# Orthogonal Polynomials and Special Functions

*SIAM Activity Group on Orthogonal Polynomials and Special Functions*

★ ★ ★ ★ **Newsletter** ★ ★ ★ ★

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## From the Editor

Beginning in September 1997 I was appointed Professor of Applied Mathematics at the Hochschule für Technik, Wirtschaft und Kultur Leipzig, Germany, Department of Computer Science, Mathematics and Natural Sciences (Informatik, Mathematik und Naturwissenschaften). My new address there is:


Prof. Dr. Wolfram Koepf
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At least for the next few months, I will, however, be still affiliated with Konrad-Zuse-Zentrum so that you can send your messages to both places, and my old WWW homepage will still be valid.

These changes are time-consuming, and I am indebted to Tom Koornwinder and Martin Muldoon who do a great job as Editors of the electronic newsletter OP-SF Net. Fortunately I could borrow much material from them. Indeed, their material was the main source for my collection today.

I would like to announce that I will resign my editorship at the end of the period of my appointment, i.e. by the end of next year, and although this is more than a year away, candidates to fill the position should get in touch with me without delay. It would be probably much easier for a new editor to do the editorial work jointly with me for the next year before taking over.

I would like to thank the Editors of SIAM Review and of the American Mathematical Monthly who gave their permission to reprint two reviews which may be of interest to our readers, one by myself (SIAM Review) on the book $A = B$ by Marko Petkovšek, Herbert S. Wilf and Doron Zeilberger, and the second one by Jet Wimp (Amer. Math. Monthly) on the book and CD-ROM versions of *Table of Integrals, Series, and Products* by Gradshteyn and Ryzhik, edited by Alan Jeffrey. Note that an independent review by Marvin Rosenblum of the CD-ROM version already appeared on p. 11 of Newsletter 7-1.

In future issues of the Newsletter I would like to incorporate more book reviews rather than just announcements. This procedure entitles the reviewers to receive free copies of the books under review. Such reviews should have more depth than announcements and might be more helpful for the readers. Hence I would like to ask everybody who is willing to write such a review to make suggestions on books to be reviewed. Then I will try to receive these books from the publishers and send them to the reviewers.

I am very happy that Dick Askey and George Gasper were able to solve one of the early problems of the Problems Section. You will find their solution of Problem 2 on p. 18.

As usual I hope you enjoy reading this issue!

September 30, 1997 Wolfram Koepf

## Message from the Chair: Charter Renewal

Dear Colleagues,

On June 12, 1997 I sent an application for renewal of our Charter (see below) to Allison Bogardo at SIAM. This was her official answer.

> Dear Charles:
>
> I am pleased to advise you that the SIAM Council and Board of Trustees approved the renewal of the charter for the SIAM Activity Group on Orthogonal Polynomials and Special Functions. The charter is for a three-year period beginning January 1, 1999 and ending December 31, 2001.
>
> We will be in touch with you in the spring of 1998 to begin work on your activity group ballot. In the meantime, if there is any other way we can be of assistance to you and your activity group, please do not hesitate to be in touch with me.
>
> Thank you.
>
> Allison Bogardo (bogardo@siam.org)

## Charter Renewal Application

This *Charter Renewal Application* (hereinafter called "RENEWAL") applies to the SIAM Activity Group on Orthogonal Polynomials and Special Functions.

The SIAM Activity Group (hereinafter called "SIAG") to which this RENEWAL applies was originally formed under the aegis of the Society for Industrial and Applied Mathematics (hereinafter called "SIAM") in July 15, 1990 by the SIAM Council and July 19, 1990 by the SIAM Board of Trustees with its initial operating period beginning Jan. 1, 1990 and ending Dec. 31, 1992.

Its charter has been renewed by the council and board two times thereafter. This SIAG has 133 members as of March 15, 1997.

According to its Rules of Procedure, the objectives of the SIAG are to promote basic research in orthogonal polynomials and special functions; to further the application of this subject in other parts of mathematics, and in science and industry; and to encourage and support the exchange of information, ideas and techniques between workers in this field, and other mathematicians and scientists.

Its proposed functions are as follows:

"The group is concerned with the following topics and their applications: general systems of orthogonal polynomials - asymptotic analysis, three-term recurrence relations and Markov processes, numerical quadrature, Julia sets, least-squares of orthogonal polynomials - harmonic analysis, approximation theory, representations of compact groups, quantum mechanics, combinatorics, coding and design theory; orthogonal polynomials in several variables - Lie groups, tomography, optics, wave functions in crystals; special functions - for example, Bessel, gamma, theta, spheroidal wave, etc., solutions of partial differential equations, harmonic analysis of noncompact groups, statistical mechanics, integral transforms, number theory.

"Activities will include dissemination of information about upcoming conferences and sponsoring special sessions at SIAM meetings. Also, the group will assist researchers in the use of symbolic computer calculations by publicizing available software for special functions. Another goal is to establish some working relationships with the various SIAM journals, especially the one on mathematical analysis, with the view of sporadically sponsoring some invited or contributed articles."

It has complemented SIAM's activities and supported its proposed functions as follows: The SIAG has sponsored a minisymposium at each of the recent annual SIAM meetings. There is a printed newsletter which has three issues annually. The electronic news service OP-SF Net is produced about six times per year. There is an ftp site for electronic circulation of preprints which is maintained by one of the SIAG's members, and it is linked to the SIAG's permanent web site. The SIAG has been involved in disseminating news about current projects of several groups of workers who are planning handbooks of special functions, in print or digital forms. These handbooks are intended to be used by the general mathematical and scientific public. The vice chair of the SIAG has had discussions with a new mathematical society dedicated to special functions, based in India, regarding possible cooperation and links.

In an effort to continue to foster activities and interaction between members of this group and the special functions community in general, this SIAG has planned and proposes the following activities: There will be a minisymposium at the July 1997 meeting in Stanford regarding the handbook projects. There will be speakers representing the NIST project revising the well-known Abramovitz and Stegun tables and the "Askey-Bateman" project, which is taking the Erdelyi-Bateman "Higher Transcendental Functions" series as its beginning point. These are both exciting projects which should lead to much more useful ways of retrieving information about and using special functions. One will be able to compute both symbolically and numerically, locate references, and find useful facts. The newsletters will continue to serve researchers and users of special functions, with useful news about conferences, book and software reviews, and pointers to the existing literature for the non-specialists. This list of instructive survey articles and texts is a new feature of the print newsletter and has been well received.

This SIAG requests that the SIAM Council and Board of Trustees renew its charter for a three-year operating period beginning Jan. 1, 1999.

Charles F. Dunkl
(cfd5z@virginia.edu)

## Reports from Meetings and Conferences

### 1. First ISAAC Conference: University of Delaware, Newark, Delaware, June 3-7, 1997

Session 13 at the First ISAAC Conference (*International Society for Analysis, its Applications and Computation*), University of Delaware, Newark, Delaware, June 3–7, 1997, was devoted to *Orthogonal Polynomials*, and was organized by Wolfram Koepf. The session took place on June 6 and June 7, and on June 6 there was also a plenary talk by Wolfram Koepf on *Orthogonal Polynomials and Computer Algebra*. Since the other sessions of the conference were relatively distant from our topic, the session had a rather firm attendance of 15 to 30.

The program follows:

**June 6**

- **André Ronveaux:** *Recurrence Relations for Connecting Coefficients Between Some Orthogonal Polynomials Families–A Simple Algorithm (Mathematica)*
- **Ivan Area**, E. Godoy, A. Ronveaux and A. Zarzo: *Inversion Problems for Classical Orthogonal Polynomials and their q-analogues*
- **A. Zarzo:** *Spectral Properties of Orthogonal Polynomials: A Computer Algebra Approach*
- **Tom H. Koornwinder**: *rec2ortho: Three-term Recurrence Relations and the Askey Scheme*
- **Doron Zeilberger** (talk presented by Tewodros Amdeberhan): *The Super-Holonomic Hierarchy*
- **Tewodros Amdeberhan:** *Computer Aided Proofs of a Determinant Identity*
- **John Majewicz:** *On a Positivity Conjecture of Richard Askey*
- **Richard A. Askey:** *Some Problems on Orthogonal Polynomials*

**June 7**

- **Alan Schwartz:** *Polynomials of Several Variables and Harmonic Analysis*
- **Natig M. Atakishiyev:** *On the Fourier-Gauss Transforms of some q-exponential and q-trigonometric Functions*
- **Charles F. Dunkl:** *Using Maple to Explore Special Functions of Several Variables*
- **Victor Adamchik:** *On Series Involving the Riemann Zeta Function*
- **Yang Chen**, Mourad Ismail: *Asymptotics of the Largest Zeros of Some Orthogonal Polynomials*
- **Kelly Roach:** *Maple and Orthogonal Polynomials*

Many participants told me that the session was a pleasure. My feeling was that the attendees liked the session a lot, and they liked the brewery in Main Street.

A Special Issue of the *Journal of Symbolic Computation* on *Orthogonal Polynomials and Computer Algebra* will be dedicated to the subject of this session and will include some of the presented lectures, see p. 16. I am very happy that Dick Askey and Tom Koornwinder have agreed to coedit this volume with me.

Wolfram Koepf
(koepf@zib.de)

### 2. SIAM Annual Meeting: Stanford, July 14-18, 1997
### Minisymposium: Handbooks for Special Functions and the World Wide Web

The *Activity Group on Orthogonal Polynomials and Special Functions* sponsored the Minisymposium *Handbooks for Special Functions and the World Wide Web* that was held on July 14, 1997 at the SIAM Annual Meeting at Stanford University. The principal handbooks on special functions, the "Bateman Project" and the NIST (formerly National Bureau of Standards) "Handbook of Mathematical Functions," are among the most useful, widely consulted technical volumes ever published, but they are now out of date, due to rapid research progress and revolutionary changes in technology. The minisymposium was organized by Dick Askey and Willard Miller, and featured talks by representatives of the groups that are proposing to update the Bateman Project (Mourad Ismail) and Abramowitz & Stegun (Dan Lozier), respectively, a presentation on the development of a Mathematica Special Functions Handbook (Olag Marichev and Paul Wellin), and an assessment of the historical influence of special functions handbooks (Dick Askey), followed by a general discussion.

Willard Miller chaired the session and started with some desirable criteria for handbooks:

1. Present, codify and organize the principal results and tools in appropriate parts of the field of special functions. Provide insight into the structure of the theory.
2. Provide easy accessibility for users.
3. Ensure long term impact. Do carefully (accurately) and guided by the most knowledgeable experts in the field. Make easy to update.

Dan Lozier talked about the planning for publication on the World Wide Web of a modernized and updated revision of the NBS Handbook of Mathematical Functions, first published in 1964.[1] The authoritative status of the original publication is to be preserved by enlisting the aid of qualified mathematicians and scientists. However, there will be increased practical emphasis on formulas, graphs

[1] see p. 22

and numerical evaluation through the provision of interactive capability to permit generation of tables and graphs on demand. The "handbook" will be available at a Web site and will involve a digital library.

Mourad Ismail discussed plans for updating the Bateman project, both to reflect progress made on topics covered in the original (written in the early 1950's) and to add topics of current importance that were not in the initial project. In particular, the plan is for major additions on one and multi-variable orthogonal polynomials and on $q$-series. Mourad emphasized the importance of involving leading experts in the project. The plan is that the update should be published in a paper version, with formulas (and corrections) available via ftp or on a Web site. One of the arguments that he presented for a paper version was that many third world mathematicians and users of mathematics would not have access to a Web-based product. Cambridge University Press has shown interest in the project and some funding has been received from the National Security Agency. Major funding is being sought from the National Science Foundation.

Oleg Marichev presented current work on the Mathematica Interactive Special Functions Handbook. The Wolfram Research representatives emphasized the importance of using recent Web-based technological developments in a handbook project. Their concept was to have a paper version of the handbook and a World Wide Web version. On the Web version search engines could be used, formulas could be down-loaded, manipulated with Mathematica, and data could be generated and plotted. Hyperlinks could be used to link formulas with the original reference where they are derived. Updating and correcting would be simple on the Web. Their message was that the Mathematica system should be used as the underlying system for an update of either the NBS or the Bateman project.

Dick Askey talked about the influence, for good or ill, of special function handbooks through the decades. Many of his examples illustrated the value of involving leading researchers in these projects. Their input is critical in deciding what material to include and how to present it.

The issues raised at the Minisymposium are of importance for all in the special functions user community. The need for updating and codifying the principal results of the theory are clear, but the sources of funding and the optimal delivery system are not yet determined. Clearly, special function handbooks will continue to be produced. If the special functions research community is to have much influence on their content, we need to resolve these issues now.

Willard Miller, Jr.
(miller@ima.umn.edu)

## Forthcoming Meetings and Conferences

### 1. Tricomi Centennial Conference, Rome, November 28-29, 1997, *and* Turin, December 1-2, 1997

*Tricomi's Ideas* and *Contemporary Applied Mathematics*: Convegno Internazionale in occasione del *Centenario della nascita di Francesco G. Tricomi*

The *Accademia Nazionale dei Lincei and the Accademia delle Scienze di Torino*, jointly with the *Universitá di Torino* and the *Politecnico di Torino*, organize an international conference to celebrate the 100th anniversary of the birth of **Francesco G. Tricomi**.

The main subjects of the conference will be: Partial Differential Equations, Singular Integral Equations, Transonic Aerodynamics, Special Functions and Ordinary Differential Equations. The first three subjects will be discussed in Rome, and the other two in Turin.

Tricomi's work on these subjects has been of fundamental importance and the conference intends to focus the influence of Tricomi's ideas in contemporary applied mathematics, giving at the same time a picture of the state of the art.

**Organizing Committee**: G. Allasia, L. Amerio, A. Conte, D. Galletto, L. Gatteschi, P. Germain, G. Grioli, E. Magenes, E. Marchi, C. Morawetz, S. Nocilla, O. Oleinik, R. Piva, G. Salvini, E. Vesentini.

**Local Organizing Committee (Turin)**: G. Allasia, S. Benenti, A. Conte, R. Conti, D. Galletto, B. Gabutti, L. Gatteschi, F. Lerda, R. Malaroda, S. Nocilla, F. Skof, E. Vesentini.

**Conference Locations**: Rome, Accademia Nazionale dei Lincei, Palazzo Corsini, Via della Lungara 10 and
Turin, Universitá degli Studi, Aula Magna, Via Verdi 8; Accademia delle Scienze di Torino, Via Maria Vittoria 3

**Program**
Friday, November 28
Accademia Nazionale dei Lincei

9.30: **Opening**

10.00: **P. Germain**, Paris, France: *Tricomi problem and fundamental solution for Tricomi problem.*

11.00: **Coffee-break**

11.15: **G. Moretti** Burlington, Vermont, USA: *Lights and shadows of transonic aerodynamics across a century.*

12.15: **D. Gottlieb**, Providence, Rhode Island, USA: *The use of special functions in the numerical solutions of nonlinear hyperbolic equations.*

15.00: **W. Wendland**, Stuttgart, Germany: *On boundary integral equations and applications.*

16.00: **Coffee-break**

16.15: **J.-C. Nedelec**, Palaiseau, France: *The use of integral equations for harmonic Maxwell equations.*

17.15: **S. Proessdorf**, Berlin, Germany: *Approximation methods for integral equations using splines and wavelets.*

Saturday, November 29
Accademia Nazionale dei Lincei

9.30: **E. I. Moiseev**, Moscow, Russia: *On spectral problems for the Tricomi equation.*

10.30: **G. Monegato**, Torino, Italy: *Numerical resolution of the generalized airfoil equation with Possio kernel.*

11.30: **Coffee-break**

11.45: **O. A. Oleinik**, Moscow, Russia: *Free boundary problems for backward parabolic equations.*

Monday, December 1
Aula Magna, Universitá di Torino

9.30: **Opening**

10.00: **A. Conte**, Torino, Italy: *Francesco G. Tricomi maestro a Torino.*

11.50: **Coffee-break**

11.20: **R. Askey**, Madison, Wisconsin, USA: *Hermite and Laguerre polynomials and extensions.*

Accademia delle Scienze di Torino

15.00: **L. Gatteschi**, Torino, Italy: *New results on some two-dimensional iterative algorithms.*

15.50: **B. C. Carlson**, Ames, Iowa, USA: *Elliptic integrals: symmetry and symbolic integration.*

16.40: **Coffee-break**

17.10: **N. Temme**, Amsterdam, The Netherlands: *Recent problems from uniform asymptotic analysis of integrals.*

Tuesday, December 2
Accademia delle Scienze di Torino

9.30: **W. Gautschi**, West Lafayette, Indiana, USA: *The incomplete gamma function since Tricomi.*

10.20: **F. W. J. Olver**, College Park, Maryland, USA: *Asymptotic and numerical solutions of linear differential equations.*

11.10: **Coffee-break**

11.40: **J. Mahwin**, Louvain la Neuve, Belgium: *The forced pendulum equation: a challenging problem for the qualitative theory of ordinary differential equations.*

15.00: **F. Lerda**, Torino, Italy: *Formally linear methods for nonlinear ordinary differential equations.*

15.50: **F. Zanolin**, Udine, Italy: *Time-maps and boundary value problems for ordinary differential equations.*

16.40: **Coffee-break**

17.10: **E. Regazzini**, Milano, Italy: *Some examples of the interplay between special functions and statistics.*

**Further information**:
e-mail: ticam@dm.unito.it
fax: +39 11 670 2878 (to Giampietro Allasia)

**Registration for Turin**: The Local Organizing Committee needs to know some preliminary data on participants.

Please ask for a registration form (with information about hotel reservation) by e-mail or fax.

Registration and hotel reservation **no later than October 31, 1997** (if possible).

Giampietro Allasia
(ticam@dm.unito.it)

### 2. Applications and Computation of Orthogonal Polynomials, Oberwolfach, Germany, March 22-28, 1998

A meeting on *Applications and Computation of Orthogonal Polynomials* will be held at Mathematisches Forschungsinstitut Oberwolfach, Germany from March 22 to March 28, 1998.

The organizers are

Walter Gautschi, West Lafayette
Gene H. Golub, Stanford
Gerhard Opfer, Hamburg

Participants of the meetings at Oberwolfach are invited personally by the director of the institute. The participation is subject to such an invitation. The e-mail address for the administration at Oberwolfach is admin@mfo.de.

Interested researchers, in particular young mathematicians, can contact the administration of the institute. Since the number of participants is restricted, not all enquiries can be considered.

URL of Oberwolfach: http://www.mfo.de/

Tom H. Koornwinder
(thk@wins.uva.nl)

### 3. Symmetries and Integrability of Difference Equations: Sabaudia, Italy, May 16-22, 1998

First Announcement of the *3rd International Interdisciplinary Meeting on Symmetries and Integrability of Difference Equations*, Sabaudia, near Rome (Italy), 16–22 May 1998.

This series of international meetings started in 1994: the first *Symmetries and Integrability of Difference Equations* (SIDE) meeting was held in Esterel, Quebec (near Montreal - Canada) and was organized under the auspices of the CRM (Centre de Recherches Mathématiques) of the

Université de Montreal. A second meeting took place in 1996 at the University of Kent in Canterbury (UK). Informations on the 1st SIDE meeting has been reported in: *Symmetries and Integrability of Difference Equations*, edited by D. Levi, L. Vinet and P. Winternitz, AMS 1996.

The meeting is intended to bring together specialists from various disciplines, all working or using methods from discrete integrable systems, i.e. systems that can be described by ordinary or partial difference equations and that allow for exact methods for their solutions. This domain forms the core of a great variety of fields, including classical and quantum physics, computer science, mathematical biology, economics, numerical analysis, difference geometry, and so on.

The SIDE meetings want to be a point of contact between researchers of various disciplines on discrete systems who otherwise would not be able to interact among themselves. The participation of young scientists is encouraged. The main topics of the present meeting will be: integrable difference equations, symmetries of ordinary and partial difference equations and reduction techniques, integrable correspondences, asymptotics and difference monodromy problems, orthogonal polynomials and $q$-special functions, discrete geometry, applications to computer science, neural networks, physical, biological, and economical systems.

As in the previous meetings of the series, to enhance the interactions and to promote informal contacts, all the participants will be accomodated under the same roof, that in this case will be the Hotel "Oasi di Kufra", as the venue where the lectures are delivered and where the social activities are organized (meals, receptions, etc.). So as not to overload the meeting with talks, while keeping the single-session format, we plan to accept a limited number of participants ($< 50$). The format of the meeting consists of a certain number of short lectures (30 minutes) and of a few longer review expositions intended for students and young researchers. A poster session will be available for presenting partial or side results.

The cost of participation will consist of a registration fee of Lit. 150000 (equivalent to $\approx$ 90 US$) if paid before March 15th, 1998, otherwise Lit. 180000 ($\approx$ 110 US$) and of a flat all-inclusive rate of Lit. 810000 ($\approx$ 480 US$) (+ Lit. 180000 for single occupancy rooms). This flat rate might be partly or completely waived for qualified participants who have no funds to support their participation if our applications for financial support will be successful. The listed prices are subject to small variations according to fluctuations of the exchange rates.

Interested persons should contact the local organizers, preferably by e-mail. Please take into account that there will be room for, at most, 20 talks (30 minutes). Consequently the scientific committee will decide whether the applicant's contribution can be accepted as a talk or as a poster.

The following speakers have already confirmed their participation:

- C. Brezinski (U. Lille - France): *Difference equations and convergence acceleration methods*
- P. Clarkson (U. Kent - UK): *Exact solutions of discrete Painlevé equations*
- V. Enolsky (U. Kiev - Ukraine): *Addition laws on hyperelliptic Jacobian and integration of discrete systems of KdV type*
- F.A. Grunbaum (U. California, Berkeley - USA) - to be announced
- R. Hirota (Waseda U. - Japan): *Discretization of integrable systems*
- M. Ismail (Univ. of South Florida - USA): *Asymptotics of recursion coefficients and zeros of orthogonal polynomials*
- I. Krichever (Columbia U. - USA) - to be announced
- T. Miwa (RIMS Kyoto - Japan): *Quantum Knizhnik-Zamolodchikov equation and its applications to integrable systems*
- L. Nieto (U. Valladolid - Spain) - to be announced
- R. Quispel (Latrobe U., Melbourne - Australia): *Symmetries and integrability of difference equations*
- A. Ramani (CNRS - France) - to be announced
- G. Rollet (U. Paris Cergy - France) - to be announced
- P. Santini (U. Catania - Italy): *Discrete geometry and integrable difference equations*
- J. Satsuma (U. Tokyo - Japan): *Nonautonomous soliton equations and their solutions*
- W. Schief (UNSW, Sydney - Australia): *The geometry of discrete integrable systems*
- V. Spiridonov (Dubna - Russia): *Self-similar potentials and q-special functions*
- Yu. Suris (U. Bremen - Germany): *Nonlocal quadratic Poisson algebras and constrained lattice KP*
- M. Tarlini (U. Firenze - Italy): *Induced representations of the Quantum Galilei Group and integrable models in 1 space dimension*
- W. Van Assche (U. Louvain - Belgium): *Non-symmetric linear difference equations for polyorthogonal polynomials*
- A.P. Veselov (U. Loughborough - UK): *Laplace - Darboux lattices and spectral theory of Schroedinger operators*
- L. Vinet (CRM - Canada): *Algebraic treatment of integrable difference equations and special functions*
- R. Yamilov (Ufa - Russia): *Formal symmetry approach to the classification and testing of integrable differential difference equations*

The local organizers are:

D. Levi, O. Ragnisco
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http://www.roma1.infn.it/~ragnisco/side/sideiii.htm

The International Scientific Committee is: F.W. Nijhoff (U. Leeds - UK) chairman, A. Bobenko (TU Berlin - Germany), J. Hietarinta (U. Turku - Finland), N. Joshi (U. Adelaide - Australia), M. Kruskal (Rutgers U. - USA), D. Levi (U. Roma 3 - Italy), V. Papageourgiou (U. Patras - Greece), C. Viallet (U. Paris VI - France), P. Winternitz (U. Montreal - Canada)

Tom H. Koornwinder
(thk@wins.uva.nl)

### 4. International Workshop on Orthogonal Polynomials: Numerical and Symbolic Algorithms, Madrid, June 29-July 2, 1998

**First announcement**

It is well-known that increasing attention has been paid in recent years to the theory of Orthogonal Polynomials. This is due, in particular, to their applications in areas like numerical integration, spectral methods, interpolation, approximation theory, etc. and also in combinatorics, mathematical physics, quantum physics, etc. For this reason the *Universidad Carlos III de Madrid* organizes an international workshop devoted to this topic every two years. The first workshop in 1992 was dedicated to Sobolev orthogonal polynomials, the second, in 1994, to polynomials orthogonal on the unit circle, and the most recent one, in 1996, to the applications of orthogonal polynomials in mathematical physics.

The main **aim** of the next (1998) Workshop is that a relatively small number of invited mathematicians discuss and review recent progress in the Theory of *Orthogonal Polynomials* with special emphasis on numerical applications and symbolic algorithms. The Workshop will take place in the main building of the Escuela Politécnica Superior, Universidad Carlos III de Madrid, Leganés (Madrid).

**The topics to be considered will be:**

1. Quadrature formulas
2. Spectral methods in boundary value problems
3. Numerical Linear Algebra
4. Symbolic algorithms and software
5. Combinatorics

It will be possible for interested participants to present their own contributions in the above mentioned areas. Because the limited number of short communications we ask participants who want to present their works to send us, as soon as possible (March 31, 1998), the abstract (no more than one page). Priority will be given to those talks closely related to the main subject of the Workshop.

**The invited speakers are:**

- Walter Gautschi (Purdue University, USA)
- Gene Golub (Stanford University, USA)
- Wolfram Koepf (Konrad-Zuse-Zentrum, Berlin, Germany)
- Yvon Maday (Université Pierre et Marie Curie, France)
- Marko Petkovšek (University of Ljubljana, Slovenia)
- Doron Zeilberger (Temple University, USA)

**The Proceedings:** We will prepare a special monograph containing the Proceedings of the Workshop.

**Registration fee:** 15.000 ptas.; includes lunch and the Proceedings.

**The Organizing Committee is:**

- M. Alfaro (Univ. de Zaragoza),
- R. Álvarez-Nodarse (Secretary) (Univ. Carlos III),
- J. Arvesú (Univ. Carlos III),
- F. Marcellán (Chairman) (Univ. Carlos III).

**The Scientific Committee is:**

- R. Álvarez-Nodarse (Univ. Carlos III),
- J. S. Dehesa (Univ. de Granada),
- E. Godoy (Univ. de Vigo),
- G.López Lagomasino (Univ. Carlos III),
- F. Marcellán (Chairman) (Univ. Carlos III) and
- A. Zarzo (Univ. Politécnica de Madrid).

**Invited Talks (60 min.):**

- Walter Gautschi, *Orthogonal Polynomials and Quadrature* and *Rational Gauss-type Quadrature Rules*
- Gene Golub, *Matrices, moments and quadrature* and *Solution of regularized systems*
- Wolfram Koepf, *Software for the Algoritmic Work with Orthogonal Polynomials and Special Functions*
- Marko Petkovsek, *(no title yet)*
- Doron Zeilberger, *The Unreasonable Power of Orthogonal Polynomials in Combinatorics, I and II*
- Yvon Maday, *(no title yet)*

To get more information please contact:

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For updated information visit the IWOP'98 WWW page http://dulcinea.uc3m.es/users/workshop/iwop98.html. On http://dulcinea.uc3m.es/users/workshop/iwop96.html you will find information about the most recent Workshop on *Orthogonal Polynomials* held in Leganés on June 24-26, 1996.

R. Álvarez-Nodarse
(renato@dulcinea.uc3m.es)

### 5. 4th International Conference on Lattice Paths Combinatorics and Applications, Vienna, Austria, July 8-10, 1998

This conference is dedicated to the Memory of T.V. Narayana. Topics to be covered by the conference include

- lattice paths and boundaries
- plane partitions
- Young tableaux
- $q$-calculus
- orthogonal polynomials
- random walk problems
- nonparametric statistical inference
- discrete distributions and urn models
- queueing theory
- analysis of algorithms

**Submission of papers:** Authors are invited to submit abstracts of at most four pages before February 1, 1998. Preferred way of submission is by sending **one** postscript file by email to boehm@isis.wu-wien.ac.at.

If an author is not able to send a postscript version of her/his extended abstract, four copies of the extended abstract should be mailed to Walter Böhm, address below. Authors are also requested to indicate how much time they will need to present their talks.

The complete versions of the papers to be presented should be received not later than July 10, 1998. After a standard refereeing process papers accepted by the scientific committee will be published in a special issue of the *Journal of Statistical Planning and Inference*.

**Location:** The conference will take place at the Institut für Mathematik of the Universität Wien. The first talk is scheduled on July 8, 1998 at 9:00 a.m.

**Organizing committee:** W. Böhm, University of Economics, Vienna, Austria; Ch. Krattenthaler, University of Vienna, Austria; S.G. Mohanty, McMaster University, Canada; K. Sen, University of Delhi, India;

**Scientific committee:** N. Balakrishnan, McMaster University, Canada; Ch. Charalambides, University of Athens, Greece; E. Csaki, Hungarian Academy of Science, Hungary; I. Gessel, Brandeis University, U.S.A.; A.W. Kemp, University of St. Andrews, Scotland; C.D. Kemp, University of St. Andrews, Scotland; S.G. Mohanty, McMaster University, Canada; H. Niederhausen, Atlantic University, U.S.A.

**Further information:** A WWW site http://www.wu-wien.ac.at/wwwu/institute/stat1/lp/lp.html has been set up for the conference which will always contain the latest state of affairs. For any further question, please just write to

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Please communicate this message also to other people which you think are interested in this conference.

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### 6. SIAM Annual Meeting 1998: Toronto, Canada, July 13-17, 1998

The 1998 SIAM Annual Meeting is scheduled for July 13–17 and will take place in Toronto. The meeting is co-chaired by Max Gunzburger (Iowa State University ), Kenneth R. Jackson (University of Toronto) and Roy Nicolaides (Carnegie Mellon University).

Martin Muldoon
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## Books and Journals

### Announcements

**1. Integral Expansions Related to Mehler-Fock Type Transforms**
**By B.N. Mandal and Nanigopal Mandal**
Pitman Research Notes in Mathematics Series 367, 144 pp., 1997, ISBN 0-582-30816-X.

(The following announcement was found on the web site: http://www.awl-he.com/advmaths/titles/367.html)

An important class of integral expansions generated by Sturm-Liouville theory involving spherical harmonics are commonly known as Mehler-Fock integral transforms. In the inverse transformation formulae, the subscript of the associated Legendre functions generally appears as the integration variable while the superscript is either zero or an integer or a complex number. There is another class of integral transforms involving associated Legendre functions for which the superscript appears as the integration variable in the inverse transformation formulae while the subscript remains fixed. This class of integral transforms and the associated integral expansions are not widely known in the literature and they are somewhat related to Mehler-Fock type transforms. In this book, a number of integral expansions of such type have been established rigorously and integral expansions of some simple functions are also obtained as applications. It is hoped that this book will motivate researchers in applied mathematics to initiate work on appropriate physical problems in continuum mechanics and electromagnetic wave diffraction involving conical, toroidal, and ellipsoidal regions.

Readership: Researchers and graduate students in applied mathematics and engineering, engineers and physicists working in electromagnetic diffraction problems and elasticity problems.

Contents:



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**2. Proceedings of the Organic Mathematics Workshop**
**Ed. by Jonathan Borwein, Peter Borwein, Rob Corless, Loki Jörgenson and Nathalie Sinclair**

Information on this publication is available at: http://www.cecm.sfu.ca/organics/contents.html.
From the mission statement:

The Organic Mathematics Project (OMP) is directed towards the exploration of the emerging network and information technologies within the context of mathematics. The end product will be a digital proceedings of the Workshop on Organic Mathematics on December 12 to 14, 1995 at Simon Fraser University.

See as an example the invited article *Pfaff's Method (III): Comparison With the WZ Method* by George Andrews.

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**3. Rivlin Festschrift**
**Special Issue of Annals of Numerical Mathematics**
**Ed. by Charles A. Micchelli**
Annals of Numerical Mathematics, Vol. 4, 1997

Annals of Numerical Mathematics, Vol. 4, 1997 is a special issue on *The heritage of P.L. Chebyshev: A Festschrift in honor of the 70th birthday of T.J. Rivlin*. The following table of contents is found at the URL: http://www.baltzer.nl/anuma/an4-97.html . A considerable part of the included papers deals with orthogonal polynomials.

- A. Cuyt and D.S. Lubinsky: A de Montessus theorem for multivariate homogeneous Padé approximants
- C. de Boor: The multiplicity of a spline zero
- M.G. de Bruin and A. Sharma: Overconvergence of some simultaneous Hermite-Padé interpolants
- R.A. DeVore, K.I. Oskolkov and P.P. Petrushev: Approximation by feed-forward neural networks
- N. Dyn: Uniqueness of least-norm generalized monosplines induced by log-concave weight-functions
- M.K. El-Daou and E.L. Ortiz: On upper bounds for the number of extrema of Chebyshev alternants
- D. Gaier: Polynomial approximation of functions continuous on $[-1, 1]$ and analytic on $(-1, 1)$
- W. Gautschi: On the computation of special Sobolev-type orthogonal polynomials
- W.M.Y. Goh, E. Schmutz and J. Wimp: On some recursive triangular systems
- A.J. Hoffman and C.A. Micchelli: On a measure of dissimilarity between positive definite matrices
- A.J. Hoffman, W.R. Pulleyblank and J.A. Tomlin: On computing $Ax$ and $\pi^T A$, when $A$ is sparse
- B. Kitchens: The dynamics of group automorphisms
- A. Kroo and D. Schmidt: A variational approach to optimizing linear functionals over Haar spaces
- A. Kroo, J. Szabados and R.S. Varga: Weighted polynomial approximation of some entire functions on the real line
- D. Leviatan and A. Shadrin: On monotone and convex approximation by splines with free knots
- A.P. Magnus: On optimal Padé-type cuts
- A.A. Melkman: Subdivision schemes with nonnegative masks converge always – unless they obviously cannot?
- C.A. Micchelli: On a measure of dissimilarity for normal probability densities
- D.J. Newman: A splitting problem
- V.Y. Pan, A. Zheng, X. Huang and Y. Yu: Fast multipoint polynomial evaluation and interpolation via computations with structured matrices
- G.M. Phillips: Bernstein polynomials based on the $q$-integers
- M.J.D. Powell: A new iterative algorithm for thin plate spline interpolation in two dimensions
- M.A. Qazi and Q.I. Rahman: The Fundamental Theorem of Linear Programming applied to certain extremal problems for polynomials
- R. Schaback: Optimal recovery in translation-invariant spaces of functions
- H.S. Shapiro: The Chebyshev constant of a linear set
- P. Vertesi: On the zeros of generalized Jacobi polynomials
- G. Wahba and Z. Luo: Smoothing spline ANOVA fits for very large, nearly regular data sets, with application to historical global climate data
- B. Weiss: Measurable entire functions
- H. Wozniakowski: Strong tractability of weighted tensor products
- Y. Xu: Summability of certain product ultraspherical orthogonal series in several variables
- Author index

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### 4. Logarithmic Potentials with External Fields
**By E. Saff and V. Totik**

Springer, Grundlehren der mathematischen Wissenschaften, Vol. 316, September 1997, DM 158, 525 pp., hardcover, ISBN 3-540-57078-0.

The following information is taken from the web site: http://www.springer.de/catalog/html-files/deutsch/math/toc/3540570780-c.html

This treatment of potential theory emphasizes the effects of an external field (or weight) on the minimum energy problem. Several important aspects of the external field problem (and its extension to signed measures) justify its special attention. The most striking is that it provides a unified approach to seemingly different problems in constructive analysis. These include the asymptotic analysis of orthogonal polynomials; the limited behavior of weighted Fekete points; the existence and construction of fast decreasing polynomials; the numerical conformal mapping of simply and doubly connected domains; generalization of the Weierstrass approximation theorem to varying weights; and the determination of convergence rates for best approximating rational functions.

Keywords: potential theory, mathematical physics. For graduate students and researchers in the above fields.

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## Reviews

### 1. $A = B$
**By Marko Petkovšek, Herbert S. Wilf and Doron Zeilberger**

AK Peters, Wellesley, 1996, $39.00. xii + 212 pp., ISBN 1-56881-063-6.

(Editor's Note: This review is reprinted with permission from SIAM Review, Volume 39, Number 3, pages 538-540, © 1997 by the Society for Industrial and Applied Mathematics.)

In their recent research, the authors of the book under review have given important contributions towards computer

proofs of hypergeometric identities. Hypergeometric identities are identites about hypergeometric sums, i.e., definite sums

$$S_n := \sum_{k\in\mathbb{Z}} F(n,k) \tag{1}$$

where the summand is a hypergeometric term with respect to both $n$ and $k$, i.e.; the term ratios

$$\frac{F(n+1,k)}{F(n,k)} \quad \text{and} \quad \frac{F(n,k+1)}{F(n,k)}$$

are rational functions in $n$ and $k$.

In the book under review, this knowledge is collected, and a nice introduction to the topic is given.

The main idea behind these computerized proofs is to detect a *holonomic recurrence equation* for the sum $S_n$ under consideration, i.e., a linear recurrence equation with polynomial coefficients. Zeilberger was the one having the idea how to adjust Gosper's algorithm on indefinite hypergeometric summation to the definite case.

Although many of these ideas can be generalized, e.g., towards the consideration of multiple sums, integrals, $q$-sums, the generation of differential rather than recurrence equations, etc., the authors are mainly concerned with the above mentioned setting.

The contents of the book follow:

Foreword: The foreword is written by Donald Knuth. He gives some examples of sums which he was investigating, and for which the new methods are great tools. The funny thing is that his main example,

$$S_n = \sum_{k\in\mathbb{Z}} \binom{2n-2k}{n-k}^2 \binom{2n}{k}^2$$

is slightly corrupted by a typographical error. This one has a recurrence equation whose printout covers a whole page, which shows the power and the pitfalls of Zeilberger's method at the same time! The sum Knuth really meant is the much more well-behaved equation

$$S_n = \sum_{k\in\mathbb{Z}} \binom{2n-2k}{n-k}^2 \binom{2k}{k}^2$$

which satisfies the simple recurrence equation

$$\begin{aligned} 0 \;=\; & (n+2)^3\, S_{n+2} - 8\,(3+2\,n)\left(2\,n^2+6\,n+5\right) S_{n+1} \\ & + 256\,(n+1)^3\, S_n\ . \end{aligned}$$

Note that an errata sheet can be found at the URL http://www.cis.upenn.edu/~wilf/AeqBErrata.html.

A Quick Start ...: Here, by a short example, it is shown how to download software in Maple and Mathematica from the World Wide Web, and how to deal with this software.

I Background

1. Proof machines: Canonical and normal forms are discussed, and it is shown how proofs can be given "by examples," using recurrence equations as normal forms. Polynomial, trigonometric, and other types of identities are discussed.

2. Tightening the target: Here the main topic of the book, the *hypergeometric identities*, are introduced. It is shown how Mathematica and Maple deal with hypergeometric sums, and WZ proof certificates (see Chapter 7) are introduced.

3. The hypergeometric database: A database of hypergeometric identities can be used to identify sums as soon as such sums are converted into hypergeometric notation. Here this conversion is considered.

II The five basic algorithms

4. Sister Celine's method: Celine Fasenmyer's method of finding a recurrence equation with respect to $n$ for a sum $S_n$ given by (1) is presented. Celine Fasenmyer uses linear algebra to detect a $k$-free recurrence equation with respect to both $n$ and $k$ for the *summand*, which afterwards is summed resulting in the recurrence equation searched for.

5. Gosper's algorithm: Gosper's algorithm finds a hypergeometric term antidifference $s_k$ for $a_k$, i.e., $s_{k+1} - s_k = a_k$, whenever such an antidifference exists. As a result, indefinite summation of hypergeometric terms can be treated algorithmically.

6. Zeilberger's algorithm: Zeilberger's algorithm uses a variant of Gosper's algorithm to determine holonomic recurrence equations for definite sums, given by (1). In most cases this recurrence equation is of lowest order. If it is of first order, then one can read off the hypergeometric term solution; if it is not, Petkovšek's algorithm, described in Chapter 8, can be used to determine such solutions if applicable.

Note that Zeilberger's algorithm in general is much faster than Celine Fasenmyer's method since its linear algebra part deals with mainly $J+1$ rather than with $(J+1)^2$ variables if $J$ denotes the order of the recurrence equation searched for.

7. The WZ phenomenon: In the cases in which Zeilberger's algorithm determines a first order recurrence equation, the WZ phenomenon occurs: such a hypergeometric identity can be proved by bringing it into the form

$$S_n := \sum_{k\in\mathbb{Z}} F(n,k) = 1\ , \tag{2}$$

and by using Gosper's algorithm to find a rational multiple $G(n,k) = R(n,k)\,F(n,k)$ of $F(n,k)$ for which

$$F(n+1,k) - F(n,k) = G(n,k+1) - G(n,k)\ . \tag{3}$$

Hence by summation $S_{n+1} - S_n = 0$ proving (2) (modulo one initial value). The rational function $R(n,k)$ is called the *WZ proof certificate*. Its knowledge makes a proof of (2) available by verifying a single rational identity.

8. Algorithm Hyper: Petkovšek's algorithm is a decision procedure to determine all hypergeometric term solutions of a given holonomic recurrence equation. It uses a representation lemma for rational functions initially due to Gosper, the *Gosper-Petkovšek representation*, in a clever way.

III Epilogue

9. An operator algebra viewpoint: The main theme of the book are holonomic recurrence equations. Using the shift operator $N\,a_n := a_{n+1}$, these can also be understood as operator equations, and one can deal with them in a non-commutative algebra where the commutator rule $Nn - nN = N$ is valid. In the given chapter this approach is considered in more detail.

In the Appendix the WWW sites and the software are discussed in more detail.

All algorithms that are discussed in the book under review are accompanied by examples and a few exercises for the reader, some of which come with solutions. Furthermore, the authors give examples for the use of Mathematica and Maple to do the computations. It is assumed that the reader has access to the World Wide Web or to other file transfer services, as well as to either Maple or Mathematica since the use of implementations of the algorithms considered seems to be a must.

The authors refer to Maple software available from Zeilberger's WWW site, and to Mathematica software due to Krattenthaler (hypergeometric database), Paule/Schorn (Gosper's and Zeilberger's algorithms) and Petkovšek (Petkovšek's algorithm). Implementational details are not discussed. Note that the Maple package `sumtools` written by the reviewer [2] comes with Maple V.4 and does also contain an implementation of both Gosper's and Zeilberger's algorithms.

The presentation of the book is charming, and it gives an excellent introduction to this modern topic. I would like to mention two minor inconveniences, though. First, the fact that the rational certificate of an application of Zeilberger's algorithm might contain poles with some obvious defects is not addressed. Second, I find it a little inconvenient that in some instances the authors use different notations at different places of the book. This might be influenced by the fact that the book forms essentially a collection of previously published material [1], [4], [5], [6], [7].

There is no need, e.g., for new notations for rising and falling factorials different from the ones given on pages 39 and 149, respectively, in the proof of the "Fundamental Theorem" on p. 66. In my opinion, this causes confusion. Similarly the footnote on p. 157 about the rising factorial notation is unnecessary since this definition is given on p. 39. Even worse, the mentioned footnote contains a *wrong* notation.

The authors mention the continuous analogues of the algorithms presented without giving the details. A forthcoming book by the reviewer [3] emphasizing the use of Maple for orthogonal polynomials and special functions will cover these topics.

One of the highlights of the presentation is the consideration of finite sums of hypergeometric terms. The authors show how Gosper's algorithm can be extended to this case. This previously unnoticed fact is rather important since summation is a linear operation, but Gosper's original algorithm is not.

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## 2. Tables of Integrals, Series and Products By I. S. Gradshteyn and I. M. Ryzhik, edited by Alan Jeffrey

Academic Press, New York, 5th Edition, 1996, Book and CD-ROM Version 1.0, ISBN 0-12-294756-8

(Editor's Note: This review is reprinted, with permission, from the American Mathematical Monthly, April 1997. We have

already published a short review by Marvin Rosenblum of the CD-ROM version in Newsletter 7-1.)

I *adore* GRJ (Gradstheyn, Ryzhik and Jeffrey), all 1200 pages of it. I have two copies: one for the office, one for home. If my upper body musculature were better developed, I would carry it to Europe with me, to Asia, to the Himalayas. Though many worthwhile identities are missing from GRJ, for instance those relating to differential and difference properties of hypergeometric functions, it can serve as a near equivalent to the combined fives volumes in the Erdélyi set [1], [2]. It's the ideal desert island book, though of course, one has to get it there, and it may not be easy explaining to one's ex-shipmates why this hefty and venerable tome should be given precedence over that moldy goatskin flask of water when planning the contents of the lifeboat.

Let me talk a bit about this volume, the book, not the CD I'm reviewing. Though called, modestly, *Table of Integrals, Series, and Products*, that doesn't begin to do justice to its contents. The appellation "sums" covers closed-form expressions of frequently occurring sums along with much material on the convergence of infinite sums. These topics, and products, functional series, asymptotic series and formulas from differential calculus, comprise the introductory chapter, the 0th chapter, of the volume. Also materializing in this chapter are some of those hoary, cabalistic functions that used to inhabit our mathematical books, but no longer do: the Gudermannian, Lobachevsky's "Angle of parallellism."

The first chapter is a treatment of elementary functions so comprehensive it will allow the owner to ditch those tattered trig tables. The second, third and fourth chapters deal with definite and indefinite integrals of elementary functions, and the fifth, sixth, and seventh deal with definite and indefinite integrals of special functions. Of course, to make the book self-contained, a discussion of special functions is required, and this the book has, in spades. Its eighth and ninth chapters comprise a wonderful 200 page treatment of all the standard higher functions, and many of the results given there are absent from the Erdélyi volumes — indeed, absent from any mathematical treatment commonly available. The formulas are a lot of fun to read, and one can, Jeopardy-like, shield from view the left hand side of an equation and ask what the right hand side represents, for instance:

ANSWER: The series

$$\sum_{k=0}^{\infty} c_k z^k, \quad |z| < 2,$$

where

$$c_{n+1} = \frac{\sum_{k=0}^{\infty} (-1)^{k+1} s_{k+1} c_{n-k}}{n+1}, \quad n = 0, 1, 2. \ldots,$$

with $c_0 = 1, s_1 = \gamma = .57721 \ldots, s_n = \zeta(n), n = 2, 3, \ldots$.

QUESTION: What is $\Gamma(z+1)$?

There are several supplementary chapters that practicing mathematicians will consider pure gold: a chapter on vector field theory, one on algebraic inequalities, one on integral inequalities, a chapter on matrices, one on determinants, one on norms, one on ordinary differential equations, and, to cap it off, a chapter containing Mellin, Fourier, and Laplace transforms; this chapter is vestigial, though, and doesn't offer a viable alternative to the standard tables [4], [5], [6].

The earlier editions had gobs of mistakes, but thanks to dedicated readers who filed their emendations with the editor, with the fifth edition most of the mistakes have been weeded out. I found one though (I almost had to to justify my credentials for writing this review.) The power $\beta$ on the right hand side of the transformation formula for the Appell function $F_4$, formula 3 on page 1083, should be $-\beta$.

Where, in its comprehensiveness, does this volume stand, in comparison to other tables? Well, it far surpasses the quaint handwritten two-volume 1965 table of integrals of Gröbner and Hofreiter [3]. The transform sections, as pointed out, suffer in comparison to other tables. It certainly contains far less material than the mammoth five-volume set by Prudnikov and others with its 3500 pages of material [7]. The scope of [7], though, probably exceeds what anyone will ever require. The reader may know of the Borges short story, "The Library of Babel" in which the author envisions a library whose first room contains a hundred or so books, the first containing a single page with the single letter "A," the second a page with the single letter "B,", etc. The second room of the library starts with a book containing a single page with the letters "AA," followed by a book containing "AB," ... . I'm sure the reader gets the idea. Any desired text you want will be somewhere in that library; the problem is only one of information retrieval. The Prudnikov volumes come close to being a mathematical equivalent to Borges' imaginary construction.

And the *presentation* of the GBJ book: the binding, the appearance, the typography, are all splendid. Remember the shabby photo offset reproductions of Russian books that held us hostage twenty or so years ago — the malodorous books on oatmeal colored paper with the English intertext and the Cyrillic formulas? Nothing could be aesthetically more distant from those books than this. Academic Press has compiled a gorgeous volume. Naturally, I can think of things that should have been included that weren't, but when I am searching for a formula vital to some research objective, its surprising how often it can be found in GRJ.

So what more remains to be said? Well, there's the old joke about the play heavily revised in tryouts in Philadel-

phia and Boston, and the producer saying to the playwright after a disastrous Broadway opening night, "You just died from improvement." Could this CD sound the death knell of a wonderful publishing concept?

On the drawing board, it must have seemed like a marvellous idea. A vastly popular enchiridion: let's make it available to anyone at the flick of a computer key. Let's make a CD out of it! Academic Press enlisted the talents of Lightbinders of San Francisco, who produced the CD using the opulent and flexible text display software called DynaText 2.3. I was impressed with the software, which might be a wonderful way to render some books computer accessible. But here, no. What went wrong? The very worst thing that could go wrong. *You can't see the formulas!* They are tiny, tiny, tiny. The liner notes for the CD cautioned that the reader should have Adobe File Manager to properly view the formulas. I ordered it. It helped not at all. One can do a mouse click in the upper right hand corner and things enlarge a bit, but not enough.

I noticed how Academic, perhaps reacting to an increasingly litigious society, had included in the flyleaf of the original book a warning:

> Academic Press and the editor have expended great effort to ensure that the material presented in this volume is correct..... However.... neither Academic Press nor the editor shall be responsible for any errors, omissions, or damages arising from the use of this volume.

If this warning was ever warranted, it was in the liner notes of this CD. Damages, indeed. I estimate the chances of retrieving a correct formula from the screen display at about one out of three, that is, for anyone lacking microfocal vision. Now I was doing all this on a Mac platform; maybe someone's DOS or Windows equation was lushly readable, elegant, utile. If so, I offer my humblest apologies to Academic; my hearty congratulations go to any such customers. They may imagine themselves fortunate but, computer karma being what it is, they'll soon enough be victims to some other piece of flawed software.

I phoned customer support at Lightbinders, and there followed one of those Kafkaesque conversations that seem to be so much a feature of the computer age. "You can display the TeX representation of an equation," the consultant pointed out. "Then you can paste the equation into a TeX document." But copying by highlighting the selection didn't work. "Hmmmm," the consultant said. "Do a copy from the file menu," he ordered. It didn't work. "HHMMMM," he said. "Do a copy by depressing the command and C keys." No luck. "HHHMMMMMMMMM," he said. "*That's strange*". There I was, facing this travesty of the desired equation, staring at me in its TeX metamorphosed form. Has the reader ever tried to reconstruct an equation from its TeX representation? Don't. Its easier to *derive* the equation. One of my favorite equations, a darling little double integral for an Appell function, had become a 16 line porridge of "{}}}{}}{" s and "//\/\/\/"s. One can print the whole page containing the equation in question, but before one would do this, one would want to know what the equation says. The pericopes on notations and index of special functions look like so much flypaper. From the customer rep I got no satisfaction, and I was reminded of those primitive tribes where half the members speak one language, the other half another.

What else could go wrong? Something else did. You cant *find* things. The list of contents is too abbreviated consisting only of the chapter headings in the book; further details require clicking the mouse on the displayed headings. Once again, a Catch-22. You have to know where something is to find it. Where should you look for the definition of Euler's constant? In the introduction? In the definition of elementary functions? In the integral tables? No, far away in Chapter 9, Special Functions, buried several layers deep. The table of contents of the book, the honest to god paper book, has to be visually scanned too, but it is all in front of you: nothing is buried under something else. After struggling with the CD-ROM, I clutched my book to my chest, praying it would never go away.

Also, the software may have corrupted my system file, entailing a trek to the computer services center four city blocks away and, after I discovered the computer hardware was intact, a trek back to reinstall the system software. However, I make no accusations. Those ethereal and subtle software incompatibilities may be the only true things of the spirit, the only mysteries, vouchsafed to to a technocratic society barreling into the 21st century. Let us revere them.

I love books, but I'm not a Luddite. I embrace software that works. Macbeth spoke of someone being yanked untimely from his mother's womb, and I think that software — market pressure, no doubt — is often yanked untimely from the designer's noggin. If you're planning on adding this CD to your library, beware. Be certain your computing platform accommodates it in a way that makes it legible, convenient and system compatible. Take nothing for granted. A hasty purchase means a wracking day on the phone to some callow customer rep in some distant part of the country, and no concomitant satisfaction.

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## Call For Papers

**1. Orthogonal Polynomials and Computer Algebra**
**Journal of Symbolic Computation: Special Issue**
**Guest Editors: R. A. Askey, W. Koepf and T. H. Koornwinder**
Academic Press, London

In the last decade major steps towards an algorithmic treatment of orthogonal polynomials and special functions (OP & SF) have been made, notably Zeilberger's brilliant extension of Gosper's algorithm on algorithmic definite hypergeometric summation. By implementations of these and other algorithms symbolic computation has the potential to change the daily work of everybody who uses orthogonal polynomials or special functions in research or applications. It can be expected that symbolic computation will also play an important role in on-line versions of major revisions of existing formula books in the area of OP & SF.

Many articles on the algorithmic treatment of orthogonal polynomials and special functions and on applications of such algorithms have been published in the meantime. But such articles are distributed widely in the literature. To collect these efforts Wolfram Koepf organized Session 13 on *Orthogonal Polynomials* (`http://www.zib.de/koepf/isaac.html`) at the *First ISAAC Congress* with the emphasis on the use of computer algebra. This congress took place at the University of Delaware, Newark, Delaware, June 3–7, 1997.

In this special issue of the *Journal of Symbolic Computation* we would like to collect articles about the interaction between computer algebra and orthogonal polynomials and special functions. Hopefully, the participants of Session 13 at the First ISAAC Congress will submit papers, but this special issue is open for everybody. Rather than a Proceedings of a session, the issue is meant as a state of the art account of this topic.

Contributions should discuss non-trivial usage of symbolic computation which significantly contributes to the theory of Orthogonal Polynomials and Special Functions. Examples of categories in which contributions may fall are:

- New symbolic algorithms for obtaining specific results in OP & SF. This may also be a presentation of a drastically improved implementation of an existing algorithm.
- Proofs aided by symbolic computation of significant new results in OP & SF which are yet untractable by purely human effort.
- New significant results in OP & SF which are finally provable without computer aid, but which would have been hard to find without experimentation by symbolic computation. The description of this experimentation should be illustrative for future efforts by others.
- Aspects of symbolic computation in connection with the production of on-line text books and formula dictionaries on OP & SF.

Authors are invited to submit their manuscripts to the Managing Guest Editor Wolfram Koepf who will handle the preparation of this special issue, preferably by e-mail in some TEX dialect, or in PostScript. All submitted papers will be refereed according to the JSC usual refereeing process (see `http://www.cis.udel.edu/~caviness/jsc.html` for information about JSC). The journal's LATEX style file can be obtained from `ftp://ftp.udel.edu/pub/jsc` (America) or `ftp://ftp.risc.uni-linz.ac.at/pub/jsc` (Europe).

**Important dates:**

| | |
|---|---|
| Deadline for submission of full papers: | 15 January, 1998 |
| Notification of acceptance/rejection: | 31 May, 1998 |
| Final revised manuscripts due: | 15 September, 1998 |
| Appearance of special issue: | 1998/1999 |

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## Software Announcements

### 1. Maple procedure rec2ortho

René Swarttouw and I have completed a major update of our Maple procedure rec2ortho (get OP families in the Askey scheme from the coefficients in the three term recurrence relation). See more information at the URL: http://turing.wins.uva.nl/~thk/rec2ortho.html

Tom H. Koornwinder
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## Problems and Solutions

Thus far 18 problems have been submitted six of which have been solved (#**1**, **4**, **6**, **7**, **10**, **14**). In this issue the solution of Problem #**2** is presented. Still unsolved are Problems #**3**, **5**, **8**, **9**, **11**, **12**, **13**, **15**, **16**, **17** and **18**.

**16. A Definite Integral.** Prove that

$$\int_0^1 \frac{\log(\pi^2+(\log x)^2)}{1+x^2}\,dx = \pi\log\frac{\sqrt{\pi/2}\,\Gamma(1/4)}{2\,\Gamma(3/4)}\ .$$

Remark: The integral is related to the Dirichlet $L$-function. The right side of this identity can be rewritten as

$$\frac{\pi\,(\gamma+2\,\log(\pi/2))}{2} - 2\,L'(1)$$

where $L(s)$ is the Dirichlet $L$-function

$$L(s) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^s}\ .$$

(Submitted on October 10, 1996)

Victor Adamchik
(victor@wolfram.com)

**17. Canonical Leibniz' Formula for Difference Operators.** Let $\Delta$ and $\nabla$ be the usual forward and backward operators. The action on a product $f(x)g(x)$ is usually written in a nonsymmetrical (and nonunique) form, for instance:

$$\begin{aligned}\Delta[f(x)g(x)] &= f(x)\Delta g(x) + g(x+1)\Delta f(x)\\ &= g(x)\Delta f(x) + f(x+1)\Delta g(x)\ .\end{aligned}$$

A (unique) canonical form, without any shift on argument $x$, is sometime preferable to the nonsymmetrical formula, like

$$\Delta[f(x)g(x)] = f(x)\Delta g(x) + g(x)\Delta f(x) + \Delta f(x)\Delta g(x)\ ,$$

or

$$\nabla[f(x)g(x)] = f(x)\nabla g(x) + g(x)\nabla f(x) - \nabla f(x)\nabla g(x)\ .$$

Iteration of $\Delta$ or $\nabla$ acting on a product of $r$ functions $f_i(x)$ can obviously be written in the canonical form

$$\Delta^n\left[\prod_{i=1}^r f_i(x)\right] = \sum_{j_1\cdots j_r=0}^{n} R_n(j_1\cdots j_r)\prod_{i=1}^r \Delta^{j_i} f_i(x)\ ,$$

where the coefficients $R_n(j_1\cdots j_n)$ are nonnegative integers, invariant under the group of permutation $P_r$; ($S_n(j_1\cdots j_r)$ appears when using the operator $\nabla^n$). Using two times the link with the shift operator $E$ ($\Delta^n = (E-1)^n, E^k = (\Delta+1)^k$), the $\Delta^n$ canonical formula can be written:

$$\Delta^n\left[\prod_{i=1}^r f_i(x)\right] = \sum_{k=0}^{n}(-1)^k\binom{n}{k}\prod_{i=1}^r\left[\sum_{j=0}^{n-k}\binom{n-k}{j}\Delta^j[f_i(x)]\right].$$

Is it possible, using this representation, to obtain coefficients $R_n$ (and $S_n$) in a closed form?

It is obvious that $R_n(0,\cdots,0) = R_n(s,0,\cdots,0) = R_n(0,s,0,\cdots,0) = \cdots = R_n(0,\cdots,0,s) = 0$, ($s = 1,\cdots,r-1$).
(Submitted on January 7, 1997)

André Ronveaux
(Andre.Ronveaux@fundp.ac.be)

**18. Maclaurin Expansion.** For $a,b\in(0,1)$ let

$$Q(a,b,r) = \frac{B(a,b)}{\log\left(\frac{c}{1-r}\right)}\,{}_2F_1\left(\begin{matrix}a,b\\a+b\end{matrix}\,\middle|\,r\right)$$

where $B(a,b)$ denotes the Beta function, and

$$c = e^{R(a,b)}\ ,\quad R(a,b) = -\Psi(a) - \Psi(b) - 2\gamma\ ,$$

$\gamma$ is Euler's constant, and

$$\Psi(z) = \frac{\Gamma'(z)}{\Gamma(z)}\ .$$

Let

$$G(a,b,r) = \frac{Q(a,b,r)-1}{1-r} = \sum_{j=0}^{\infty} d_j\,r^j\ .$$

Is it true that all $d_j > 0$?

This question arose in connection with Theorem 1.4 in Trans. Amer. Math. Soc. 347 (1995), 1713–1723, which is a refinement of Ramanujan's asymptotic formula for the zero-balanced hypergeometric function ${}_2F_1$.
(Submitted on March 24, 1997)

Matti Vuorinen
(mv@geom.Helsinki.FI)

I am very happy that Dick Askey and George Gasper were able to solve one of the early problems of the Problems Section. George Gasper sent me their solution, and he wrote in his e-mail message: "When Eugene Tomer and I started the new 'Problems' section, both he and I submitted one problem to start it off with. Little did I realize that it would take five years before a solution would be submitted to Problem 2."

Here is their solution.

**Problem 2**: Is it true that

$$x^2 t^x {}_2F_1(x+1, x+1; 2; 1-t)$$

is a convex function of $x$ whenever $-\infty < x < \infty$ and $0 < t < 1$?

Submitted by George Gasper, August 19, 1992.

**Solution to Problem 2**
by Richard Askey and George Gasper
(askey@math.wisc.edu and george@math.nwu.edu)

Let $0 < t < 1, -\infty < x < \infty$, and set

$$f_t(x) = x^2 t^x {}_2F_1(x+1, x+1; 2; 1-t). \tag{4}$$

By using the binomial theorem to expand $t^x = (1-(1-t))^x$ in powers of $1-t$, it follows that

$$f_t(x) = x^2 \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{(-x)_j (1+x)_k (1+x)_k}{j!\, k!\, (2)_k} (1-t)^{j+k}$$

$$= x^2 \sum_{n=0}^{\infty} \frac{(-x)_n}{n!} (1-t)^n \, {}_3F_2(-n, 1+x, 1+x; 2, 1+x-n; 1)$$

after setting $j = n-k$ and changing the order of summation. Then, application of the $b = -n$ case of the transformation formula [1, 3.8(1)] to the above ${}_3F_2$ series yields the expansion formula (generating function)

$$f_t(x) = x^2 \sum_{n=0}^{\infty} F_n(x) (1-t)^n \tag{5}$$

with

$$F_n(x) = {}_3F_2(-n, 1+x, 1-x; 2, 1; 1), \quad n = 0, 1, \ldots . \tag{6}$$

From (6) it is clear that each $F_n(x)$ is a polynomial of degree $n$ in $x^2$, and hence, by (5), that $f_t(x)$ is an even function of $x$. Also, computations of the coefficients of the polynomials $F_n(x)$ for many values of $n$ suggest that each $F_n(x)$ is an absolutely monotonic function (one whose power series coefficients are nonnegative) of $x$. Then (5) would imply that $f_t(x)$ is an absolutely monotonic function of $x$ and, consequently, a convex function of $x$ when $0 < t < 1$. Thus it suffices to prove that each $F_n(x)$ is an absolutely monotonic function of $x$.

First observe that

$$F_n(x) = \frac{1}{n!\,(n+1)!} S_n(-x^2; 1, 1, 0), \tag{7}$$

where

$$\frac{S_n(x^2; a, b, c)}{(a+b)_n (a+c)_n} = {}_3F_2(-n, a+ix, a-ix; a+b, a+c; 1), \tag{8}$$

$n = 0, 1, \ldots$ , are the continuous dual Hahn polynomials, see [4, §1.3]. Since the polynomials $\{S_n(y; a, b, c)\}_{n \geq 0}$ are orthogonal on the interval $(0, \infty)$ with respect to a positive weight function when $c \geq 0$ and either $a, b > 0$ or $a, b$ are complex conjugates with positive real parts, the zeros $y_{k,n}(a, b, c), 1 \leq k \leq n$, of each $S_n(y; a, b, c)$ are positive, and hence

$$S_n(x^2; a, b, c) = C_n(a, b, c) \prod_{k=1}^{n} \left( y_{k,n}(a, b, c) - x^2 \right)$$

where, by inspection of the right side of (8), $C_n(a, b, c) > 0$. Therefore, with the above-mentioned restrictions on $a, b, c$ each

$$S_n(-x^2; a, b, c) = C_n(a, b, c) \prod_{k=1}^{n} \left( y_{k,n}(a, b, c) + x^2 \right) \tag{9}$$

is an absolutely monotonic function of $x$, and hence, by (7) and (5), $F_n(x)$ and $f_t(x)$ are absolutely monotonic functions of $x$, which completes the proof. From this proof it is clear that the function $t^x {}_2F_1(x+1, x+1; 2; 1-t)$ is also an absolutely monotonic, and hence convex, function of $x$. For a discussion of the origin of this problem, see page 11 of the Fall 1994 issue of this Newsletter (note that in the fourth displayed equation $(K_{i(x+iy)}(a))^2$ should read $(K_{ix}(a))^2$ ).

**Remark 1.** Formula (5) can also be derived by first applying the Pfaff-Kummer transformation formula [3, (1.5.5)] to the ${}_2F_1$ series in (4) with the additional restriction that $|(1-t)/t| < 1$, using the binomial theorem to expand each of the $t^{-j-1}$ powers of $t$ in powers of $1-t$, changing the order of summation, and then employing analytic continuation to remove the $|(1-t)/t| < 1$ restriction.

**Remark 2.** From (9) and an extension of the derivation of (5) it follows that the generating function (cf. [4, (1.3.6)],

[5, p. 398])

$$(1-t)^{x-c}\,{}_2F_1(a+x,b+x;a+b;t)$$

$$=\sum_{n=0}^{\infty}\frac{S_n(-x^2;a,b,c)}{n!\,(a+b)_n}\,t^n,\quad 0\le t<1,$$

is an absolutely monotonic function of $x$ when $c\ge 0$ and either $a,b>0$ or $a,b$ are complex conjugates with positive real parts, and that the generating function (cf. [2, 19.10 (25)], [4, (1.3.9)])

$$e^t\,{}_2F_2(a+x,a-x;a+b,a+c;-t)$$

$$=\sum_{n=0}^{\infty}\frac{S_n(-x^2;a,b,c)}{n!\,(a+b)_n\,(a+c)_n}\,t^n,\quad 0\le t<\infty,$$

is an absolutely monotonic function of $x$ when $a,b,c>0$.

**Remark 3.** Similarly, additional absolutely monotonic functions can be obtained by using the generating functions for the Wilson $W_n(x^2;a,b,c,d)$ and the Askey-Wilson $p_n(x;a,b,c,d|q)$ polynomials in [4, §1.1 and §3.1], and their limit cases, along with suitable changes in variables. In particular, it follows from [4, (1.1.2) and (1.1.6)], [6, (1.2) and (2.4)] that the generating function

$${}_2F_1(a+x,b+x;a+b;t)\,{}_2F_1(c-x,d-x;c+d;t)$$

$$=\sum_{n=0}^{\infty}\frac{W_n(-x^2;a,b,c,d)}{n!\,(a+b)_n\,(c+d)_n}\,t^n,\quad 0\le t<1,$$

is an absolutely monotonic function of $x$ when $\mathrm{Re}(a,b,c,d)>0$ and non-real parameters occur in conjugate pairs with $a+b>0$ and $c+d>0$.

### References


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[2] Erdélyi, A. et al.: *Higher Transcendental Functions*, Vol. III, McGraw-Hill, New York–London, 1955.

[3] Gasper, G. and Rahman, M.: *Basic Hypergeometric Series*, Encyclopedia of Mathematics and Its Applications, Vol. 35, Cambridge University Press, 1990.

[4] Koekoek, R. and Swarttouw, R.F.: The Askey-scheme of hypergeometric orthogonal polynomials and its $q$-analogue, Delft University of Technology, Faculty of Technical Mathematics and Informatics, Report 94-05, 1994; revised version of February 20, 1996 obtainable by anonymous ftp from: unvie6.un.or.at, directory siam/opsf_new/koekoek_swarttouw, file koekoek_swarttouw1.ps, or via the World Wide Web: ftp://unvie6.un.or.at/siam/opsf_new/koekoek_swarttouw/koekoek_swarttouw1.ps

[5] Letessier, J. and Valent, G.: Dual birth and death processes and orthogonal polynomials, SIAM J. Math. Anal. 46 (1986), 393–405.

[6] Wilson, J.A.: Asymptotics for the ${}_4F_3$ polynomials, J. Approx. Theory. 66 (1991), 58–71.


## Miscellaneous

**1. Student membership in SIAM**

The following information may be of interest to students and postgraduates.

Students can become a member of SIAM for $20/year, with free membership in one activity group.

Postgraduates, during three consecutive years after receiving their highest degree, can become members of SIAM for $45/year.

e-mail: join@siam.org

Tom H. Koornwinder
(thk@wins.uva.nl)

**2. Revising the 1991 Math. Subject Classification**

The following message was sent on June 19, 1997 to Prof. Keith Dennis (Executive Editor Mathematical Reviews) and Prof. Bernd Wegner (Chefredakteur, Zentralblatt für Mathematik).

> Dear Editors,
>
> In reply to your call for comments and suggestions for the revision of the Mathematics Subject Classification I send you here a large number of suggestions from the community of Orthogonal Polynomials and Special Functions. The SIAM Activity Group on Orthogonal Polynomials and Special Functions has solicited reactions via the electronic Newsletter OP-SF Net (freely available for all interested people) and the printed Newsletter of the Activity Group (only for members). On the basis of the reactions we received I have made a comprehensive proposal for changes of those parts of the 1991 Classification which deal with Orthogonal Polynomials and Special Functions (in particular part 33). A preliminary version of this final proposal I recently discussed with Prof. Richard Askey, and he agreed with it.
>
> Experts from our Activity Group are available if you need further feed-back in the area of Orthogonal Polynomials and Special Functions during your revision process.
>
> With kind regards,
>
> Tom Koornwinder

**about 33**

- 33C45, change into: Orthogonal polynomials and functions of hypergeometric type (Jacobi, Laguerre, Hermite, Askey scheme, etc.; see 42C05 for general orthogonal polynomials and functions)

- add: 33C47 Other special orthogonal polynomials and functions
- 33C50: change into: Orthogonal polynomials and functions in several variables expressible in terms of special functions in one variable
- add: 33C52 Orthogonal polynomials and functions associated with root systems
- 33C55, change into: Spherical harmonics
  Motivation: ultraspherical polynomials unrelated to spherical harmonics are covered by 33C45; spherical functions (on Gelfand pairs) are covered by 33C80
- add: 33C67 Hypergeometric functions associated with root systems
- 33C80, change into: Connections with groups, algebras and related topics
- 33D10: skip this
  Motivation: we know theta functions but we do not know basic theta functions
- 33D15, change into: Basic hypergeometric functions in one variable, ${}_r\phi_s$
- 33D20: skip this
  Motivation: studying ${}_2\phi_1$ immediately gives rise to studying more general ${}_r\phi_s$
- 33D45, change into: Basic orthogonal polynomials and functions (Askey-Wilson polynomials, etc.)
- add: 33D50 Orthogonal polynomials and functions in several variables expressible in terms of basic hypergeometric functions in one variable
- add: 33D52 Basic orthogonal polynomials and functions associated with root systems (Macdonald polynomials, etc.)
- 33D55: skip this
  Motivation: Basic spherical functions in the sense of spherical functions on quantum groups are covered by 33D80. If basic spherical harmonics mean the elements in irreducible subspaces of the algebra of polynomials on a quantum sphere then these are also covered by 33D80.
- add: 33D67 Basic hypergeometric functions associated with root systems
- 33D80, change into: Connections with quantum groups, Chevalley groups, $p$-adic groups, Hecke algebras and related topics
- add: 33E12 Mittag-Leffler functions and generalizations
- add: 33Fxx Computational aspects
- add: 33F05 Numerical approximation [See also 65D20]
- add: 33F10 Symbolic computation (Zeilberger algorithm, etc.) [See also 68Q40]

**about 34**

- 34B30, change into: Special equations (Mathieu, Hill, Bessel, Painlevé, etc.)

**about 40**

- add: 40A27 Explicit summation of series
- add: 40A29 Explicit computation of integrals
- add: 40A40 Generating functions [See also 05A15]
- add: 40B10 Rearrangements of explicit multiple series
- add: 40B15 Multiple integrals (should also be assigned at least one other classification number in this section)

**about 42**

- 42C05, change into: Orthogonal functions and polynomials in one variable, general theory [See also 33C45, 33C47, 33D45]
- add: 42C07 Orthogonal functions and polynomials in several variables, general theory [See also 33C50, 33C52, 33D50, 33D52]
- add: 42C40 Wavelets
- add: 42C45 Biorthogonal families of functions

**about 65**

- 65D20, change into: Computation of special functions, construction of tables [See also 33F05]

**about 68**

- 68Q40, add to "See also" a ref to 33F10 (warning: 68Q40 gives a ref to 16-08, but 16-08 does not exist)

As an answer I received the following letter

> Dear Professor Koornwinder,
>
> Thank you very much for the carefully considered suggestions for revision of the MSC you have sent from the OP-SF group. You have clearly not only tried to improve the scientific quality of the scheme, but also been mindful of the technical constraints that we at MR and Zbl work under in revising something long used for database access.
>
> Speaking only directly in connection with the area 33 (Special Functions) which I am myself involved with editorially, I can say that I hope we shall be able to adopt almost all your suggestions. I would like to take a later opportunity to respond to some of your proposals, and to provoke more expert input to our revision process.
>
> Thank you again for your, and your group's, willingness to help with MSC2000.
>
> Best regards,
>
> Patrick Ion
> Associate Editor
> Mathematical Reviews
> P. O. Box 8604
> Ann Arbor, MI 48107-8604

Tom H. Koornwinder
(thk@wins.uva.nl)

### 3. Bibliography on Orthogonal Polynomials

In Newsletter 7-2, p. 15, Dick Askey wrote about the *Bibliography on Orthogonal Polynomials* published in 1940 by Shohat, Hille and Walsh. Professor Keith Dennis, managing editor of Math. Reviews, has proposed preparing an electronic version of this bibliography. We quote the following from a message he sent us last January.

"Let me clarify a bit how I was proposing that Mathematical Reviews be involved in the project. First of all I should say that from my point of view the main goal is to get as much bibliographic information as possible into a reasonably standard format. Ideally this would mean

1. placing the data in appropriate fields (author, title, journal, volume, year, etc.);
2. identifying the journal or series;
3. identifing the author;
4. classifying the information using a uniform scheme;
5. providing a review or abstract of the article.

In the case of the Shohat bibliography I think that the detailed classification given there could be used to create a sort of combination of 4 & 5.

"This is not exactly the same format as that for current MR data, but it is about as close as one can get without a lot of additional effort.

"My original intent was that Mathematical Reviews, with support from the mathematical community, would carry out the details of putting the data in an appropriate form. That is, from the point of view of putting the existing bibliography into usable form, no additional person would be necessary to direct the project. However, if one wanted to update this, or integrate it with current MR or Zbl, or if one wanted to assign current MSC classifications, then extra help would certainly be necessary.

"My original estimate of $5000 would be to carry out most of the project: The book is 200 pages long but with very intricate classifications that would have to be carefully proofread. The actual keyboarding & proofreading can no doubt be done for quite a bit less. However, I wanted to include the extra work to identify the journals & individuals (as they are already grouped this way, this would mean merging with the MR individual database) and writing scripts to derive a written description of the classification for each item.

"I was further proposing that MR take whatever steps necessary to deal with any copyright problems.

"We have experience with a number of keyboarding companies (e.g., the reviews from 1940 through 1979 are currently in the process of being converted to text form) and this might well be the cheapest input method. However, due to the fine detail in the classification, I believe that we will also want to have it proofread again, but fortunately it is only 200 pages long.

"So what I was proposing was that community raise the necessary funds for the data entry as described above.

. . .

"I would propose that we attempt to start with a smaller project, converting the Shohat bibliography into electronic form. Then if this raises sufficient interest in the community, perhaps an editorial group could be put together to update it in a useful way."

The officers of our Activity Group have discussed the plans of Keith Dennis for an elctronic update of the Shohat bibliography on orthogonal polynomials. Our conclusion is that we are sympathetic concerning these proposals, but that we do not see it as a top priority for our group. We feel that providing bibliographic information of recent work on OP & SF (cf. for instance the *Compiled Booklist* in the Newsletter 7-3), should have a higher priority for us. Therefore we have written to Prof. Dennis that our Activity Group will not contribute financially to his project. Of course, this leaves open the possibility for individual members, that they may contribute.

Tom H. Koornwinder
(thk@wins.uva.nl)

**4. Editorial Policy of SIAM Journal on Mathematical Analysis**

The following is endorsed by all officers of our Activity Group.

Traditionally, SIAM Journal on Mathematical Analysis has been closest of all SIAM journals to the scientific purposes of our Activity Group. From the start of this journal in the early seventies onward, many papers on OP & SF have appeared in SIAM J. Math. Anal. including some of the best papers in our area. A typical example is Ian Macdonald's paper *Some conjectures for root systems* in Vol. 13, 1982, pp. 988–1007, which has had an enormous impact.

The editorial policy of SIAM J. Math. Analysis is given by the following lines (see http://www.siam.org/journals/sima/mapol.htm):

"The SIAM Journal on Mathematical Analysis focuses on those parts of classical and modern analysis that have direct or potential application to the natural sciences and engineering. Papers fall into two broad categories, the first being those that analyze interesting problems associated with realistic mathematical models for natural phenomena. The second category includes those papers which contribute in a substantial way to the general, analytical information and techniques which are likely to bear upon such models."

Implementation of this policy has become more strict. This can be seen from:

- inspection of the tables of contents
- recent experiences that papers on OP & SF of a more theoretical nature were rejected as a matter of policy rather than because of the quality of the contributions
- Charles Dunkl having quit from the Editorial board and not being succeeded by a specialist in theoretical aspects of OP & SF.

We are aware that our Activity Group, more than the other Activity Groups in SIAM, houses members whose research is motivated by theory rather than by application. Still, the potential of applications (in one, ten or fifty years) is an important aspect of OP & SF. Anyhow, for an Activity Group within SIAM, it is unfortunate if an important part of the membership cannot publish in any SIAM journal.

According to Nico Temme, who is at present a member of the Editorial Board of SIAM J. Math. Anal., there is a good chance that a theoretical paper with enough mathematical quality and novelty will improve its suitability for SIAM J. Math. Anal. if the author takes a few sentences or paragraphs to address the possibility of specific applications, with a few examples.

We would suggest our readers to keep trying to submit papers (keeping Nico's advice in mind), but to report to us (the officers of the Activity Group) if your paper is rejected just because it is out of scope. Such reports will be collected by Tom Koornwinder (thk@wins.uva.nl). We will be able to read about positive experiences in the list of accepted papers (http://www.siam.org/journals/sima/maarts.htm).

In about one year we intend to write an evaluation.

Tom H. Koornwinder
(thk@wins.uva.nl)

### 5. An OP-SF listserv?

It has been suggested that an OPSF listserv should be started. None such exist now to my knowledge. I believe it would not be difficult to have one set up at some university with an address of the form OPSF-L@.... Such a listserv could be useful in promoting discussion and posing questions in the general areas of orthogonal polynomials and special functions. A selection of contributions could be included in OP-SF NET. It has been suggested that we might create an automatic way to subscribe and unsubscribe to the listserv and OP-SF NET. To make it workable, the listserv would probably have to be "unmoderated" which means that the Group would have no control over what might appear there. Some have observed that this could lead to abuse. In any case, such a listserv need not have any official connection to our Activity Group, though it could be publicized in the Group's media. Of course if it used the acronym OPSF it would be associated with the Group, in people's minds, in any case. Let me know what you think of the idea of starting such a listserv; your comments can be sent to muldoon@yorku.ca. If it seems to be a good idea, would anyone like to volunteer to set it up?

Martin Muldoon
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### 6. A Belated Tribute to O. Bottema

In a contribution published in the June issue of this Newsletter [3] I have presented an application of Laguerre polynomials in classical mechanics, "which seems to have been overlooked so far." I should have added "by myself"! John Boersma from the Technische Universiteit Eindhoven has drawn my attention to Oene Bottema's work on this subject. More than half a century ago Bottema has treated the problem of a compound pendulum by Laguerre polynomials [1]. Later, in the early sixties, he proposed two problems for the Problem Section of the Dutch Archive for Mathematics [2], which are related to his previous paper.

In fact Bottema has anticipated the main result of my article, along with some modifications of the problem. An interesting detail concerns the transition to the continuously distributed mass. Bottema speculates that the eigenfrequencies of a multiple pendulum of fixed length might increase monotonically as the number of masses tends to infinity, but he had not succeeded in proving it. Figure 3 of my article shows why: Only the two lowest eigenfrequencies share this monotonical behaviour.

After all, the problem of the multiple pendulum offers an interesting application of Laguerre polynomials, which is worthwhile to be studied further. I thank John Boersma for calling my attention to Bottema's original contribution.

#### References

[1] Bottema, O.: Die Schwingungen eines zusammengesetzten Pendels. Eine Anwendung der Laguerreschen Polynome. Jahresberichte der deutschen Mathematiker-Vereinigung **42**, 1933, 42–60.

[2] Bottema, O.: Problems No. 42 and No. 163, Nieuw Archief voor Wiskunde (3) **8**, 1960, 48, and **11**, 1963, 52–53. See also: Wiskundige Opgaven met de Oplossingen, Deel **21**, 1960–1964, 41–45 and 158–161.

[3] Braun, M.: Laguerre Polynomials and the Vibrations of a Multiple Pendulum. SIAM OP-SF Newsletter **7**(3), June 1997, 17–20.

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### 7. Bill Miller appointed IMA Director

Bill Miller, our Group's Program Director, has been chosen to succeed Avner Friedman as Director of the Institute for Mathematics and its Applications (IMA), University of Minnesota, for a five-year term beginning September 1, 1997. The IMA was established in 1982 by the U.S. National Science Foundation, as a result of a national competition. Its mission is to close the gap between theory and its applications by identifying problems and areas of mathematical research needed in other sciences and by encouraging the participation of mathematicians in these areas of application.

Professor Miller is a mathematical physicist. His research involves the use of symmetry methods, in particular Lie groups and Lie algebras, in the analysis of the structure of physical theories. He has written extensively on topics in special function theory, separation of variables, and quantum algebras.

Willard Miller has had a long association with the IMA. He was the head of the School of Mathematics at Minnesota in 1979 when the proposal for the IMA was made, and he played a major role in bringing the IMA to Minnesota. He became Associate Director of the IMA in 1987 and served in that position for seven years, before becoming Associate Dean for Finance and Planning of the Institute of Technology at the University of Minnesota.

We congratulate Bill on this important appointment and wish him every success in his leadership of the IMA.

(Much of the above information is taken from the IMA web site: http://www.ima.umn.edu/).

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### 8. NIST Digital Library of Mathematical Functions

A major new World Wide Web site for special functions and their applications is being planned at the National Institute of Standards and Technology, provided adequate financial and other resources are secured. The new site will be called

the NIST Digital Library of Mathematical Functions. See http://math.nist.gov/DigitalMathLib for current information about the project.

NIST is the new name for the old National Bureau of Standards, and the core component of the DLMF will be a thorough revision of Abramowitz and Stegun, Handbook of Mathematical Functions (with Formulas, Graphs, and Mathematical Tables), NBS Applied Mathematics Series 55, US Government Printing Office, 1964. The procedures used to generate and validate the original handbook will be adapted and strengthened to ensure a successor of the highest possible quality.

The static content of the DLMF will be stored in a computer database at NIST, accessible at the web site and by CD-ROM. The ability will be provided to copy formulas, graphs and numerical data into local computer files in formats appropriate for subsequent word, numerical, symbolic and graphical processing.

Application components in technical fields where special functions are important are a prominent part of the DLMF concept. For example, such a component could include sample problems and their solutions in terms of special functions. Basic mathematical information resident in the core component will be extracted and adapted to the notation and definitional conventions of the technical field. Components for electromagnetism and quantum physics are included in the NIST plan as prototypes for other application components.

A later stage of development is envisioned in which computational services are provided by NIST on request from web users. For example, a user would be able to specify a set of numerical values for the arguments and parameters of a special function, together with a minimum precision requirement. If the requisite computational resources are available to the DLMF, then the corresponding set of function values would be computed and delivered to the user. Other potential services-on-demand are preparation of graphs and computer algebraic processing.

Recent events include:

- The DLMF initiative was presented on July 14 at the SIAM Annual Meeting in the OPSF-sponsored Minisymposium on *Handbooks for Special Functions and the World Wide Web.* A corresponding NIST report will be available soon at the web site.
- A workshop was held July 28-30 at NIST to refine and develop the DLMF concept. About a third of the 30 participants were prominent researchers invited from outside NIST. In addition to lectures, lively and useful discussions took place on several general topics: approach to the project, organization of the project, funding possibilities, new chapter layout, roles and applications, and future phases. A summary NIST report will be available soon at the web site.

The web site gives an e-mail address where comments can be sent on any aspect of the project. Also, any site visitor can subscribe to a mailing list to receive occasional e-mail from NIST announcing significant new developments.

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**9. Krawtchouk Polynomials Home Page**

Recently I created a web site, devoted to the Krawtchouk orthogonal polynomials: http://www.isir.minsk.by/~zelenkov/physmath/kr_polyn . All your proposals, additions, corrections will be appreciated. Thank you in advance.

Besides M. Krawtchouk's biography and the polynomial properties I would like to include references to papers in which these polynomials are investigated or used.

If you want your article to be mentioned please e-mail me the bibliographic data, abstract and (if possible) the text in LaTeX format and/or the appropriate hyperlink.

Note that my WWW homepage is http://www.isir.minsk.by/~zelenkov.

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## How to Contribute to the Newsletter

Send your Newsletter contributions directly to the *Editor:*

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preferably by e-mail, and in LaTeX format. Other formats are also acceptable and can be submitted by e-mail, regular mail or fax.

**Deadline for submissions to be included in the February issue 1998 is January 15, 1998.**

Back issues of the Newsletter can be obtained from http://www.zib.de/koepf/siam.html .

The Activity Group also sponsors an electronic news net, called the **OP-SF Net**, which is transmitted periodically by SIAM. The Net provides a rather fast turnaround compared to the Newsletter. To *receive* transmissions, just send your name and e-mail address to poly-request@siam.org (as with other nets, nonmembers can also receive the transmissions). Your OP-SF Net *contributions* should be sent to poly@siam.org. Please note that submissions to OP-SF Net are automatically considered for publication in the Newsletter, and vice versa, unless the writer requests otherwise.

The Net is organized by Tom Koornwinder (thk@wins.uva.nl) and Martin Muldoon (muldoon@yorku.ca). Back issues of OP-SF Net can be obtained by anonymous ftp from `ftp.wins.uva.nl`, in the directory

pub/mathematics/reports/Analysis/koornwinder/opsfnet.dir

or by WWW at the addresses

http://turing.wins.uva.nl/~thk/opsfnet/

http://www.math.ohio-state.edu/JAT

Martin Muldoon, moreover, manages our home page
http://www.math.yorku.ca/Who/Faculty/Muldoon/siamopsf/
on World Wide Web. Here you will find also a WWW version of the OP-SF Net. It currently covers the topics

- Conference Calendar
- Books, Conference Proceedings, etc.
- Compendia, tools, etc.
- Compiled booklist on OP-SF
- Meeting Reports
- Projects
- Problems
- Personal, Obituaries, etc.
- History
- Positions available
- Miscellaneous
- Memberlist
- Links to WWW pages of interest to members

## Activity Group: Addresses

The *SIAM Activity Group on Orthogonal Polynomials and Special Functions* consists of a broad set of mathematicians, both pure and applied. The Group also includes engineers and scientists, students as well as experts. We now have around 140 members scattered about in more than 20 countries. Whatever your specialty might be, we welcome your participation in this classical, and yet modern, topic. Our WWW home page http://www.math.yorku.ca/Who/Faculty/Muldoon/siamopsf/ is managed by Martin Muldoon (muldoon@yorku.ca).

The **Newsletter** is a publication of the *SIAM Activity Group on Orthogonal Polynomials and Special Functions*, published three times a year. To receive the Newsletter, you must first be a member of SIAM so that you can join the Activity Group. The annual dues are $96 for SIAM plus $10 for the Group; students pay $20/year with free membership in one activity group; postgraduates can become members of SIAM for $45/year. To join, contact:

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