Orthogonal Polynomials and Special Functions

SIAM Activity Group on Orthogonal Polynomials and Special Functions

⋆ ⋆ ⋆ ⋆ **Newsletter** ⋆ ⋆ ⋆ ⋆

Published Three Times a Year — October 1998 — Volume 9, Number 1

Contents

From the Editors

As already announced, this is my last issue as Editor of the Newsletter. On this occasion I would like to thank everybody who contributed to the Newsletter in the past three years. It was my pleasure to collect the essential information for the Newsletter, to find reviewers, reporters and authors of articles, and to put these things together. I hope you found the Newsletter interesting, entertaining and useful.

Beginning next year, we will have both new officers of our SIAM Activity Group and new editors of our Newsletter. I thank Renato and Rafael again for their readiness to follow me as Editors; furthermore I thank them for their help with the last two issues.

Concerning the election for officers that takes place this fall, let me point to the following letter that Tom Koornwinder sent at the beginning of June:

> I am writing on behalf of the nominating committee for selecting candidates for office for the SIAM Activity Group on Orthogonal Polynomials and Special Functions. This committee consists of George Gasper, Martin Muldoon, Charles Dunkl, Willard Miller, Nico Temme and myself. We have put together the following slate:
>
> Chair:
>
> Daniel W. Lozier
> National Institute of Standards and Technology
> Gaithersburg, MD, USA
> dlozier@nist.gov

The purpose *of the Activity Group is*

—to promote basic research in orthogonal polynomials and special functions; to further the application of this subject in other parts of mathematics, and in science and industry; and to encourage and support the exchange of information, ideas, and techniques between workers in this field, and other mathematicians and scientists.

Vice-Chair:

1. Walter Van Assche
Katholieke Universiteit Leuven
Leuven, Belgium
Walter.VanAssche@wis.kuleuven.ac.be

2. Rupert Lasser
GSF-National Research Center for Environment and Health
Institute for Biomathematics and Biometry
Ingolstädter Landstr. 1
85764 Neuherberg
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martina.probst@gsf.de

Secretary:

1. Charles F. Dunkl
University of Virginia
Charlottesville, VA, USA
cfd5z@virginia.edu

2. M. Lawrence Glasser
Clarkson University
Potsdam, NY, USA
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Program Director:

1. Francisco Marcellán
Univ. Carlos III de Madrid
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2. Peter A. McCoy
US Naval Academy
Annapolis, MD, USA
pam@sma.usna.navy.mil

All proposed persons have been contacted by us, and they are willing to be a candidate for the office mentioned.

There will be elections for vice-chair, secretary, program director. A ballot will be mailed to all members this summer and those elected will hold office for a three-year period beginning January 1, 1999.

In particular, there will be no contest for election as Chair since there is only one candidate; on behalf of all members I wish our new Chair Dan Lozier as well as the other elected officers good luck for the next three years.

Each candidate was asked to provide *a statement on issues you think the activity group should be addressing and directions you think it should be taking in the future.* Here are these statements:

Chair: Daniel W. Lozier

"Orthogonal polynomials and special functions are important because of their utility in applied mathematics, statistics, and all scientifically based disciplines. They provide both qualitative insight into the behavior of solutions and a firm foundation for solving problems computationally. As much as any field of mathematics, ours is distinguished by the relevance of its history and the applicability of its current research frontiers. In my opinion, the activity group should build on its already substantial achievements in bringing together researchers and users of orthogonal polynomials and special functions through meetings, publications, and, where it can be done appropriately and effectively, modern electronic media such as the World Wide Web. I will work in these directions with the elected officers and general membership of the activity group."

Vice Chair: Rupert Lasser

"To extend the application of Orthogonal Polynomials and Special Functions in various fields of mathematics, e.g., approximation theory, harmonic analysis, stochastic processes. Furthermore, to find new topics in particular in life sciences, in which Orthogonal Polynomials and Special Functions can play a vital role. Lastly to intensify the contacts of the European members of the Activity Group with those of the U.S."

Vice Chair: Walter Van Assche

"The activity group should continue the OPSF electronic newsletter and the printed newsletter, which were the two most visible activities during the past years. In addition to the minisymposia during the SIAM annual meetings, there should be other workshops and short (one or two days) meetings on Orthogonal Polynomials and Special Functions on both sides of the Atlantic (North America and Europe), at least once every year. Furthermore, we should not neglect other parts of the world and there should be a meeting on another continent during the next three years (preferably Africa or Asia). At present there is no SIAM journal publishing a reasonable amount of material of interest to our activity group (up to a few years ago this was the case for SIAM J. on Mathematical Analysis). I will try to work towards a better representation of our research in SIAM."

Secretary: Charles F. Dunkl

"I hope the group can continue to maintain the high level of quality of its two newsletters, print and electronic. I would like SIAM to form cooperative and reciprocal agreements with scientific societies in other countries, allowing a more inexpensive way to join the activity group. An obvious benefit would be to encourage more international membership. It seems to me that there is a decline in graduate enrollments in American mathematics departments; the group should try to encourage more young people to do research in our area."

Secretary: M. Lawrence Glasser

"The OPSF Activity Group should continue to serve as a resource center for this area of mathematics: a coordinating body for conferences and regional meetings, an archive for appropriate books and papers, and a forum for workers in this and related fields. I hope to see increased participation in an augmented Problems and Solutions department of the Newsletter to compensate for its loss from the SIAM Review. The group should encourage and spearhead the preparation and updating of resources such as the Bateman Project in print and other media."

Program Director: Francisco Marcellán

"1. - Continue the promotion of our SIAG inside SIAM. The aim is to increase the number of our members, the dissemination of OPSF Newsletter, the minisymposia during the SIAM annual meetings, and a better representation of our research from a qualitative point of view.

"2. - The organization of SIAG thematic workshops (three days) every year in both sides of the Atlantic (North America and Europe) because of the experience of our members in organizing such events. The aim is the attraction of young researchers to our SIAG and the interface with researchers of other domains interested in our subject.

"3. - The organization of a OPSF summer school (seven days) every two years in a developing country. The aim is to promote the research in OPSF there with successful results for the participants. Because of previous relations, Latin American countries and Asia in a first step and African countries in a second step would be good places to realize it."

> **Program Director: Peter A. McCoy**
>
> "The Orthogonal Polynomials and Special Functions Activity Group will be strengthened by a program that enhances global interdisciplinary relations between academics, laboratories and industry. This connectivity broadens the scope of the discipline and expands working relationships between its members. I believe that establishment of a biannual meeting and workshop would serve the purpose of focusing the discipline on particular areas and setting the direction for their study. These initiatives would be highlighted in our Newsletter, on the OP-SF Net, and implemented through minisymposia at the SIAM annual meeting. The initiatives will also be coordinated with special sessions of other societies."

Finally, I would like to apologize for the poor quality of the last issue of the Newsletter. It was printed on only one side of the paper and the pages were not stapled in the usual way. This was the result of an internal problem at the SIAM headquarters, and had nothing to do with our new co-editors.

Here is the letter of explanation from Vickie Kearn who is the responsible SIAM person.

> Dear Wolfram:
>
> When your newsletter came in I was out of the office for a week with the flu. The person who processed the newsletter was new and did not understand the procedure.
>
> I apologize for this and can assure you that the procedure is understood and will be correct in the future.
>
> Vickie

September 30, 1998

Wolfram Koepf
(koepf@imn.htwk-leipzig.de)

As Wolfram Koepf already pointed out, we have collaborated with him in editing the present issue, the last one for him as editor. Again, a lot of material was collected from OP-SF NET. We thank all the people who have submitted items for this issue, specially the Meetings reports.

As usual we hope you find this issue interesting and useful, and remember that you can send items for future issues to either of us.

September 30, 1998

Renato Álvarez-Nodarse
(nodar@math.uc3m.es)
Rafael J. Yáñez
(ryanez@ugr.es)

Reports from Meetings and Conferences

1. VIIth International Scientific Krawtchouk Conference: Kiev, Ukraine, May 14-16, 1998

The *7th International Krawtchouk Conference* took place in Kiev, Ukraine, from May 14 to May 16, 1998. Below are some titles of the reports related to orthogonal polynomials, special functions and integral transforms.

- M. Khomenko, M. Krawtchouk's background
- V. Zelenko, Recent development of M. Krawtchouk's ideas: related articles
- Yu. Bily, M. Krawtchouk on international mathematical forums
- M. Babyuk, Integral Hankel type transforms of the 1st kind and spectral parameter in a boundary condition
- N. Virchenko, About integral equations with generalized Bessel type functions
- V. Gaidei, New generalization of integral transform of the Bessel type
- V. Zelenkov, V. Savva, Orthogonal polynomials as a tool to solve differential equations describing multilevel systems dynamics
- V. Korolyuk, Stochastic Krawtchouk polynomials
- A. Mazurenko, V. Savva, Discrete variable polynomials: Analog of the Christoffel formula and its application to solve some differential equations
- Yu. Mamteev, V. Stukalina, T. Hoochraeva, Features of an algorithm for calculating the modified function by recurrence relations
- M. Mironenko, Pair adder equation in periodic contact problems
- A. Mironov, On the integral equations for the Riemann function
- G. Prizva, Generalization of classical orthogonal polynomials of discrete variable
- E. Seneta, Characterization of Markov chains by orthogonal polynomial systems

- S. Tsurpal, Interaction of simple single waves with a structure as Chebyshev-Hermite functions of any index in the materials with microstructure
- O. Manzyi, Decomposition of the ratio of Appell hypergeometric functions F_3 into the ramified chain fraction

The 8th Conference is to be held in May 2000.

Vadim Zelenkov
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2. International Workshop on Orthogonal Polynomials: Numerical and Symbolic Algorithms, Madrid, June 29-July 2, 1998

A four-day *International Workshop on Orthogonal polynomials: Numerical and Symbolic Algorithms* hosted by the Departamento de Matemáticas, Universidad Carlos III de Madrid, took place during June 29-July 2, 1998.

A total of 72 participants, 47 Spanish ones from 14 different institutions and 25 foreign ones from 23 different places, were engaged in friendly discussions during the whole meeting. There were 6 invited lectures by Walter Gautschi (Purdue University, USA), Gene Golub (Stanford University, USA), Wolfram Koepf (Hochschule für Technik, Wirtschaft und Kultur Leipzig, Germany), Yvon Maday (Université Pierre et Marie Curie, France), Marko Petkovšek (University of Ljubljana, Slovenia) and Doron Zeilberger (Temple University, USA)

The sessions were completed by 24 half-hour communications given by María Álvarez de Morales, Manuel Bello, Andrei B. Bogatyrev, Francisco Cala Rodríguez, Daniela Calvetti, Cecilia Costa, María Victoria Fernández-Muñoz, Esther García Caballero, Peter Kravanja, Stanisław Lewanowicz, Guillermo López Lagomasino, Pedro López, Miguel Lorente, Juan C. Medem, Lionello Pasquini, Carmen Perea Marco, Lothar Reichel, Paolo E. Ricci, Jorge Ruano, Ahmed Salam, Javier Segura, Hossain O. Yakhlef and Rafael J. Yáñez.

All speakers were kindly invited to submit written versions of their talks for the proceedings of the meeting which will be published as a special issue of *Electronic Transactions on Numerical Analysis* (ETNA) (see the URL site `http://etna.mcs.kent.edu/`).

The Organizing Committee was: Manuel Alfaro (Univ. de Zaragoza), Renato Álvarez-Nodarse (Secretary) (Univ. Carlos III), Jorge Arvesú (Univ. Carlos III), and Francisco Marcellán (Chairman) (Univ. Carlos III).

On behalf of the Organizing Committee I want also to thank some other people who have collaborated with the Organization Committee. They are Esteban Moro Egido, Antonio Pastor, Niurka Rodríguez Quintero, Enrique San Millán from the Universidad Carlos III, Rafael Yáñez from Universidad de Granada and Alejandro Zarzo from Universidad Politécnica de Madrid. The workshop was sponsored by Departamento de Matemáticas de la Universidad Carlos III de Madrid, Vicerrectorado de Investigación de la Universidad Carlos III de Madrid, INTAS, Comunidad de Madrid (Consejería de Educación y Cultura) and the Ministerio de Educación y Cultura of Spain (CICyT). Finally, we thank Compaq and Addlink for helping us with the hardware and software respectively. To all these institutions our most sincere acknowledgement.

Below we give the impressions of two of the participants (Marcel de Bruin and Doron Zeilberger)

Renato Álvarez-Nodarse
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From June 29 to July 2 the biannual *International Workshop on Orthogonal Polynomials* was held at the University Carlos III de Madrid in Leganés (near Madrid).

This year the emphasis was on *Numerical and symbolic algorithms* as was clearly indicated by the choice of invited lecturers, each giving two one hour lectures:

1. Walter Gautschi (Orthogonal polynomials and quadrature, Gauss Quadrature for rational functions).
2. Gene Golub (Bounds for the entries of matrices with applications to pre-conditioning, Inverting shape from moments).
3. Wolfram Koepf (Software for the algorithmic work with orthogonal polynomials and special functions).
4. Yvon Maday (The basic spectral element and mortar element methods for elliptic problems, The spectral element method for resolution of the Stokes and Navier-Stokes problems).
5. Marko Petkovšek (Linear operators and compatible polynomial bases).
6. Doron Zeilberger (The unreasonable effectiveness of orthogonal polynomials in combinatorics).

Furthermore there were 24 short communications on different subjects and the meeting was concluded with a problem session.

The mathematical contents and the weather were important ingredients to make this IWOP a great success, but the most important factor was the excellent organization! Without wanting to underestimate the work done by all of the organizers (much in the background) special thanks go to Renato Álvarez-Nodarse and Rafael Yáñez.

We have something to look forward to in two years again.

Marcel G. de Bruin
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This was one of the most exciting and enjoyable conferences that I have ever participated in. The organizers, Paco Marcellán and Renato Álvarez-Nodarse did a superb job in making everyone comfortable. Renato's boundless energy, enthusiasm and friendliness, and Paco's leadership are an excellent recipe for a great conference.

Most of the talks, both invited and contributed, were first-rate. It was remarkable to see the breadth and depth of the Spanish school of orthogonal polynomials and special functions, many of whose members are academic descendants of Paco Marcellán.

Another unique feature of this meeting was its combining numeric and symbolic computations. The older tradition of numeric computation, and the younger tradition of symbolic computations have completely different cultures, concerns, and methodology. I am sure that they both can benefit from a cross-fertilization. By choosing three of the invited speakers (Golub, Gautschi, Maday) from the former camp, and the other three (Koepf, Petkovšek, Zeilberger) to be from the later, the participants and speakers each learned much more than they would in yet another specialized meeting in their field.

I myself, who is almost ignorant of numerical analysis, learned so much from Gautschi's and Golub's talk, and even from Maday's 300 words per minute talk.

The conference also fostered many personal discussions. In particular, I had a fascinating discussion with Marcel de Bruin (who was the only one, incidentally, to hand in the homework that I have assigned during my talk), about determinants. He turned out to be a real determinant-whiz, and later has sent me little-known, but very interesting, papers by van der Corput (in Dutch!).

On a more personal note, I was happy to meet Gene Golub for the first time, after I have heard so much about him from our mutual friend Marvin Knopp. I was also happy to meet the famous Walter Gautschi, who was the first to believe de Branges!

As the years go on, the memories of most conferences blend into a dull continuum. Not this one! It will always be remembered as a very happy singularity.

Doron Zeilberger
(zeilberg@euclid.math.temple.edu)

3. Minisymposium on Problems and Solutions in Special Functions
SIAM Annual Meeting 1998: Toronto, Canada, July 13-17, 1998

On July 14, 1998, our Activity Group sponsored a Minisymposium *Problems and solutions in Special Functions* (Organizers: Willard Miller, Jr. and Martin E. Muldoon) at the SIAM Annual Meeting in Toronto. The organizers recognized that by providing concrete and significant problems, the problem sections in journals such as SIAM Review and the American Mathematical Monthly have been influential in advancing mathematical research and have played a role in attracting young people to the mathematical profession. At a time when the SIAM Review is phasing out its problem sections (see Newsletter 8.2, pp. 19–21) it seemed appropriate to assess the history and impact of the problems sections and their future evolution.

Cecil C. Rousseau, University of Memphis offered a retrospective on the 40-year history of the SIAM Review Problems and Solutions Section, based on his experience as a collaborating editor and then as an editor of the Section. We learned that of the 777 problems proposed, 329 were starred (no solution submitted by the proposer). The title most used was "A definite integral" while the keywords occurring most frequently were "integral" (131 times), "inequality" (47), "identity" (33), "series" (25) and "determinant" (24). The most frequent problem proposers were M. S. Klamkin (46), M. L. Glasser (38), D. J. Newman (24) and L. A. Shepp (20).

Cecil chose a specific issue (April, 1972) and mentioned Problem 72-6 by Paul Erdős (*A solved and unsolved graph coloring problem*) that provided the first contact between Erdős and the Memphis graph theory group (Faudree, Ordman, Rousseau, Schelp), and in that way led to more than 40 joint papers involving Erdős and the members of this group. He mentioned Problem 72-9 (*An extremum problem*) by Richard Tapia, who, coincidentally, was honored on the same day as our Minisymposium by a Minisymposium for his 60th birthday. In the same issue, the solution to Problem 71-7 (*Special subsets of a finite group*) was the very first publication by Doron Zeilberger. Rousseau himself had a solution of Problem 71-13 proposed by L. Carlitz, which called for a proof that a certain integral involving the product of Hermite polynomials was nonnegative. At the time, Rousseau looked for, but did not find, a combinatorial interpretation of the integral that would immediately imply its nonnegativity. That there is such an interpretation was shown by Foata and Zeilberger in 1988. Later in the discussion, Rousseau mentioned that problems sometimes get repeated in spite of the best efforts of the editors; for example, Problem 95-6 repeats part of Problem 75-12 but that he had found the relevant double integral later in Williamson's Calculus (6th ed), 1891!

Otto G. Ruehr, Michigan Technological University, discussed the forty-year history of the Section with particular attention to the second half. He offered an anecdotal description of the trials, tribulations and satisfactions of being editor. Special attention was paid to problems in classical analysis, particularly those relating to orthogonal polynomials and special functions. He regretted that some problems he had proposed (73-12, 84-11) attracted only one solution other than that of the proposer. Sometimes, sheer luck played a role as in a solution of his which depended on the relatively sharp inequality $27e^2 < 200$. In spite of the best editorial efforts, errors often crept in.

In the very last issue which contained problems a complicated asymptotic expression (Problem 97-18) was correct except for an error in sign! Nevertheless, it led to collaboration between one of the proposers (D. H. Wood) and J. Boersma.

Otto mentioned that, very appropriately, the last issue (December 1998) of the Section will be dedicated to its founding editor, Murray S. Klamkin. In some brief remarks, Murray discussed some highlights and problems such as *A network inequality* and (the very first) Problem 59-1 *The ballot problem*, Proposed by Klamkin and Mary Johnson. This has not been solved in the general case.

Willard Miller, Jr., University of Minnesota, spoke on *The Value of Problems Sections in Journals*. He stressed their importance in getting young people interested in mathematics and as a place where a person not expert in an area can get their feet wet. He offered Doron Zeilberger as someone who exemplified the value of problem sections. Bill mentioned that by participation in problems sections you can get established researchers in other areas interested in what you have to say. People see that a problem is hard and when the solution comes out they are interested in it and are challenged to find a better proof.

Richard Askey, University of Wisconsin, was unable to attend the Minisymposium but submitted a written statement, some of which was read by Bill Miller, and which offered some thoughts about problems and the role that a problem section can play in a scientific journal.

Askey's first example was on the generalization to Jacobi polynomials of an inequality for trigonometric functions. Rather than writing a one page paper, he decided to submit it as a problem to have people work on it. Unfortunately his plan failed. Nobody else submitted a solution because he had not been explicit enough about a limiting case which would be more familiar to readers.

Askey also described some of the history (including an incorrect published solution) of a problem where it was required to show that the sum from 1 to n of

$$(-1)^{k+1}\left(\frac{\sin(kx)}{k\,\sin x}\right)^{2m}$$

is positive for all real $x, m = 1, 2, \ldots$.

Askey described his favorite SIAM Review problem as Problem 74-6 (*Three multiple integrals*) submitted by a physicist, M.L. Mehta. It called for the evaluation of a multidimensional normal integral. "I spent many hours on this problem, unsuccessfully. Eventually, a multidimensional beta integral which Atle Selberg had evaluated about 1940, and published a derivation of in 1944, came to light. Then it was easy to prove the Mehta-Dyson conjecture, as Dyson realized once Bombieri told him of Selberg's result. I heard about this from George Andrews, who was in Australia at the time, and he heard of it from Kumar, a physicist there. I worked out what should happen in a q-case, and published my conjectures in SIAM Journal of Mathematical Analysis. All of these conjectures have now been proven. Ian Macdonald heard about Selberg's result from someone in Israel, and he came up with some very significant conjectures about other q-beta integrals. He had been working on questions like this for root systems, and his conjecture for a constant term identity for $BC(n)$ was equivalent to Selberg's result. Some of this would have been done exactly as it was without Mehta's problem in SIAM Review, but I doubt that I would have appreciated the importance of Selberg's result as rapidly if I had not spent so much time on the Mehta-Dyson conjecture."

In the general discussion which followed there was mention of "opsftalk" the discussion forum for this Activity Group. It was generally agreed that it could not replace Problem Sections of the kind being discussed both because of the limited readership and the fact that it is restricted to orthogonal polynomials and special functions.

Dick Askey had cautioned: "I am afraid that having a problem section only on line will lead to a restricted group of readers, those with a love of problems for their own sake, and not reach the wider group of mathematicians, applied mathematicians, and scientists who could use some of the results in these problem sections."

A wide-ranging discussion continued informally between those attending. Some of the points raised in these discussions follow:

It was felt that it was very important to stress that any web initiative for a Problem Section should cover all areas. It would be unsatisfactory to have separate operations for say, the various SIAM activity groups.

There was some skepticism about the web proposal. In particular, the importance of careful editing was stressed. It is a commonplace that much material on the web is sloppy and done in a hurried manner. It will be very important to make sure that the present proposal is carefully monitored. There was also a sense that "putting it on the web" is sometimes offered as a panacea for all sorts of information distribution without a realistic understanding of the work involved.

Nevertheless, the advantages of speed and access which are provided by a web site are eagerly anticipated by those interested in preserving and enhancing the SIAM Review Problem Section.

There should be a part of the web initiative devoted to problems suitable for high school students. This has the potential to greatly broaden the audience for the problem sections and to attract more young people to mathematics research.

The web pages should be divided into two parts. Part

A would contain the problems and refereed solutions, and would be comparable to what appears in the SIAM Review now (but with hyperlinks and other bells and whistles). Part B would be more informal. It would contain proposed solutions (before they have been fully refereed), comments on the solutions and other comments and background information related to the problems. Part B would be more timely. The editor would still control what is posted in Part B but wouldn't vouch for the accuracy of all proofs. Part B would be more lively, and give a better indication of how mathematics research is actually carried on. Part A would be more polished.

There should be some way to archive in print the problems and solutions of Part A. Perhaps a volume could be produced every few years.

SIAM should refer routinely to the website in the Review, say a paragraph in each issue.

Once the website is well launched, there should be an article about the project in the SIAM Newsletter.

Martin Muldoon
(muldoon@yorku.ca)

4. International Congress of Mathematicians: Berlin, Germany, August 18-27, 1998

ICM'98 (the International Congress of Mathematicians) was held during August 18-27, 1998 in Berlin, Germany. Orthogonal Polynomials and Special Functions were not a major theme at this Congress, but some was represented, scattered over the sections:

7. Lie groups and Lie algebras
8. Analysis
11. Mathematical physics
15. Numerical analysis and scientific computing
16. Applications

Here is a (probably not exhaustive) list.

- Fields Medal Winners:

  One of the winners, Richards E. Borcherds, has obtained generalizations of Macdonald identies in connection with generalized Kac-Moody algebras (just as the Macdonald identities follow from denominator identities for affine Kac-Moody algebras).

- Plenary Lectures:

  I.G. Macdonald, "Constant term identities, orthogonal polynomials and affine Hecke algebras"

- Invited Section Lectures:

  Ivan V. Cherednik, "From double Hecke algebra to analysis" (contains some one-variable q-identities for which Cherednik would like to hear from others a classical proof)

  Barry McCoy, "Rogers-Ramanujan identities: A century of progress from mathematics to physics"

  Percy Alec Deift, "Uniform asymptotics for orthogonal polynomials" (very recommended)

  Leslie Frederick Greengard, "A new version of the fast Gauss transform" (uses Hermite polynomials)

- Short Communications and Poster Sessions:

  N. Jing, "Quantized Kac-Moody algebras and symmetric functions"

  Boris Rubin, "Fractional calculus and wavelet transforms in integral geometry"

  Ahmed I. Zayed, "Wavelets in closed form"

  Thomas C. Kriecherbauer, "Asymptotics of orthogonal polynomials via integrable methods" (relates to Deift's lecture)

  Margit Rösler, "Positivity of Dunkl's intertwining operator"

  Raoul R.F.G. Gloden, "Proprietes des polynômes orthogonaux. Développements de cas particuliers"

  Kathy A. Driver, "Zeros of hypergeometric functions"

  William C. Connett, "Measure algebras that have oblate spheroidal wave functions as characters"

  Alan L. Schwartz, "Hypergroups and their maximum subgroup"

  Andreas Ruffing, "On Schrödinger-Hermite operators in lattice quantum mechanics"

  Vitaly Tarasov, "q-Hypergeometric solutions of the quantized Knizhnik-Zamolodchikov equation"

All abstracts are available at the ICM'98 web site

http://elib.zib.de/ICM98

There you will also find a link to a site from which you can download the files of the papers of the Invited Section Lectures.

Tom H. Koornwinder
(thk@wins.uva.nl)

Forthcoming Meetings and Conferences

1. Fifth International Conference on Approximation and Optimization in the Caribbean: Guadeloupe, French West Indies, March 29-April 2, 1999

The complete announcement appeared in Newsletter 8.3, pp. 6–8.

Scientific Program:

1. **Tutorials:** *Wavelets Methods for Numerical Simulation* by A. Cohen and Y. Meyer (France), *Convex Analysis and Nonsmooth Optimization* by J. Borwein (Canada).

2. **Invited talks:** A. P. Araujo (Brazil), H. Attouch (France), A. Bensoussan (France), P.-L Butzer (Germany), F. Clarke (France), I. Ekeland (France), C.C. Gonzaga (Brazil), T. Ichiishi (U.S.A.), A. Ioffe (Israel), E. Saff (U.S.A.), S. Smale (Hong-Kong), H. Stahl (Germany), W. Van Assche (Belgium).

Contributions, Submission and Program Committee: Applicants to the tutorials should send a short CV via e-mail to:

appopt5@univ-ag.fr, subject: tutorial

Contributors are invited to submit abstracts in TEX or LATEX via e-mail to:

appopt5@univ-ag.fr, subject: abstract

Participants can also propose a mini-symposium on a specific topic with 4-5 speakers. A proposal for a mini-symposium, stating the theme, the list of speakers and the abstracts, should be sent via e-mail to:

appopt5@univ-ag.fr, subject: mini-symposium

The **deadline** for applications to the tutorials and for submissions of contributions is 30 October 98. Admission in tutorials and acceptance of abstracts or mini-symposia will be notified by 15 December 98.

Research results which are obtained from joint Caribbean projects and which involve young researchers are especially welcomed. We intend to publish the proceedings of the conference in a special volume of the Caribbean Journal of Mathematics and Computing Sciences (CJMCS).

Program Committee Chair: J. Guddat

- *Approximation:* D. Hinrichsen (Germany), D. Lubinsky (South Africa), F. Marcellan (Spain), W. Roemisch (Germany), H. Wallin (Sweden)
- *Optimization:* J.-B. Hiriart-Urruty (France), P. Kall (Switzerland), B.S. Mordukhovich (U.S.A.), J. Stoer (Germany), M. Tapia (U.S.A.)
- *Mathematical Economics:* B. Cornet (France), C. Herrero (Spain), E. Jouini (France), H. Keiding (Denmark), V. Vasilev (Russia)

To get more information please contact:

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For updated information visit the Conference WWW page
http://www.cepremap.cnrs.fr/conferences/appopt5.html

Francisco Marcellán
(pacomarc@ing.uc3m.es)

2. International Conference on Rational Approximation: Antwerp, June 6-11, 1999

The following information is taken from
http://win-www.uia.ac.be/u/icra99/

The conference will focus on rational approximation theory in the broadest sense, including all computational aspects and applications. Contributions are welcomed on the subjects of Padé Approximation, Continued Fractions, Orthogonal Polynomials and Rational Approximation in general.

More details can be found at the URL http://win-www.uia.ac.be/u/icra99/ or using the e-mail address icra99@uia.ua.ac.be.

Organizing Committee: Annie Cuyt (Annie.Cuyt@uia.ua.ac.be), Brigitte Verdonk (verdonk@uia.ua.ac.be).

Scientific Committee: Adhemar Bultheel (Leuven), Annie Cuyt (Antwerpen), Alphonse Magnus (Louvain La Neuve), Jean Schmets (Liège), Jean-Pierre Thiran (Namur), Marc Van Barel (Leuven), Paul Van Dooren (Louvain La Neuve), Brigitte Verdonk (Antwerpen).

Invited Speakers: George A. Baker Jr. (Los Alamos), Peter Borwein (Simon Fraser University), Peter R. Graves-Morris (Bradford), William B. Jones (University of Colorado at Boulder), George Labahn (Waterloo), Lisa Lorentzen (Trondheim), Doron S. Lubinsky (University of Witwatersrand), Hans J. Stetter (Wien).

Proceedings: Kluwer Academic Publisher and Baltzer Science Publishers have agreed to publish the proceedings of this international conference, which will be distributed over issues of the following three journals:

- Numerical Algorithms
- Reliable Computing
- Acta Applicandae Mathematicae

The participants who wish to contribute to the proceedings, should indicate on their registration form to which journal they want to submit their paper.

Registration: The registration fee of 9500 BEF includes lunch on campus from Monday to Friday, all social events, the conference dinner, transportation from and to town, the coffee-breaks and a copy of the proceedings. Payment for registration should be done by bank transfer on account number 001-3214170-54 of CANT, University of Antwerp (UIA), Universiteitsplein 1, B-2610 Antwerp, Belgium. Be sure to mention "ICRA99".

Deadlines: The deadlines for registration and submission of abstracts is April 1, 1999 and for submission of papers July 1, 1999.

For further information, please, contact

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Wolfram Koepf
(koepf@imn.htwk-leipzig.de)

3. International Workshop on Special Functions: Hong Kong, June 21-25, 1999

An *International Workshop on Special Functions* will take place on June 21-25, 1999 at the *City University of Hong Kong*. The main focus will be on *Asymptotics*, *Harmonic Analysis*, and *Mathematical Physics*. For details, see http://www.cityu.edu.hk/ma/conference/iwsf/.

Objective: The purpose of the conference is to provide a forum for an exchange of ideas among experts in various topics listed below. It also aims at disseminating information on recent advances made in these areas.

Plenary Speakers: K. Aomoto (Nagoya University, Japan), R. Askey (University of Wisconsin-Madison, USA), T. Baker (University of Melbourne, Australia), C. Berg (University of Copenhagen, Denmark), C. Dunkl (University of Virginia, USA), G. Gasper (Northwestern University, USA), W. Gautschi (Purdue and ETH (Zürich), Switzerland), E. Koelink (Universiteit van Amsterdam, The Netherlands), A. McBride (University of Strathclyde, UK), F. Olver (University of Maryland, USA), R. O'Malley (University of Washington, USA), E. Opdam (University of Leiden, The Netherlands), R. Simion (George Washington University, USA), D. Stanton (University of Minnesota, USA), N. Temme (CWI, Amsterdam,The Netherlands), A. Terras (University of California, San Diego, USA), V. Totik (Jozsef Attial University, Hungary), L. Vinet (CRM, Université de Montréal, Canada), R. Wong (City University of Hong Kong), Y. Xu (University of Oregon, USA)

Session Topics: Asymptotics, Classical Special Functions, Combinatorics, Harmonic Analysis and Quantum Groups, Mathematical Physics and PDEs, Orthogonal Polynomials.

Organizing Committee: Charles Dunkl, U. of Virginia, USA; Mourad Ismail, U. of South Florida, USA; Roderick Wong, City U. of Hong Kong.

Call for Papers: Titles and abstracts of contributed papers must be received by January 31, 1999. The abstracts should be preferably typed in LaTeX, not to exceed one page, and sent to the Workshop Secretary (see address below) by e-mail.

Information: Colette Lam, IWSF '99 Workshop Secretary, Department of Mathematics, 83 Tat Chee Avenue, Kowloon, Hong Kong; phone: +852 2788-9816, fax: +852 2788-8561; e-mail: malam@cityu.edu.hk; workshop e-mail: hkconf99@weyl.math.virginia.edu.

Charles F. Dunkl
(cfd5z@virginia.edu)

4. ICIAM '99: Edinburgh, July 5-9, 1999

The following is taken mostly from the conference web site: http://www.ma.hw.ac.uk/iciam99/

"The Fourth International Congress on Industrial and Applied Mathematics will be held in Edinburgh, the capital of Scotland, from 5th to 9th of July 1999.

"It will be jointly organised by the *Institute of Mathematics and its Application* and the *International Centre for Mathematical Sciences*, with the involvment of *Mathematics Departments at Edinburgh University* and *Heriot-Watt University.*

"Previous Congresses met in Paris in 1987, Washington, DC in 1991, and Hamburg in 1995. Broad developments and the latest advances in industrial and applied mathematics will be presented. Cross disciplinary themes within mathematics, between mathematics and other disciplines, and between mathematics and particular industries will be covered."

SIAM is one of the Member Societies of the *Committee for International Conferences on Industrial and Applied Mathematics* (CICIAM).

The web site includes information on mini-symposia. A mini-symposium is a session of 3-5 speakers focusing on a single topic and lasting for two hours. The organiser of a mini-symposium invites the speakers and decides on the topics to be addressed. The deadline date for mini-symposium proposals is 30 September 1998. Our Activity Group sponsored a mini-symposium at the 1995 ICIAM in Hamburg. Suggestions of minisymposia to be sponsored by us at the 1999 meeting should be sent to our Program Director, Willard Miller (miller@ima.umn.edu).

Martin Muldoon
(muldoon@yorku.ca)

5. Analytic Methods of Analysis and Differential Equations: Minsk, Belarus, September 14-18, 1999

The *Institute of Mathematics of Belarusian National Academy of Sciences* and *Belarusian State University* (BSU) together with *Moscow State University* and the *Computer Center of the Russian Academy of Sciences* will organize the International Conference "Analytic Methods of Analysis and Differential Equations" (AMADE) on September 14-18, 1999, in Minsk, Belarus. The arrival and departure days are September 13 and 19.

The Section Titles of AMADE:

1. Integral Transforms and Special Functions.
2. Differential Equations and Applications.
3. Integral, Difference, Functional Equations and Fractional Calculus.

The length of plenary invited lectures is 45 min, reports 20 min, and short communications 10 min. The publication of the abstracts is planned. The Proceedings of Conference are supposed to be published in the Journal "Integral Transforms and Special Functions".

Organizing Committee:

Chairmen: academician I.V. Gaishun (Belarus), academician V.A. Il'in (Russia) and Rector of BSU A.V. Kozulin.

Vice-Chairmen: V.V. Gorokhovik (Belarus), A.A. Kilbas (Belarus), V.I. Korzyuk (Belarus) and A.P. Prudnikov (Russia).

Secretaries: M.V. Dubatovskaya (Belarus), S.V. Rogosin (Belarus).

Members: P. Antosik (Poland), C. Brezinski (France), L. Debnath (USA), I.H. Dimovski (Bulgaria), L. Gatteschi (Italy), J. Gilewicz (France), H.-J. Glaeske (Germany), R. Gorenflo (Germany), V.I. Gromak (Belarus), N.A. Izobov (Belarus), N.K. Karapetyants (Russia), V.S. Kiryakova (Bulgaria), O.I. Marichev (USA), E.I. Moiseev (Russia), A.F. Nikiforov (Russia), O.A. Oleinik (Russia), O.A. Repin (Russia), V.N. Rusak (Belarus), M. Saigo (Japan), S.G. Samko (Portugal), A.A. Sen'ko (Belarus), H.M. Srivastava (Canada), B. Stankovic (Yugoslavia), P.K. Suetin (Russia), N.A. Virchenko (Ukraine), P.P. Zabreiko (Belarus), A.H. Zemanian (USA), E.I. Zverovich (Belarus), L.A. Yanovich (Belarus), N.I. Yurchuk (Belarus).

Our address:

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amade99@mmf.bsu.unibel.by

Let us know by the end of December, 1998, about your intention to participate in the Conference. Please send the following information:

1. Name, Surname
2. Affiliation and Position
3. Mailing Address and Telephone (FAX)
4. e-mail
5. Section Title
6. Title of Report

The second announcement with preliminary list of participants, registration fee, travelling information, accommodation, etc. will be sent in March of 1999.

Sergei Rogosin
(rogosin@mmf.bsu.unibel.by)

6. Fifth International Symposium On Orthogonal Polynomials, Special Functions And Their Applications: Patras, Greece, September 20-24, 1999

On September 20-24, 1999 the fifth international symposium on *Orthogonal Polynomials, Special Functions and their Applications* (OPSFA, in short) will be held in Patras, Greece, at the Department of Mathematics, University of Patras.

The OPSFA follows the European Conferences of Bar-Le-Duc (1984), France; Segovia (1986), Spain; Erice (1990), Italy; Evian (1992), France; and also Granada (1991, VII SPOA), Spain; Delft (1994, in honor of Thomas Jan Stieltjes Jr. (1856-1894)), Netherlands; and Sevilla (1997, VIII SPOA), Spain.

Main subjects: The main subjects of OPSFA include:

- Zeros of Orthogonal Polynomials and Special Functions.
- Asymptotics of Orthogonal Polynomials and Special Functions.
- Orthogonal Polynomials, Special Functions and Functional Analysis.
- Continued Fractions, Orthogonal Polynomials and Special Functions.
- Complex Orthogonal Polynomials.
- Computational Problems on Orthogonal Polynomials and Special Functions.
- Potential Theory.
- Orthogonal Polynomials, Special Functions and Symbolic Computer Algebra.
- Structural Properties of Orthogonal Polynomials.
- Spectral Properties of Orthogonal Polynomials.
- Orthogonal Polynomials and Sobolev Spaces.
- Orthogonal Polynomials and Moment Problems.
- Differential Operators and Orthogonal Polynomials.
- Orthogonal Polynomials and Special Functions in connection with Quantum Groups.
- Special Functions and Orthogonal Polynomials in Physics and Engineering.
- Approximation Theory and Orthogonal Polynomials.
- Mathematical Modelling of Natural Phenomena.

Local Organizing Committee: E. K. Ifantis, C. G. Kokologiannaki, P. D. Siafarikas

Scientific Committee: Walter Van Assche (Belgium), Marcel de Bruin (Holland), Evangelos Ifantis (Greece), Andrea Laforgia (Italy), Lance Littlejohn (USA), Paco Marcellán (Spain), Martin Muldoon (Canada), Panayiotis Siafarikas (Greece).

The scientific program is currently being elaborated by the scientific committee. It consists of some plenary lectures and short communications (20 minutes). The second circular will give detailed information about it.

The cost of attendance is expected to be very reasonable. The following estimates are subject to change but it is anticipated that the registration fee will be around 50.000 drachmas (1$=300 drachmas approx.), which includes the admission to the Symposium, a copy of the book of abstracts, a copy of the Proceedings, reception and participation in some social events (welcome drink, a Greek evening, a visit to ancient Olympia, etc.).

To help us with the organisation of the Symposium, we would appreciate if you, already at this early stage, could indicate your potential attendance by completing the preliminary registration at our web site or by contacting us. If you are interested in being invited to participate or in receiving subsequent circulars, please fill out the preregistration form (at our web site or obtainable from us) and return it as soon as possible and, in any case, not later than October 31, 1998 to the Symposium Mailing Address.

The Symposium will be held at the building of Department of Mathematics of the University of Patras. The Department is located at the University Campus, 7 km from downtown of the city of Patras and 3 km from Rio region, where there are many hotels which the participants could choose to stay. More details will be given in the next circulars.

Access to Patras is easy; it lies along the National Road that connects Athens with Patras (220 km). For more information see also "how to reach Patras" at our website.

Mailing address:

Fifth international Symposium OPSFA
Department of Mathematics
Prof. P. D. Siafarikas
University of Patras
Patras 26500 Greece
Fax: +(3) 061 997169
E-Mail: OPSFA@math.upatras.gr
WWW: http://www.math.upatras.gr/opsfa/

Panos D. Siafarikas
(panos@math.upatras.gr)

Books and Journals

Book Announcements

1. Orthogonal Functions, Moment Theory, and Continued Fractions
Edited by William B. Jones and A. Sri Ranga

Marcel Dekker, Inc., New York, Lecture Notes in Pure and Applied Mathematics 99, 1998, 440 pp., $ 165, ISBN 0-8247-0207-7

This announcement is from http://www.dekker.com

Description: This valuable collection of articles outlines an array of recent work on the analytic theory and potential applications of continued fractions, linear functionals, orthogonal functions, moment theory, and integral transforms.

Describes links between continued fractions, Padé approximation, special functions, and Gaussian quadrature!

Featuring the insights of nearly 30 contributors, Orthogonal Functions, Moment Theory, and Continued Fractions

- analyzes the asymptotic behavior of continued fraction coefficients for the Binet and gamma functions
- details new results on orthogonal Laurent polynomials
- computes special functions in the complex domain using continued fractions
- uses the Freud conjecture to analyze the coefficients of Stieltjes continued fractions for the first time
- profiles new results using Szegő polynomials and their application to frequency analysis
- develops new results on strong moment theory and orthogonal rational functions using finite Blaschke products
- proves that a two-parameter subfamily can subsume a four-parameter family of twin-convergence regions for continued fractions
- and more!

Including over 1600 equations, references, and drawings, *Orthogonal Functions, Moment Theory, and Continued Fractions* is suitable for pure and applied mathematicians, numerical analysts, statisticians, theoretical and mathematical physicists, chemists, electrical engineers, and upper-level undergraduate and graduate students in these disciplines.

Contents:

Wolfram Koepf
(koepf@imn.htwk-leipzig.de)

2. International Conference on Harmonic Analysis
By K.A. Ross e.a. (eds.)

Birkhäuser, 1998. 256 pages, ISBN 3-7643-3943-8

Table of Contents:

Kenneth Ross
(ross@math.uoregon.edu)

3. Fractional Order Integral Transforms of Hypergeometric Type
By N. Virchenko and V. Tsarenko

Kiev, 1995, 216 pages, ISBN 5-7702-1101-6, in Russian

This book deals with the theory and apparatus of new integral transforms (the fractional G-transforms) with kernels which are transcendental solutions of differential equations of hypergeometric type.

Following this is a development and research in the theory of integral operators, integral equations with Gauss hypergeometric function which correspond to different special cases of parameters and variables.

The main titles of the sections are as follows:

Chapter 1. Integral transforms of the fractional order connected to orthogonal polynomials.

1. Some information on the theory of orthogonal polynomials.
2. Integral transforms of fractional order.
3. Basic fractional operational calculus.
4. Some applications of integral fractional G-calculus.

Chapter 2. Integral transforms connected to the hypergeometric function ${}_2F_1(a, b; c; z)$.

1. Application of classical methods for reception of the inversion formulae.
2. Method of fractional integro-differentiation.

Vadim Zelenkov
(zelenkov@gray.isir.minsk.by)

4. The Askey-scheme of hypergeometric orthogonal polynomials and its q-analogue
By Roelof Koekoek and René F. Swarttouw

Delft University of Technology, Faculty of Information Technology and Systems, Department of Technical Mathematics and Informatics, Report no. 98-17, 1998.

Recently a completely revised and updated version of our report appeared.

A PostScript-file named `DUT-TWI-98-17.ps.gz` can be obtained by using ftp: `ftp://ftp.twi.tudelft.nl/` at the directory `TWI/publications/tech-reports/1998/`

This revised version includes a description of all families of hypergeometric orthogonal polynomials appearing in the Askey-scheme (named after Richard A. Askey) and in the $q-$analogue of this scheme. For all families of these (basic) hypergeometric orthogonal polynomials we give

- the definition in terms of hypergeometric functions
- the second order differential or difference equation
- some generating functions

and also the (limit) relations between the families of orthogonal polynomials appearing in both schemes. Further we updated the list of references and added the following formulas for each family of (basic) hypergeometric orthogonal polynomials :

- the three term recurrence relation for the monic orthogonal polynomials (with leading coefficient equal to 1)
- forward and backward shift operators
- Rodrigues-type formula

More information (including a link to this ftp-address) can be found at the web page http://aw.twi.tudelft.nl/~koekoek/research.html

Roelof Koekoek
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René F. Swarttouw
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5. Srinivasa Ramanujan, a Mathematical Genius
By K. Srinivasa Rao

EastWest Books, Madras, 1998, xii+231 pp., ISBN: 81-86852-14-X

Table of Contents:

Tom H. Koornwinder
(thk@wins.uva.nl)

Book Reviews

1. Representation of Lie groups and Special Functions, Vols 1,2,3
By N. Ja. Vilenkin and A. U. Klimyk
Translated from the Russian by V. A. Groza and A. A. Groza

Kluwer Acad. Publ., Dordrecht, $804.50 (set).
Vol. 1: Simplest Lie groups, special functions and integral transforms, vol. 72, 1991, xxiv + 608 pp., $408.00, ISBN 0-7923-1466-2;
Vol. 2: Class I representations, special functions, and integral transforms, vol. 74, 1992, xviii + 607 pp., $397.00, ISBN 0-7923-1492-1;
Vol. 3: Classical and quantum groups and special functions, vol. 75, 1992, xx + 634 pp., $397.00, ISBN 0-7923-1493-X

(Editor's Note: This review appeared in the Bulletin of the AMS Vol 35, number 3, July 1998, pages 265-270, see http://www.ams.org/bull/1998-35-03/, and is reprinted with permission from the AMS.)

The book under review deals with the interplay between two branches of mathematics, namely representation theory of groups and the theory of special functions. Both fields go back over a century, and their intimate connection has been observed since the forties and fifties. Pioneering work was done by Bargmann [2], Gel'fand & Šapiro [6] and Wigner [23]. They linked the representation theory of the Lorentz group and of the rotation group in three-dimensional space to hypergeometric functions and Jacobi polynomials. Since then an enormous amount of work has been done on this subject, also motivated by physical models. The state of the art in the sixties has been given in the books by Talman [17] and, in particular, Vilenkin [19], while Miller's book [14] exposed a very different approach. Some less comprehensive books or edited volumes have appeared afterwards in which newer developments are discussed, for instance [1], [3], [4], [8], [9], [15], [18], [22]. In 1990 (English translation in 1995) Vilenkin and Klimyk wrote a pleasant, relatively short introduction [20] to the subject, taking in account modern developments. But none of these books has the wide scope of the three-volume set under review, which is the successor of Vilenkin's influential book [19].

In the preface the authors state that their aim is "to summarize the development of the theory and to outline its future development." This is certainly a challenging task. On the one hand, if one stays within the scope of the paradigms of the pioneers, the technical complexity of cases being studied has enormously increased. On the other hand there have been many developments in the interaction between special functions and algebraic structures which do not easily fit into the old paradigms. We mention special functions related to any of the following structures: Jordan algebras, symmetric groups and Chevalley groups, p-adic groups, discrete subgroups (relation with number theory), infinite dimensional limits of classical groups, affine Lie algebras, root systems, quantum groups, Hecke algebras, association schemes, hypergroups, combinatorics. Some of these new interpretations of special functions are in the volumes under review, as we will indicate below. In our possibly biased view, developments on special functions related to root systems (Heckman-Opdam polynomials, Macdonald polynomials) and their interpretations on Hecke algebras (via Dunkl-Cherednik operators) and on quantum groups have been in particular spectacular; see for instance Macdonald's Bourbaki lecture [13] and Noumi & Sugitani [16]. The authors of the present volumes have treated some of these last topics in a volume 4, called *Recent advances* [21].

The notion of special functions is not precisely defined, but we tend to think of special functions as functions that occur in solutions of specific problems and satisfy many explicit properties, in particular a rich collection of formulas. Turán and Askey have suggested calling them useful functions. As a typical example one might think of the Bessel function, which arises as a solution of the Laplace operator in cylindrical coordinates. Another example is the Jacobi polynomial, which also yields a solution of the Laplace operator but now in spherical coordinates. The Bessel function and the Jacobi polynomial can both be expressed in terms of (generalized) hypergeometric functions, and this feature holds for almost all special functions considered in this book. Jacobi polynomials form a system of orthogonal polynomials, and from a Bessel function one can build a generalized orthogonal system described by the Hankel transform pair. Such orthogonalities, which occur for many special functions, give rise to generalized Fourier analysis and suggest a link with harmonic analysis on groups.

The groups that play a role are usually Lie groups. Already three-dimensional groups like $SU(2)$, $SO(3)$, $SL(2,\mathbb{R})$, $ISO(2)$ and the Heisenberg group allow interpretations of many familiar one-variable special functions. On higher dimensional analogues of these groups like $SU(n)$ and $SL(n,\mathbb{R})$ one finds interpretations of the same special functions for more general parameter values and interpretations of special functions of more complex nature. Typically, on such a Lie group G, one considers some canonical decomposition of G in terms of certain subgroups, and one takes a coordinate system on G corresponding to this decomposition. Then one considers the irreducible representations (irreps) of G in a suitable basis, behaving nicely w.r.t. a subgroup H involved in the decomposition of G; one looks at the matrix elements as functions in the coordinates; and one tries to recognize the matrix elements as (products of) special functions. Most commonly, one does this for the spherical functions on G, i.e., for matrix elements which are, as functions on G, left and right invariant w.r.t. H. After having established expressions for matrix elements of irreps, one can use the group to find properties of the special function involved. For compact groups one finds orthogonality relations for the special functions from Schur's orthogonality relations. This works also for square integrable representations of non-compact groups, but in general one finds transform pairs in the non-compact case. This is essentially the computation of the Plancherel formula. Also, from the homomorphism property of the representations it is possible to derive additional formulas for the special functions. These are only a few of the many properties which can be obtained from their interpretation on the group. It is important to know that this is not a one-way influence. Specific properties of the special functions involved are sometimes needed to establish theorems on the group level. For instance, Harish-Chandra [7] conjectured the explicit Plancherel measure for the spherical Fourier transform on a non-compact symmetric space by using spectral analysis of the hypergeometric differential operator. Later he found a complete proof.

Another important way to link special functions to representations of Lie groups is the following. If the same representation has two explicit realizations in terms of (generalized) orthonormal bases, then the transition matrix is orthogonal and we obtain orthogonality relations if the matrix elements can be calculated explicitly in terms of special functions. Typical examples of such constructions are the Clebsch-Gordan coefficients and the Racah coefficients connecting different orthonormal bases in tensor product representations.

The interaction between group representations and special functions can be approached from at least three different points of view:

(i) Start with a special group. Consider different kinds of special functions occurring on it. This usually gives rise to relationships between these special functions. For instance, Jacobi polynomials and Hahn polynomials live on $SU(2)$ as matrix elements of irreps and as Clebsch-Gordan coefficients, respectively. As a consequence, products of Hahn polynomials occur as expansion coefficients in the expansion of a product of Jacobi polynomials in terms of other Jacobi polynomials; see §8.3.6 of the volumes under review.

(ii) Start with a special function. Find interpretations for it of various kinds on several groups. Then try to find a conceptual explanation which links these interpretations together. For instance, Krawtchouk polynomials live on $SU(2)$, again as matrix elements (see §6.8.1) and on wreath products of symmetric groups as spherical functions (see §13.1.4). A conceptual link between the two interpretations was given in Koornwinder [11].

(iii) Start with a general structure in the context of group representations. Find properties involving special functions which fit into this structure. For instance, consider (zonal) spherical functions on Gelfand pairs; see §17.2. Spherical functions satisfy many nice properties. Whenever one has an interpretation of a special function as a spherical function, then one should try to rephrase these nice properties in terms of the special function.

In our opinion, the third approach should get the most emphasis. Of course, the cases where elegance in group representations and in special functions happily meet do not exhaust everything of interest in special functions. Many important formulas for special functions may be derived in a shorter or longer, but not very illuminating, way from the group interpretation, but may possibly have a shorter derivation just from the analytic definition of the special function. We think that for these cases one should be pragmatic and give the shortest derivation.

Let us now discuss the volumes under review in more detail. The authors have succeeded in writing an encyclopedic treatise containing a wealth of identities on special functions. The main examples fit into the methods sketched above. But there is also a chapter in volume 3 on the quantum $SU(2)$ group, or better on the quantized universal enveloping algebra $U_q(\mathfrak{sl}(2,\mathbb{C}))$. There is also some information on the symmetric group, on groups over finite fields and over the p-adics, on affine Lie algebras and on modular forms.

A typical chapter starts with a discussion of the Lie group and Lie algebra involved, while introducing suitable bases for the Lie algebra and corresponding one-parameter subgroups of the Lie group. This then gives suitable coordinates on the Lie group like the Euler angles on $SU(2)$. Next the representation theory is discussed: constructions, irreducibility and intertwiners. The special functions are then brought into play, and usually the remainder of such a chapter is on the special functions involved. Then the role of the group is pushed into the background and the special functions take a predominant role. Some special function identities derived do not involve any group theoretic considerations.

As mentioned previously, the three-volume set is the successor of Vilenkin's 1965 book [19], so a comparison is in order. Vilenkin's book [19] is about the size of one of these volumes. The subject of the chapters of volume 1 overlaps with Chapters 1–8 of [19], although the material is expanded, which is particularly true for Chapter 8 in volume 1 on Clebsch-Gordan and Racah coefficients, one of the fields of expertise of the second author [9] (see also Chapter 18 in volume 3). Also new compared to [19] is the consideration of the transition matrices for irreps of $SL(2,\mathbb{R})$ in Chapter 7. Volume 2 has some overlap, but much less so, with Chapters 9–11 of [19]. So the new material is mainly contained in volumes 2 and 3.

Chapters 13, 14 and 19 deviate from the others in the sense that the groups considered are not Lie groups. In Chapter 13 discrete groups are considered. First there are two examples of finite groups: the symmetric group (but not any other Weyl group) related to Hahn and Krawtchouk polynomials and finite groups of Lie type related to basic (or q-)hypergeometric polynomials. It is a pity that the authors' notation for basic hypergeometric series has a meaning which is different from the usual one; see the standard reference by Gasper and Rahman [5]. Also in Chapter 13 there is a section on the p-adic number field and related Γ and B-functions and on SL_2 over the p-adics, but not on the spherical functions on a group of p-adic type; see Macdonald [12]. Chapter 14 contains a discussion of the quantum $SL(2,\mathbb{C})$ group and its relation to basic hypergeometric orthogonal polynomials. The chapter does not treat the important interpretation of Askey-Wilson polynomials as spherical functions or matrix elements on the quantum $SL(2,\mathbb{C})$ group; see for instance Koelink's survey lectures [10]. Finally, Chapter 19 contains an introduction to affine Lie algebras and modular forms.

The present book does not always follow a clear philosophy about what should be obtained from the group context and what can be derived as well analytically (see our point of view above). For example, the second order differential equation for the Jacobi polynomials is first derived in §6.7.5 by composition of two ladder operators acting on matrix elements of irreps of $SU(2)$. These ladder operator actions are derived in a rather complicated way, with the group playing some role, but the opportunity is missed to make a link with the ladder operator action of the Lie algebra in §6.2.2. The most conceptual interpretation of the second order differential equation, from the action of the Casimir operator, occurs only 30 pages later, in a short remark in §6.10.3. For an example of another kind, the first order divided difference formula for Racah polynomials is given (not quite correctly) in §8.5.4. The straightforward and short proof from the ${}_4F_3$ formula for Racah polynomials is not mentioned. Instead a proof is indicated which uses formulas which are obtained from the group context, but which does not look very conceptual and which is tedious in computational details.

We consider this three-volume set as a reference work rather than a book which one will use for learning the subject. With this in mind there are some comments to be made on the presentation. Apart from a 5-page Chapter 0 in volume 1 there is no motivation given. One would expect that at least for each chapter a short introduction would be in place. The table of contents is extraordinarily long, giving the full list of sections and subsections of each chapter. On the other hand, the index is too restricted, while we badly missed a cumulative index in volume 3. There is no hierarchy with important results being formulated as theorems and less important results as propositions, apart from sections 18.6–18.7, which have been written by A. V. and L. V. Rozenblyum. Almost nowhere in the text are references given to the literature. The lists of references at the end of each of the three volumes altogether contain 845 items. At the end of volume 3 the authors have indicated the primary and secondary references for each chapter, but without specifying the particular section or subsection to which the reference applies. We missed more verbose notes at the end of each chapter. Navigation through the volumes is further complicated because the thousands of formulas have a simple numbering starting with 1 in each new subsection, while the running heads mention only the chapter number, not the number of the section or subsection. Tables and diagrams are almost absent. They would have made the book more accessible. Although a geometric approach is sometimes emphasized, this is not supported by pictures.

The book contains many errors; we think too many. Most of them are harmless, but one should not blindly trust the formulas in the book. Apparently there has been no double or triple checking of the formulas. We have found a few conceptual mathematical errors. For producing a full list of errata one has to read and check computations in all 1800 pages, which we have not done.

In view of the price of the books it is likely that only the wealthiest libraries and individuals can afford to buy them. For this price we would have expected greater care in editing on the part of the publisher than was evident from the books.

The conclusion is that the authors have done a great job in compiling this encyclopedic treatise and that this three-volume set will become a standard reference for special functions and group representations. Needless to say, these books form a worthwhile addition to any mathematics library. But with some more time and effort, both from the authors and the publisher, the result could have been much better. However, for anybody starting to learn the subject, Vilenkin's older book [19], possibly complemented with [20], is still the best buy.

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2. Conformal Invariants, Inequalities, and Quasiconformal Maps
By Glen D. Anderson, Mavina K. Vamanamurthy and Matti K. Vuorinen

Canadian Mathematical Society, Series of Monographs and Advanced Texts, Wiley, New York, 1997, 505 pp., $ 79.95, ISBN 0-471-59486-5

This book deals with the estimation and evaluation of conformal and quasiconformal invariants. A function $f : D \to \mathbb{R}^n$ of a region $D \subset \mathbb{R}^n$ is called conformal if it is angle-preserving. Hence informally it maps small disks to small disks. In two dimensions a map is conformal if and only if it is analytic with zero-free derivative.

As an extension, $f : D \to \mathbb{R}^2$ of a region $D \subset \mathbb{R}^2$ is called K-quasiconformal, if it maps small disks to small ellipses such that the local distortion ratio remains bounded by a constant K. This terminology can be extended to higher dimensions.

For analytic functions of the unit disk $\mathbb{D}$, Schwarz' lemma holds: If $f : \mathbb{D} \to \mathbb{C}$ is analytic with $f(0) = 0$ and $|f(z)| \le 1$, then $|f(z)| \le |z|$. As an extension, the quasiconformal Schwarz lemma states that if f is a K-quasiconformal map of the unit ball B^n into itself with $f(0) = 0$, then $|f(x)| \le \varphi_{K,n}(|x|)$ for all $x \in B^n$ where $\varphi_{K,n}$ is a certain special function depending only on K and n.

In particular, for $n = 2$, one has

$$\varphi_{K,2}(r) = \mu^{-1}\left(\frac{\mu(r)}{K}\right)$$

where

$$\mu(r) = \frac{\pi}{2}\,\frac{\mathcal{K}(\sqrt{1-r^2})}{\mathcal{K}(r)}\ ,$$

and

$$\mathcal{K}(r) = \int_0^1 \frac{dt}{\sqrt{(1-t^2)(1-r^2t^2)}}$$

is the complete elliptic integral.

This shows how special functions come into the scenery. To obtain sharp bounds in such inequalities, one is forced to study special function like the complete elliptic integral rather carefully.

Therefore, the authors begin by defining and studying special functions, in particular hypergeometric functions and complete elliptic integrals. Then the theory of conformal and quasiconformal mappings is developed. With the aid of elliptic integrals important special conformal and quasiconformal mappings are defined and studied.

Finally, bounds for distortion functions, inequalities for conformal and quasiconformal invariants and other estimates are given. Some of these estimates are rather sharp; mostly they are the currently best-known ones.

In my opinion, this is a very interesting book for the reader interested in conformal and quasiconformal mappings who is willing to study special functions rather deeply. Indeed, without these studies, the results of the book cannot be obtained.

Contents:

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3. Special Functions and Differential Equations By K. Srinivasa Rao, R. Jagannathan, G. Vanden Berghe & J. Van der Jeugt (Eds.)

Allied Publishers, New Delhi, 1998, xiv+486 pp., ISBN 81-7023-764-5

This volume contains the Proceedings of a workshop held at the *Institute of Mathematical Sciences* in Chennai (formerly known as Madras), India, from January 13 to January 24, 1997.

The workshop was devoted to Special Functions and their q-generalizations as well as to Differential Equations and Numerical Methods. Most of the 46 contributions which appear in this volume are focused in the study of special functions and their applications in mathematical and theoretical physics. Topics such as the Lie theoretic approach, root systems, quantum groups, Hecke algebras, the general theory of orthogonal polynomials and computer algebra methods for finding hypergeometric identities as well as the applications of special functions in engineering are represented.

A key-note address by R. P. Agarwal gives an interesting survey on special functions in India during the past century. The contributions of the Indian teams in several domains such as Integral Transform Calculus, Generalized Hypergeometric Functions, Orthogonal Polynomials and the theory of Lie Groups and Algebras are recognized and they show the powerful and creative activity of the Indian mathematicians. This historical survey is welcome in order to disseminate such contributions and stimulate a stronger connection between researchers in the fields.

Among the papers included in the book, I recommend the following five papers:

1. Tom H. Koornwinder, *Special functions associated with root systems: A first introduction for non-specialists*

2. F. Calogero, *Generalized Lagrangian interpolation, finite-dimensional representations of shift operators, remarkable matrices, trigonometric and elliptic identities*

3. K. Srinivasa Rao, *Angular momentum coefficients and generalized hypergeometric functions*

4. L. Lapointe and L. Vinet, *Operator construction of the Jack and Macdonald symmetric polynomials*

5. W. Van Assche, *Approximation theory and analytic number theory*

which gives very impressive sample of the power of the tools of orthogonal polynomials and special functions in several areas of Mathematics.

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Software Announcements

1. Revised Maple Packages on Hypergeometric and q-Hypergeometric Summation

Revised versions of our Maple packages ‘`hsum.mpl`‘ and ‘`qsum.mpl`‘ on hypergeometric and q-hypergeometric summations coming with the book *Wolfram Koepf: Hypergeometric Summation, Vieweg, Braunschweig/Wiesbaden, 1998* that was announced in Newsletter 8.3, p. 8, are available now from the web site
www.imn.htwk-leipzig.de/~koepf/research.html .

They contain implementations of Gosper's, Zeilberger's, Petkovšek's and related algorithms and their q-analogues, respectively. For details you should consult the accompanying book which is distributed by the AMS (www.ams.org/bookstore).

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Problems

Thus far 19 problems have been submitted seven of which had been solved in previous issues (#**1**, **2**, **4**, **6**, **7**, **10**, **14**) Still unsolved are Problems #**3**, **5**, **8**, **9**, **11**, **12**, **13**, **15**, **17**, **18** and **19**. This time neither a new problem nor a solution have been submitted. Since our policy is to reprint every unsolved problem three times, we reprint Problem 19.

19. Uniform Bounds for Shifted Jacobi Multiplier Sequences. For Fourier series the following is immediate: Suppose the real or complex sequence $\{m_k\}$ generates a bounded operator on $L^p(\mathbf{T})$, $1 \le p \le \infty$, i.e., for polynomial f

$$\left\| \sum m_k \hat{f}_k e^{ik\varphi} \right\|_{L^p(\mathbf{T})} \le \|m\|_{M^p(\mathbf{T})} \left\| \sum \hat{f}_k e^{ik\varphi} \right\|_{L^p(\mathbf{T})} ,$$

then one has for the shifted sequence $\{m_{k+j}\}_{k\in\mathbf{Z}}$ that

$$\sup_{j\in\mathbf{N}_0} \|\{m_{k+j}\}\|_{M^p(\mathbf{T})} \le C\,\|m\|_{M^p(\mathbf{T})} , \ 1 \le p \le \infty . \qquad (1)$$

Looking at cosine expansions on $L^p(0,\pi)$ one easily derives the analog of (1) via the addition formula

$$\cos(k \pm j)\theta = \cos k\theta \cos j\theta \mp \sin k\theta \sin j\theta$$

provided the periodic Hilbert transform is bounded, i.e., for $1 < p < \infty$. More generally, by Muckenhoupt's transplantation theorem [2, Theorem 1.6],

$$\left(\int_0^\pi \left| \sum m_{k+j}\, a_k\, P_k^{(\alpha,\beta)}(\cos\theta) \right|^p \sin^{2\alpha+1}\frac{\theta}{2} \cos^{2\beta+1}\frac{\theta}{2}\, d\theta \right)^{1/p}$$

$$\equiv \left(\int_0^\pi \left| \sum m_{k+j}\, b_k\, \phi_k^{(\alpha,\beta)}(\cos\theta) \right|^p w_{\alpha,\beta,p}(\theta)\, d\theta \right)^{1/p}$$

$$\approx \left(\int_0^\pi \left| \sum m_{k+j}\, b_k\, \cos k\theta \right|^p w_{\alpha,\beta,p}(\theta)\, d\theta \right)^{1/p},$$

where $P_k^{(\alpha,\beta)}$ are the Jacobi polynomials, $\phi_k^{(\alpha,\beta)}(\cos\theta)$ are the orthonormalized Jacobi functions with respect to $d\theta$, and

$$w_{\alpha,\beta,p}(\theta) = \sin^{(2-p)(\alpha+1/2)}\frac{\theta}{2} \cos^{(2-p)(\beta+1/2)}\frac{\theta}{2} .$$

Therefore, the above argument for cosine expansions also applies to Jacobi expansions provided the periodic Hilbert transform is bounded with respect to the weight function $w_{\alpha,\beta,p}$; hence, the analog of (1) holds for Jacobi expansions when

$$\frac{2\alpha+2}{\alpha+3/2} < p < \frac{2\alpha+2}{\alpha+1/2} , \qquad \alpha \ge \beta \ge -\frac{1}{2} .$$

(i) Can the above p-range be extended? By Muckenhoupt [2, (1.3)], a fixed shift is bounded for all p, $1 < p < \infty$.

(ii) Consider the corresponding problem for Laguerre expansions (for the appropriate setting see [1]); a fixed shift is easily seen to be bounded for all $p \ge 1$.

Both questions are of course trivial for $p = 2$ since $\ell^\infty = M^2$ by Parseval's formula.

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(Submitted on May 19, 1998)

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Miscellaneous

1. SIAM Student Paper Prizes

The annual SIAM Student Paper Prizes will be awarded during the 1999 SIAM Annual Meeting, May 12-15, at the Radisson Hotel in Atlanta, Georgia.

If you are a student or know of a student who would like to take part in the competition, here are the details:

The authors of the three best papers in applied and computational mathematics written by students and submitted to SIAM will present their papers at the meeting and each will receive a $1,000 cash prize as well as gratis registration for the meeting. There is no provision for travel expenses associated with the prize.

Papers must be singly authored and not previously published or submitted for publication to be eligible for consideration. To qualify, authors must be students in good standing who have not received their PhDs at the time of submission.

In submitting their work for publication, authors are asked to consider SIAM journals. However, student paper prize winners are not guaranteed publication in any SIAM journal; all papers submitted to SIAM journals are subject to the same refereeing process and standards.

Submissions must be received in the SIAM office on or before February 1, 1999.

Submissions, which must be in English, can be sent by regular mail or fax. Each submission must include (1) an extended abstract NOT LONGER THAN 5 PAGES (including bibliography); (2) the complete paper, which will be used solely for clarification of any questions; (3) a statement by the student's faculty advisor that the paper has been prepared by the author indicated and that the author is a student in good standing; (4) a letter by the student's faculty advisor describing and evaluating the paper's contribution; and (5) a short biography of the student.

Submissions will be judged on originality, significance, and quality of exposition.

The winners will be notified by March 15, 1999.

Please direct your submission and any questions you may have to A. Bogardo at SIAM, 3600 University City Science Center, Philadelphia, PA 19104-2688; telephone (215) 382-9800; e-mail to bogardo@siam.org.

Allison Bogardo
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2. Connection and Linearization pPoblems using the `Navima` Algorithm

For about two years a European group has been collaborating on *connection* problems

$$P_n(x) = \sum_{m=0}^{n} C_m(n) Q_m(x)$$

and on (generalized) *linearization* problems

$$\prod_r P_{i_r}(x) = \sum_m L_m(i_1, \ldots, i_r) Q_m(x)\,.$$

Our approach to these problems is based on a template developed by our group and called the `Navima` algorithm (for **NA**mur-**VI**go-**MA**drid). Under certain general conditions on polynomials $P_n(x)$ and $Q_m(x)$, the algorithm provides a recurrence relation, in m only, for the connection and linearization coefficients. The algorithm, very easy to apply, is described on the web page of NAVIMA at the following address

http://www.uvigo.es/webs/t10/navima

This web page also contains information on new directions to be explored in the future.

Essentially, $P_n(x)$ should satisfy a differential, difference or q-difference equation of any order and $Q_m(x)$ must be a classical (or semi-classical) orthogonal polynomial family with respect to the corresponding operator (differential, difference or q-difference). In many cases the recurrence relation given by the `Navima` algorithm for the connection or linearization coefficients is solved using the powerful symbolic algorithms of Gosper, Zeilberger, Petkovšek etc. At this moment 14 papers, available on request, are already published and cover a large spectrum of problems including new connection formulas, asymptotics, hypergeometric relations, limit relations between families etc. which can be used in Mathematical Physics, in Combinatorics, in Group Representation Theory etc.

Everybody is welcome to the NAVIMA site. NAVIMA people can be also be reached at

`navima@uvigo.es`

to get more information, to make suggestions or to submit specific problems.

André Ronveaux
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3. Mathematica Special Functions Poster

At the recent ICM'98 in Berlin, the Mathematica booth displayed a giant poster on "The Mathematical Functions of Mathematica". It has four panels:

- Elliptic functions
- Elementary functions
- Hypergeometric functions
- Zeta, Mathieu, and other functions

The full poster, or parts of it, are available for free from Mathematica (in limited amount).

See http://www.wolfram.com/icm/poster.html

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4. MathCD 1998

The following is taken from
http://topo.math.u-psud.fr/~lcs/MathCD.html

A CD-ROM for Mathematics: Call for Submissions

Dear Fellow Mathematicians,

We are currently editing a CD-ROM devoted chiefly to freely distributable material of interest to mathematicians. Called MathCD 1998, it will be published in Fall 1998 by a French non-profit organization. Hopefully this CD-ROM will be the first of a series.

CD-ROM technology is unrivalled for low-cost high-volume distribution. For a unit cost of a few dollars, one can, by exploiting efficient formats, distribute a quantity of information that would fill several hundred books.

Our first priority has been to provide the electronic mathematics journals with a durable distribution medium to complement the volatile internet.

Our second priority is to improve the organisation and distribution of mathematical software produced by individual mathematicians.

Much space will be available for further material that will hopefully make this CD-ROM something of a mathematician's almanac and vademecum: various readers (viewers), choice pieces of the classical literature, monograph reprints, math memorabilia, math folklore, electronic graphics, programming tools, manuscript preparation tools, etc.

We solicit from our mathematical colleagues proposals of material to distribute on CD-ROM. We hope that each proposing mathematician (be he an author or an observer) will be willing to write a useful signed review and arrange the material for the CD-ROM. He or she should, at very least, put us in contact with the author, and indicate other qualified experts.

It is expected that authors will, in most cases, retain copyright and provide their own copyright notices within their contributions, while giving the editors specific permission for publication on the CD-ROM. Contributions exploitable in many computer environments will be preferred; but those specific to DOS, MSWindows or Macintosh will also be very acceptable. Commercial material and advertizing may be accepted under conditions to be negotiated, and the editors' intent here will be to reduce the price at which the CD-ROM can be distributed.

After the closure of MathCD 1998, submissions will be welcomed for future editions.

Laurent Siebenmann (editor in chief)
Krzysztof Burdzy (editor for electronic journals)
Richard Palais (editor for math software)
Xah Lee (associate editor for math software)

Tom H. Koornwinder
(thk@wins.uva.nl)

5. PhD Student Position at University of Amsterdam

The Korteweg-de Vries Institute for Mathematics of the University of Amsterdam has available in the near future a few four-year assistant positions for PhD students. Prospective candidates should have completed a PhD thesis by the end of this period. See http://turing.wins.uva.nl/~thk/links/aioen.html for details. Thesis work may be done in a large number of different fields of mathematics. To a large extent, selection of candidates will be based on the quality of their work for the master's degree.

One of the possible fields for a prospective candidate is in the area of OP & SF, with me as an adviser. Possible directions are:

- computer algebra algorithms for special functions
- special functions associated with root systems
- q-special funcions in connection with quantum groups

If you are interested, please contact me.

Tom H. Koornwinder
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6. Rota's "Ten Math problems I will never solve"

The 2/98 issue of the journal *Mitteilungen der Deutschen Mathematiker-Vereinigung* has a *Sonderbeilage zum ICM'98 in Berlin* called *Zukunft der Mathematik*. One of the contributors (in English) is Gian-Carlo Rota with the title "Ten Mathematics Problems I will never solve" (Invited address at the joint meeting of the American Mathematical Society and the Mexican Mathematical Society, December 6, 1997). His fourth Problem is "A unified theory of special functions", his eighth Problem is "Confluent symmetric functions".

Concerning Problem 4 he has a funny comment: Lately, q-analogs have come into high fashion. They have been ennobled by the name "quantum groups", even though they are neither quantum nor groups. Thirty years ago, those half dozen of us who worked on q-analogs were looked at with deep suspicion. More than sixty years ago, the Reverend F.H. Jackson, who was at that time probably the only person working on q-analogs, stormed out of the lecture room when someone in the audience made an unpleasant comment on q-analogs, and he never finished delivering his lecture on the q-analog of the gamma function.

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7. Migration of Preprint Archive to xxx

Dear Colleagues,

Electronic publishing is making fast progress and gets gradually better organized.

Further to discussions in the OPSF-Newsletter and the OPSF-Net, the OPSF-FTPSITE will be "frozen" effectively immediately. Following suggestions in the message from Greg Kuperberg below, no new submissions should be made to the OPSF-FTPSITE. In January 1999, the OPSF-FTPSITE will be disconnected entirely from the WWW.

All papers at the OPSF-FTPSITE were archived as submitted. You are kindly invited to inspect your paper(s) at the LANL Mathematics E-print Archive. You can find these papers by visiting the URL http://front.math.ucdavis.edu and then typing in the search window the author's name followed by a space and the (unquoted) string "op-sf".

If you discover any problem in the postscript format of your paper(s), you may wish to send an e-mail message about such problems straight to www-admin@xxx.lanl.gov (as explained at http://front.math.ucdavis,edu/people.html). The staff of the LANL Mathematics E-print Archive has kindly agreed to take care of them.

I look forward reading your papers at the LANL Mathematics E-print Archive.

Thank you for your patience and cooperation.

Best wishes,

Hans J. Haubold
(haubold@ekpvs2.dnet.tuwien.ac.at)

Hi folks. The OP-SF archive has been incorporated at Los Alamos and the archive at the URL ftp://unvie6.un.or.at/siam/opsf_new/00index.html should be retired. It should at least be "frozen", meaning put into a state where no further changes will be made and no new submissions will be accepted. Alternatively you could disallow public access entirely. Either way you could put up a notice redirecting readers and contributors to

http://front.math.ucdavis.edu/math.CA
http://xxx.lanl.gov/form/math.CA

The migrated papers are listed all together at the URL:

http://front.math.ucdavis.edu/search/op-sf&num=200

You should appoint people to inspect the postscript of these papers. About 1/3 of the papers initially failed autocompilation, and there may be a dozen or two with subtle compilation errors due to unpredictable changes in macro packages and so forth. If you make a list of problem cases, xxx staff will fix them.

Greg Kuperberg
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Karl Theodor W. Weierstrass
Life and Work

by U. Skornik
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19 February 1997 marked the hundredth anniversary of the death of the great German mathematician, the father of classical mathematical analysis and theory of Abelian and special functions, Karl Theodor Wilhelm Weierstrass. He was born on 31 October 1815, the first child of Wilhelm Weierstrass, secretary of the mayor of Ostenfelde and

Theodora Vonderforst.

Nothing in his early life indicated that he would become a famous mathematician. Karl attended the Catholic Gymnasium in Paderborn from 1829 to 1834 when he entered the University of Bonn in order to follow a course in public finance, economics and administration. This choice, far from his own interests, was dictated by his father with the result that, after four years spent on fencing, drinking and mathematics, Karl returned home without having taken any examinations. The years in Bonn, however, were not entirely wasted. It was during the stay there that Weierstrass attended lectures of the famous geometer Plücker, studied "Mécanique Céleste" by Laplace, "Fundamenta nova" by Jacobi and extended his knowledge by an accidentally found transcript of lectures on elliptic functions by Gudermann. As a last resort Karl was sent in 1839 to the Theological and Philosophical Academy at Münster where he was to prepare for a career as a secondary school teacher. He attended lectures on elliptic functions given by Cristof Gudermann. The theory of these functions was initiated by Gauss and developed by Abel and Jacobi. In the early XIXth century Abel considered the elliptic integral of the form

$$\alpha = \int_0^x \frac{dt}{\sqrt{(1-c^2t^2)(1+e^2t^2)}}$$

and the function $x = \varphi(\alpha)$, inverse to the integral. The function $\varphi(\alpha)$ is called an elliptic function and, when extended to the whole complex plane, gives a doubly periodic function. Jacobi based his theory on the integral

$$u = \int_0^\varphi \frac{d\theta}{\sqrt{1-k^2\sin^2\theta}},$$

with a parameter k, $0 < k < 1$, called the *modulus of the elliptic integral*. The elliptic functions obtained from that integral were called modular functions by Gudermann. It was a new theory and Gudermann was the first after Jacobi to give lectures on the subject.

On 2 May 1840 Weierstrass was given problems for his final examinations; one of them was posed by Gudermann in response to a special wish of his student and concerned elliptic functions. In autumn 1840 Weierstrass presented the results of his research on the decomposition of modular functions. He expressed Jacobi modular functions as ratios of entire functions whose power series coefficients are polynomials in k^2. In memory of Abel, he called them *Al functions*. Next, Weierstrass introduced his famous σ-functions, which differ from Al functions by a multiplier.

His dissertation contained significant new material and could ensure an academic position for him in Germany or elsewhere. It is not known why this work, highly prized by Gudermann, was not published until 54 years later in the first volume of Weierstrass's Collected Papers. Instead of gaining mathematical fame, Weierstrass, after passing the second oral part of his examinations in spring 1841, worked for 14 years as a secondary school teacher.

After a probationary year in Münster Weierstrass worked at the Catholic gymnasium (a high level high school) in Deutsch-Krone (West Prussia) from 1842 to 1848 and then in a similar school in Braunsberg (East Prussia) from 1848 to 1855. He taught not only mathematics and physics but also German, botany, geography, history, gymnastics and calligraphy. In 1844 Weierstrass took part in a course for gymnastics teachers in Berlin. During this time he visited the famous geometer Steiner. This, however, did not change his situation. In Deutsch-Krone Weierstrass had neither access to the mathematical literature, nor the possibility of exchanging ideas with any other mathematician. He felt isolated and filled his life with work. During those years he developed the theory of Abelian functions which form a larger class than the elliptic functions. Abelian integrals are defined like elliptic integrals by

$$u = \int_o^v R(t, \sqrt{f(t)})dt = I(v)$$

where $R(x,y)$ is a rational function of x and y, except that the function f is of a very general type which includes all polynomials. Inversion then yields Abelian functions, just as elliptic functions arise from the inversion of elliptic integrals, i.e. the integrals where $f(t)$ is a polynomial.

During Weierstrass' probationary year at the gymnasium in Münster he worked on three papers. The first "Darstellung einer analytischen Funktion, deren absoluter Betrag zwischen zwei gegebenen Grenzen liegt" contained a proof of Cauchy's integral theorem without the use of double or surface integrals. He also introduced Laurent series and arrived at the Laurent theorem two years before it was officially published in Comptes Rendus by Cauchy. In that work Weierstrass expressed complex numbers in form

$$r\frac{1+\lambda i}{1-\lambda i},$$

where r is the absolute value and λ is a real number given by $\lambda = \tan\frac{\theta}{2}$, where θ is an amplitude of the complex number. The paper "Zur Theorie der Potenzreihen" dates from autumn 1841. In that work Weierstrass introduced the notion of uniform convergence and examined series in several variables. In the next paper "Definition analytischer Funktionen einer Veränderlichen vermittelst algebraischer Differentialgleichungen", dating from 1842, he proved the theorem of Cauchy concerning the solution of systems of differential equations

$$\frac{dx_i}{dt} = G_i(x_1, \ldots, x_n)$$

with the initial conditions $x_i(0) = a_i$, where the G_i are polynomials. In that paper Weierstrass described the process of analytic continuation of power series. Those papers were published only in the first volume of his Collected Papers in 1894 but they make it clear that, already in 1842, he was in full possession of all the methods and ideas which allowed him to construct his theory of functions. Unfortunately, his first published paper "Bemerkung über die analytischen Fakultäten" appeared in a supplement to the school report of the year 1842 in Deutsch-Krone and received little attention. The same happened to another paper on Abelian functions which was published in 1848 in the Braunsberg school prospectus. Weierstrass would have remained unnoticed but for the fact that in summer 1853 during his stay in Münster he read Gudermann's opinion of his dissertation for the first time. Already in 1840 Cristof Gudermann not only recognised Karl's rare talent but also placed him among famous discoverers and suggested that his student should work at a university rather than as a secondary school teacher. This note encouraged Weierstrass to publish his paper "Zur Theorie der Abelschen Funktionen". Its appearance in 1854 in Crelle's Journal caused a sensation in the mathematical world. The consequences were amazing. The first recognition was the award of an honorary doctorate by the University of Königsberg. Then the Prussian ministry of education gave him a year's paid leave from the Braunsberg gymnasium to enable him to concentrate on his research.

Weierstrass gained enough confidence to apply for the post at the University in Breslau which was vacated by Kummer's appointment as professor in Berlin. It may sound strange that Weierstrass's application was rejected because of Kummer. The reason was that Kummer, who spent 13 years teaching in Breslau, intended to take Weierstrass to Berlin. He applied on 12 June 1856 to the university in Berlin with a request for a post for Weierstrass. He was not successful on that occasion but his eventual success gave to mathematics in Berlin three great names: Weierstrass, Kummer and Kronecker.

Weierstrass's "Theorie der Abelschen Funktionen" published in Crelle's Journal in 1856 contained results of his dissertation. D. Hilbert considered these results concerning the solution of the Jacobi inversion problem for the hyperelliptic integral the greatest achievement of analysis. This publication was a turning point in the life of Karl Weierstrass. He became famous abroad and the Austrian government made inquiries about him through Alexander von Humboldt. This pushed von Humboldt to action and on 1 July 1856 Weierstrass was appointed professor at the Industry Institute in Berlin.

In September 1856 Weierstrass and Kummer went to Vienna. There, Graf Thun offered Weierstrass 2000 Gulden and a professorship at an Austrian university of his choice. Weierstrass declined that offer but it became clear to Kummer that if they wanted to keep the great mathematician in Germany he had to take action again.

Shortly afterwards as a result of Kummer's efforts, Weierstrass was appointed Associate Professor at the University in Berlin. In November 1856 he became a member of the Berlin Academy. From then until 1890 he lectured on a great variety of topics, including periodic lectures on elliptic functions, lectures on geometry, and on mechanics. The famous mathematical seminar which he initiated together with Kummer in 1861 attracted international interest. From 1862 it was customary to make awards to the best participants. It is worth mentioning that the first mathematical-physical seminar was founded in Königsberg in 1834 by Jacobi, Neumann and Sohnke, the second in Halle in 1838 and the third in Göttingen in 1850.

Weierstrass' heavy work load resulted in a breakdown in his health in December 1861. He returned to teaching after a year but he lectured from a sitting position with a student writing the necessary text on the blackboard. For the rest of his life he suffered recurring bouts of bronchitis and phlebitis, but his determination kept him teaching and pursuing his research. In his lectures Weierstrass initiated the logical and rigorous development of analysis starting with his own construction of the real number system. He established the ε, δ notions in the concept of continuity and convergence, uniform convergence, absolute value, neighbourhood $(a-\delta, a+\delta)$ of a point a, and many others. He rejected intuitive arguments which were still prevalent among many contemporary mathematicians. It was Weierstrass who in his lectures of 1862 gave the famous example of a continuous nowhere differentiable function. In that year he first developed in his lectures the theory of the $\gamma(u)$ and $\sigma(u)$ functions. His famous approximation theorem appeared in connection with the heat equation and was published in July 1885 in the Proceedings of the Meetings of the Berlin Academy of Sciences. He also contributed to the development of the Calculus of Variations on which he lectured in 1879. His main lectures however concentrated on Abelian functions. His periodically presented lectures included "Introduction to the Theory of Analytic Functions", "The Theory of Elliptic Functions", once approached from the point of view of differential equations, another time from the point of view of the theory of functions, "Application of Elliptic Functions to Geometry and Mechanics", "Application of Abelian Functions to Geometric Problems and the Calculus of Variations". His lectures drew audience of up to 250, among them over 100 future professors, including S. Kovalevsky, Schwarz, Fuchs, G. Mittag-Leffler, L. Koenigsberger, H. Minkowski, and Cantor.

Weierstrass became the leading influence on the mathematical world. He obtained high recognition throughout Europe. The Swedish mathematician Mittag-Leffler mentions a nice anecdote. When he got a scholarship to study abroad in 1873 he went to Paris where Hermite greeted him with the words "Vous avez fait erreur, Monsieur, vous auriez dû suivre les cours de Weierstrass a Berlin". And of course Mittag-Leffler followed Hermite's advice and went to Berlin.

The most remarkable among Weierstrass's students was Sonia Kovalevsky, the daughter of a Russian artillery general. Women were not allowed to study in Russia so in 1868 she contracted a marriage of convenience to a young paleontologist, Vladimir Kovalevsky, and the couple went to study abroad. They first enrolled at the University of Heidelberg but after two years they separated. Vladimir went to Jena and Sonia travelled to Berlin hoping to attend Weierstrass's lectures. Unfortunately, there was a ban on women students at the University in Berlin and her application was rejected. So she went straight to Weierstrass with her Heidelberg references. He gave her some problems to solve and her solutions and enthusiasm impressed him so he decided to teach her privately. During the four years that she spent in Berlin, she became his close friend and an irreplaceable partner for scientific discussions. We do not know much about Weierstrass' private life. Sonia, however, the second woman (after Maria Skłodowska Curie) to hold a university post, a novelist and a revolutionary, became a wonderful topic for many biographies and novels. Weierstrass regarded her as his most talented student. In fact during her stay in Berlin she produced three outstanding papers; on differential equations, on Abelian integrals and on Saturn's rings, and managed to obtain a doctorate from the University of Göttingen. However, Weierstrass, much to his regret, could not secure a job for her in Germany and she had to return to Russia.

Their friendship, although based on purely scientific interaction, became the subject of rumour. Weierstrass felt deeply hurt by persistent insinuations surrounding his student and her mathematical achievements. On return to Russia her interest in mathematics ceased. She couldn't work at the University in Petersburg because her qualifications were not recognised. She returned to family life, gave birth to a daughter in 1878 and together with her husband, Vladimir, tried to make money as an estate entrepreneur, but this venture ended in bankruptcy.

Weierstrass and Sonia Kovalevsky corresponded fairly regularly from the time they met till her death in 1891. His letters and encouragement to take up mathematics again inspired her and helped her through financial difficulties and her husband's suicide in 1883. In 1884 Mittag-Leffler succeeded in getting her a post as lecturer at the University in Stockholm. In 1888 she achieved great success, when her famous paper *"On the rotation of a solid body about a fixed point"* won the French Academy's Bordin prize.

Sadly, Weierstrass burned all Sonia's letters after her early death. His letters, however, survived and are preserved at the Mittag-Leffler Institute in Djursholm, Sweden. The vast correspondence to Mittag-Leffler, H. Schwarz, Paul du Bois-Reymond, L. Koenigsberger, Riemann, L. Fuchs and Sonia, which contains mostly mathematical problems, also illuminates Weierstrass's life. He was considered successful but, in fact, his life was full of sufferings and personal problems. He did not marry after an engagement in Deutsch-Krone was broken due to the unfaithfulness of the fiancee, according to his brother, Peter. His career in Berlin had hardly started when his health collapsed. Moreover, towards the end of his life, rather than enjoying fame and appreciation, he felt isolated. This was because of a conflict with the mathematician and philosopher Leopold Kronecker.

Weierstrass and Kronecker were friends for more than twenty years, sharing many fruitful mathematical discussions and ideas. Unfortunately, at the end of 1870's their views on mathematics, especially the foundations of mathematics, diverged gradually. Weierstrass's work on limits and convergence led him to develop a theory of irrational numbers based on convergent sequences of rationals which he called "aggregates". His student, Georg Cantor, founded the theory of transfinite numbers which caused a revolution in mathematical thought. Before Cantor, mathematicians had accepted the notion of the infinite in the situation of a sequence *"tending to infinity"*, but were not prepared to accept an actual infinity *per se*. Cantor's great achievement was to introduce this precise concept, in fact a whole class of different infinities – that corresponding to the countable sets such as the natural numbers, that of the continuum of real numbers, and so on. For Kronecker, the type of non-constructive reasoning used by Weierstrass and Cantor was deeply suspect. His famous dictum, *"God made the integers, all the rest is the work of man"*, pronounced at the Berlin Congress in 1886, encapsulated his philosophy, and he envisaged the inclusion of essentially the whole of mathematics within the terms of arithmetic.

Kronecker did not hide his objections to the work of Weierstrass and Cantor and criticized them openly in front of students. No wonder that this mathematical conflict turned into personal quarrels. Towards the end of the 1880's Weierstrass evidently admitted that his long friendship with Kronecker was over though Kronecker himself seemed unaware of it. Weierstrass even considered the possibility of leaving Berlin for Switzerland to avoid the continuing conflict; but, since he did not wish his successor at the university to be chosen by Kronecker, he decided to stay. It became clear to him, however, that if he did not publish his lectures and works his achievements might fall into oblivion.

His worry was probably exaggerated because his students were spread all over Europe and continued his research. His greatest successor was H. Poincaré in Paris. Nevertheless the situation in Berlin was tense. In 1885 a special commission responsible for editing his works was set up. Weierstrass himself was no longer able to supervise the whole process so his students undertook the task of gathering and polishing up his lectures based on their own notes or transcripts. Weierstrass intended to give to mathematics an extensive treatment of analysis, clear and complete. So Knoblauch and Hettner were to prepare the theory of Abelian functions, his greatest aim in life. His dream was not fulfilled, however, and the draft was highly unsatisfying in his opinion and not up to the standard he expected. The lack of precision and printing errors worried him too. The first volume appeared in 1894 and contained his collected papers. The second was printed in 1895 but the next five volumes were published between 1902 and 1927, after his death. It is worth mentioning that it is to Weierstrass that we owe the publication of the collected papers of Jacobi, Dirichlet, Steiner and the letters of Gauss. In his old age he worked as an editor to supplement his salary which was not sufficient to maintain his family. He lived in Berlin with his sisters Elise and Klara who kept house, his father who died in 1869 and his uncle's grandson whom he took custody of as a two-year-old in 1884. He also took care of Borchardt's six children after his closest friend's death in 1880.

In 1892 Weierstrass received the Helmholtz medal and in 1895 he was awarded the Copley Medal, the highest honour of the Royal Society of London. In the same year he celebrated his 80th birthday among his students. He spent his last three years in a wheelchair and died on 19 February 1897, after an inflammation of the lungs.

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