```
> read "hsum9.mpl";
```

Package "Hypergeometric Summation", Maple V - Maple 9

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```
> summand:=pochhammer(1/2,j)*pochhammer(alpha/2+1,n-j)*pochhammer((alpha+3)/2,n-2*j)*pochhammer(alpha+1,n-2*j)/(j!*pochhammer((alpha+3)/2,n-j)*pochhammer((alpha+1)/2,n-2*j)*(n-2*j)!)*hyperterm([2*j-n,n-2*j+alpha+1,(alpha+1)/2],[alpha+1,(alpha+2)/2],x,k);
```

$$summand := \mathrm{pochhammer}\left(\frac{1}{2}, j\right)\mathrm{pochhammer}\left(\frac{\alpha}{2}+1, n-j\right)\mathrm{pochhammer}\left(\frac{\alpha}{2}+\frac{3}{2}, n-2j\right)\mathrm{pochhammer}(\alpha+1, n-2j)\,\mathrm{pochhammer}(2j-n, k)\,\mathrm{pochhammer}(n-2j+\alpha+1, k)\,\mathrm{pochhammer}\left(\frac{\alpha}{2}+\frac{1}{2}, k\right)x^k \Big/ \left(j!\,\mathrm{pochhammer}\left(\frac{\alpha}{2}+\frac{3}{2}, n-j\right)\mathrm{pochhammer}\left(\frac{\alpha}{2}+\frac{1}{2}, n-2j\right)(n-2j)!\,\mathrm{pochhammer}(\alpha+1, k)\,\mathrm{pochhammer}\left(\frac{\alpha}{2}+1, k\right)k!\right)$$

Zeilberger's algorithms yields the recurrence equation for the summand of the outer sum:

```
> RE:=sumrecursion(summand,j,S(k));
```

$$RE := x\,(\alpha+1+2k)\,(-n+k)\,(k+\alpha+2+n)\,S(k) - (k+1)\,(\alpha+1+k)\,(\alpha+3+2k)\,S(k+1) = 0$$

Hence the sum is a multiple of

```
> hyp:=hypergeom([-n,n+2+alpha,(alpha+1)/2],[alpha+1,(alpha+3)/2],x);
```

$$hyp := \mathrm{hypergeom}\left(\left[-n, \alpha+2+n, \frac{\alpha}{2}+\frac{1}{2}\right], \left[\alpha+1, \frac{\alpha}{2}+\frac{3}{2}\right], x\right)$$

We compute the initial value S(k=0) by the same algorithm:

```
> sumrecursion(subs(k=0,summand),j,init(n));
```

$$(\alpha+2+n)\,\mathrm{init}(n) - (1+n)\,\mathrm{init}(1+n) = 0$$

Hence S(0) is given as

```
> S0:=pochhammer(2+alpha,n)/n!;
```

$$S0 := \frac{\mathrm{pochhammer}(2+\alpha, n)}{n!}$$

>