should send me an e-mail message (haubold@ekpvs2.dnet.tuwien.ac.at), and I will air mail them the desired books.

Furthermore I would like to ask whether somebody knows recent articles in which Meijer's G-functions or Fox' H-functions are used in connection with problems in astrophysics or cosmology. Any information in this direction would be appreciated.

Hans Haubold
(haubold@ekpvs2.dnet.tuwien.ac.at)

3. End of Problem Section in SIAM Review

We heard from Joop Boersma (Technical University Eindhoven) of a report that SIAM Review is considering dropping its problem section. Boersma would regret this very much. According to him the problem section provides a forum where problems from Applied Analysis, in particular, can be brought to the attention of a wider audience. He asks whether our Activity Group can intervene.

Intermediately, the recent issue Vol. 39, Number 1 of SIAM Review arrived. It opens with an item "Changes planned for SIAM Review". Beginning in 1999 SIAM Review will consist of five sections:

- Survey and Review
- Problems and Techniques
- Selected papers from SIAM's research journals
- Education
- Book reviews

No problem section is mentioned. The second section, where you might expect a problem section, will consist of papers. These plans seem to be definite. The blueprint was approved by SIAM's council and Board of Trustees in July 1996. Questions about the new content or structure of SIAM Review can be addressed to Mary Rose Muccie, SIAM Editorial Director (muccie@siam.org).

Opinions among officers of the Activity Group on this matter vary. Some opinions you find below. Possibly we will react to SIAM's Editorial Director Mary Rose Muccie, but first we would like to get some further opinions from our membership. Please send us your opinion. Here are extracts from comments by Tom Koornwinder, Martin Muldoon and Nico Temme.

Tom Koornwinder: "Boersma raises a valuable role for the present problem section in SIAM Review: that researchers can call the help of other specialists in such a way (for instance applied mathematicians calling the help of specialists in special functions). However I think that it is better not to hide such questions in a problem section, but rather call it a section on research questions. One other point about this problem section is that it also gave some more publicity about Special Functions to the SIAM community (almost every issue of the problem section had at least one problem on SF). In a sense that was good, because OP & SF does not get much other coverage in SIAM News and SIAM Review. On the other hand, by the very nature of a problem section, the aspects of Special Functions being treated there emphasize the "special" and the formula aspect of Special Functions, much less the qualitative and conceptual sides of the field. In this way such a problem section can also help to maintain or strengthen existing prejudices against special functions."

Martin Muldoon: "The problems [in SIAM Review] are much too difficult. I suspect that many other readers would agree and that may explain why the Section may be chopped. I believe that its main strength, compared to problem sections in other journals, is that many of the problems arise from applications. While it is true that it provides a way for applied mathematicians to call on experts, in practice it is surely much too slow for this. A member once told me that he thought our Newsletter should not have a Problem Section since it competes with and takes problems away from SIAM Review. I answered that I thought that were more than enough good problems to go around."

Nico Temme: "My opinion about problem sections in journals is that the reader should have a fair chance, and that much detailed and technical expertise should not be needed to solve the problems. In this sense it is good to have a few places where one can find problems. Nieuw Archief voor Wiskunde offers only problems of which the solutions are known. But they had difficult years some time ago to get good editors.

"In some cases the problems are too difficult for a non-introduced reader. This is acceptable when the solution is not known.

"Usually I am not motivated to solve the first category. It is more "educational" and it consumes too much time. When I see a problem with a $*$ in SIAM Review (which means an "open problem") I become interested.

"I think that there should be a place for both categories, and that it should be clear where to find both categories. SIAM Review has a long tradition on this, and the journal is available in many places.

"I think that having a problem section in our Newsletter for open problems is important and of interest to many readers."

Tom H. Koornwinder
(thk@wins.uva.nl)

4. Revising the 1991 Mathematics Subject Classification

In the Newsletter 6-3, June 1996, topic 2 on p. 18, the revision of the 1991 AMS Mathematics Subject Classification was announced. We called then for suggestions for revision concerning the sections on Orthogonal Polynomials and Special Functions. Some comments were published in the last issue. Intermediately the following further comments were submitted:

Rudolf Gorenflo (gorenflo@math.fu-berlin.de):

Herewith I propose to include (maybe 33 C is best suited) as an additional item with extra classification number the term "Mittag-Leffler functions and generalized Mittag-Leffler functions".

Comment: Such functions have been introduced already at the beginning of this century. In recent years, they are finding more and more applications in the treatment of integro-differential equations with power-functions as convolvers and in applications in visco-elasticity, theory of generalized Brownian motion, control theory, and in other problems of processes with memory. For people interested I can easily provide references.

Per W. Karlsson (karlsson@mat.dth.de)

I have two more comments upon the AMS-scheme.

1. From time to time rearrangements of (multiple) series with "arbitrary" terms are considered in the literature. In most such papers, emphasis is upon the formulas themselves, not upon convergence. Thus, No. 40 is not really appropriate as it stands. On the other hand, some Pochhammer symbols would quite often be involved, and so one may use—reluctantly—33C or 33D. This is not quite satisfactory, and I feel that a new box would be advantageous, e.g., 40??? – Rearrangements of multiple series.
2. In 05A15, generating functions are mentioned. However, they often pop up in other areas than combinatorics. A separate box for them is suggested, for instance somewhere in No. 40.

Wolfram Koepf
(koepf@zib.de)

5. Telegraph Equation

I have been working on several aspects of the telegraph equation (TE); a simple form is: $u_{xy}+u=0$. I am looking for researchers working on the TE who can help me in finding applications or are interested in joint work. My questions are:

(a) Has the investigation of solutions that are periodic on $y=-x$ any interest for somebody, taking into account the almost periodic character of its traces on, for example, the line $y=0$?

(b) More general: investigation of the almost-periodic traces of periodic solutions on some lines.

(c) Construction of the solution with given global properties in the incomplete rectangular boundary-value problem.

(d) Solvability of some overfilling rectangular boundary-values problems.

(e) The main general point in the possible applications is that one may consider all such problems from the point of view of electric problems (cut-in and cut-off alternating current, for example).

Andrew Bakan
(andrew@bakan.kiev.ua)

6. Call for Nominations: Pólya Prize

The Polya Prize:

SIAM will present the award at the 1998 SIAM Annual Meeting in Toronto, Canada, July 13–17. The award honors the memory of George Polya and will be given for a notable contribution in one of the following areas: approximation theory, complex analysis, number theory, orthogonal polynomials, probability theory, or mathematical discovery and learning.

Eligibility:

There are no restrictions except that the prize is broadly intended to recognize specific work.

Description of Award:

The award will consist of an engraved medal and a $20,000 cash prize.

Nominations:

A letter of nomination, including a description of achievement(s), should be sent by October 1, 1997, to:

> Professor Harry Kesten Chair, Polya Prize Selection Committee c/o Allison Bogardo SIAM 3600 University City Science Center Philadelphia, PA 19104-2688 Telephone: (215) 382-9800 Fax: (215) 386-7999 E-mail: bogardo@siam.org

Other members of the selection committee are Lennart Carleson (Royal Institute of Technology, Stockholm), Barry Mazur (Harvard University), Paul Nevai (The Ohio State University), and Andrew Yao (Princeton University).

Allison Bogardo
(bogardo@siam.org)

7. Call for Nominations: DiPrima Prize

The DiPrima Prize:

SIAM will present the award at the 1998 SIAM Annual Meeting in Toronto, Canada, July 13–17. The award honors the memory of Richard C. DiPrima, long-time Chair of the *Department of Mathematical Sciences* at *Rensselaer Polytechnic Institute* and past-president and energetic supporter of SIAM. The award will be based on an outstanding doctoral dissertation in applied mathematics.

Eligibility:

The award, based on Ph.D. research in applied mathematics (defined as those topics covered in SIAM journals or series) is made to a young scientist. The Ph.D. thesis and all other Ph.D. requirements should have been completed in the time period from July 1, 1995 to June 30, 1997. The Ph.D. degree must be awarded by December 31, 1997.

Description of the Award:

The award will consist of a certificate and a cash prize of $1,000. The SIAM president will notify the recipient of the award in advance of the award date and invite the recipient to attend the annual meeting to receive the award.

Nominations:

Nominations, along with a copy of the dissertation (in English), should be sent by November 30, 1997 to:

> Professor Gilbert Strang
> Chair, DiPrima Prize Selection Committee
> c/o Allison Bogardo
> SIAM
> 3600 University City Science Center
> Philadelphia, PA 19104-2688
> phone: +1-215-382-9800
> fax: +1-215-386-7999
> e-mail: bogardo@siam.org

Members of the selection committee are Philip Holmes (Princeton University), Gilbert Strang (MIT), and Shmuel Winograd (IBM Research Center).

Allison Bogardo
(bogardo@siam.org)

Laguerre Polynomials and the Vibrations of a Multiple Pendulum

by M. Braun
(braun@mechanik.uni-duisburg.de)

Laguerre polynomials can be used to solve the Schrödinger equation for a particle in the Coulomb field [1, page 125]. A cursory glance over the relevant literature suggests that it is probably the only application of these polynomials in physics.

The purpose of this note is to present an application of Laguerre polynomials even in classical mechanics, which seems to have been overlooked so far. It will be shown that the natural frequencies of a regular $n$fold pendulum are directly related to the zeros of the Laguerre polynomial L_n. Also the amplitudes at which the several mass points oscillate in the corresponding vibrational mode can be expressed in terms of Laguerre polynomials. Whatever is known about the behavior of Laguerre polynomials can be associated with the vibrations of the regular $n$fold pendulum. Tricomi's asymptotic expansion, for instance, provides an approximation for the natural frequencies.

1. Equations of Motion

The regular $n$fold pendulum (Figure 1) consists of n equal point masses m attached to a flexible, inextensible and massless string of length ℓ at equal distances $a = \ell/n$. The mass points are numbered consecutively, from number 0 at the free end to

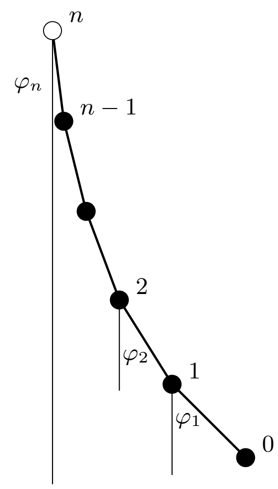

Figure 1: Regular n-fold pendulum

number $n-1$ next to the fixed suspension, which is given the number n. The configuration of the pendulum is described by n generalized coordinates $\varphi_1, \varphi_2, \dots, \varphi_n$, where the angle φ_k measures the inclination of the leg joining the mass points k and $k-1$ with respect to the vertical.

Lagrange's equations of the second kind are used to describe the dynamics of the pendulum. To this end the kinetic and potential energies of the whole system have to be expressed in terms of the generalized coordinates φ_k and their time derivatives $\dot{\varphi}_k$. In a single mass point k the kinetic and potential energies

$$\begin{aligned} T_k &= \frac{1}{2} m a^2 \sum_{i,j=k+1}^{n} \dot{\varphi}_i \dot{\varphi}_j \cos(\varphi_i - \varphi_j), \\ U_k &= mga \sum_{i=k+1}^{n} (1 - \cos\varphi_i) \end{aligned}$$

are stored, where g denotes the gravitational acceleration. By collecting the contributions of all mass points one obtains the Lagrangian, i. e. the difference of total kinetic and potential energies,

$$\begin{aligned} L &= \frac{1}{2} m a^2 \sum_{i,j=1}^{n} \min(i,j)\, \dot{\varphi}_i \dot{\varphi}_j \cos(\varphi_i - \varphi_j) - \\ &\quad - mga \sum_{i=1}^{n} i(1 - \cos\varphi_i). \end{aligned} \tag{1}$$

The whole dynamics of the pendulum is governed by Lagrange's equations of the second kind

$$\frac{\mathrm{d}}{\mathrm{dt}} \left(\frac{\partial L}{\partial \dot{\varphi}_i} \right) - \frac{\partial L}{\partial \varphi_i} = 0, \quad i = 1, 2, \dots, n.$$

By carrying out the appropriate differentiations the nonlinear equations of motion for the regular $n$fold pendulum are established in the form

$$\begin{aligned} a^2 \sum_{k=1}^{n} \min(i,k) \left[\ddot{\varphi}_k \cos(\varphi_i - \varphi_k) + \dot{\varphi}_k^2 \sin(\varphi_i - \varphi_k) \right] + \\ + agi \sin\varphi_i = 0, \qquad i = 1, 2, \dots, n, \end{aligned} \tag{2}$$

where the common factor m has been dropped. A closed-form solution of the *nonlinear* equations is not available, of course, except for the trivial case $n = 1$ of a single pendulum.

2. Small Vibrations

In the vicinity of the stable equilibrium configuration $\varphi_k = 0$, $k = 1, 2, \dots, n$, the motion of the pendulum can be described by a linearized system of equations. Let $\boldsymbol{\varphi} = [\varphi_1, \varphi_2, \dots, \varphi_n]^{\mathrm{T}}$ denote the column vector of the generalized coordinates. Then the linearized system assumes the form

$$\boldsymbol{M}\ddot{\boldsymbol{\varphi}} + \boldsymbol{C}\boldsymbol{\varphi} = 0.$$

The symmetric and positive definite mass matrix $\boldsymbol{M}$ is composed of elements $M_{ik} = a^2 \min(i,k)$, and the matrix of restitutional forces $\boldsymbol{C}$ assumes diagonal form with entries $C_{ii} = iga$.

Small vibrations of the pendulum are described by

$$\boldsymbol{\varphi} = \widehat{\boldsymbol{\varphi}} \sin\omega t,$$

where the frequency ω and the amplitude vector $\widehat{\boldsymbol{\varphi}}$ have to be determined from the eigenvalue problem

$$(\boldsymbol{C} - \omega^2 \boldsymbol{M})\widehat{\boldsymbol{\varphi}} = 0. \tag{3}$$

This can be turned into an *ordinary* eigenvalue problem of the form

$$(\boldsymbol{A} - \lambda \boldsymbol{I})\widehat{\boldsymbol{x}} = 0. \tag{4}$$

To preserve symmetry the mass matrix is subjected to a Cholesky decomposition $\boldsymbol{M} = \boldsymbol{U}^{\mathrm{T}}\boldsymbol{U}$, and the upper triangular Matrix $\boldsymbol{U}$ is used to generate a new amplitude vector

$$\widehat{\boldsymbol{x}} = \boldsymbol{U}\widehat{\boldsymbol{\varphi}}.$$

Explicitly, its components are given by,

$$\widehat{x}_i = a \sum_{k=i+1}^{n} \widehat{\varphi}_k, \quad i = 0, 1, 2, \dots, n-1, \tag{5}$$

and can be interpreted as linear approximations of the horizontal displacement amplitudes of the mass points. Premultiplication of (3) by $(a/g)\boldsymbol{U}^{-\mathrm{T}}$ yields the desired form (4) of the eigenvalue problem with

$$\boldsymbol{A} = \frac{a}{g} \boldsymbol{U}^{-\mathrm{T}} \boldsymbol{C} \boldsymbol{U}^{-1}, \quad \lambda = \frac{a}{g} \omega^2. \tag{6}$$

The free vibrations of the regular $n$fold pendulum are thus determined by the eigenvalue problem (4) for the matrix

$$\boldsymbol{A} = \begin{pmatrix} 1 & -1 & & & \\ -1 & 3 & -2 & & \\ & -2 & 5 & \ddots & \\ & & \ddots & \ddots & -(n-1) \\ & & & -(n-1) & 2n-1 \end{pmatrix}. \tag{7}$$

The eigenvalues λ are connected with the natural frequencies ω by $(6)_2$, and the eigenvectors $\widehat{\boldsymbol{x}}$ determine the corresponding modes of vibration.

The generic law for the entries of the matrix $\boldsymbol{A}$ can be read off immediately from (7). Abandoning matrix notation we can formulate the explicit equation

$$-i\widehat{x}_{i-1} + (2i+1)\widehat{x}_i - (i+1)\widehat{x}_{i+1} = \lambda \widehat{x}_i \tag{8}$$

reproducing the ith row of the eigenvalue problem (4). The equation holds for $i = 1, 2, \dots, n-2$ and has to be modified to include also the first and last rows. This can be done by appending two components $\widehat{x}_{-1}$ and $\widehat{x}_n$ to the amplitude vector $\widehat{\boldsymbol{x}}$. The additional component $\widehat{x}_{-1}$ at the lower end has no physical relevance and may be chosen arbitrarily, since it is multiplied by zero. The new component at the upper end, however, has to satisfy the condition

$$\widehat{x}_n = 0, \tag{9}$$

in order to make (8) valid for $i = n-1$. This last component can be interpreted as the horizontal displacement amplitude of point n, which represents the fixed suspension.

Thus the eigenvalue problem has been reformulated as a set of equations (8) for $i = 0, 1, 2, \ldots, n-1$ with the additional condition (9).

3. Solution of the Eigenvalue Problem in Terms of Laguerre Polynomials

Laguerre polynomials satisfy the recurrence relation

$$-iL_{i-1}(x) + (2i+1)L_i(x) - (i+1)L_{i+1}(x) = xL_i(x).$$

This equation coincides with the generic equation (8) expressing the ith row of the eigenvalue problem. One simply has to identify x with λ and $L_i(x)$ with $\widehat{x}_i$. Hence,

$$\widehat{x}_i = L_i(\lambda), \quad i = 0, 1, 2, \ldots, n,$$

satisfies the generic equations (8) identically in λ. However, the last of these equations, for $i = n-1$, holds only with the proviso that the auxiliary component $\widehat{x}_n$ vanishes, as stated in (9). Therefore, since $\widehat{x}_n = L_n(\lambda)$, the additional equation

$$L_n(\lambda) = 0 \tag{10}$$

has to be satisfied. According to $(6)_2$ this equation determines the natural frequencies ω of the regular $n$fold pendulum: *The k-th natural frequency ω_k of the small vibrations of an n-fold pendulum is related to the k-th zero $\lambda_{n,k}$ of the Laguerre polynomial L_n by*

$${\omega_k}^2 = \frac{g}{a}\,\lambda_{n,k}. \tag{11}$$

In the corresponding mode of vibration the horizontal displacements of the n mass points have the amplitudes

$$\widehat{x}_i = AL_i(\lambda_{n,k}), \quad i = 0, 1, 2, \ldots, n-1, \tag{12}$$

where $A = \widehat{x}_0$ denotes the amplitude of the free end of the pendulum.

The original amplitudes $\widehat{\varphi}_i$ can easily be recovered from the displacement amplitudes $\widehat{x}_i$. According to (5) the angle amplitudes are

$$\widehat{\varphi}_i = \frac{A}{a}\Big(L_{i-1}(\lambda_{n,k}) - L_i(\lambda_{n,k})\Big).$$

Also the total energy of the pendulum can be expessed in terms of the Laguerre polynomials: From either the kinetic or the potential energy one obtains

$$E = \frac{mgA^2}{2a}\,\lambda_{n,k}\sum_{i=0}^{n}\Big(L_i(\lambda_{n,k})\Big)^2,$$

where the zero term for $i = n$ has been added for convenience. According to the summation formula of Christoffel-Darboux [3, page 214] the sum of squared Laguerre polynomials is

$$\sum_{i=0}^{n} L_i^2(x) = (n+1)\left(L_n'(x)L_{n+1}(x) - L_n(x)L_{n+1}'(x)\right),$$

and the derivative of the Laguerre polynomial L_n can be expressed as

$$L_n'(x) = \frac{n}{x}\Big(L_n(x) - L_{n-1}(x)\Big)$$

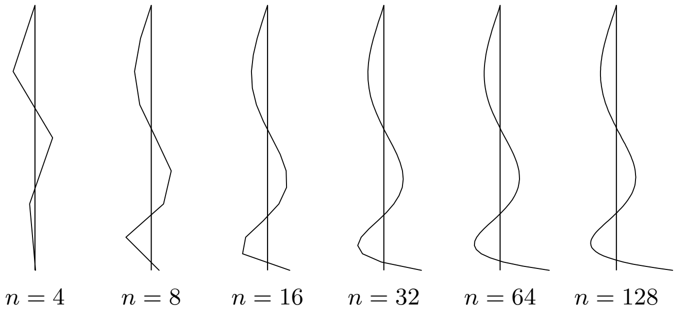

Figure 2: The 4th mode of vibration for different degrees of freedom n

[3, page 215]. Actually these formulas have to be evaluated only at the zeros $x = \lambda_{n,k}$ of L_n. Thus the energy expression is reduced to

$$E = -\frac{mgA^2}{2a}\,n(n+1)L_{n-1}(\lambda_{n,k})L_{n+1}(\lambda_{n,k}). \tag{13}$$

Since, at a zero of L_n, the two neighboring polynomials L_{n-1} and L_{n+1} have always different signs, the energy is in fact positive. The energy expression (13) can be used to normalize the amplitude vector with respect to the energy. This is done in Figure 2, which shows the amplitudes corresponding to the 4th mode of an $n$fold pendulum for various degrees of freedom n.

Whatever is known about Laguerre polynomials can be applied, via the established analogy, to the vibrations of the regular $n$fold pendulum. According to Tricomi [3, page 223] the kth zero of the Laguerre polynomial L_n can be approximated by the asymptotic formula

$$\lambda_{n,k} = \frac{j_{0,k}^2}{2(2n+1)}\left(1 + \frac{j_{0,k}^2 - 2}{12(2n+1)^2}\right) + O(n^{-5}), \tag{14}$$

where $j_{0,k}$ denotes the kth zero of the Bessel function J_0. This formula can be used to approximate the natural frequencies ω of an $n$fold pendulum. The leading term

$$\omega_k^{(\infty)} = \frac{j_{0,k}}{2}\sqrt{\frac{g}{\ell}} \tag{15}$$

gives the eigenfrequencies of a flexible string with *uniformly distributed* mass [2, pages 19–23].

Figure 3 shows the dependence of the natural frequencies of a multiple pendulum on the number n of point masses, which are spaced at equal distances over the total length ℓ. The circles represent computed values of the eigenfrequencies, and the solid lines correspond to the approximation based on Tricomi's formula (14). Except for the lower modes at high degrees of freedom the coincidence between exact and approximate values is quite good.

The characteristic polynomial of the tridiagonal matrix $\boldsymbol{A}$ is representable in the form

$$\det(\boldsymbol{A} - \lambda\boldsymbol{I}) = n!L_n(\lambda). \tag{16}$$

From the explicit representation of the Laguerre polynomials [3, page 213] the principal invariants of the matrix $\boldsymbol{A}$ are obtained as

$$I_k(\boldsymbol{A}) = \binom{n}{k} \frac{n!}{(n-k)!}\,.$$

An interesting property can be derived from

$$\sum_{k=1}^{n} \frac{1}{\lambda_{n,k}} = \frac{I_{n-1}}{I_n} = n.$$

Reformulating this equation in terms of the natural frequencies of the $n$fold pendulum one obtains

$$\sum_{k=1}^{n} \frac{1}{{\omega_k}^2} = \frac{\ell}{g}\,. \tag{17}$$

It means that *the sum of squared periodic times corresponding to all vibrational modes of the multiple pendulum is independent of the number n of masses.* This remarkable property remains valid even in the limit $n \to \infty$. From the asymptotic behavior (15) of the frequencies we regain the formula

$$\sum_{k=1}^{\infty} \frac{1}{j_{0,k}^2} = \frac{1}{4} \tag{18}$$

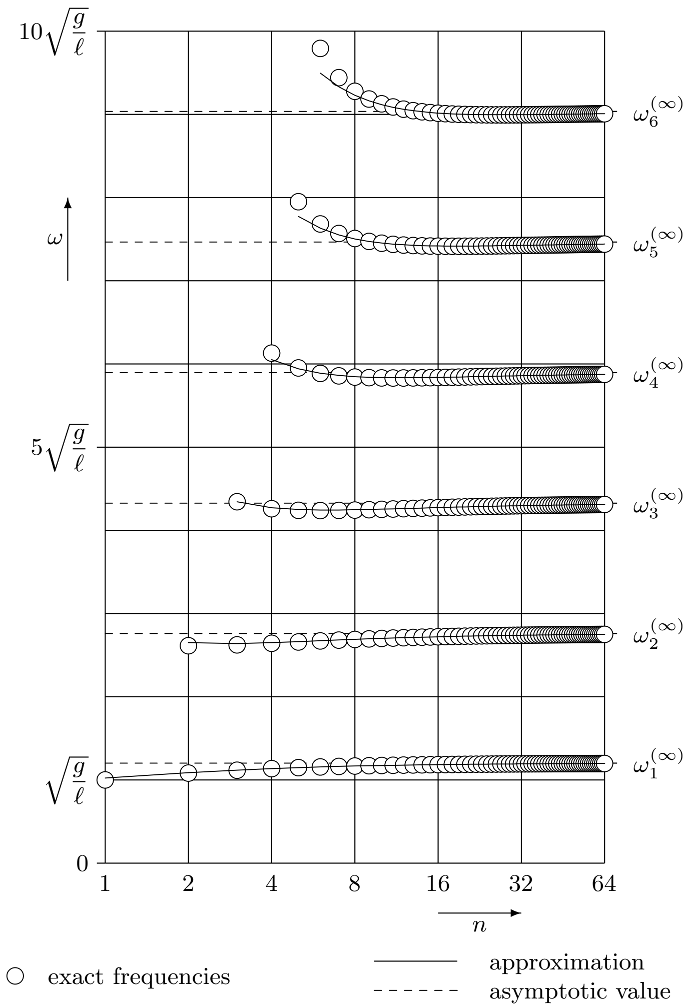

Figure 3: Natural frequencies of the $n$fold pendulum

concerning the zeros $j_{0,k}$ of the Bessel function J_0. It should be noted that the property (17) still holds for the case of an irregular $n$fold pendulum with different masses and nonequidistant spacing, but this lies outside the scope of Laguerre polynomials.

4. Concluding Remarks

The eigenvalue problem describing the natural vibrations of a multiple pendulum can be solved in terms of Laguerre polynomials. This connection allows to transfer results from the theory of Laguerre polynomials to the vibration problem.

The established analogy is restricted to the case of a multiple pendulum with equal masses and equidistant spacing. If, for instance, a bigger mass is attached to the lower end of the otherwise regular pendulum, the correspondence breaks down. Also a pendulum composed of a chain of rods or other rigid bodies leads to a different set of polynomials, which do not coincide with any of the well-known orthogonal polynomials.

[1] L. D. Landau and E. M. Lifschitz: *Lehrbuch der theoretischen Physik. Band III: Quantenmechanik.* Dritte, berichtigte Auflage. Akademie-Verlag, Berlin, 1967.

[2] P. Hagedorn: *Technische Schwingungslehre.* Band 2: Lineare Schwingungen kontinuierlicher mechanischer Systeme. Springer-Verlag, Berlin · Heidelberg · New York, 1989.

[3] F. G. Tricomi: *Vorlesungen über Orthogonalreihen.* Zweite, korrigierte Auflage, Springer-Verlag, Berlin · Heidelberg · New York, 1970.

How to Contribute to the Newsletter

Send your Newsletter contributions directly to the *Editor:*

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preferably by e-mail, and in LaTeX format. Other formats are also acceptable and can be submitted by e-mail, regular mail or fax.

Deadline for submissions to be included in the October issue 1997 is September 15, 1997.

The Activity Group also sponsors an electronic news net, called the **OP-SF Net**, which is transmitted periodically by SIAM. The Net provides a rather fast turnaround compared to the Newsletter. To *receive* transmissions, just send your name and e-mail address to poly-request@siam.org (as with other nets, nonmembers can also receive the transmissions). Your OP-SF Net *contributions* should be sent to poly@siam.org. Please note that submissions to OP-SF Net are automatically considered for publication in the Newsletter, and vice versa, unless the writer requests otherwise.

The Net is organized by Tom Koornwinder (thk@wins.uva.nl) and Martin Muldoon (muldoon@yorku.ca). Back issues of OP-SF Net can be obtained by anonymous ftp from `ftp.wins.uva.nl`, in the directory

pub/mathematics/reports/Analysis/koornwinder/opsfnet.dir

or by WWW at the addresses

ftp://ftp.wins.uva.nl/pub/mathematics/reports/Analysis/koornwinder/opsfnet.dir

http://www.math.ohio-state.edu/JAT

Martin Muldoon, moreover, manages our home page

http://www.math.yorku.ca/Who/Faculty/Muldoon/siamopsf/

on World Wide Web. Here you will find also a WWW version of the OP-SF Net. It currently covers the topics

- Conference Calendar
- Books, Conference Proceedings, etc.
- Compendia, tools, etc.
- Meeting Reports
- Projects
- Problems
- Personal, Obituaries, etc.
- History
- Positions available
- Miscellaneous

Activity Group: Addresses

The *SIAM Activity Group on Orthogonal Polynomials and Special Functions* consists of a broad set of mathematicians, both pure and applied. The Group also includes engineers and scientists, students as well as experts. We now have around 150 members scattered about in more than 20 countries. Whatever your specialty might be, we welcome your participation in this classical, and yet modern, topic. Our WWW home page http://www.math.yorku.ca/Who/Faculty/Muldoon/siamopsf/ is managed by Martin Muldoon (muldoon@yorku.ca).

The **Newsletter** is a publication of the *SIAM Activity Group on Orthogonal Polynomials and Special Functions*, published three times a year. To receive the Newsletter, you must first be a member of SIAM so that you can join the Activity Group. The annual dues are $93 for SIAM plus $10 for the Group. To join, contact:

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