

# Orthogonal Polynomials and Special Functions

*SIAM Activity Group on Orthogonal Polynomials and Special Functions*

★ ★ ★ ★

Newsletter

★ ★ ★ ★

---

Published Three Times a Year

June 1999

Volume 9, Number 3

---

## Contents

From the Editors .....	1
Message from the Chair .....	1
Reports from Meetings and Conferences .....	2
Forthcoming Meetings and Conferences .....	5
Books and Journals .....	12
Problems and Solutions .....	14
Miscellaneous .....	16
How to Contribute to the Newsletter .....	16
Activity Group: Addresses .....	17

In this issue we include several reports on past meetings as well as a large number of announcements of forthcoming events. We thank the authors of these items. Also a new subsection devoted to questions is added to the Problem Section.

As usual we hope that you find this issue interesting and useful, and remind you that you can send item for future issues to either of us.

June 1, 1999

Renato Álvarez-Nodarse  
(ran@cica.es)

Rafael J. Yáñez  
(ryanez@ugr.es)



## From the Editors

**T**his is the second issue of the Newsletter for 1999. A glance at it and the previous issue gives one an idea of the high level of activity in our area of research, considering the number of meetings, conferences and workshops directly related to orthogonal polynomials and special functions which are organized around the world.

## Message from the Chair

Let me open by congratulating Dick Askey for his election to the National Academy of Sciences. Watch for an account of his accomplishments that led to this honor in an upcoming issue of SIAM News.

The SIAM Annual Meeting in Atlanta was quite successful from my point of view. First, the Minisymposium on Orthogonal Polynomials: Theory and Applications, organized by Jeff Geronimo, featured seven stimulating talks; see the report by

---

*SIAM Activity Group*

---

on  
*Orthogonal Polynomials and Special Functions*

△

Elected Officers

DANIEL W. LOZIER, *Chair*

WALTER VAN ASSCHE, *Vice Chair*

FRANCISCO MARCELLÁN, *Program Director*

CHARLES DUNKL, *Secretary*

Appointed Officers

RENATO ÁLVAREZ-NODARSE, *Co-Editor of the  
Newsletter*

RAFAEL J. YÁÑEZ, *Co-Editor of the Newsletter*

MARTIN E. MULDOON, *Webmaster*

△

THE PURPOSE of the Activity Group is

*—to promote basic research in orthogonal polynomials and special functions; to further the application of this subject in other parts of mathematics, and in science and industry; and to encourage and support the exchange of information, ideas, and techniques between workers in this field, and other mathematicians and scientists.*

---

Charles Dunkl in this issue of the Newsletter. Second, the informal poolside reception, sponsored by the Activity Group and paid for by SIAM, was pleasant though lightly attended. Thanks to Coley Lyons of the SIAM staff for arranging this! I would like to try it again at the next Annual Meeting because I think it is a good way to promote the Activity Group among SIAM attendees who could benefit from more exposure to orthogonal polynomials and special functions.

Third, we had a very useful officer's meeting, attended by Walter Van Assche, Charles Dunkl, Peter McCoy and myself, in which we considered the Activity Group program from now until the next SIAM Annual meeting. Unfortunately Paco Marcellan, our Program director, was not present but I will see him at the upcoming International Workshop on Special Functions in Hong Kong. The major activity is likely to be a minisymposium at the SIAM Meeting. Peter McCoy volunteered to work on this and to be the co-chair. If you have ideas for a topic, or would like to assist

in some way, please contact one of the Activity Group officers.

I had a discussion with Gil Strang, President of SIAM. He is very supportive of our Activity Group. He has mentioned the impact of special functions in some of his recent columns in SIAM News, and I am grateful for his support. Also, I felt that his Town Meeting, in which he presented views and invited discussion on future directions for SIAM, was valuable. One emphasis was on international activities, an area in which our Activity Group is already a leader.

Please note that the subscription lists for OPSF-Net and OPSF-Talk have been transferred from Tom Koornwinder's institution to mine. See Tom's discussion of OPSF-Talk in this issue of the Newsletter. For full information on the new setup, see [math.nist.gov/opsfnet](http://math.nist.gov/opsfnet) and [math.nist.gov/opsftalk](http://math.nist.gov/opsftalk). Archives are available from these sites, or directly at [math.nist.gov/opsfnet/archive](http://math.nist.gov/opsfnet/archive) and [math.nist.gov/opsftalk/archive](http://math.nist.gov/opsftalk/archive). If you have comments or suggestions for improvements, please send them to me. For OPSF-Net, as before, mirror archives are available in Europe and at the Journal of Approximation Theory.

Daniel Lozier  
([dlozier@nist.gov](mailto:dlozier@nist.gov))

## Reports from Meetings and Conferences

### 1. INTAS workshop "Constructive complex analysis", Leuven, Belgium, February 3-5, 1999.

This was already the fourth workshop in the framework of an INTAS project, with participating research groups from Spain (Universidad de Granada and Universidad Carlos III de Madrid), Belgium (Katholieke Universiteit Leuven) and the 'new independent states' (Steklov Mathematical Institute, Keldysh Institute of Applied Mathematics, Moscow State University in Moscow, and State Technical University in Nizhnii Novgorod). The project ended in April so that part of this workshop was used to present the results obtained by the various research groups, but there was also some room for new research problems and future plans.

There were about 30 participants, mainly from Spain, Belgium and Russia. Each day consisted of expository

talks and workshops, allowing participants to do some collaborative work in small groups). On the first day, Francisco (Paco) Marcellán (the activity group's program director) talked about *semiclassical orthogonal polynomials and electrostatic properties of their zeroes*, Antonio Durán lectured about his results on *indeterminate matrix moment problems*, Guillermo López presented several results regarding the *asymptotics of Stieltjes polynomials*, and Andrei Martínez showed the latest results on *entropy of Gegenbauer polynomials*, which was one of the main research problems which we wanted to investigate in the INTAS project.

The second day was entirely devoted to talks from the Russian participants of the project. Alexander (Sasha) Aptekarev and Vladimir Buyarov first explained how one can get *asymptotics for polynomials with varying weights*. Vladimir Sorokin then gave a very nice talk on *Angelesco-Nikishin systems in number theory*, which deals with simultaneous rational approximation in a very general setting, with its applications in irrationality and transcendence proofs for various real constants. Finally Sergei Suetin talked about *sets of minimal capacity and asymptotics of the generalized Akhiezer polynomials*.

The final day started with a talk by Arno Kuijlaars on *the generic behaviour of equilibrium measures*, explaining how one can get information of the support of the equilibrium measure for various external fields, a problem of interest not only for orthogonal polynomialists but also for random matrix theory and Toda lattices. Jorge Arvesú talked about work in progress on *discrete multiple orthogonal polynomials*, Renato Álvarez-Nodarse (one of our newsletter editors) and his wife Niurka Rodríguez Quintero (no, she's not the Rodrigues formula person) presented some of their work on *linearization and connection problems for discrete and  $q$ -polynomials*, with emphasis on computer algebra and algorithmic aspects, while Jorge Sánchez-Ruiz tried to find explicit analytic expressions for *continuous hypergeometric linearization*.

I enjoyed very much driving the participants around from their hotel to the university and to the airport, with the minibus that I rented for the occasion, which was not particularly easy since there was a lot of construction going on just in front of the hotel.

Walter Van Assche  
(walter@wis.kuleuven.ac.be)

## 2. Fifth International Conference on Approximation and Optimization in the Caribbean: Guadeloupe, French West Indies, March 29-April 2, 1999

The 5th International Conference on Approximation and

Optimization in the Caribbean was held during March 29-April 2, 1999 in Pointe-à-Pitre, Guadeloupe. This conference is part of a series initiated in Havana in 1987 and 1993 and continued every two years in a different country of the Caribbean area: Puebla (Mexico) in 1995 and Caracas (Venezuela) in 1997. Participation in these meetings has increased both because of interest in overlapping research in domains like Optimization Theory (under the influence of very powerful teams from Germany and France with an important cooperative experience with people in Latin America), Approximation Theory (with the important leadership of the Cuban and Russian school) and Mathematical Economics, as well as the support of Mathematics in an emerging geographic area. For the latter reason, Plenary Lectures and Tutorial Courses (Wavelets by Albert Cohen; Convex Analysis and Nonlinear Optimization by Jonathan Borwein) were organized in order to present the state of the art in the subject. 150 people from Venezuela, Cuba, Mexico, Guatemala, Brazil, Chile, Colombia, Costa Rica, Venezuela as well from USA, Canada, Morocco and Europe attended the Conference.

Orthogonal Polynomials and Special Functions were represented in the framework of Approximation Theory, with two Plenary Lectures by Walter Van Assche (Leuven) and Ed Saff (USF, Tampa). Walter presented some recent results of joint work with A. Kuijlaars on Inverse and Direct Problems for Orthogonal Polynomials and application in solving dynamical systems (Toda and Kac-Van Moerbeke lattices) while the lecture by Ed was devoted to Discrete Minimal Energy Problems on the Sphere. Ed showed an amazing set of slides and asked questions about soccer. (Bernd Schuster, a very famous German player was almost unknown to the participants when Ed showed a picture of him.) A Parallel session took place each day with contributions on Asymptotics of Orthogonal Polynomials for Freud weights (K. McLaughlin), Matrix Orthogonal Polynomials (A. J. Duran, F. Marcellán), Operator Theory and Orthogonal Polynomials (F. Szafraniec), Orthogonal Polynomials in Functional Analysis (R. Lasser, R. Girgensohn) Asymptotics of Sobolev Orthogonal Polynomials (H. Pijeira, G. López-Lagomasino), Moment Theory (A. Bakan), Padé and Rational Approximation (E. Acosta, F. Cala, M. Bello, L. Piñeiro), Polynomial inverse images of intervals (V. Totik) Approximation in the Complex domain (J. P. Thiran, L. Baratchart) and Harmonic Analysis (W. Urbina).

The weather was excellent and the participants enjoyed the wonderful Caribbean beaches. Social activities such as excursions around the islands (note that Guadeloupe consists of Basse Terre plus Grand Terre, two connected islands), a welcome party and a dinner with Caribbean music and dancing completed a very friendly atmosphere.

Francisco Marcellán  
(pacomarc@elrond.uc3m.es)

### 3. Second Workshop on Orthogonal Polynomials: Approximation and Harmonic Analysis. Ballenstedt, Germany, April 23–26, 1999

This workshop was the successor of a workshop which took place in Travemünde, Germany, in April 1998, see <http://www.gsf.de/institute/ibb/prestin/orthopol.html>

The very old castle in Ballenstedt was the pleasant location for a wonderful workshop on orthogonal polynomials. Forty-four participants from eight countries, but mostly from Germany, attended the workshop. The Conference languages were German and English. I had not been aware of how many German researchers are working in this field, and I was very favourably impressed!

The invited speakers were Hrushikesh N. Mhaskar, Walter Gautschi, Ajit I. Singh, Paul L. Butzer, Charles F. Dunkl and Ryszard Szwarc.

Although the schedule was rather dense—there were 31 talks presented in 3 1/2 days—the auditorium was always full. Fortunately, there was still time for a joint social event: On Sunday afternoon, most of the participants took a walk to one of the neighboring towns, and for the return trip a steam engine with a regular schedule was used. This was a remarkable trip!

To give a hint about the interesting topics of the workshop I would like to give a (rather personal) selection: Paul L. Butzer spoke about Bernoulli functions, and presented connections with the Riemann zeta function; Hypergeometric representations of positive definite 1-radial functions were given by Wolfgang zu Castell; Charles F. Dunkl gave an introductory talk about orthogonal polynomials of several variables that are connected with symmetry groups; Roland Girgensohn presented Schauder bases for  $C[-1, 1]$  consisting of orthogonal polynomials; The indeterminate Hamburger moment problem was discussed by Caroline Lasser; Andreas Ruffing treated the  $q$ -Hermite polynomials II in connection with the  $q$ -Heisenberg algebra, and showed that their orthogonality measure is not uniquely determined; Gerhard Schmeißer spoke about the location of the zeros of polynomials depending on the coefficients of an orthogonal expansion; Nonnegative linearization coefficients for arbitrary, and in particular for discrete orthogonal polynomial systems were discussed by Ryszard Szwarc. The main emphases of the talks were orthogonal polynomials of one variable.

The program, list of participants, and time table of the workshop as well as pictures of participants of the workshop as well as of the steam engine can be found at the URL <http://www.gsf.de/institute/ibb/prestin/work2.html>

I would like to thank the organizers, especially Jürgen Prestin, for the excellent organization, and Frank Filbir for the selection of the well-suited location. I will remember this event as a special one!

Wolfram Koepf  
([koepf@imn.htwk-leipzig.de](mailto:koepf@imn.htwk-leipzig.de))

The second workshop on Orthogonal Polynomials in Ballenstedt was an event with a very pleasant and cooperative atmosphere. Specific topics and new ideas for concrete work in the theory of this subject were discussed, as well as other areas where orthogonal polynomials can play an important role. Here is a (personal) selection of some possible future research subjects:

- Develop Poisson summation formulas in the context of orthogonal polynomials and their continuous analogs, and derive related sampling theorems. The work of P.L. Butzer and M.H. Annaby can provide useful guidelines.
- Find new specific examples for translation operators connected with orthogonal polynomials, as in the work of R. Szwarc. Another class of interesting examples comes from the 1-radial functions presented by W. zu Castell.
- Investigate in detail orthogonal polynomial systems with respect to special orthogonality measures. Examples of such systems connected with multiresolution analysis were presented by W. Gautschi; other examples concerning the  $q$ -Hermite polynomials were given by A. Ruffing.
- Continue investigations on multivariable orthogonal polynomials. The polynomials connected with the differential-difference operators of Ch. F. Dunkl are particularly interesting from various points of view.
- Transfer more of the powerful results of functional analysis and operator theory to the field of orthogonal polynomials, as in the work of H.N. Mhaskar and B. Beckermann. Similarly, Th. Kriecherbauer reported on the application of results from the theory of dynamical systems (in particular, on the Riemann-Hilbert problem) to orthogonal polynomials.

Finally we thank all participants for their contributions and we hope to continue this fruitful endeavour by similar activities in the future.

Jürgen Prestin  
([prestin@gsf.de](mailto:prestin@gsf.de))

#### 4. Minisymposium; Orthogonal polynomials; Theory and Applications. Atlanta, May 12-15, 1999

The 1999 SIAM Annual Meeting was held at the Sheraton Atlanta Hotel, May 12-15, in Atlanta, Georgia. As part of the meeting Jeffrey Geronimo of Georgia Tech organized a minisymposium on "Orthogonal Polynomials: Theory and Applications". There were seven talks in two sessions. Thursday (May 13), Jeff Geronimo started the program and lectured about multiwavelets and multiresolution analysis; some of the results were joint work with G. Donovan and D. Hardin. Then Percy Deift of the Courant Institute discussed asymptotics for orthogonal polynomials whose weight function was of the type  $\exp(-Q(x))$ ,  $Q$  being a polynomial of even degree, by means of the Riemann-Hilbert method. T. Kriecherbauer, K. McLaughlin, S. Venakides and X. Zhou had collaborated with Deift on this work. Walter van Assche of Leuven (who is vice-chair of the Activity Group) talked on multiple orthogonal polynomials (that is, orthogonal for two different weights) associated to modified Bessel functions of the second kind.

Friday (May 14) there were four talks; your reporter discussed examples of Calogero-Moser systems and orthogonal polynomials with hyperoctahedral (Weyl groups of type B) symmetry, and the problems involved in trying to construct complete orthogonal bases of the associated spherical harmonics. Sergei Suslov of Arizona State U. talked about a novel type of Fourier series associated to  $q$ -analogues of sine and cosine (some of this was joint work with J. Bustoz and W. Gosper). H. Woerdeman of the College of William and Mary talked on joint work with Geronimo about expressing a positive trigonometric polynomial of two variables as the squared absolute value of another polynomial, in connection with autoregressive filters. Gerard Teschl of Vienna concluded the program with his lecture on inverse spectral theory of Jacobi operators; the information about the operator which is contained in the support of the spectral measure.

Dan Lozier (chair of the A.G.) participated in the Poster Session, with an update on the Handbook project at NIST (National Institute for Science and Technology). Bruce Fabijonas, also of NIST, talked in a Contributed Papers session about numerical calculation of modified Bessel functions (software which could become part of the Handbook accessories).

As often happens at large annual meetings of scientific societies, the attendance at a parallel session was disappointingly small; it ranged from 12 to 20 at the O.P. minisymposium. One notes there were twelve parallel minisymposia Thursday afternoon.

The invited lecture by Herbert Wilf on asymptotic oscillations and binary partitions of integers certainly was of interest to the special functions people, notably the asymptotics and combinatorics experts.

The Friday evening pool side reception organized by Dan Lozier provided good conversation and collegiality, but unfortunately reflected the low level of attendance at the minisymposium.

Saturday the activity group officers (named above) met to discuss plans. One point worth making here: the OP-SF Net and the printed Newsletter are an excellent way of promulgating information about upcoming conferences, and we urge the world-wide community of OP&SF people to use these news media for both announcing conferences and planning programs to avoid scheduling conflicts.

Charles F. Dunkl  
(cfd5z@virginia.edu)

### Forthcoming Meetings and Conferences

#### 1. International Workshop on Special Functions: Hong Kong, June 21-25, 1999

An *International Workshop on Special Functions* will take place on June 21-25, 1999 at the *City University of Hong Kong*. The main focus will be on *Asymptotics*, *Harmonic Analysis*, and *Mathematical Physics*. For details, see <http://www.cityu.edu.hk/ma/conference/iwsf/>.

**Objective:** The purpose of the conference is to provide a forum for an exchange of ideas among experts in various topics listed below. It also aims at disseminating information on recent advances made in these areas.

**Plenary Speakers:** K. Aomoto (Nagoya University, Japan), R. Askey (University of Wisconsin-Madison, USA), T. Baker (University of Melbourne, Australia), C. Berg (University of Copenhagen, Denmark), C. Dunkl (University of Virginia, USA), G. Gasper (Northwestern University, USA), W. Gautschi (Purdue and ETH (Zurich), Switzerland), E. Koelink (Universiteit van Amsterdam, The Netherlands), A. McBride (University of Strathclyde, UK), F. Olver (University of Maryland, USA), R. O'Malley (University of Washington, USA), E. Opdam (University of Leiden, The Netherlands), D. Stanton (University of Minnesota, USA), N. Temme (CWI, Amsterdam, The Netherlands), A. Terras (University of California, San Diego, USA), L. Vinet (CRM, Université de Montréal, Canada), R. Wong (City University of Hong Kong), Y. Xu (University of Oregon, USA)

**Session Topics:** Asymptotics, Classical Special Functions, Combinatorics, Harmonic Analysis and Quantum Groups, Mathematical Physics and PDEs, Orthogonal

Polynomials.

**Organizing Committee:** Charles Dunkl, U. of Virginia, USA; Mourad Ismail, U. of South Florida, USA; Roderick Wong, City U. of Hong Kong.

**Information:** Colette Lam, IWSF '99 Workshop Secretary, Department of Mathematics, 83 Tat Chee Avenue, Kowloon, Hong Kong; phone: +852 2788-9816, fax: +852 2788-8561; e-mail: malam@cityu.edu.hk

Charles F. Dunkl  
(cfd5z@virginia.edu)

## 2. 1999 IMACS Conference on Applications of Computer Algebra, El Escorial, Spain, June 24-27, 1999

The following information has been taken from the WWW site <http://math.unm.edu/ACA/1999.html>. Further and detailed information can be obtained in the same site.

Computer Algebra or Symbolic Computing is a young field of research in-between Computing and Mathematics. Merging both mentalities (one theoretical and the other closer to the real world) provides a fertilized soil for the growth of new useful and exciting applications. Although the same kind of tools are in the background, a broad variety of fields are covered, as the sessions organized at IMACS-ACA '98 show: Robotics, High Energy Physics, Industry, Algebra and Geometry, Artificial Intelligence, Education, Biology...

This will be the 5th IMACS-ACA. Previous ones have taken place in Albuquerque, New Mexico (USA) 1995; Linz (Austria) 1996; Wailea, Maui, Hawaii (USA) 1997 and Prague (Czech Republic) 1998. The evolution of the number of talks and attendees indicate a rapidly increasing interest in this particular conference.

Probably the two most important conferences about Computer Algebra nowadays are the much older ISSAC and IMACS-ACA. IMACS-ACA is getting nearly as many attendees as ISSAC. Moreover, as far as we know, IMACS-ACA is the only conference in the world specifically devoted to applications of this field.

In previous years organizations and companies as important as the America National Science Foundation (NSF) and National Security Agency (NSA), Czech Ministry of Education, Texas Instruments, Wolfram Research... have sponsored this conference.

Renato Álvarez-Nodarse  
(ran@cica.es)

## 3. Methods and applications of asymptotic analysis. Edinburgh, July 5-9, 1999

Adri Olde Daalhuis (Edinburgh) and Nico Temme (Amsterdam) are organizing a mini-symposium on asymptotics during ICIAM 99 in Edinburgh, 5-9 July 1999 entitled "Methods and applications of asymptotic analysis".

Asymptotic analysis is an important tool in many branches of mathematics, physics and other application areas. When solving problems described in terms of differential equations or integrals, it is often required to obtain qualitative information on the solution of the problem. Another area of interest is the study of the behavior of certain special functions for large values of one or several (complex) parameters and the application of uniform asymptotic expansions in computing special functions for a large domain of the parameters. This mini-symposium presents several actual problems in which asymptotics plays an important role.

Speakers:

1. Roger J. Hosking, James Cook University, Australia.  
Title: Asymptotic evaluation of Fourier integral solutions for the response of a flexible plate to a moving load
2. David Kaminski, University of Lethbridge, Canada  
Title: Hills and valleys at infinity for the steepest descent method
3. Jose L. Lopez, Universidad de Zaragoza, Spain  
Title: Uniform asymptotic expansions of Bernoulli and Euler polynomials.
4. Richard B Paris, University of Abertay, Dundee, Scotland  
Title: The asymptotic expansion of Gordeyev's integral
5. Alastair Wood, Dublin City University, Ireland  
Title: Asymptotically assisted numerics in MHD stability problems

The ICIAM 99 web site at [www.ma.hw.ac.uk/iciam99](http://www.ma.hw.ac.uk/iciam99) contains up-to-date information about the Congress. (See also OP-SF NET 5.5, Topic #4)

Nico M. Temme  
(nicot@cwi.nl)

## 4. 1999 MAA North Central Section Summer Seminar: Calculus and Counting, University of Minnesota - Duluth, USA, August 9-13, 1999

The Mathematical Association of America has made the following announcement concerning a seminar this August

sponsored by the North Central Section. 1999 MAA North Central Section Summer Seminar: Calculus and Counting.

**Principal Lecturer:** Richard Askey (University of Wisconsin–Madison)

**The Program:** Counting aspects of binomial coefficients date back over 2000 years, and Pascal's triangle more than 1000. In spite of their great age, new aspects of binomial coefficients have been discovered in each of the past four centuries. Newton discovered the infinite series version of the binomial theorem over 300 years ago. Euler discovered the gamma function and how it could be used to evaluate the beta integral. There is a strong connection between the binomial theorem and the beta integral. There are extensions to Gaussian binomial coefficients and both commutative and non-commutative forms of the  $q$ -binomial theorem. There are also multivariate extensions of both the sums and integrals.

Many of these extensions will be explored in this seminar, which will present the historical development of binomial coefficients from their origins to the present day. Sessions for a small number of contributed presentations will be available, as well.

**The Principal Lecturer:** Professor Richard Askey is Professor of Mathematics at the University of Wisconsin, Madison. He received his PhD in 1961 from Princeton University. He has held visiting positions at the University of Paris, the University of Minnesota, the University of Strasbourg, the Mathematical Institute in Budapest and the Mathematical Center in Amsterdam. His research interests are in special functions, especially orthogonal polynomials and  $q$ -series. He is the author of over 150 research papers, books and expository articles and has delivered more than 50 invited lectures.

**Conference Information:** Scheduled during the traditional "dog-days" of summer, this conference provides the perfect opportunity to visit one of the most picturesque cities in the country. Duluth attracts thousands of tourists each year with its cool summer temperatures, beautiful sights and nearby wilderness areas. For entertainment and recreation, Duluth offers many fine restaurants; numerous parks and golf courses; marine, railroad, and art museums; boat cruises and railroad excursions; sailboat rental and charter fishing; and ready access to canoeing, camping, and hiking. The conference coincides with a Duluth Yacht Club race, and concludes just as Duluth's Bayfront Blues Festival begins.

Conference activities include a welcoming reception, a conference banquet, ample time to enjoy the sights, and some very stimulating mathematics.

The UMD campus offers excellent single and double occupancy dorm rooms and two or three bedroom furnished apartments at very reasonable rates. The Best Western

Hotel (located on the shore of Lake Superior, and a mile from the UMD campus) has set aside a block of rooms with special conference rates. A campus meal plan is also available to conference participants.

Online information is available at:  
<http://www.d.umn.edu/math/maa99.html>.

**Address inquiries to:** 1999 MAA Summer Seminar, Department of Mathematics and Statistics, 140 Campus Center, University of Minnesota - Duluth, Duluth, Minnesota 55812.

Martin Muldoon  
(muldoon@yorku.ca)

### 5. Third International Workshop "Transform Methods & Special Functions" (TMSF), Blagoevgrad, AUBG, Bulgaria, August 13-20, 1999

Institute of Mathematics & Informatics - Bulgarian Academy of Sciences, Institute of Applied Mathematics & Informatics - Technical University Sofia, American University in Bulgaria are organizing: The Third International Workshop "Transform Methods & Special Functions, AUBG '99"

The 3rd Workshop "TMSF" is a continuation of the Workshops "TMSF, Sofia '94" and "TMSF, Varna '96". It will take place in the town of Blagoevgrad (situated 100 km south of Sofia), with the kind assistance and co-organization of the American University in Bulgaria (AUBG; <http://www.aubg.bg>). The working days of the meeting will be 14–19 August 1999, arrivals on 12–13 August and departure on 20 August.

**Organizing Committee:** Profs. Peter Rusev (IMI-BAS), Shyam Kalla (Kuwait Univ.), Ivan Dimovski (IMI-BAS), Asso. Profs. Virginia Kiryakova (IMI-BAS), Lyubomir Boyadjiev (TU-Sofia).

**Topics:** Integral Transforms, Special Functions, Series Expansions, Fractional Calculus, Algebraic Analysis, Generalized Functions, Operational Calculus, Univalent Functions; Applications of these topics to Complex Analysis, Differential and Integral Equations, Mathematical Physics; Related problems.

**Proceedings:** The Proceedings "TMSF, AUBG '99" will be published as a special issue of the new international journal *Fractional Calculus & Applied Analysis* (FCAA Journal). All the submitted papers will pass through peer-reviewing process before their acceptance. The deadline for presentation, together with source files, will be during the meeting (13-20 August, 1999) but earlier submissions (hard-copies by post and ASCII files by e-mail, or on a

diskette) will facilitate us and speed the process of reviewing / acceptance and final preparation of the volume. For any inquires about the "FCAA Journal" and submission of papers, please contact Dr. Virginia Kiryakova (Managing Editor of "FCAA Journal"). Instructions to authors for the "FCAA Journal" can be seen and download from the Journal's web sites: <http://www.diogenes.bg/fcaa> and <http://www.math.bas.bg/~fcaa>, or obtained by e-mail upon request.

**Participation fees:** USD 140 (or USD 120, if sent in advance (before June 15, 1999) for participants in scientific program, and USD 70 for accompanying persons. These fees include: organizational and administrative expenses, reservation fees, postage, transportation Sofia-Blagoevgrad-Sofia (in groups by mini-buses), brochures with Abstracts and Scientific Program and other conference materials (except for the volume of the Proceedings), coffee breaks, social program - welcome party, excursion to several interesting places in Rila and Pirin Mountains, etc. The additional Fees for Publication in the Proc. "TMSF, AUBG'99" and obtaining a copy of this volume, are USD 40.

Please note that the meeting has no budget and sponsors and the Organizers have no funds to support any participants, even the invited lecturers. The participation fees are due from all the participants, to ensure the meeting's organization and to cover all the arising expenses.

It would be a great help, if some of the participants could send us the participation fees in advance (before June 15, 1999).

**Accommodation:** Basically, the participants will be accommodated in the "Alen Mak" Hotel located in front of the AUBG, both at a beautiful square in the very center of the town. Cheaper accommodation is possible in the student dormitories of South-West University in Blagoevgrad, a 10-15 minute walking distance from AUBG. Meals (lunch and supper) will be arranged on the basis of full board in the canteen of AUBG. The prices (per person/per night) are: "Alen Mak" Hotel: USD 35 (single room), USD 25 (1 bed in double room) (these prices include breakfast in the hotel) Student Dormitory: USD 5 Meals: USD 10/day (lunch and supper in AUBG)

**Further Contacts:** Second Announcement is available at <http://www.math.bas.bg/~tmsf/tmsf99.htm> In case of confirmed interest official letter or letter of acceptance (for the purposes of obtaining grants, visas, etc) will be sent. All further correspondence and informations will be by e-mail.

Please, always provide us with current e-mail address for correspondence and keep us in touch at: [tmsf@math.bas.bg](mailto:tmsf@math.bas.bg), or [virginia@diogenes.bg](mailto:virginia@diogenes.bg). All the details on "TMSF'AUBG'99" (incl. announce-

ment, sample files for abstracts and papers, information for the previous meetings "TMSF'94", "TMSF'96" and their proceedings) are available at our web-site, <http://www.math.bas.bg/~tmsf>.

**Addresses for Contact:** "TMSF" Workshops (P. Rusev, I. Dimovski, V. Kiryakova)

Institute of Mathematics & Informatics - Bulg. Acad. Sciences Sofia 1090, Bulgaria Fax: (+359) 2 971 3649  
E-mail: [tmsf@math.bas.bg](mailto:tmsf@math.bas.bg) (Workshop),  
[virginia@diogenes.bg](mailto:virginia@diogenes.bg) (V. Kiryakova)  
Web-site: <http://www.math.bas.bg/~tmsf>

**IMPORTANT NOTE:** Deadline for sending Participation Fees in advance: June 15, 1999 (for payment details, contact us)

Virginia Kiryakova  
([fcaa@math.bas.bg](mailto:fcaa@math.bas.bg), [virginia@diogenes.bg](mailto:virginia@diogenes.bg))

## 6. DMV-Jahrestagung 1999: Mainz, Germany, September 5-11, 1999

The yearly meeting of the German Mathematical Society takes place in Mainz from 5 to 11 September 1999. I would like to announce that Bernd Sturmfels is invited to present a plenary talk at this conference with title *Gröbner bases and hypergeometric functions..* Further informations can be obtained from <http://www.mathematik.uni-mainz.de/DMV99>.

Wolfram Koepf  
([koepf@imn.htwk-leipzig.de](mailto:koepf@imn.htwk-leipzig.de))

## 7. Conference on Analytic Methods of Analysis and Differential Equations, Minsk, Belarus, September 14-18, 1999

### First Announcement

The Institute of Mathematics of the Belarusian National Academy of Sciences and the Belarusian State University (BSU) together with Moscow State University and the Computer Center of the Russian Academy of Sciences will organize the International Conference "Analytic Methods of Analysis and Differential Equations" (AMADE) on September 14-18, 1999, in Minsk, Belarus. The arrival and departure days are September 13 and 19.

### Section Titles:

1. Integral Transforms and Special Functions.
2. Differential Equations and Applications.
3. Integral, Difference, Functional Equations and Frac-



tional Calculus.

The length of plenary invited lectures is 45 min, reports - 20 min, and short communications - 10 min. The publication of the abstracts is planned. The Proceedings of Conference are supposed to be published in the Journal "Integral Transforms and Special Functions".

**Organizing Committee:** Chairmen: Academician I.V. Gaishun (Belarus), Academician V.A. Il'in (Russia) and Rector of BSU A.V. Kozulin. Vice-Chairmen: V.V. Gorokhovik (Belarus), A.A. Kilbas (Belarus), V.I. Korzyuk (Belarus) and A. P. Prudnikov (Russia).

**Secretaries:** M.V. Dubatovskaya (Belarus), S.V. Rogosin (Belarus).

**Members:** P. Antosik (Poland), C. Brezinski (France), L. Debnath (USA), I. H. Dimovski (Bulgaria), L. Gatteschi (Italy), J. Gilewicz (France), H. -J. Glaeske (Germany), R. Gorenflo (Germany), V. I. Gromak (Belarus), N. A. Izobov (Belarus), N. K. Karapetyants (Russia), V. S. Kiryakova (Bulgaria), O. I. Marichev (USA), E. I. Moiseev (Russia), A. F. Nikiforov (Russia), O. A. Oleinik (Russia), O. A. Repin (Russia), V. N. Rusak (Belarus), M. Saigo (Japan), S. G. Samko (Portugal), A. A. Sen'ko (Belarus), H. M. Srivastava (Canada), B. Stankovic (Yugoslavia), P. K. Suetin (Russia), N. A. Virchenko (Ukraine), L. A. Yanovich (Belarus), N. I. Yurchuk (Belarus), P. P. Zabreiko (Belarus), A. H. Zemanian (USA), E. I. Zverovich (Belarus).

**Our address:**

Belarus, 220050 Minsk 50, Fr. Skaryny Avenue 4, Belarusian State University, Department of Math & Mech, AMADE;

**e-mail addresses:** amade99@im.bas-net.by and amade99@mmf.bsu.unibel.by

Tom H. Koornwinder  
(thk@wins.uva.nl)  
Martin Muldoon  
(muldoon@yorku.ca)

**8. Computer Algebra in Fundamental and Applied Researches and Education, Minsk, Belarus, September 20-24, 1999**

(This information is taken from the WWW site <http://www.bsu.unibel.by/2310/gloweb/cas.htm>)

The Second International Scientific Conference: Computer Algebra in Fundamental and Applied Researches and Education will be held in Minsk, Belarus, from September 20 -24, 1999. The main goal of the conference is to provide a forum for exchange of ideas how to obtain the mathematical knowledge on different levels of education and scientific investigations based on computer

algebra methods.

For further information contact with

Dr. Yuri Poznjak,  
Organization Committee CAS-99  
Belarusian State University,  
Skarina Ave. 4, 220050 Minsk Belarus

Tel: (+375) 17 226-58-95  
Fax: (+375) 17 220-77-52

E-mail: cas@cit.bsu.unibel.by,  
poznjak@cit.bsu.unibel.by

Renato Álvarez-Nodarse  
(ran@cica.es)

**9. Fifth International Symposium On Orthogonal Polynomials, Special Functions And Their Applications (OPSFA) in Honor of Theodore Chihara: Patras, September 20-24, 1999**

Dear colleague and friend,

The Department of Mathematics of the University of Patras is delighted to host the Fifth International Symposium on Orthogonal Polynomials, Special Functions and their Applications (OPSFA, for short), which will be held for the first time in Patras in the period September 20 - 24, 1999. The Scientific Committee has decided that the Symposium should be dedicated to Professor Theodore Chihara. Based on the overwhelming response to the first circular, we expect about 150 scientists from all over the world. If you would like to give a talk, please indicate its title on the registration form. We shall do our best to set up a balanced program. The city of Patras, with its friendly people, its numerous sights and its diverse surroundings, has much to offer to create a pleasant and enjoyable atmosphere surrounding the Symposium.

We look forward to seeing you in Patras in September.

The scientific program includes plenary lectures and research seminars. An approximate time table will be given in the third circular, which will be sent to all those who register for the Symposium. The final program will be distributed at the registration desk on Sunday 19 and Monday 20 September.

**Plenary Lectures and Research Seminars**

There will be 10 plenary lectures and about 100 research seminars or contributed talks organized into parallel sessions. The time assigned for a plenary lecture is 60 minutes (50 minutes for presentation and 10 minutes for discussion). Research seminars are assigned 25 minutes (20 minutes for presentation and 5 minutes for discussion). All participants are invited to submit a 25 - minute research

seminar.

The plenary lectures are as follows:

- D. Bessis (France): On the application of Moment Methods to non-linear partial differential equations of parabolic type in arbitrary dimensions.
- T. Chihara (USA): 45 years of Orthogonal Polynomials: a View from the Wings.
- J. Dehesa (Spain): Quantum Entropies and Orthogonal Polynomials.
- A. Elbert (Hungary): Some recent results on the zeros of Bessel functions and Orthogonal Polynomials.
- N. Everitt (U.K.) - Orthogonal polynomials, linear differential equations and the Kramer sampling theory.
- W. Gautschi (USA) - Orthogonal polynomials and quadrature.
- T. Kriecherbauer (Germany) - A Riemann-Hilbert approach to asymptotic questions in Orthogonal Polynomials.
- A. Kuijlaars (Belgium) - Asymptotics of Orthogonal Polynomials with Slowly Decaying Weights.
- V. N. Sorokin (Russia) - Applications of simultaneous orthogonal polynomials in number theory, theoretical physics and dynamical systems
- R. Wong (Hong Kong) - Asymptotics and Orthogonal Polynomials.

## Abstracts

Participants who wish to present a research talk, are asked to send a title and an abstract written in English before May 30. The abstract should not exceed one page but should have a minimum of 15 lines with the description of the topic covered in a comprehensive manner. Since the abstracts will be reproduced directly in the book of abstracts, their typographical quality should be as good as possible.

The invited speakers are also requested to send a one page abstract of their presentation.

## Publications

The book of abstracts will be distributed to the participants upon registration at the Symposium desk. The Proceedings of the Symposium will be published in Journal of Computational and Applied Mathematics.

## Registration, fees and accommodation

Please complete the **Registration Form** (available from the organizers) and return it before May 30 to the Symposium Mailing Address (below).

NOTE: (US)\$1 = 300 drachmas, approximately

The following fees are applicable for the Symposium:

Participants: 50000 drachmas  
 Students: 20000 drachmas  
 Accompanying persons: 20000 drachmas

After May 30, all participants and their companions must pay an additional fee of 10000 drachmas for late registration. Participant and student fees include:

- Admission to the Symposium
- Symposium documents
- Book of Abstracts
- Symposium Proceedings
- Official Reception
- Greek evening
- Guided visit to ancient Olympia
- Other social events are being considered

Accompanying persons' fee includes:

- Official Reception
- Greek evening
- Guided visit to ancient Olympia
- Other social events are being considered

Payment of the registration fee should be made in Greek currency by bank-to-bank transfer to bank account no. 131 01 0091 79 92 of Agricultural Bank of Greece, Patras branch. A copy of the corresponding bank receipt should be sent together with the Registration Form. In each payment the name(s) of the participant(s) must be stated.

Cancellations should be made to the local organizing committee in writing. The following, rules will apply.

cancellation received before July 15, 1999: 80% refund  
 cancellation received before August 30, 1999: 50% refund  
 cancellation received after August 30, 1999: no refund

The registration / information desk will be open on:

Sunday, September 19, from 16:00 - 21:30 Other days, from 9:00 to 18:00

Accommodation: All participants will be lodged in rooms at some of the hotels of Patras, which are near the Symposium Site or in the center of Patras. (Details from the organizers.) On the registration form, please put the hotels in order of preference. Because the number of available rooms in each hotel is limited, we will reserve your rooms strictly in order of response. The cost of accommodation be paid directly to the hotel. Remark: We intend to arrange your transportation from your hotels to the Symposium Site when this is not provided by the hotels.

**Official Invitations.** In special cases, the Organizing Committee is prepared to send a personal invitation for participation to the Symposium. It should be understood

that such an invitation is not a commitment on the part of the organizers to provide any financial support.

**Symposium Site.** The Symposium will be held at the Conference and Cultural Hall of the University of Patras, located at the University Campus (well connected by bus to Patras city center.) The city and its surroundings are special attractions. Patras may be reached easily by car, bus (3 hours from Athens by National Road) train (3.5 hours from Athens) and ship (especially for people coming from Italy).

**Deadlines.** Registration must be received before May 30. Abstracts must be received before May 30.

**Scientific Committee:** Walter Van Assche (Belgium), Marcel de Bruin (Holland), Evangelos Ifantis (Greece), Andrea Laforgia (Italy), Lance Littlejohn (USA), Paco Marcellan (Spain), Martin Muldoon (Canada), Panayiotis Siafarikas (Greece).

**Local Organizing Committee:** Evangelos Ifantis (Greece), Chrisoula Kokologiannaki (Greece), Panayiotis Siafarikas (Greece)

#### Further Communications

There will be a third (final) circular in June sent only to those who will have submitted the registration form. We suggest that you make your registration

#### Mailing address:

Fifth international Symposium OPSFA  
Department of Mathematics  
Prof. P. D. Siafarikas  
University of Patras  
Patras 26500 Greece  
Fax: +(3) 061 997169  
E-Mail: OPSFA@math.upatras.gr

WWW site: <http://www.math.upatras.gr/opsfa/>

Panos D. Siafarikas  
(Panos@math.upatras.gr)

(On behalf of the Organizing Committee)

#### 10. Workshop on Contemporary Problems in Mathematical Physics, Institut de Mathématiques et de Sciences Physique (IMSP), Cotonou, République du Bénin (Africa), October 31st - November 7th, 1999.

A Workshop on Contemporary Problems in Mathematical Physics will take place at The *Institut de Mathématiques et de Sciences Physique* (Cotonou, République du Bénin), from October 31st to November 7th, 1999. L'Institut de Mathématiques et de Sciences Physique

(L'IMSP), as one of the Graduate school of Université Nationale de Bénin, organizes a third Cycle in Mathematics and Physics since 1989. L'IMSP was the first African Institution initiated with the support of the Third World Academy of Science, and affiliated in 1988 to the ICTP (International Center for Theoretical Physics-Trieste, Italy) created by the late Nobel laureate Abdus Salam. PhD students coming from many countries around Bénin and being bachelor in Mathematics or Physics, are selected every two years, in order to start a DEA (Diplôme d'Etudes Approfondies), followed by the preparation during at least two more years of a PhD Dissertation in IMSP and abroad (mainly Europe and North America). One of the orientation, inside Mathematical Physics covers the topic of "Special Functions and Orthogonal Polynomials".

The main objective of the workshop is to contribute to the development of a **critical mass** of researchers in Africa in such a dynamic area as mathematical physics. The main **aim** of the workshop is to bring together specialists from diverse topics in Theoretical and Mathematical Physics. It will offer to the international scientific community the possibility of exchanging useful information on contemporary problems in these fields through direct interaction. The workshop will also provide young African researchers with an opportunity to know each other and to initiate scientific cooperation. Therefore, the workshop will help to strengthen research capacity and revitalize activities in mathematical physics in African universities. The workshop will include invited review talks and contributed communications. The invited review talks will be presented in plenary sessions whereas the contributed papers will be presented in three working groups corresponding to the three main topics.

#### Main topics:

1. Coherent States, Wavelets and Geometric Methods in Theoretical Physics
2. Quantum Field Theory, Atomic and Molecular Physics
3. Operator Theory and Orthogonal Polynomials

The official languages of the workshop are English and French.

The activity is open to scientists from all countries. Travel and subsistence expenses of the participants should be covered by the home institution. However, limited funds will be made available for scientists from African countries. Participants interested in presenting a paper are encouraged to send a camera ready copy. The deadline for submission of papers and/or of requests for financial support is July 15th, 1999. Applications should be submitted to:

Prof. M. N. Hounkonnou (Workshop in COPROMAPH)  
Institut de Mathématiques et de Sciences Physiques

(IMSF)

B. P. 613, Porto-Novo, République du Benin

Phone number: (00) 229 22 24 55

E-mail: hounkon@syfed.bj.refer.org

The local **Organizing Committee** is: Prof. M. N. Hounkonnou, (Unité de Recherche en Physique Théorique), Institut de Mathématiques et de Sciences Physiques, Porto-Novo-Benin (E-mail: hounkon@syfed.bj.refer.org) and Prof. W. A. Lester, Department of Chemistry, University of California, Berkeley (E-mail: walester@cchem.berkeley.edu)

The **Scientific Advisory Committee** is: S. T. Ali (Canada), J.P. Antoine (Belgium), A. Banyaga (USA), A. Bellemans (Belgium), S. Belmechi (France), X. Chapuisat (France), J. M. Combes (France), J. P. Gazeau (France), G. Ciccotti (Italy), C. S. Diatta (Sénégal), J. P. Ezin (Benin), C. Goudjo (Benin), M. N. Hounkonnou (Benin), R. Kerner (France), W. Koepf (Germany), W. A. Lester (USA), M. Mareschal (CECAM), A. Msezane (USA), R. Murenzi (USA), A. Ronveaux (Belgium), J. P. Ryckaert (Belgium), J. Shabani (UNESCO), B. Torrèsani (France).

## Application Form

WORKSHOP ON CONTEMPORARY PROBLEMS IN MATHEMATICAL PHYSICS (October 31st - November 7th, 1999)

Last name: \_\_\_\_\_

First name(s): \_\_\_\_\_

Nationality: \_\_\_\_\_

Professional address: \_\_\_\_\_

\_\_\_\_\_

Phone, fax: \_\_\_\_\_

E-mail: \_\_\_\_\_

Address for correspondence (if it is different from the above): \_\_\_\_\_

\_\_\_\_\_

Academic qualification: \_\_\_\_\_

Field of research interest: \_\_\_\_\_

Chosen working group at this workshop: \_\_\_\_\_

Accompanied by (specify): \_\_\_\_\_

Please check the appropriate spot:

- I can find funds for the expenses:

- I am requesting financial support for:

half travel: \_\_\_\_\_

full travel: \_\_\_\_\_

living expenses: \_\_\_\_\_

I shall submit a paper: Yes \_\_\_ No \_\_\_

Registration fees:

1. Participants from developed countries: US\$150

2. Participants from developing countries: US\$50

Date: \_\_\_\_\_

Signature: \_\_\_\_\_

André Ronveaux  
(Andre.Ronveaux@fundp.ac.be)

## 11. Workshop on the Minimal Energy Problems, City University of Hong Kong, Hong Kong, November 8-12, 1999.

A workshop on Minimal Energy Problems will be held from November 8-12, 1999, at the City University of Hong Kong, as part of a series of Foundations of Computational Mathematics workshops to be held this fall.

The meeting will emphasize research work related to distributing points on a sphere and on general Riemann surfaces, discrepancy results (e.g. for Fekete points), potential theoretic tools and applications to orthogonal polynomials, random matrices, integrable systems, etc.

For further information, contact Ed Saff (esaff@math.usf.edu) or Arno Kuijlaars (arno@wis.kuleuven.ac.be), or consult the workshop home page <http://www.math.usf.edu/FoCM99>

Arno Kuijlaars  
(arno@wis.kuleuven.ac.be)

## Books and Journals

### Book Announcements

#### 1. Orthogonal Polynomials and Random Matrices: A Riemann-Hilbert Approach Percy Deift

Courant Lecture Notes: A New Series Published by the Courant Institute, New York University. 1999, 273 pp., USA\$20, ISBN 0-9658703-2-4.

The following information is taken from the URL site: [www.cims.nyu.edu/lecnotes/](http://www.cims.nyu.edu/lecnotes/) and the OP-SF Talk [www.nist.gov/itl/div896/emaildir/opsftalk/maillist.html](http://www.nist.gov/itl/div896/emaildir/opsftalk/maillist.html)

These notes expand on a set of lectures at the Courant Institute in 1996-1997 on Riemann-Hilbert problems, orthogonal polynomials, and random matrix theory. The main goal of the course was to prove universality for a variety of statistical quantities arising in the theory of random matrix models. The main ingredient in the proof is the steepest descent method for oscillatory Riemann-Hilbert problems introduced earlier by the author and Xin Zhou.

Contents:

1. Riemann-Hilbert Problems
2. Jacobi Operators

3. Orthogonal Polynomials
4. Continued Fractions
5. Random Matrix Theory
6. Equilibrium Measures
7. Asymptotics for Orthogonal Polynomials
8. Universality

Renato Álvarez-Nodarse  
(ran@cica.es)

**2. "Self-Similar Systems", Proceedings of the International Workshop (July 30 - August 7, Dubna, Russia, 1998). Edited by V.B. Priezhev and V.P. Spiridonov**

(JINR, E5-99-38, Dubna, 1999), 404 pp. ISBN 5-85165-525-9

This book consists of three parts:

I. Wavelets and their applications. II. Self-structuring, quasicrystals, nonlinear equations. III. Orthogonal polynomials and special functions.

The orthogonal polynomials session of the workshop was dedicated to the centenary of the birth of the Soviet mathematician and mechanicist Ya. L. Geronimus. The contents of the corresponding part of the proceedings is given below. Complete information on the book is available on the web-page: <http://thsun1.jinr.ru/meetings/98/sss/proceedings.html>

**Contents of the Part III**

1. Golinskii L., On the scientific legacy of Ya.L. Geronimus (to the hundredth anniversary)
2. Andrievskii V.V. and Blatt H.-P., Local discrepancy theorems for the distribution of zeros of polynomials
3. Aptekarev A.I. and Kaliaguine V.A., On a generalization of the nonlinear Lengmuir chains
4. Bangerezako G. and Magnus A.P., The factorization method for the semi-classical polynomials
5. Belogradov A., Semi-classical orthogonal polynomials and integrable chains
6. Mantica G., Generalized Bessel functions: theoretical relevance and computational techniques
7. Marcellan F. and Petronilho J., Orthogonal polynomials and polynomial mappings on the unit circle
8. Nikiforov A. and Skachkov M., Methods for computing the Hahn polynomials
9. Osilenker B.P., Generalized trace formula for symmetric Legendre-Sobolev orthogonal polynomials
10. Rodriguez J.M., Alvarez V., Romera E., and Pestana D., Generalized weighted Sobolev spaces
11. Spiridonov V. and Zhedanov A., Self-similarity, spectral transformations and orthogonal and biorthogonal polynomials
12. Stahl H., Rational best approximants in the Hardy space  $H^2$
13. Steinbauer R., Considerations on Schur parameters with constant blocks
14. Suetin P.K., Classical orthogonal polynomials in the theory of Schrodinger equation
15. Van Assche W., Zero distribution of orthogonal polynomials with asymptotically periodic varying recurrence coefficients

V. Spiridonov  
(svp@thsun1.jinr.ru)

**Book Reviews**

**1. Hypergeometric Summation**

**By Wolfram Koepf**

Verlag Vieweg, Braunschweig/Wiesbaden, 1998, ISBN 3-528-06950-3

The book under review deals with the very successful set of algorithms for hypergeometric summation which came to maturity during the last decade. This development started with Gosper's algorithm for indefinite summation in 1978, and got a decisive impetus by Zeilberger's algorithm for definite summation in 1990. Closely related is the WZ-method, while Petkovšek's algorithm provides a useful complement. These algorithms were extended to the  $q$ -case and (in some cases) to multiple summation. Also, analogous algorithms were developed for the case where one or both variables become continuous and derivatives and integrals replace differences and summations. Implementations of the algorithms in many different computer algebra systems were given, in particular in Maple and Mathematica. The success of these implementations has been enormous. It is a serious option nowadays to replace compendia of formulas for hypergeometric functions, or their electronic versions containing static formulas, by electronic utilities where the formula of desired type is produced by an implemented algorithm. A further, very attractive property of the Gosper-Zeilberger algorithm is that it not just produces an explicit sum evaluation or recurrence (if it exists), but also provides a few simple data

(proof certificate) which give all ingredients for a short proof of the identity. These developments might revolutionize the way mathematics will be done in future, as is frequently argued by Doron Zeilberger in an eloquent but provoking way.

An account of these algorithms addressed to a wide audience was published in 1996 by Petkovšek, Wilf and Zeilberger in the book  $A=B$ . The book under review is very much related to  $A=B$  in scope and aimed audience, but in many respects it is also different. Let me first discuss the similarities. Both books can be read with few preliminaries. They do not require earlier acquaintance with special functions or computer algebra algorithms or with computer algebra systems in practice. In both books the “five basic algorithms” by Sister Celine, Gosper, Zeilberger, Wilf-Zeilberger and Petkovšek take a prominent place (Koepf treats W-Z before Zeilberger’s algorithm). Each of the two books gives many worked out sessions with Maple and Mathematica (in Koepf’s book only Maple). Both books effectively bring the reader from scratch to a good understanding and knowledge of these algorithms and to a practical ability to use them. However, none of the books gives fully rigorous proofs that the algorithms are valid. Both books point to websites from where implementations by the author(s) of the algorithms can be downloaded.

As for the differences, the style in  $A=B$  is looser, and the message that this topic is great fun is effectively sent to the reader. Standard hypergeometric notation is introduced in both books, and regularly used by Koepf, while  $A=B$  gives most sums with terms in the form of products of binomial coefficients. Koepf pays more attention to subtle aspects of the algorithms (for instance zeros occurring in the denominator), but he usually does not discuss these subtleties in an exhaustive way (see for instance the discussion about zeros in the denominator on various places in Ch. 6 dealing with the Wilf-Zeilberger method). Thus, if one wants to be definitely convinced that the algorithm is correct, one has to go to Koepf’s implementation, and check the details of his Maple code. Koepf gives more extensions of the algorithms than  $A=B$ . Almost each chapter ends with a short discussion of the  $q$ -analogue. He discusses the extended Gosper algorithm and WZ method (for instance where in the evaluation of a definite sum different parities of  $n$  yield different analytic expressions). Very valuable additions, compared to  $A=B$ , are Koepf’s chapters 10–13 on, respectively, Differential equations for sums, Hyperexponential antiderivatives, Holonomic equations for integrals, Rodrigues formulas and generating functions. Koepf has a wealth of exercises at the end of each chapter, many more than in  $A=B$ . Koepf’s accompanying Maple source file `hsum.mpl` bundles all relevant procedures for single summation and single integration ( $q = 1$ ), while his file `qsum.mpl` gives many procedures for the  $q$ -case. These files are somewhat more comprehensive than Doron Zeil-

berger’s Maple source files `EKHAD` and `qEKHAD`. A very nice service of Koepf is that Maple worksheets for the various Maple sessions described in his book are available from his website.

So, as a conclusion the book under review, together with the accompanying free software, can be very much recommended for self-study, for reference, and for usage in classroom and student seminars.

**NOTE:** Free accompanying software can be downloaded from

<http://www.vieweg.de/welcome/downloads/supplements.htm>

or from

<http://www.imn.htwk-leipzig.de/~koepf/research.html>

Tom H. Koornwinder  
([thk@wins.uva.nl](mailto:thk@wins.uva.nl))

## Problems and Solutions

Thus far 20 problems have been submitted seven of which have been solved in previous issues. Still unsolved are Problems #3, 5, 8, 9, 11, 12, 13, 15, 17, 18, 19 and 20. This time no new problems have been submitted.

**19. Uniform Bounds for Shifted Jacobi Multiplier Sequences.** For Fourier series the following is immediate: Suppose the real or complex sequence  $\{m_k\}$  generates a bounded operator on  $L^p(\mathbf{T})$ ,  $1 \leq p \leq \infty$ , i.e., for polynomial  $f$

$$\left\| \sum m_k \hat{f}_k e^{ik\varphi} \right\|_{L^p(\mathbf{T})} \leq \|m\|_{M^p(\mathbf{T})} \left\| \sum \hat{f}_k e^{ik\varphi} \right\|_{L^p(\mathbf{T})},$$

then one has for the shifted sequence  $\{m_{k+j}\}_{k \in \mathbf{Z}}$  that

$$\sup_{j \in \mathbf{N}_0} \|\{m_{k+j}\}\|_{M^p(\mathbf{T})} \leq C \|m\|_{M^p(\mathbf{T})}, \quad 1 \leq p \leq \infty. \quad (1)$$

Looking at cosine expansions on  $L^p(0, \pi)$  one easily derives the analog of (1) via the addition formula

$$\cos(k \pm j)\theta = \cos k\theta \cos j\theta \mp \sin k\theta \sin j\theta$$

provided the periodic Hilbert transform is bounded, i.e., for  $1 < p < \infty$ . More generally, by Muckenhoupt’s transplanted theorem [2, Theorem 1.6],

$$\begin{aligned} & \left( \int_0^\pi \left| \sum m_{k+j} a_k P_k^{(\alpha, \beta)}(\cos \theta) \right|^p \sin^{2\alpha+1} \frac{\theta}{2} \cos^{2\beta+1} \frac{\theta}{2} d\theta \right)^{1/p} \\ & \equiv \left( \int_0^\pi \left| \sum m_{k+j} b_k \phi_k^{(\alpha, \beta)}(\cos \theta) \right|^p \omega_{\alpha, \beta, p}(\theta) d\theta \right)^{1/p} \\ & \approx \left( \int_0^\pi \left| \sum m_{k+j} b_k \cos k\theta \right|^p \omega_{\alpha, \beta, p}(\theta) d\theta \right)^{1/p}, \end{aligned}$$

where  $P_k^{(\alpha,\beta)}$  are the Jacobi polynomials,  $\phi_k^{(\alpha,\beta)}(\cos \theta)$  are the orthonormalized Jacobi functions with respect to  $d\theta$ , and

$$w_{\alpha,\beta,p}(\theta) = \sin^{(2-p)(\alpha+1/2)} \frac{\theta}{2} \cos^{(2-p)(\beta+1/2)} \frac{\theta}{2}.$$

Therefore, the above argument for cosine expansions also applies to Jacobi expansions provided the periodic Hilbert transform is bounded with respect to the weight function  $w_{\alpha,\beta,p}$ ; hence, the analog of (1) holds for Jacobi expansions when

$$\frac{2\alpha + 2}{\alpha + 3/2} < p < \frac{2\alpha + 2}{\alpha + 1/2}, \quad \alpha \geq \beta \geq -\frac{1}{2}.$$

(i) Can the above  $p$ -range be extended? By Muckenhoupt [2, (1.3)], a fixed shift is bounded for all  $p$ ,  $1 < p < \infty$ .

(ii) Consider the corresponding problem for Laguerre expansions (for the appropriate setting see [1]); a fixed shift is easily seen to be bounded for all  $p \geq 1$ .

Both questions are of course trivial for  $p = 2$  since  $\ell^\infty = M^2$  by Parseval's formula.

**References**

[1] Gasper, G. and W. Trebels: On necessary multiplier conditions for Laguerre expansions, *Canad. J. Math.* 43 (1991), 1228 – 1242.  
 [2] Muckenhoupt, B.: Transplantation Theorems and Multiplier Theorems for Jacobi Series, *Memoirs Amer. Math. Soc.*, Vol. 64, No. 356, Providence, R.I., 1986.

(Submitted on May 19, 1998)

George Gasper  
(george@math.nwu.edu)

Walter Trebels  
(trebels@mathematik.tu-darmstadt.de)

**20. Question about Elliot's formula** Generalization of Legendre's identity for complete elliptic integrals

Let  $E, K$  be the complete elliptic integrals. Then

$$K'E + KE' - KK' = \pi/2 \quad (*)$$

This is the special case  $p = r = -a + 1/2, q = c + a - 3/2$  in Elliott's identity (see Erdelyi e.a., *Higher Transcendental Functions*, Vol. 1, p. 85):

$$\begin{aligned} &F(p + \frac{1}{2}, -r - \frac{1}{2}, 1 + p + q; z)F(-p + \frac{1}{2}, r + \frac{1}{2}; 1 + q + r; 1 - z) \\ &+ F(p + \frac{1}{2}, -r + \frac{1}{2}, 1 + p + q; z)F(-p - \frac{1}{2}, r + \frac{1}{2}; 1 + q + r; 1 - z) \\ &- F(p + \frac{1}{2}, -r + \frac{1}{2}, 1 + p + q; z)F(-p + \frac{1}{2}, r + \frac{1}{2}; 1 + q + r; 1 - z) \\ &= \frac{\Gamma(p + q + 1)\Gamma(q + r + 1)}{\Gamma(p + q + r + \frac{3}{2})\Gamma(q + \frac{1}{2})} \quad (**) \end{aligned}$$

Question 1. Is there a counterpart of Legendre's identity (\*) for incomplete elliptic integrals?

Question 2. The Elliott identity (\*\*) provides a generalization of the identity (\*) to hypergeometric functions. The only handbook where I have seen this identity is Bateman vol. I. Has Elliott's identity been used or mentioned elsewhere in papers/books?

Question 3. Are there generalizations of the Elliott identity (\*\*) to the  ${}_pF_q$  case or to other generalizations of hypergeometric functions?

Matti Vuorinen  
(vuorinen@csc.fi)

**Questions**

In this section we will include some questions raised by various people which have appeared elsewhere. These questions will appear only once, so please send the answer directly to the author.

**1. Orthogonal polynomials with order-dependent argument**

(appeared on opsfnct)

Can anyone tell me if there is any literature on orthogonal polynomials of order-dependent argument, i.e. a complete set of orthogonal polynomials  $P_n(x, n)$ , in particular of the form  $P_n(nx)$ , regarding such fundamental concepts as generating functions, weights, Rodrigues formulas, etc. I am interested in Hermite and Laguerre specifically, but any info anyone could give me regarding this, I would be interested.

Tom Hoover  
(bx238@freenet.buffalo.edu)

**2. Laguerre Polynomials in Two Variables**

(appeared on NA Digest, V. 99, # 20)

I am looking for information on approximating functions in two variables by Laguerre polynomials in  $(0, \infty) \times (0, \infty)$ . Can anyone provide me with references?

In particular, I am interested in the convergence of the  $n$ -th interpolating polynomial to the function in the appropriate weighted norm (on a weighted  $L^2$  space). The proof in one variable is established (since 1991 at least).

Thank you very much to anyone who can help me on this issue.

K. Feigl  
(feigl@ifp.mat.ethz.ch)

## Miscellaneous

### 1. Student Travel Awards for 1999 Conferences

During 1999, SIAM has made a number of awards for \$300 to support student travel to some SIAM conferences (see the previous Newsletter (Vol. 9 number 2, February 1999). It is still possible to get funds for the

- Sixth SIAM Conference on Geometric Design, November 2-5, Albuquerque, NM

The awards are to be made from the SIAM Student Travel Fund, created in 1991 and maintained through book royalties donated by generous SIAM authors.

Any full-time student in good standing is eligible to receive an award plus gratis meeting registration. Top priority will be given to students presenting papers at the meeting, with second priority to students who are co-authors of papers to be presented at the meetings. Only students traveling more than 100 miles to the meetings are eligible for the awards.

An application for a travel award must include:

1. A letter from the student describing his/her academic standing and interests, his/her expected graduation date and degree, advisor's name, and, if available, a URL for a working Web page.
2. A one-page vita that includes the student's research interests, projects, and papers published.
3. A detailed letter from the student's faculty advisor indicating why the student is deserving of receiving a travel award and any special circumstances.
4. If applicable, the title(s) of the paper(s) to be presented (co-authored) by the student at the meeting.

Applications should be sent to the SIAM office (Attention: SIAM Student Travel Awards), 3600 University City Science Center, Philadelphia, PA 19104-2688. Students also may apply by e-mail to bogardo@siam.org or by fax to 215-386-7999.

Complete applications must be received at the SIAM office no later than TWO MONTHS before the first day of the meeting for which support is requested.

Winners will be notified FIVE WEEKS before the first day of the meeting. Checks for the awards will be given to the student awardees when they arrive at the given meeting and pick up their registration packet at the SIAM Registration Desk.

Allison Bogardo  
(bogardo@siam.org)

### 2. Combinatorics Net

I wish to inform you that Annals of Combinatorics (Springer) is building up a web site: <http://www.combinatorics.net/>, which is called the Combinatorics Net, this site is also intended

to serve the combinatorics community. It includes a useful Who's Who in Combinatorics list.

Bill Chen  
(chen@t7.lanl.gov)

### 3. opsftalk listserv taken over by Dan Lozier

Up to now the listserv opstalk has been supervised and moderated by me at the University of Amsterdam via a major-domo program. It is a service of the SIAM Activity Group on Orthogonal Polynomials and Special Functions. From now on this service will be continued by Dan Lozier (e-mail: [dlozier@nist.gov](mailto:dlozier@nist.gov)). Dan has copied the present mailing list, so no action by subscribers is required, unless they want to unsubscribe.

All information about opsftalk can now be found in

<http://math.nist.gov/opsftalk/>

This information includes the following:

To subscribe to OPSF-Talk, send a message to list-proc@nist.gov with

subscribe opsftalk xxxxx

on a single line in the message body, where in place of xxxxx you put your name. To cancel your subscription, send "unsubscribe opsftalk" to the same address.

You can post messages by sending mail to [opsftalk@nist.gov](mailto:opsftalk@nist.gov). Your message will be forwarded to the moderator, who will usually pass it to everybody on the opsftalk list. It will also be added automatically to an archive. You can see all opsftalk messages (going back to January 1998) in

<http://www.nist.gov/itl/div896/emaildir/opsftalk/maillist.html>

The archive of postings to opsftalk at <http://www.findmail.com/listsaver/opsftalk/> will no longer be updated, and will probably soon be removed completely.

I am grateful to Dan that he is willing to continue this. I think a listserv like this can potentially be a valuable medium for providing and getting information, posing questions and discussing topics which are of immediate interest for our specialism. To be honest, during the first 15 or 16 months of its existence, postings were not yet up to my expectations.

Tom H. Koornwinder  
(thk@wins.uva.nl)

---

### How to Contribute to the Newsletter

Send your Newsletter contributions directly to one of the Co-editors:

Renato Álvarez-Nodarse  
Departamento de Análisis Matemático  
Universidad de Sevilla



Apdo 1160, E-41080 Sevilla, Spain  
 phone: +34-954-55-79-97  
 fax: +34-95-455-79-72  
 e-mail: ran@cica.es, renato@gandalf.ugr.es

or

Rafael J. Yáñez  
 Departamento de Matematica Aplicada  
 Universidad de Granada  
 E-18071 Granada, Spain  
 phone: +34-958-249947  
 fax: +34-958-242862  
 e-mail: ryanez@ugr.es



preferably by e-mail, and in L<sup>A</sup>T<sub>E</sub>X format. Other formats are also acceptable and can be submitted by e-mail, regular mail or fax.

**Deadline for submissions to be included in the October issue 1999 is September 15, 1999.**

Back issues of the Newsletter can be obtained from <http://www.imm.htwk-leipzig.de/~koepf/siam.html>.

The Activity Group also sponsors an electronic news net, called the **OP-SF Net**, which is transmitted periodically by SIAM. The Net provides a rather fast turnaround compared to the Newsletter. To receive transmissions, just send your name and e-mail address to [poly-request@siam.org](mailto:poly-request@siam.org) (as with other nets, nonmembers can also receive the transmissions). Your OP-SF Net *contributions* should be sent to [poly@siam.org](mailto:poly@siam.org). Please note that submissions to OP-SF Net are automatically considered for publication in the Newsletter, and vice versa, unless the writer requests otherwise.

The Net is organized by Martin Muldoon ([muldoon@yorku.ca](mailto:muldoon@yorku.ca)). Back issues of OP-SF Net can be obtained by anonymous ftp from [ftp.wins.uva.nl](ftp:wins.uva.nl), in the directory [pub/mathematics/reports/Analysis/koornwinder/opsfnet.dir](ftp://pub/mathematics/reports/Analysis/koornwinder/opsfnet.dir) or by WWW at the addresses

<http://turing.wins.uva.nl/~thk/opsfnet/>  
<http://www.math.ohio-state.edu/JAT>

Martin Muldoon also manages our home page <http://www.math.yorku.ca/siamopsf/>

on World Wide Web. Here you will find also a WWW version of the OP-SF Net. It currently covers the topics

- Conference Calendar
- Books, Conference Proceedings, etc.
- Compendia, tools, etc.
- Compiled booklist on OP-SF
- Meeting Reports
- Projects
- Problems
- Personal, Obituaries, etc.
- History
- Positions available
- Miscellaneous
- Memberlist
- Links to WWW pages of interest to members

**Activity Group: Addresses**

The *SIAM Activity Group on Orthogonal Polynomials and Special Functions* consists of a broad set of mathematicians, both pure and applied. The Group also includes engineers and scientists, students as well as experts. We now have around 140 members scattered about in more than 20 countries. Whatever your specialty might be, we welcome your participation in this classical, and yet modern, topic. Our WWW home page <http://www.math.yorku.ca/siamopsf/> is managed by Martin Muldoon ([muldoon@yorku.ca](mailto:muldoon@yorku.ca)).

The **Newsletter** is a publication of the *SIAM Activity Group on Orthogonal Polynomials and Special Functions*, published three times a year. To receive the Newsletter, you must first be a member of SIAM so that you can join the Activity Group. The annual dues are \$96 for SIAM plus \$10 for the Group; students pay \$20/year with free membership in one activity group; postgraduates can become members of SIAM for \$45/year. To join, contact:

*Society for Industrial and Applied Mathematics*  
 3600 University City Science Center  
 Philadelphia, PA 19104-2688  
 phone: +1-215-382-9800    [service@siam.org](mailto:service@siam.org)

*Address corrections:* Current Group members should send their address corrections to Marta Lafferty ([lafferty@siam.org](mailto:lafferty@siam.org)). Please feel free to contact any of the Activity Group Officers. Their addresses are:

Daniel W. Lozier — *Chair of the Activity Group*  
National Institute of Standards and Technology  
100 Bureau Drive  
Gaithersburg, MD 20899-0001 USA  
phone: (301) 975-2706 fax: (301) 990-4127  
e-mail: dlozier@nist.gov  
WWW: <http://math.nist.gov/mcsd/Staff/DLozier/>

Walter Van Assche — *Vice Chair*  
Department of Mathematics  
Katholieke Universiteit Leuven  
Celestijnenlaan 200B  
B-3001 Heverlee  
Belgium  
phone: +32-16-327051 fax: +32-16-327998  
e-mail: walter@wis.kuleuven.ac.be  
WWW: <http://www.wis.kuleuven.ac.be/wis/applied/walter.html>

Francisco Marcellán — *Program Director*  
Departamento de Matemáticas  
Escuela Politécnica Superior  
Universidad Carlos III, Ave. de la Universidad  
Edificio Sabatini, E-28911  
Leganés, Madrid, Spain  
phone: +34-91-624-94-42 fax: +34-91-624-91-51  
e-mail: pacomarc@ing.uc3m.es

Charles Dunkl — *Secretary*  
Department of Mathematics  
University of Virginia, Charlottesville, VA 22903  
phone: +1-804-924-4939 fax: +1-804-982-3084  
e-mail: cfd5z@virginia.edu  
WWW: <http://www.math.virginia.edu/~cfd5z/home.html>

Renato Álvarez-Nodarse — *Editor of the Newsletter*  
Departamento de Análisis Matemático  
Universidad de Sevilla, Apdo. 1160, E-41080  
Sevilla, Spain  
phone: +34-95-455-79-97 fax: +34-95-455-79-72  
e-mail: ran@cica.es, renato@gandalf.ugr.es  
WWW: <http://gandalf.ugr.es/~renato/>

Rafael J. Yáñez — *Editor of the Newsletter*  
Departamento de Matemática Aplicada  
Universidad de Granada  
E-18071 Granada, Spain  
phone: +34-58-242941 fax: +34-58-242862  
e-mail: ryanez@ugr.es  
WWW: <http://gandalf.ugr.es/~ryanez>

Martin E. Muldoon — *Webmaster*  
Department of Mathematics & Statistics  
York University  
North York, Ontario M3J 1P3, Canada  
phone: +1-416-736-5250 fax: +1-416-736-5757  
e-mail: muldoon@yorku.ca  
WWW: <http://www.math.yorku.ca/~muldoon/>