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# **Orthogonal Polynomials and Special Functions**

SIAM Activity Group on Orthogonal Polynomials and Special Functions

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## The News

In this Winter Edition we talk about the upcoming election of officers for our Activity Group, and remind you to cast your vote. Then on account of a slow response, the call for an informal member survey is repeated, and likewise for the logo design. In the Meetings and Conferences section you will find a number of activities associated with the Stieltjes commemoration of 1994. And finally, best wishes for a Happy New Year!

An updated member list will be mailed out with the Spring Edition. A few corrections to the member list have already been received, but we ask again that you check your current listing carefully for accuracy and omissions. If anything untoward is found, you can inform either SIAM directly (service@siam.org), or Charles Dunkl (cfd5z@virginia.edu). (Charles will forward your information to SIAM headquarters where the list is compiled.) If you have recently acquired an e-mail address, please be sure to provide it.

For the next member list a different font will be employed, namely a font that distinguishes between the letter l and the number 1, thus eliminating confusion over e-mail addresses.

It appears that Jet Wimp was the only participant from the U.S. at the Evian conference. Claude Brezinski has just sent this report:

Preceded by the symposia of Bar-le-Duc in 1984, Segovia in 1986, and Erice in 1990, the 4th International Symposium on Orthogonal Polynomials and Their Applications was held in Evian, France, from October 19 to 23, 1992. More than 70 participants attended, mainly from Europe. Many interesting talks were given and the participants enjoyed a very friendly atmosphere. The symposium was dedicated to Professor Luigi Gatteschi from the University of Torino in Italy. The proceedings will be published as a special volume of the Journal of Computational and Applied Mathematics.

The upcoming congresses will be held in Delft, The Netherlands, for the Stieltjes commemoration; in Patras, Greece; and possibly in Logan, Utah.

#### **Activity Group Election**

Since we are due for an election (this one should be painless), on 18 September 1992, SIAM President Robert O'Malley, Jr. appointed a committee to nominate candidates for office for the SIAM Activity Group on Orthogonal Polynomials and Special Functions. The committee is made up of the present four officers, namely Charles Dunkl, Jet Wimp, George Gasper, and Mourad Ismail; in addition there is Walter Gautschi (Purdue), Willard Miller, Jr. (Univ. Minnesota), and Audrey Terras (Univ. Calif. San Diego). Then on 3 November the committee submitted its list of candidates. A ballot is in the mail please remember to vote.

A couple of the candidates are from outside the United States, as the committee felt that the group's important role in multinational communication would be strengthened by having some officers from outside the U.S. We have just received word that Martin Muldoon (York Univ., Toronto) and Mizan Rahman (Carlton Univ., Ottawa) have each agreed to stand for election to Program Director. Moreover, Tom Koornwinder (Univ. Amsterdam) and Walter Van Assche (Katholieke Univ., Leuven, Belgium) have each agreed to stand for election to Secretary.

## Meetings and Conferences

1. Tom Koornwinder informs us that a one day symposium will be held at CWI (formerly called the Centre for Mathematics and Computer Science), in Amsterdam, on 8 January 1993. The subject will be "Asymptotics and Special Functions" and it will mark N.M. Temme's 25th year at CWI. Among the speakers will be F.W.J. Olver (Maryland), R.B. Paris (Dundee), and A. Olde Daalhuis (CWI). The organizers are M. Bakker (miente@cwi.nl) and T.H. Koornwinder (thk@fwi.uva.nl).

2. A conference on "Nonlinear Numerical Methods and Rational Approximation" will be held at the University of Antwerp from September 5 to 11, 1993. Registration will cover housing, participation, and a copy of the conference proceedings—which will be similar to the 1987 proceedings. Special attention will be paid to housing and to social events for the participants.

The emphasis will be on Padé approximation, rational interpolation, rational approximation, continued fractions, and orthogonal polynomials. Each of these topics will be introduced by a one hour survey lecture. Also welcome are contributions on multivariate or multidimensional problems, error analysis, software development, and applications. Participants are invited to present a 20 minute research talk.

The invited speakers will be A. Gonchar (Moscow), M. Gutknecht (Zürich), W.B. Jones (Boulder, USA), D. Lubinsky (Witwatersrand, South Africa), and E. Saff (Tampa, USA).

For more information contact the organizer, who is

Annie Cuyt uyt@wins.uia.ac.be Department of Mathematics & C.S. University of Antwerp B-2610 Wilrijk-Antwerp, Belgium (32) 3 820-2407 Fax: (32) 3 820-2244

3. We now have the dates for the conference on Applications of Hypergroups and Related Measure Algebras. They are: 31 July to 6 August 1993. The venue is Seattle and the contact is

Alan L. Schwartzhypergrp@arch.umsl.eduDepartment of Mathematics & C.S.University of Missouri-St. LouisSt. Louis, MO 63121-4499(314) 553-6343Fax: (314) 553-5415

\_\_\_\_ SIAM Activity Group \_\_\_\_

on Orthogonal Polynomials and Special Functions

## CHARLES DUNKL, Chair JET WIMP, Vice Chair MOURAD E.H. ISMAIL, Program Director GEORGE GASPER, Secretary

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THE PURPOSE of the Activity Group

is to promote basic research in orthogonal polynomials and special functions; to further the application of this subject in other parts of mathematics, and in science and industry; and to encourage and support the exchange of information, ideas, and techniques between workers in this field, and other mathematicians and scientists.

4. In Leiden, and also in Toulouse, the 100th anniversary of T.J. Stieltjes' premature death will be commemorated on 31 December 1994. A number of activities are planned at both locations. Thomas Jan Stieltjes was born in 1856, in Zwolle, Holland. An astronomer who became a mathematician, he developed interests in analysis and in number theory. T.J. Stieltjes eventually became a professor at the University of Toulouse and he was a friend of Charles Hermite.

First of all, in April 1994 the Congress of Wiskundig Genootschap (Dutch Mathematical Society) will be held in Leiden, where Richard Askey will give the Stieltjes Lecture. Contact:

Prof. G. van Dijk Rijksuniversiteit Leiden Afdeling Wiskunde en Informatica P.O. Box 9512 2300 RA Leiden, The Netherlands

Then in Toulouse there will be a colloquium, of perhaps three days, in the spring of 1995, where it has been agreed that the academic year 1994-1995 will be known as "l'année Stieltjes". The focus will be on continued fractions and moment problems, orthogonal polynomials, Laplace transforms, the Riemann hypothesis, and other topics. This will have a somewhat historical character. There will also be an emphasis on the work of Stieltjes in both the graduate and undergraduate seminars. Contact:

Prof. J.-B. Hiriart-Urruty Groupe d'Histoire des Mathématiques de l'Université Paul Sabatier 118, Route de Narbonne 31062 Toulouse, France Preceding these activities in Leiden and Toulouse, a new edition of the collected works of Stieltjes will appear at the beginning of 1993. It will be edited by G. van Dijk and published by Springer-Verlag. In addition to containing all the Stieltjes papers (as in the first edition published by Noordhoff in 1914/1918), there will be a short biography of Stieltjes, an English translation of his important paper "Recherches sur les fractions continues", and four other contributions:

"The Impact of Stieltjes' Work on Continued Fractions and Orthogonal Polynomials", by W. Van Assche

"The Stieltjes Integral and Its Influence on Analysis", by W.A.J. Luxemburg

"The Contributions of Stieltjes to Number Theory", by F. Beukers

"Stieltjes and The Riemann Hypothesis", by H.J. te Riele

Also, The University of Leiden has taken the initiative to create a "Stieltjes Institute" of mathematics. This will be a joint effort involving several Dutch Universities.

Finally, G. van Dijk is preparing an article on the life and work of Stieltjes for The Mathematical Intelligencer.

## The Minisymposium (correction)

The title of Ram P. Srivastav's talk at the Minisymposium on Special Functions and Their Applications, which took place at the SIAM annual meeting in July, was incorrectly listed in the Fall Edition of the Newsletter as "Weber's Integral Theorem and Singular Solutions for Mixed Boundary Value Problems of Elasticity". This title was inadvertently taken from the Final Program given out at the meeting.

The correct title is "Some Identities Involving Products of Distances Between the Zeros of the Chebyshev Polynomials".

### Abstracts of Contributed Papers

1. The following paper will appear in an upcoming issue of Astrophysics and Space Science.

"Analytic Stellar Structure", by Hans J. Haubold † and Arak M. Mathai ‡.

## Abstract

We construct models of the internal structure of solar type stars using analytic methods. Since a detailed stellar model is desired, we have assumed a nonlinear analytic density distribution containing two parameters. These parameters control the shape of the distribution and the rate of decrease of the density from the center to the surface.

Hydrostatic equilibrium and energy conservation determine the conditions for the gravitationally stabilized stellar fusion reactor. The special function  $_2F_1$  appears in the evaluation of the mass, pressure, and luminosity. Some expressions can be represented in terms of special functions of many variables of the Lauricella type.

The authors have just attended the Second United Nations/European Space Agency Workshop on Basic Space Science for the Benefit of Developing Countries, co-organized by The Planetary Society, in cooperation with the Governments of Costa Rica and Columbia.

†Outer Space Division, United Nations, New York, NY 10017.

<sup>‡</sup>Department of Mathematics and Statistics, McGill University, Montreal, Canada H3A 2K6.

2. The following has been submitted for publication in the treatise *Contemporary Aspects of Fourier Analysis* (to be published by Academic Press). The editors are Wm. O. Bray, W.R. Madych, and C.V. Stanojevic.

"Using sums of squares to prove that certain entire functions have only real zeros", by George Gasper\*.

## Abstract

It is shown how sums of squares of real valued functions can be used to give new proofs of the reality of the zeros of the Bessel functions  $J_{\alpha}(z)$  when  $\alpha \geq -1$ , the confluent hypergeometric functions  $_{0}F_{1}(c;z)$  when c > 0 or 0 > c > -1, Laguerre polynomials  $L_{n}^{\alpha}(z)$  when  $\alpha \geq -2$ , and Jacobi polynomials  $P_{n}^{(\alpha,\beta)}(z)$  when  $\alpha \geq -1$  and  $\beta \geq$ -1. Besides yielding new inequalities for  $|F(z)|^{2}$ , where F(z) is one of these functions, the derived identities lead to inequalities for  $\partial |F(z)|^{2}/\partial y$  and  $\partial^{2} |F(z)|^{2}/\partial y^{2}$ , which also give new proofs of the reality of the zeros.

\*Department of Mathematics, Northwestern University, Evanston, IL 60208

## Problems

Submitted problems may range from elementary to advanced, either solved or unsolved. This gives several possibilities since the proposer need not disclose whether or not the solution is known. The first two problems appeared in the last issue of the Newsletter and they are repeated here.

1. Prove or disprove

$$\prod_{n=1}^{\infty} \frac{\cos\left(t/2^n\right) - \cos\left(\pi 3^n/5\right)}{1 - \cos\left(\pi 3^n/5\right)} = \frac{4\cos(t)}{\sqrt{5} + 5} + \frac{1}{\sqrt{5}}$$

Submitted by R. William Gosper, August 13, 1992.

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2. Is it true that

$$x^{2}t^{x} {}_{2}F_{1}(x+1, x+1; 2; 1-t)$$

is a convex function of x whenever  $-\infty < x < \infty$  and 0 < t < 1?

Submitted by George Gasper, August 19, 1992.

3. The following Toeplitz matrix arises in several applications. Define for  $i \neq j$ 

$$A_{ij}(\alpha) = \frac{\sin \alpha \pi (i-j)}{\pi (i-j)},$$

and set  $A_{ii} = \alpha$ . Conjecture: the matrix

$$M = (I - A)^{-1}$$

has positive entries. A proof is known for  $0 < \alpha \le 1/2$ . Can one extend this to  $0 < \alpha < 1$ ?

Submitted by Alberto Grünbaum, November 3, 1992.

#### Solutions

Solutions should appear eventually, whereupon they will be printed together with the original statement of the problem.

#### Survey of Members (continued)

The informal survey of member interests announced in the Fall Edition will be continued. Unfortunately, we did not hear from sufficiently many members to arrive at a meaningful result, perhaps because September is too busy a time for most academics. Please let us know what your interests are. Of course a few of us have interests that are known to just about everybody, while still others have interests that are not so well known, or perhaps still developing. Your timely response will contribute to the success of our activity group.

As mentioned in the Fall Edition, therefore, we seek answers to the following two questions. Do you apply the special functions or is your interest theoretical, perhaps both, or neither? Or something else? Please send your reply or comments to Charles Dunkl.

## Contest for the Logo Design (continued)

The logo contest will also be continued, more or less for the same reasons given in the above survey of members. The following is repeated from the Fall Edition.

We considered adopting a logo of some kind, so at the business meeting in Los Angeles it was decided to hold a Logo Design CONTEST. The contest will be open to all current members of the group, except for those on the editorial committee of the **Newsletter** since they will be the judges.

The winner will be awarded a *prize*, of course a book on orthogonal functions, namely

Orthogonal Functions by GIOVANNI SANSONE Dover reprint, 1991.

Please send your entries, along with any comments or suggestions, to the Editor at the address below.

## About the Newsletter

The Newsletter is a quarterly publication of the SIAM Activity Group on Orthogonal Polynomials and Special Functions. To join the Group, please contact

Society for Industrial and Applied Mathematics

3600 University City Science Center Philadelphia, PA 19104-2688

(215) 382-9800 service@siam.org

Like all newsletters, this one needs some input from the members it is supposed to serve. For example how about submitting an abstract of your latest paper, or of your upcoming talk; or how you plan to spend the summer, who is visiting from where, and things like that. Please send your contributions for the Spring Edition by 1 February, 1993 to the Editor

Eugene Tomer 1691 –11<sup>th</sup> Avenue San Francisco, CA 94122 Tel/Fax: (415) 665-9555 etomer@netcom.com

Eugene will be travelling during December and January. If you prefer, you may send contributions to the Chair of the Activity Group

Charles Dunkl Department of Mathematics University of Virginia Charlottesville, VA 22903 (804) 924-4939 Fax: (804) 982-3084 cfd5z@virginia.edu

Along about the middle of February the draft of the Spring Edition will be sent to SIAM headquarters for reproduction and mailing.